The observation of $Z_c(3900)$ by the BESIII Collaboration in the invariant mass spectrum of $J/\psi \pi^\pm$ in $e^+e^- \rightarrow J/\psi \pi^+\pi^-$ at the center of mass 4.26 GeV suggests the existence of a charged $D\bar{D}^*$ molecular state with $I(J^P) = 1(1^+)$, which could be an isovector brother of the famous $X(3872)$ and an analogue of $Z_b(10610)$ claimed by the Belle Collaboration. We demonstrate that this observation provides strong evidence that the mysterious $Y(4260)$ is a $D\bar{D}_1(2420) + D\bar{D}_1(2420)$ molecular state. Especially, we show that the decay of this molecule naturally populates low momentum $\bar{D}D^*$ pairs and leads unavoidably to a cusp at the $D\bar{D}^*$ threshold. We discuss the signatures that distinguish such a $D\bar{D}^*$ cusp from the presence of a true resonance.

During the past years, the experimental observation of a large number of so-called $X$, $Y$, $Z$ states has initiated tremendous effort to unravel their nature beyond the conventional quark model. Especially, the confirmation of signals in charged channels would be direct evidence for exotic states. For instance, the Belle Collaboration reported signals for $Z_c(4430)$ in $\psi' \pi^\pm$, and $Z_1(4050)$ and $Z_2(4250)$ in $\pi^\pm$ in $B$ meson decays [1]. However, an enhancement in the same mass range was interpreted as a reflection by BABAR [2]. The more recent experimental results for charged bottomonium states $Z_b(10610)$ and $Z_b(10650)$, located close to the $B^*B$ and $B^*B^*$ thresholds, respectively, by the Belle Collaboration [3] seem to be the first strong evidence for QCD “exotics” in the heavy quark sector. In this context the recent report of an enhancement in the $J/\psi \pi^\pm$ invariant mass distribution around 3900 MeV, right at the $D\bar{D}^*$ threshold, by the BESIII Collaboration [4] clearly reinforces the existence of such an unusual phenomenon. This state, called $Z_c(3900)$ below, might be an isovector partner of the well established $1^{++}$ isoscalar $X(3872)$ [5], but with $I^G(J^{PC}) = 1^+(1^-)$ for the neutral state.

In this work we demonstrate that, if $Y(4260)$ is a $D\bar{D}_1 + c.c.$ molecule (below we use $D\bar{D}_1$ as a short notation), the appearance of an enhancement around 3900 MeV in the $J/\psi \pi^\pm$ invariant mass distribution can be shown to be natural. Here $D_1$ refers to the narrow axial vector $D_1(2420)$ ($\Gamma = 27 \pm 3$ MeV) with $I(J^P) = (1/2)(1^+)$ [5]. In this sense the observation of the charged $Z_c(3900)$ state by BESIII in $Y(4260) \rightarrow J/\psi \pi^\pm \pi^\pm$ provides very strong evidence for the molecular nature of $Y(4260)$. We also discuss whether the observed enhancement can be interpreted purely as a cusp or whether the inclusion of explicit poles in the $D\bar{D}^*$ system is necessary. Before we go into details of our calculations we first briefly review the status of $Y(4260)$.

The most mysterious fact about $Y(4260)$ is not that its mass does not agree with what is predicted by the potential quark model. Instead, as a charmonium state with $J^{PC} = 1^{--}$, it is only “seen” as a bump in its two pion transitions to $J/\psi$, but not in any open charm decay channel like $D\bar{D}$, $D\bar{D}^*$ + c.c., $D^*D^*$, and $D_1D_1^*$, or other tens of measured channels. In fact, the cross section line shapes of the $e^+e^-$ annihilations into $D^{(*)}$ meson pairs appear to have a dip at its peak mass 4.26 GeV instead of a bump.

In the vector sector one should recognize that the $D\bar{D}_1(2420)$ is the first open charm relative $S$-wave channel coupled to $J^{PC} = 1^{--}$, and the nominal threshold is only about 29 MeV above the location of the $Y(4260)$. The possibility that $Y(4260)$ may be a bound system of $D\bar{D}_1$ and $D^*D_0$ was investigated in Refs. [6-9]. It should be stressed, however, that broad components [the widths of $D_0$ and $D_1(2430)$ are as large as 300 MeV] cannot produce narrow resonances [10]. We thus do not consider the $D^*D_0$ or $D^*_1D_1(2430)$ component here, but focus on the assumption that the $Y(4260)$ is a bound system of only $D\bar{D}_1(2420)$.

In the literature many other solutions were proposed for the $Y(4260)$ (see Ref. [11] for a recent review). Based on the data for the $\pi\pi$ spectrum of $Y(4260) \rightarrow J/\psi \pi\pi$, Dai et al. [12] concluded that the even lower threshold $\chi_{c0}\omega$ would have the largest coupling to $Y(4260)$, while the $D\bar{D}_1$ coupling to $Y(4260)$ turned out to be negligible. However, the observation of the enhancement at the $D\bar{D}^*$ threshold in the $J/\psi \pi^\pm$ invariant mass spectrum actually rules out such a scenario and suggests that the underlying dynamics should be more sensitive to the $D\bar{D}_1$ threshold.

In this work we do not try to identify the mechanisms that lead to the formation of $Y(4260)$ as a molecular state, but study the consequences of the assumption that this state is dominantly a $D\bar{D}_1$ bound system. We argue that the interpretation of $Y(4260)$ as a relative $S$-wave $D\bar{D}_1$ system is able to accommodate nearly all the present observations for $Y(4260)$. Especially, its absence in various open charm decay channels mentioned above and the observation of...
$Z_c(3900)$ in $Y(4260) \to J/\psi \pi \pi$ can be naturally understood.

The heavy quark spin symmetry also implies the presence of a $D^*D_1$ and $\bar{D}^*D_2$ component in the wave function. However, they will be neglected here since their corresponding thresholds are almost 200 MeV above the mass of $Y(4260)$. Meanwhile, we note that in a more microscopic treatment there should be heavier spin partners of $Y(4260)$ that should contain the mentioned constituents prominently. We will briefly come back to this issue at the end of this Letter.

Based on the above picture, the pertinent diagrams to be calculated are shown in Fig. 1. The central goal of our study is to pin down the structure of $Y(4260)$ and identify the quantitative importance of the $\bar{D}D^*$ cusp from the loop diagrams of Figs. 1(b)–1(d) in order to understand whether the BESIII spectra call for the additional inclusion of an explicit pole diagram as depicted in the diagram of Fig. 1(a).

It should be stressed that the vicinity of the $\bar{D}D_1$ threshold to 4.26 GeV favors the formation of low momentum $\bar{D}D^*$ pairs, which may lead to the formation of the $Z_c(3900)$ bound systems. The reason is that the $\bar{D}D_1$ intermediate state as well as the $\bar{D}D^*$ intermediate state can be simultaneously close to their mass shells—all with relative $S$ waves. This gives rise to a triangle singularity studied in a different context in Refs. [14,15]. Such a two-cut condition strongly enhances the corresponding matrix elements. In this sense the $\bar{D}D_1$ intermediate system provides an ideal doorway state for a low momentum $\bar{D}D^*$ system. There is a series of $S$-wave open charm thresholds with $J^P_{\text{PC}} = 1^{--}$ around 4.26 GeV, i.e., $\bar{D}D_1(2420), \bar{D}D_1(2430), \bar{D}^*D_0, \bar{D}_0D^*_1, \bar{D}_1D_1$, and $\bar{D}^*D_1(2420)$. However, all of them, except for $\bar{D}D_1(2420)$, are either far away from the observed physical $Y(4260)$, or too broad to make a bound state [5]. Thus, we do not take them into account explicitly here.

Note that in order to formulate the problem as an effective field theory the power counting of Refs. [16,17] needs to be adapted to the present situation. We leave this to be reported in a subsequent work and focus here on a more phenomenological investigation.

A complete calculation also needs the inclusion of the $\pi \pi$ final state interaction (FSI) for which we adopt a parametrization scheme with coupled channel unitarity [13,18]. Since the pion pairs are in the isoscalar channel and the invariant mass of the pion pairs covers a range from the two pion threshold to more than 1 GeV, the $S$-wave $\pi \pi$ FSI is expected to play an important role in this region. We can isolate the $\pi \pi$ $S$-wave contributions in the $\pi \pi$ center of mass frame as follows:

$$\mathcal{M} = \mathcal{M}_S + \mathcal{M}_{\text{non-}S}. \tag{1}$$

After including the $\pi \pi$ $S$-wave FSI, the amplitude becomes

$$\mathcal{M} = \mathcal{M}_S + \mathcal{M}_{\text{non-}S}. \tag{2}$$

where $T_{\pi \pi-\pi \pi}$ is the $\pi \pi$ elastic scattering amplitude [18] with the $KK$ threshold appropriately considered and $\alpha(s_{\pi \pi})$ is a polynomial function of the $\pi \pi$ invariant mass squared $s_{\pi \pi}$ [13]

$$\alpha(s_{\pi \pi}) = \frac{c_1}{s_{\pi \pi} - M_{\pi \pi}^2/2} + c_2 + c_3 s_{\pi \pi}. \tag{3}$$

where the Adler zero pole $m_{\pi \pi}^2/2$ is present in order to cancel the Adler zero hidden in $T_{\pi \pi-\pi \pi}$. The following parameters, $c_1 = 0.23 \text{ GeV}^2$, $c_2 = -1.07$, and $c_3 = 1.15 \text{ GeV}^{-2}$ are adopted for a reasonable description of the experimental data.

Since $Z_c(3900)$ has the same quantum numbers as $Z_b$, most interactions needed for this work can be taken from Ref. [17]. Here we only present the interactions between a $P$-wave charmed meson and other fields since they play a crucial role in this work. The heavy quark spin symmetry allows heavy mesons to form spin doublet super fields distinguished by their light degrees of freedom $s_i = s_q + l$ with $s_q$ the light quark spin and $l$ the orbital angular momentum. For $l = 1$ it can be classified into two super fields [19,20]. The interaction terms relevant for this work read

$$\mathcal{L}_Y = iy(\bar{D}^*_1 Y Y^T D^1_1 - \bar{D}^1_1 Y^T D^*_1) + \text{H.c.}, \tag{4}$$

$$\mathcal{L}_{D_1} = i \frac{\lambda^T}{f_{\pi}} [3 D^i_{1a} (\phi^i_{ab} \phi_{ab}) D^i_{1b} - D^1_{1a} (\phi^i_{ab} \phi_{ab}) D^*_1 + 3 \bar{D}^i_{1b} (\phi^i_{ab} \phi_{ab}) D^i_{1a} - D^*_1 (\phi^i_{ab} \phi_{ab}) D^1_{1a}] + \text{H.c.}. \tag{5}$$

where $D$ ($D^1$) and $\bar{D}$ ($\bar{D}^1$) contain the annihilation (creation) operators for $c\bar{q}$ and $\bar{c}q$ fields, respectively. The analogous conventions are applied to $D^*$ and $D_1$. The interactions contain five coupling constants in total. Since we focus on the shape of the invariant mass distributions only, all couplings that are common to all diagrams are not relevant. In this sense the only free parameters that

FIG. 1. The Feynman diagrams for $Y(4260) \to \bar{D}D_1 + \text{c.c.} \to J/\psi \pi \pi$ and $h_1 \pi \pi$ considered in this work.

$$\mathcal{M} = \mathcal{M}_S + \mathcal{M}_{\text{non-}S}. \tag{1}$$
influence the invariant mass distributions are the mass of \( Z_c(3900) \) and the coupling constant \( g_Z \) for \( Z_c \bar{D} D^* \). Meanwhile, we adopt a Breit-Wigner propagator for the \( Z_c(3900) \).

For simplicity in this exploratory study we treat \( Y(4260) \), \( D_s(2420) \), \( D \), and \( D^* \) as stable states. Their masses are taken from the Particle Data Group [5]. By assuming that \( Y(4260) \) is dominated by the \( \bar{D} D_s(2420) \) molecule component, we can estimate the \( YD_{s1} \) coupling by Weinberg’s compositeness theorem [21,22], i.e.,

\[
y^2/4\pi = 4(m_{D} + m_{D_s})^{5/2}/\sqrt{2\delta E/m_{D}m_{D_s}} \approx 17\text{GeV}^2.
\]

This predicts the dominant decay of \( Y(4260) \rightarrow \bar{D} D^* \pi + \text{c.c.} \), via the intermediate state \( \bar{D} D_s(2420) + \text{c.c.} \) of which the partial width is larger than 40 MeV. This value is consistent with the total width measured for \( Y(4260) \). At this moment, we do not pursue a perfect fit of the experimental data but demonstrate the importance of the proposed mechanisms in the description of the qualitative feature of the data. In fact, with only one parameter, \( g_Z = 0.7 \sim 1 \) GeV\(^{-1/2} \), the data can be described well in our scenario.

The numerical results for the \( J/\psi \pi \) and \( \pi \pi \) invariant mass spectra from \( Y(4260) \rightarrow J/\psi \pi \pi \) are shown in Fig. 2. The dashed lines denote the results from the box diagrams, i.e., Figs. 1(b)–1(d), while the dotted lines denote the exclusive contributions from the \( Z_c(3900) \) pole diagrams, i.e., Fig. 1(a). In our case the box diagrams play a role as background terms with respect to the \( Z_c(3900) \) pole diagrams. However, as shown by the dashed lines, an explicit enhancement around 3.9 GeV in the \( J/\psi \pi \) spectrum can be produced because of the nearly on-shell two-cut condition. The solid lines in Fig. 2 show the result of the sum of all diagrams in Fig. 1. The \( Z_c(3900) \bar{D} D^* \) coupling is chosen in order to reproduce the qualitative features of the data. The inclusion of the \( Z_c(3900) \) actually broadens and enhances the \( \bar{D} D^* \) threshold enhancement and gives rise to the detailed structures in the \( \pi \pi \) invariant spectrum. There are signatures that can be identified for the \( \pi \pi \) production mechanism. An analysis of the relative partial waves between the two pions suggests that in addition to the \( S \) wave other higher partial waves, such as \( D \) wave, are also contributing to the \( \pi \pi \) productions. Meanwhile, the dominance of the relative \( S \) wave \( \pi \pi \) in the lower invariant mass region results in the broad bump above the \( \pi \pi \) threshold and flattened dip around 0.5–0.6 GeV. This structure is driven by the box diagrams after the \( \pi \pi \) FSI is properly included. As a contrast, the exclusive contributions from the \( Z_c(3900) \) pole amplitude do not produce an obvious structure in the \( \pi \pi \) spectrum. It is essential to recognize that the dip structure around 1 GeV in Fig. 2(b) is due to the presence of the \( K K \) threshold in the \( \pi \pi \) FSI. The exact \( K K \) threshold should be located around 0.986 GeV. However, the data show that the dip position is slightly shifted to be higher than 1 GeV. In our calculation such a shift is due to the contributions from higher partial waves.

For the \( J/\psi \pi \pi \) spectrum, one can see that even without the explicit inclusion of \( Z_c(3900) \), two structures appear at the same masses in the \( J/\psi \pi \pi \) spectrum as in the BESIII data. Note that by charge conjugation invariance the \( J/\psi \pi \pi \) spectrum is identical with that for \( J/\psi \pi^+ \). The cusp at \( M_D + M_{D'} \approx 3.876 \) GeV in Fig. 2(a) marks the \( \bar{D} D^* \) threshold. The lower bump between 3.4 and 3.6 GeV in the \( J/\psi \pi^+ \) invariant mass distribution comes from the interference between the conjugate diagrams where either a \( \pi^- \) or a \( \pi^+ \) is emitted first. Therefore, the lower bump is simply a reflection of the narrow structure at 3.9 GeV. One also notices that the box diagrams are the main contributions as a background to the \( J/\psi \pi \pi \) spectrum away from the \( Z_c(3900) \) pole.

In Fig. 1, we also include the production channel for \( Y(4260) \rightarrow h_c \pi \pi \). Similar to the discussions of \( Z_b \rightarrow h_b \pi \), this channel is ideal for disentangling the molecular nature of the intermediate \( Z_c(3900) \): the power counting analysis in Ref. [17] shows that the triangle transition \( Z_b \rightarrow h_b(mP) \pi \) is not suppressed compared to \( Z_b \rightarrow Y(nS) \pi \), although the decay is via a \( P \) wave. This explains why the branching ratios for \( Z_b \rightarrow h_b(mP) \pi \) are compatible with those for \( Z_b \rightarrow Y(nS) \pi \). A similar phenomenon occurs here. In addition, higher loop contributions are suppressed in \( Z_b \rightarrow h_b(mP) \pi \), while they are not suppressed for \( Z_b \rightarrow Y \pi \) [17]. The analogous pattern is expected for the \( Z_c \) decays.

In Fig. 3, we present our prediction for the invariant mass spectra of \( h_c \pi \) and \( \pi \pi \) in \( Y(4260) \rightarrow h_c \pi \pi \) including the \( \pi \pi \) FSI. Similar to the \( J/\psi \pi \pi \) channel, a very pronounced peak right at the \( \bar{D} D^* \) threshold appears in the \( h_c \pi \) invariant spectrum. Interestingly, due to the limited phase space in this decay channel, its kinematic reflection...
is significantly shifted. It is located at higher invariant masses and even submerged by the $\bar{D}D^*$ threshold enhancement. With the parameters fixed in $Y(4260) \to J/\psi \pi \pi$, it shows that the enhancement at 3.9 GeV produced by the box diagrams is not as significant as that by the explicit $Z_c(3900)$ pole. Meanwhile, the $\pi\pi$ spectrum at lower mass regions is sensitive to the underlying dynamics. This feature is quite different from that observed in the $J/\psi \pi \pi$ channel. Experimental data for this channel will allow clear evidence for the request or elimination of the $Z_c(3900)$ resonance contribution. Also, different from the $J/\psi \pi \pi$ transition, the predicted $\pi\pi$ spectrum shown in Fig. 3(b) does not have the $\bar{K}K$ threshold discontinuation due to the limited phase space. We find that the $S$-wave $\pi\pi$ amplitude still plays an important role in the $h_\pi\pi$ channel with the $\pi\pi$ in a $P$ wave relative to the recoiled $h_c$.

We stress that in order to understand the gross features of the data, the diagrams in Fig. 1 can also be replaced completely or in parts by tree level diagrams (see, e.g., Refs. [23,24]). Then, however, the underlying dynamics would explain the large deficit in width due to the limited phase space. We find that it is the $\bar{D}D_1$ molecule nature of $Y(4260)$ that provides a natural explanation for the appearance of $Z_c(3900)$ in the $Y(4260)$ decays, and for other detailed features of the spectra.

In this work we propose that $Y(4260)$ is dominantly a $\bar{D}D_1(2420)$ molecule and identify a unique mechanism, namely the presence of the two-cut condition, which plays an essential role in $Y(4260) \to J/\psi \pi\pi$ and $h_\pi\pi$. We demonstrate that without introducing any drastic assumption, the molecular nature of $Y(4260)$ as a bound state of $\bar{D}D_1$ can naturally explain the observation of an enhancement around 3.876 GeV in the $J/\psi \pi$ invariant mass spectrum. The reflection of this peak matches the experimental data well and provides strong evidence for the molecular nature of $Y(4260)$. We also demonstrate that for a more detailed description of the data the need for an explicit $Z_c(3900)$ pole seems to be necessary.

We stress the following important consequences of this prescription that will be reported in a subsequent paper:

(i) Although the nominal $\bar{D}D_1(2420)$ threshold is higher than the mass of $Y(4260)$, the threshold of $\bar{D}D^*\pi$ (then decaying to $\bar{D}D\pi\pi$) is much lower. Dominant decays of $Y(4260)$ into $\bar{D}D^*\pi$ should be regarded as a natural consequence and would explain the large deficit in width between the total width and its decays into $J/\psi \pi\pi$ and $J/\psi K\bar{K}$. We also predict an asymmetric spectral shape for the decay of $Y(4260)$ into $\bar{D}D^*\pi$. This can be clarified by an energy scan around the nominal $Y$ mass [analogous to what happens to the shape of $Y(4660)$ when being viewed as the $\psi f_0(980)$ molecule [25]].

(ii) A possible reason why $Z_c(3900)$ does not show up as significantly as the $X(3872)$ in $B$ decays is that the $X(3872)$ might be produced via its small $c\bar{c}$ component, which is absent in $Z_c$. Here, $Z_c(3900)$ does appear as a result of the proposed molecular nature of $Y(4260)$, which actually allows the $S$-wave $\bar{D}D^*$ pairs to be copiously produced.

(iii) One might also expect a cusp or possible resonance structure at the $\bar{D}D^*\pi$ threshold in the decay of $\psi(4415)$, if this state is assumed to be a molecule driven by the nearby $\bar{D}D_1$ threshold.

(iv) The exact mechanism at work here might also be the reason for the appearance of the $Z_b$ and $Z'_b$ states in $Y(5S)$ decays, if we assume that $Y(5S)$ has a sizeable $BB_1$ component. Note that the relative $S$-wave threshold for $BB_1$ is only about 120 MeV above the $Y(5S)$ mass.

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