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Redundancy of the off-shell parameters in chiral effective field theory with explicit spin-3/2 degrees of freedom

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In this note we prove to all orders in the small scale expansion that all off-shell parameters which appear in the chiral effective Lagrangian with explicit $\Delta(1232)$ isobar degrees of freedom can be absorbed into redefinitions of certain low-energy constants and are therefore redundant.

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I. INTRODUCTION AND SUMMARY

Chiral effective field theory (EFT) with explicit $\Delta(1232)$ isobar degrees of freedom is usually formulated utilizing the Rarita-Schwinger formalism (for pioneering works, see [1, 2]). In this framework, the spin-3/2 field is represented by a vector-spinor ψ_μ (here and in what follows, we omit the spinor indices). It is well known how to write down the most general free Lagrangian for ψ_μ which describes the proper number of degrees of freedom [3]. The unphysical spin-1/2 degrees of freedom are projected out in the resulting free equations of motion. It is considerably more difficult to ensure the decoupling of the unphysical degrees of freedom in the case of interacting spin-3/2 fields. An elegant way to achieve this goal is to require that all interactions have the same type of gauge invariance as the kinetic term of the spin-3/2 field [4]. For a different strategy based on analyzing the consistency conditions within the canonical Hamilton formalism see Ref. [5]. This requirement of gauge invariance is, however, not compatible with the non-linear realization of the chiral symmetry by Coleman, Callan, Wess and Zumino [6, 7], which is commonly adopted in chiral effective field theories and ensures the chiral invariance of the effective Lagrangian on a term-by-term basis. Recently we have shown that any bilinear coupling of a massive spin-3/2 field can be brought into a gauge invariant form suggested by Pascalutsa by means of a non-linear field redefinition [8]. A similar statement can also be made for linear couplings of the Δ to a nucleon field [9]. The equivalence of the two formulations by means of the non-linear field redefinition implies that S-matrix elements can be calculated from the standard effective Lagrangian with the chiral symmetry being realized on a term-by-term basis using *naive* Feynman rules (i. e. ignoring the field-dependent determinants in the path-integral measure which may arise in the constrained quantization procedure).

An important question which still needs to be addressed in this context is related to the role of the so-called off-shell parameters which accompany every interaction term in the most general effective chiral Lagrangian with explicit $\Delta(1232)$ isobar fields, see section II for details. Here, an important observation was made by Ellis and Tang [10, 11] who showed that all three off-shell parameters appearing in the leading $\pi N\Delta$ and $\pi\Delta\Delta$ Lagrangians are redundant as they can be absorbed into other parameters in the Lagrangian, see also [9]. These findings were confirmed in the explicit calculation by Fettes and Meißner [12] based on the so-called small scale expansion (SSE) [2]. To the best of our knowledge, no rigorous proof of the redundancy of the off-shell parameters in chiral EFT to all orders in the SSE for arbitrary processes has been given in the literature. In this work we fill this gap and demonstrate explicitly that all terms in the effective chiral Lagrangian which give rise to the explicit dependence on the off-shell parameters can be eliminated by means of an appropriate field redefinition. Our work is organized as follows. In section II we provide basic definitions and explain our notation. The proof of the redundancy of the off-shell parameters in chiral EFT, which is the main result of this work, is presented in section III. Some useful relations for the spin-3/2 differential operator which are needed in the proof are derived in the appendix.

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II. BASIC DEFINITIONS AND NOTATION

The free Lagrangian for a massive spin–3/2 field representing the delta isobar can be written in the form

$$\mathcal{L}_0 = \bar{\psi}(i\partial\!\!\!/ - m)\psi, \tag{2.1}$$

where $\psi \equiv \psi^i_\mu$ is a conventional Rarita-Schwinger spinor. Here and in what follows, we make use of the following short-hand notation in order to keep the presentation compact

$$\begin{aligned}\bar{\psi}O\psi &= \bar{\psi}^\mu O_{\mu\nu}\psi^\nu = \bar{\psi}^\mu_i O^{ij}_{\mu\nu}\psi^\nu_j,\\ \bar{\psi}GN &= \bar{\psi}^\mu G_\mu N = \bar{\psi}^\mu_i G^i_\mu N \ ,\end{aligned} \tag{2.2}$$

and do not write the isospin and Dirac indices explicitly. The derivative operator that appears in the above equation has the form

$$\partial\!\!\!/^{ij}_{\mu\nu} = \gamma_{\mu\nu\alpha}\partial^\alpha\delta^{ij} \tag{2.3}$$

while the mass operator reads

$$m^{ij}_{\mu\nu} = \delta^{ij} m\gamma_{\mu\nu}. \tag{2.4}$$

The γ-tensors in the above equations are defined according to

$$\begin{aligned}\gamma_{\mu\nu} &= \frac{1}{2}[\gamma_\mu, \gamma_\nu] \ ,\\ \gamma_{\mu\nu\alpha} &= \frac{1}{2}\{\gamma_{\mu\nu}, \gamma_\alpha\} \ ,\end{aligned} \tag{2.5}$$

where the γ_μ are the Dirac matrices. Notice that the quantities $\gamma_{\mu\nu}$ and $\gamma_{\mu\nu\alpha}$ are completely antisymmetric with respect to the Dirac indices. We emphasize that the free Lagrangian is sometimes written in a more general form[1], see e.g. Ref. [3]:

$$\mathcal{L}^{(A)}_0 = \bar{\psi}^\mu_i \delta^{ij}\Big[(i\partial\!\!\!/ - m)g_{\mu\nu} + iA(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu) + \frac{i}{2}(3A^2 + 2A + 1)\gamma_\mu\partial\!\!\!/\gamma_\nu + m(3A^2 + 3A + 1)\gamma_\mu\gamma_\nu\Big]\psi^\nu_j, \tag{2.6}$$

where $A \neq -1/2$ is an arbitrary real parameter, see also [15] for a related recent discussion. As pointed out in Ref. [16], the above Lagrangian can be rewritten in the form given in Eq. (2.1) if one redefines the spin-3/2 field as follows

$$\psi^\mu_i \to \left(g^{\mu\nu} - \frac{1+A}{2}\gamma^\mu\gamma^\nu\right)\psi^\nu_i \ . \tag{2.7}$$

We will, therefore, adopt the form given in Eq. (2.1) in the following.

III. REDUNDANCY OF THE OFF-SHELL PARAMETERS IN CHIRAL EFT

The explicit inclusion of the delta isobar into an effective pion-nucleon field theory is motivated by its strong coupling to the $\pi N\gamma$ system as well as the small value of the delta-nucleon mass splitting $\Delta = 293$ MeV $\simeq 2M_\pi$. Notice, however, that contrary to the pion mass M_π, the delta-nucleon mass splitting does not vanish in the chiral limit. Consequently, such an extended EFT with explicit Δ isobar degrees of freedom does not have the same chiral limit as QCD [13]. One can set up a consistent power counting scheme by treating the mass–splitting $\Delta \equiv m_\Delta - m_N$ as an additional small parameter besides the external momenta Q and the pion mass M_π which is referred to as the small-scale expansion, see [2] for more details (note that we use the symbol Δ for both the delta field and the mass–splitting, but it is always clear from the context what is meant). Any matrix element or transition current then possesses a low-energy expansion of the form

$$\mathcal{M} = \epsilon^n\mathcal{M}_1 + \epsilon^{n+1}\mathcal{M}_2 + \epsilon^{n+2}\mathcal{M}_3 + \ldots \ , \tag{3.1}$$

[1] For a corresponding expression in d dimensions see Ref. [14].

where the power n depends on the process under consideration and ϵ collects the small parameters

$$\epsilon \in \left\{ \frac{Q}{\Lambda_\chi}, \frac{M_\pi}{\Lambda_\chi}, \frac{\Delta}{\Lambda_\chi} \right\} . \tag{3.2}$$

Here, $\Lambda_\chi \sim 1$ GeV refers to the chiral symmetry breaking scale.

The most general effective Lagrangian for pions, nucleons and the delta isobar can be written in the following symbolic form

$$\mathcal{L}_{\pi\Delta} + \mathcal{L}_{\pi N\Delta} = \bar{\psi}(i\not{D} - m + V)\psi + \bar{\psi}GN + \bar{N}\bar{G}\psi + \ldots , \tag{3.3}$$

where N is the nucleon field and the ellipses refer to terms which do not involve the delta isobar and are irrelevant for our work. The covariant derivative of the spin-3/2 field has the form

$$[i\not{D}]^{ij}_{\mu\nu} = i\,\gamma_{\mu\nu\alpha}(D^{ij})^\alpha = i\,\gamma_{\mu\nu\alpha}\left[\partial^\alpha \delta^{ij} + (\Gamma^{ij})^\alpha\right] \tag{3.4}$$

where Γ^{ij}_α is the so-called chiral connection whose explicit form can be found in Ref. [2]. The matrices V, G and $\bar{G}$ in Eq. (3.3) incorporate all possible local terms allowed by the symmetry requirements. A finite number of terms contribute at a given order ϵ^n. Utilizing the formalism by Coleman, Callan, Wess and Zumino [6, 7], one simply needs to write down all possible isospin-invariant terms[2] constructed from the various building blocks which transform covariantly under global chiral transformations in order to incorporate the non–linearly realized chiral symmetry. The explicit form of the various building blocks which depend on external sources and, in a nonlinear way, on pion fields as well as their transformation rules under chiral rotations, parity inversion, hermiticity and charge conjugations are not relevant for our work and can be found in [2].

For the following discussion, we require the pertinent counting rules for the building blocks of the chiral effective Lagrangian. We employ the scheme of Ref. [2]. We emphasize that

$$m,\ \psi,\ N,\ \gamma_\mu,\ D_\mu \sim \mathcal{O}(\epsilon^0) \tag{3.5}$$

whereas

$$i\not{D} - m \sim \mathcal{O}(\epsilon)\,. \tag{3.6}$$

and

$$[D_\mu, D_\nu] \sim \mathcal{O}(\epsilon^2)\,. \tag{3.7}$$

Notice further that the interaction terms V, G or $\bar{G}$ in Eq. (3.3) have a small-scale expansion of the form

$$V = \sum_{n=1}^{\infty} V^{(n)}\,, \qquad G = \sum_{n=1}^{\infty} G^{(n)}\,, \qquad \bar{G} = \sum_{n=1}^{\infty} \bar{G}^{(n)}\,, \tag{3.8}$$

with the superscript referring to the power of ϵ. In the following, we will also frequently use the short-hand notation for an interaction operator W:

$$W^{(\le n)} \equiv \sum_{i=1}^{n} W^{(i)}. \tag{3.9}$$

All Rarita-Schwinger fields appearing in the interaction terms of the effective Lagrangian are accompanied by matrices

$$\theta_{\mu\nu}(z) = g_{\mu\nu} + z\gamma_\mu\gamma_\nu\,, \tag{3.10}$$

where z denotes an off-shell parameter which governs the coupling of the (off shell) spin-1/2 components to a given operator. For example, the interaction $\bar{\psi}V\psi$ can be written as a series of interactions with different off-shell parameters

$$\bar{\psi}V\psi = \sum_i \bar{\psi}\Theta(z_i')V_i\Theta(z_i)\psi + \text{h.c.}. \tag{3.11}$$

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[2] Here and in the following, we consider the two-flavor case of up and down quarks.

One can switch to the gauge invariant formulation suggested by Pascalutsa [4] employing the non-linear field redefinition given explicitly in Ref. [8]

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m + V)\psi \quad \to \quad \mathcal{L}' = \bar{\psi}\left(i\partial\!\!\!/ - m + i\partial\!\!\!/ \frac{1}{m}V\frac{1}{m}\left[1-(i\partial\!\!\!/ + m)\frac{1}{m}V\frac{1}{m}\right]^{-1} i\partial\!\!\!/\right)\psi\,. \tag{3.12}$$

where the inverse mass operator in d dimensions has the following form

$$\left[\frac{1}{m}\right]^{ij}_{\mu\nu} = -\frac{1}{m}\delta^{ij}\left(g_{\mu\nu} + \frac{1}{1-d}\gamma_\mu\gamma_\nu\right). \tag{3.13}$$

Clearly, the last term in the brackets in Eq. (3.12) should be understood in terms of the Taylor expansion giving rise to an infinite series of local interactions. The more general case of bilinear and linear couplings of the spin-3/2 fields can be treated analogously, see Ref. [8]. The advantage of the new formulation is that all couplings of the spin-3/2 fields have the same constraints as in the free field theory. In the absence of the chiral symmetry one would immediately conclude that the dependence of the off-shell parameters can be completely absorbed into redefinition of the LECs entering the Lagrangian $\mathcal{L}'$. It is not obvious that this statement also applies to *chiral* effective Lagrangians since chiral symmetry is not realized on a term-by-term basis in $\mathcal{L}'$ any more.

In the following, we will prove that the off-shell parameters entering the effective chiral Lagrangian are indeed redundant and can be eliminated by suitably chosen field redefinitions. To this end we first prove a lemma which allows to rewrite the interaction terms in the effective Lagrangian at a fixed order in the small scale expansion in a more convenient way by making use of the equations of motion. For doing that, we have to be more specific regarding the form of the interaction terms entering the effective Lagrangian as compared to the previous discussion. This will require to introduce further interaction operators $\bar{V}$ and W in $\mathcal{L}_{\pi\Delta}$ and H and $\bar{H}$ in $\mathcal{L}_{\pi N\Delta}$ in addition to the ones which have already been defined in Eq. (3.3).

Lemma. Let $V, \bar{V}, W, H, \bar{H}, G, \bar{G}$ be local interaction operators which appear in the most general chiral effective Lagrangian for pions, nucleons and the delta isobar with the chiral symmetry implemented on the term-by-term basis. Then, the following relations are true:

$$\begin{aligned}
\mathcal{L}_1 &= \bar{\psi}(i D\!\!\!\!/ - m + V^{(n)} + \bar{V}^{(n)} + W^{(\leq n)})\psi + \bar{\psi}G^{(\leq n)}N + \bar{N}\bar{G}^{(\leq n)}\psi + \mathcal{O}(\epsilon^{n+1}) \\
&\simeq \bar{\psi}(i D\!\!\!\!/ - m + i D\!\!\!\!/ \frac{1}{m}V^{(n)} + \bar{V}^{(n)}\frac{1}{m} i D\!\!\!\!/ + W^{(\leq n)})\psi + \bar{\psi}G^{(\leq n)}N + \bar{N}\bar{G}^{(\leq n)}\psi + \mathcal{O}(\epsilon^{n+1}),
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
\mathcal{L}_2 &= \bar{\psi}(i D\!\!\!\!/ - m + W^{(\leq n)})\psi + \bar{\psi}(H^{(n)} + G^{(\leq n)})N + \bar{N}(\bar{H}^{(n)} + \bar{G}^{(\leq n)})\psi + \mathcal{O}(\epsilon^{n+1}) \\
&\simeq \bar{\psi}(i D\!\!\!\!/ - m + W^{(\leq n)})\psi + \bar{\psi}(i D\!\!\!\!/ \frac{1}{m}H^{(n)} + G^{(\leq n)})N + \bar{N}(\bar{H}^{(n)}\frac{1}{m} i D\!\!\!\!/ + \bar{G}^{(n)})\psi + \mathcal{O}(\epsilon^{n+1}),
\end{aligned} \tag{3.15}$$

where $\mathcal{L}_i \simeq \mathcal{L}_j$ means that the theories based on the Lagrangians $\mathcal{L}_i$ and $\mathcal{L}_j$ lead to the same S-matrix (provided the low-energy constants entering $\mathcal{O}(\epsilon^{n+1})$-terms are re-defied appropriately).

Proof. The lemma is proved by applying to the left-hand side of Eq. (3.14) the field transformations

$$\psi \to \psi + \frac{1}{m}V^{(n)}\psi, \quad \bar{\psi} \to \bar{\psi} + \bar{\psi}\bar{V}^{(n)}\frac{1}{m},$$

and to the left-hand side of Eq. (3.15) the field transformations

$$\psi \to \psi + \frac{1}{m}H^{(n)}N, \quad \bar{\psi} \to \bar{\psi} + \bar{N}\bar{H}^{(n)}\frac{1}{m}, \tag{3.16}$$

where we made use of the counting rules in Eq. (3.5). These field redefinitions do not change the S-matrix by virtue of the equivalence theorem [6, 17].

Notice that applying the previous lemma twice one immediately obtains the following useful relation:

$$\begin{aligned}
&\bar{\psi}(i D\!\!\!\!/ - m + V^{(n)} + \bar{V}^{(n)} + W^{(\leq n)})\psi + \bar{\psi}G^{(\leq n)}N + \bar{N}\bar{G}^{(\leq n)}\psi + \mathcal{O}(\epsilon^{n+1}) \\
\simeq\; &\bar{\psi}\left(i D\!\!\!\!/ - m + i D\!\!\!\!/ \frac{1}{m}(V^{(n)} + \bar{V}^{(n)})\frac{1}{m} i D\!\!\!\!/ + W^{(\leq n)}\right)\psi + \bar{\psi}G^{(\leq n)}N + \bar{N}\bar{G}^{(\leq n)}\psi + \mathcal{O}(\epsilon^{n+1})\,.
\end{aligned} \tag{3.17}$$

With these preparations we are now in the position to prove the redundancy of the off-shell parameters in chiral EFT.

Theorem. The interactions $\bar\psi V\psi$, $\bar\psi W\psi$, $\bar\psi XN$, $\bar N\bar X\psi$, $\bar\psi YN$, $\bar N\bar Y\psi$ of the form

$$\begin{aligned} V_{\mu\nu}^{ij} &= \gamma_\mu O_\nu^{ij} + \bar O_\mu^{ij}\gamma_\nu , \\ X_\mu^i &= \gamma_\mu Q^i, \quad \bar X_\mu^i = \bar Q^i \gamma_\mu , \\ W_{\mu\nu}^{ij} &= D_\mu^{il} P_\nu^{lj} + \bar P_\mu^{il} D_\nu^{lj} , \\ Y_\mu^i &= D_\mu^{il} S^l, \quad \bar Y_\mu^i = \bar S^l D_\mu^{li} , \end{aligned} \tag{3.18}$$

with arbitrary interaction operators O, $\bar O$, P, $\bar P$, Q, $\bar Q$, S, $\bar S$ which appear in the most general chiral effective Lagrangian for pions, nucleons and the delta isobar do not affect S-matrix elements.

Proof. We will prove the theorem by induction making use of the following relations for the spin-3/2 differential operator which are derived in appendix:

$$\begin{aligned} \left[\left(i\not{D}\frac{1}{m}\right)^2\right]_{ij}^{\mu\nu}\gamma_\nu &= \gamma_\nu \left[\left(\frac{1}{m}i\not{D}\right)^2\right]_{ij}^{\nu\mu} = -\frac{1}{2m^2}\frac{d-2}{d-1}\gamma^{\mu\alpha\beta}\left[D_\alpha, D_\beta\right]_{ij} = \mathcal{O}(\epsilon^2), \\ \left[\left(i\not{D}\frac{1}{m}\right)^2\right]_{il}^{\mu\nu} D_\nu^{lj} &= \frac{1}{2m}\left[i\not{D}\frac{1}{m}\right]_{il}^{\mu\lambda}\gamma_{\lambda\alpha\beta}\left[D^\alpha, D^\beta\right]_{lj} - \frac{1}{2m^2}\frac{d-2}{d-1}\gamma^{\mu\alpha\beta}\left[D_\alpha, D_\beta\right]_{il} D_\nu^{lj}\gamma^\nu = \mathcal{O}(\epsilon^2), \\ D_\nu^{il}\left[\left(\frac{1}{m}i\not{D}\right)^2\right]_{lj}^{\nu\mu} &= \frac{1}{2m}\gamma_{\lambda\alpha\beta}\left[D^\alpha, D^\beta\right]_{il}\left[\frac{1}{m}i\not{D}\right]_{lj}^{\lambda\mu} - \frac{1}{2m^2}\frac{d-2}{d-1}D_\nu^{il}\gamma^\nu\gamma^{\mu\alpha\beta}\left[D_\alpha, D_\beta\right]_{lj} = \mathcal{O}(\epsilon^2). \end{aligned} \tag{3.19}$$

Let us now show by induction that the interaction

$$Z_{\mu\nu}^{ij} = V_{\mu\nu}^{ij} + W_{\mu\nu}^{ij}. \tag{3.20}$$

in the bilinear couplings of the delta isobar can be transformed away from the effective Lagrangian via an appropriately chosen field redefinition.

Induction start. The most general bilinear interaction terms at first order in the small scale expansion (apart from the interactions entering the covariant derivative in the kinetic term) can be written in the form $\bar\psi(Z^{(1)} + R^{(1)})\psi$, where the operator $R^{(1)}$ does not involve the structures present in $Z^{(1)}$, i. e. does not contain contributions which vanish for on-shell spin-3/2 fields and is, therefore, free of the off-shell parameters. Applying the previous lemma several times we get

$$\begin{aligned} \mathcal{L} &= \bar\psi(i\not{D} - m + Z^{(1)} + R^{(1)})\psi + \bar\psi G^{(1)}N + \bar N\bar G^{(1)}\psi + \mathcal{O}(\epsilon^2) \\ &\simeq \bar\psi\left(i\not{D} - m + \left(i\not{D}\frac{1}{m}\right)^2 Z^{(1)}\left(\frac{1}{m}i\not{D}\right)^2 + R^{(1)}\right)\psi + \bar\psi G^{(1)}N + \bar N\bar G^{(1)}\psi + \mathcal{O}(\epsilon^2) \\ &= \bar\psi(i\not{D} - m + R^{(1)})\psi + \bar\psi G^{(1)}N + \bar N\bar G^{(1)}\psi + \mathcal{O}(\epsilon^2) \end{aligned} \tag{3.21}$$

In the last step we used

$$\left(i\not{D}\frac{1}{m}\right)^2 Z^{(1)}\left(\frac{1}{m}i\not{D}\right)^2 = \mathcal{O}(\epsilon^3).$$

Induction assumption. We assume that the operator $R^{(\le n)}$ does not include structures present in $Z^{(\le n)}$ and that the equality

$$\begin{aligned} \mathcal{L} &= \bar\psi(i\not{D} - m + Z^{(n)} + R^{(\le n)})\psi + \bar\psi G^{(\le n)}N + \bar N\bar G^{(\le n)}\psi + \mathcal{O}(\epsilon^{n+1}) \\ &\simeq \bar\psi(i\not{D} - m + R^{(\le n)})\psi + \bar\psi G^{(\le n)}N + \bar N\bar G^{(\le n)}\psi + \mathcal{O}(\epsilon^{n+1}) \end{aligned} \tag{3.22}$$

holds true.

Induction step. By the induction assumption, we can write the Lagrangian in the form

$$\mathcal{L} \simeq \bar\psi(i\not{D} - m + Z^{(n+1)} + R^{(\le n+1)})\psi + \bar\psi G^{(\le n+1)}N + \bar N\bar G^{(\le n+1)}\psi + \mathcal{O}(\epsilon^{n+2})$$

where the operator $R^{(\leq n+1)}$ does not include structures of the form $Z^{(\leq n+1)}$. Applying again the previous lemma several times we get

$$\begin{aligned}
\mathcal{L} &\simeq \bar{\psi}(i\not{D} - m + Z^{(n+1)} + R^{(\leq n+1)})\psi + \bar{\psi}G^{(\leq n+1)}N + \bar{N}\bar{G}^{(\leq n+1)}\psi + \mathcal{O}(\epsilon^{n+2}) \\
&\simeq \bar{\psi}(i\not{D} - m + \left(i\not{D}\frac{1}{m}\right)^2 Z^{(n+1)} \left(\frac{1}{m}i\not{D}\right)^2 + R^{(\leq n+1)})\psi + \bar{\psi}G^{(\leq n+1)}N + \bar{N}\bar{G}^{(\leq n+1)}\psi + \mathcal{O}(\epsilon^{n+2}) \\
&= \bar{\psi}(i\not{D} - m + R^{(\leq n+1)})\psi + \bar{\psi}G^{(\leq n+1)}N + \bar{N}\bar{G}^{(\leq n+1)}\psi + \mathcal{O}(\epsilon^{n+2}) \,, \qquad (3.23)
\end{aligned}$$

where in the last step we used the relation

$$\left(i\not{D}\frac{1}{m}\right)^2 Z^{(n+1)} \left(\frac{1}{m}i\not{D}\right)^2 = \mathcal{O}(\epsilon^{n+3}) \,. \qquad (3.24)$$

In the same way one can show that the interactions $X, \bar{X}, Y, \bar{Y}$ can be transformed away which completes the proof of the theorem.

As an application, consider the (leading) dimension-one effective Lagrangian describing the coupling of the massive spin-3/2 fields to the pions which has the form [2]

$$\mathcal{L}^{(1)}_{\pi\Delta} = \bar{\psi}^{\mu}_i \left([i\not{D}]^{ij}_{\mu\nu} - m^{ij}_{\mu\nu} - \frac{g_1}{2} g_{\mu\nu} \not{u}^{ij}\gamma_5 - \frac{g_2}{2}\left(\gamma_\mu u^{ij}_\nu + u^{ij}_\mu \gamma_\nu\right) - \frac{g_3}{2}\gamma_\mu \not{u}^{ij}\gamma_5\gamma_\nu \right) \psi^{\nu}_j \,, \qquad (3.25)$$

where the g_i are LECs, $u^{ij}_\mu = u_\mu \delta^{ij}$, $u_\mu = i(u^\dagger \partial_\mu u - i\partial_\mu u^\dagger)$ and the matrix $U(x) = u^2(x)$ collects the pion fields, see [2] for more details. Notice that g_2 and g_3 describe pion couplings to off-mass-shell components of the spin-3/2 fields. It immediately follows from our analysis that both the g_2- and g_3-terms are redundant, i.e. they do not contribute to S-matrix elements calculated from Eq. (3.25) (and higher-order counter terms) using naive Feynman rules.

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APPENDIX A: PROPERTIES OF SPIN-3/2 DIFFERENTIAL OPERATOR

In this appendix we derive the relations for the spin-3/2 differential operator in Eq. (3.19). We first contract the inverse of the mass operator with the Dirac-matrix γ^μ leading to:

$$\gamma_\nu \left[\frac{1}{m}\right]^{\nu\alpha}_{ij} = -\frac{1}{m}\gamma_\nu \left(g^{\nu\alpha} + \frac{1}{1-d}\gamma^\nu\gamma^\alpha \right) \delta_{ij} = -\frac{1}{m}\,\frac{1}{1-d}\gamma^\alpha \delta_{ij}. \qquad (A.1)$$

Multiplying this result with the operator $i\not{D}$ we obtain

$$\gamma_\nu \left[\frac{1}{m}\right]^{\nu\alpha}_{ik} [i\not{D}]^{kj}_{\alpha\beta} = -\frac{1}{m}\frac{1}{1-d}\gamma^\alpha\gamma_{\alpha\beta\lambda} iD^\lambda_{ij} = -\frac{1}{m}\frac{d-2}{1-d}\gamma_{\beta\lambda} iD^\lambda_{ij} = \frac{1}{m^2}\frac{d-2}{1-d} iD^\lambda_{ik} m^{kj}_{\lambda\beta}. \qquad (A.2)$$

In the second step of the last equation we used the identity

$$\gamma^\alpha \gamma_{\alpha\beta\lambda} = (d-2)\gamma_{\beta\lambda}. \qquad (A.3)$$

Multiplying Eq. (A.2) with the inverse of the mass operator from the right we get

$$\gamma_\nu \left[\frac{1}{m} i\not{D} \frac{1}{m}\right]^{\nu\alpha}_{ij} = \frac{1}{m^2}\frac{d-2}{1-d} iD^\alpha_{ij}. \qquad (A.4)$$

Finally, multiplication of the last equation with the operator $i\not{D}$ from the right yields

$$\gamma_\nu \left[\left(\frac{1}{m} i\not{D} \right)^2 \right]^{\nu\mu}_{ij} = \frac{1}{m^2} \frac{d-2}{1-d} i D^{ik}_\alpha \gamma^{\alpha\mu\beta} i D^{kj}_\beta = -\frac{1}{2m^2} \frac{d-2}{d-1} \gamma^{\mu\alpha\beta} [D_\alpha, D_\beta]_{ij}. \tag{A.5}$$

where the commutator of the differential operators is given by

$$[D_\alpha, D_\beta]_{ij} = D^{ik}_\alpha D^{kj}_\beta - D^{ik}_\beta D^{kj}_\alpha. \tag{A.6}$$

All other relations given in Eq. (3.19) can be derived in a completely analogous way.

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