Thermodynamic, thermoelectric, and magnetic properties of FeSb$_2$: A combined first-principles and experimental study

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We analyze the thermodynamic, magnetic, and transport properties of the narrow band-gap semiconductor FeSb$_2$ using density functional theory calculations corroborated by nuclear inelastic spectroscopy and ultrasound experiments. The vibrational properties (phonon spectrum, density of states, heat capacity) and elastic constants are computed through response function calculations and are in good agreements with the measurements. The electron-phonon coupling effects are also studied. The estimations of linewidth broadening due to electron-phonon coupling along the high-symmetry directions in the first Brillouin zone are given. The linewidth broadening reaches the largest value for Fe optical modes in the vicinity of the $X[0.5,0,0]$ point. The broadening, when compared to those obtained at the other symmetry points, differs by up to two orders of magnitude. From the Boltzmann theory applied to our electronic band structure, we investigate the electrical transport properties. It is found that a purely electronic structure description is incompatible with the record value of the Seebeck coefficient experimentally observed at $T \approx 12$ K. The diamagnetic to paramagnetic crossover at a temperature around 100 K is also described from the calculation of the magnetic susceptibility, and results compare well with experiment.

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I. INTRODUCTION

The study of the thermodynamic, thermoelectric, and magnetic properties of iron diantimonide FeSb$_2$ has been the subject of intense research investigations.1–4 The particular interest in this material is due to its interesting low-temperature physical properties and the considerable potential applications, the most promising of which is in the development of cryorefrigeration because of its large thermoelectric power at low temperature.4

FeSb$_2$ crystallizes in the loellingite (orthorhombic marcasite type) structure with Fe ions surrounded by deformed Sb octahedra$^5$ and contains two formula units per unit cell. The low-temperature state is a semiconductor with an extremely narrow indirect band gap, which promotes strong correlation effects between $d$ and conduction-band states.6–9 The minimum direct gap value extracted from reflectivity measurements$^{10,11}$ ranges between 30 and 40 meV. Recent band-structure calculations$^{12}$ have shown that it is possible to properly describe the indirect semiconducting gap of FeSb$_2$ by going beyond standard density functional theory (DFT) methods for instance using GW calculations.

The physical properties of FeSb$_2$ are similar in many respects to those of iron silicide FeSi.$^{13,14}$ The most remarkable observed similarities are the temperature-dependent magnetic susceptibility,$^{15,16}$ $\chi(T)$ and the strong anisotropic electrical transport properties.$^{17–19}$ For a magnetic field applied along the $c$ axis, the system evolves from a diamagnetic state toward a paramagnetic state through a crossover temperature at about 100 K. The electrical resistivity exhibits a semiconducting behavior along the $a$ and $c$ axes, whereas, for the $b$ axis, a semiconducting-metallic crossover temperature of 40 K is observed. Among the open questions regarding the temperature dependence of the physical properties of FeSb$_2$, most attention has recently been focused on the origin of the huge low-temperature thermopower. Of particular interest is the issue of whether the thermopower is purely electronic, as is usually assumed, or if there are some other important effects due to phonon drag,$^{20}$ for instance. The physical origin of this phenomenon is in fact not yet clarified. Adding to properties observed earlier, Bentien et al.$^{21}$ recently found that FeSb$_2$ exhibits a record Seebeck coefficient $S \approx -45\,000 \mu$V K$^{-1}$ at $T \approx 12$ K, and the thermoelectric power factor $PF = S^2\rho^{-1}$, where $\rho$ represents the electrical resistivity, can reach a value of $\approx 2300 \mu$WK$^{-2}$ cm$^{-1}$. This value is 65 times larger than the reference value measured for Bi$_2$Te$_3$-based thermoelectric materials. However, the figure of merit $ZT = S^2\rho^{-1}k^{-1}$ is rather low, due to the large value of the thermal conductivity$^{12}$ $k = 250$ WK$^{-1}$m$^{-1}$ at $T \approx 12$ K.

Herein, we combine experimental and theoretical investigation of transport, magnetic, and vibrational properties of FeSb$_2$. We analyze the transport coefficients within an electronic-structure picture where the contribution of the lattice thermal conductivity $k_L$ to $ZT$ is neglected (low-temperature approximation). Because the electrical transport properties of narrow band-gap semiconductors are very sensitive to chemical substitution,$^{22}$ we have investigated the effect of donor impurity doping on the Seebeck coefficient. The exponentially activated paramagnetic susceptibility$^{23}$ with increasing temperature is highlighted. Regarding the vibrational properties, few studies have been performed so far,$^{24,25}$ and the results were presented from the point of view of its absorption.
The sample for the resonance ultrasound spectroscopy was a nonenriched sample prepared in a similar way followed by spark plasma sintering pressing. A pellet was pressed by heating to 600 °C in 5 min at 60 MPa and held at 600 °C for 10 min. Before pressing a small impurity of Sb was observed by x-ray diffraction, after pressing a tiny amount of Fe was observed, however the content was too small for quantitative refinements.

The partial density of phonon states (DPS) of Sb and Fe were obtained by nuclear inelastic scattering (NIS) measurements. The technique and the extraction of the DPS from the NIS spectra are explained in Refs. 35–37. The measurements with the $^{57}$Fe and the $^{121}$Sb resonance took place on the 95% $^{57}$Fe enriched powder sample at the ESRF ID18 and ID22N beamlines, respectively, in 16 bunch mode, and at 50 and 67 K, respectively, in order to reduce the multiphonon contributions. The resolution was 0.7 and 1.3 meV full width at half maximum (FWHM) for measurements with the $^{57}$Fe and $^{121}$Sb resonance, respectively.

The resonant ultrasound spectroscopy (RUS) response was measured with a homemade inset for a QD-PPMS. The measurements were done on a polished 2.205(2) × 2.118(2) × 1.784(2) mm$^3$ polycrystalline sintered powder sample upon cooling between 300 and 10 K. It was verified by Laue diffraction that the sample is polycrystalline. The density of the sample was ∼98% of the x-ray density. Only 20 resonance frequencies could be reliably determined and modeled using a finite element algorithm for a rectangular parallelepiped with a 10th-order polynomial fit for the displacements. The root-mean-square deviation of the fits was of ∼0.8%, i.e., somewhat large. A too small number of resonance lines had a significant $c_{11}$ contribution, thus, it can not be reliably determined. The temperature dependence of the $c_{44}$ component was obtained through fits and then verified by the temperature variation of particular resonance frequencies, which depend by more than 95% on $c_{44}$. The weakness in the modeling likely comes from texture in the sample caused by the sintering process.

### III. RESULTS AND DISCUSSIONS

#### A. Electronic band structure

The GGA electronic band structure is displayed in Fig. 1. The maximum of the valence band is at the $R[0.5,0.5,0.5]$ point and the minimum of the conduction band is found in the line between the points $\Gamma$ and $Z[0 0 0.5]$. The semiconducting optical gap was estimated at 35 meV from reflectivity measurements; this is consistent with the calculated direct gap of 33 meV, despite the well-known underestimation of the band gap within DFT (see right inset to Fig. 1). The density of states in the vicinity of the Fermi level is largely dominated by the Fe-$d$ states, however, a substantial contribution from the Sb-$p$ states is observed. The projected DOS show that the two major peaks at the top of the valence band and the bottom of the conduction band have mainly $d_{z^2}$ and $d_{x^2−y^2}$ characters. These two enhanced narrow bands and the small band gap suggest strong electronic correlation effects that may be responsible for the magnetic and transport anomalies in FeSb$_2$ at low temperature.
from a constant value $= -0.510^{-5}$ emu/mol at low temperatures (diamagnetic region) toward positive values (paramagnetic region) above $T = 80$ K.

**B. Magnetic susceptibility**

The study of the magnetic susceptibility reveals an interesting diamagnetic to paramagnetic transition at temperature around 100 K, as experimentally observed. Figure 2 displays the magnetic susceptibility $\chi$ as a function of temperature. The magnetic susceptibility is calculated from the electronic density of states $g(\epsilon)$ (Fig. 1, right panel) and the derivative of the Fermi distribution function $f(\epsilon,\mu,T)$ with respect to energy

$$\chi = -2\mu_o\mu_R^2 \int g(\epsilon)[\partial f(\epsilon,\mu,T)/\partial \epsilon]d\epsilon + \chi_0,$$

where $\mu_o$ and $\mu_R$ are the magnetic permittivity of vacuum and the Bohr magneton, respectively. The calculation of the derivative $\partial f(\epsilon,\mu,T)/\partial \epsilon$ requires the knowledge of the chemical potential $\mu$ as a function of electronic temperature, which is evaluated from the conservation of total electrons number $N_e = \int g(\epsilon)f(\epsilon,\mu,T)\,d\epsilon$ is an additional constant term due to the core diamagnetism. Below 40 K, $\chi$ is temperature independent. The diamagnetism ($\chi < 0$) is progressively reduced with rising temperature leading to the paramagnetic behavior above 80 K ($\chi > 0$). The small crossover temperature $T \approx 80$ K is attributed to the narrow band gap. FeSb$_2$ can be presented as a Kondo insulator system where the electronic structure is characterized by two dominant bands at the vicinity of the Fermi level (see Fig. 1), with a given width of $W$, separated by a gap. Raising the temperature leads to delocalization of electrons from lower bands to upper bands and activates the Pauli magnetic susceptibility as shown in Fig. 2. The same scenario has been found to be the underlying mechanism of phase changes observed in the related system FeSi.

**C. Transport properties**

As previously mentioned, the calculation of transport coefficients was performed by using the semiclassical Boltzmann theory applied to the GGA electronic band structure described in Sec. III A.

**1. Seebeck coefficient**

Figure 3 shows the Seebeck coefficient $S$ as function of temperature. A spectacular drop is observed at low temperatures, yielding a minimum value of $S_{\text{max}} \approx -800$ $\mu$V K$^{-1}$ at $T \approx 15$ K. The enhanced peak is followed by a rapid increase of $S$ with $T$ toward positive values above $T \approx 25$ K. Whereas a fairly good agreement is obtained with experimental results, $S_{\text{max}}$ is rather different from the record value $S_{\text{max}} \approx -45000$ $\mu$V K$^{-1}$ measured by Bentien et al. along the $c$ direction. Our calculations also show that $S$ is relatively

FIG. 3. Seebeck coefficient $S$ vs temperature $T$. The inset shows the low-temperature curve between 0 and 40 K for $S$. 

FIG. 2. The temperature evolution of the magnetic susceptibility compared with experimental results extracted from Ref. 3. $\chi$ evolves from a constant value $= -4.510^{-5}$ emu/mol at low temperatures (diamagnetic region) toward positive values (paramagnetic region) above $T = 80$ K.
larger in the \( c \) than in the \( a \) or \( b \) directions. The directional dependence of \( S \) is not given here.

However, the observed record value of \( S(T) \) can not be quantitatively reproduced by a purely electronic-structure picture, or by a classical description of electron diffusion in a nondegenerate system. This implies that other contributions must be present. Although many aspects may suggest that the phonon drag effect plays a dominant role in the large enhancement of \( S \) at \( T \approx 12 \) K, it was also shown in a comparative study on related systems such as RuSb\(_2\) that the role of phonon drag is minor.

2. Donor doping

The quality of the sample used for experimental measurements is known to have a large influence on the value of \( S_{\text{max}} \). The measured values of \( S_{\text{max}} \) depend on the purity of the polycrystal; similar measurements with a less pure polycrystal have led to much smaller values. Our observations indicate that \( S \) is very sensitive to the value of the chemical potential \( \mu \) as is shown in Fig. 4. The value of \( \mu \) depends on the concentration of charge carriers and hence on the purity of the material. In the intrinsic case, where the number of electrons excited to the conduction band \( n \) is equal to the number of holes \( p \), left behind, the chemical potential is \( \mu \approx \Delta/2 + (3/4)k_B T \ln(m_p/m_e) \), where \( m_p \) and \( m_e \) are, respectively, the effective masses of holes and electrons. In this regime, \( \mu \) is displaced from the middle of the band gap by a term that depends on temperature and the ratio of the effective masses. The point of charge neutrality at \( T = 0 \) K is very close to the midgap point. For nonzero temperatures, the donor and acceptor sites can be thermally ionized. In the extrinsic case for finite dopings, the charge neutrality condition \( n + N_d^- = p + N_a^+ \) governs the carrier concentration, where \( N_d^- = N_a[1 - f(\Delta - E_d)] \) and \( N_a^+ = N_d[1 - f(\Delta - E_a)] \) are the concentration of ionized acceptors and donors. \( N_a \) and \( N_d \) are the number of acceptors and donors, \( E_a \) and \( E_d \) the corresponding energy levels, and \( f \) the Fermi function.

In the presence of donor impurities, \( n = p + N_d^+ \) corresponding to \( \mu > 0 \) and \( S < 0 \) in Fig. 4, the chemical potential is \( \mu = (\Delta/2 - E_{\text{imp}}/2) - k_B T \ln(n/N_d^+) \), where \( E_{\text{imp}} = \Delta/2 - E_d \) is the impurity activation energy. The enhancement of the Seebeck coefficient at a given temperature depends on whether the additional carriers bring \( \mu \) toward its optimal value \( \mu_{\text{opt}} \) or not. We found \( \mu_{\text{opt}} = 0.21 \) meV for \( T = 16 \) K corresponding to a carrier density of \( n = 3.2 \times 10^{15}/\text{cm}^3 \). This optimum chemical potential, which maximizes \( S \), is located above the midgap point. The optimum carrier density
found in Ref. 12 corresponds to \( n = 1.3 \times 10^{16}/\text{cm}^3 \) at \( T = 220 \) K. It appears, therefore, that the combination of an optimal doping and small resistivity allows us to maximize the thermopower \( S \).

3. Resistivity

Figure 5 shows the variation of the electrical resistivity \( \rho \) with temperature. \( \rho(T) \) is evaluated assuming a temperature-independent relaxation time \( \tau \) of electrons. For the estimation of \( \tau \), we fit the experimental data reported in Fig. 2 of Ref. 40 by adjusting the value of \( \rho \) at low temperature. We find \( \tau = 0.225 \) ps. A step further toward a complete theoretical description of the transport properties is to go beyond the constant relaxation-time approximation by taking into account the anisotropic nature of the electron scattering by phonons and the temperature dependence of \( \tau \).

Below \( T = 400 \) K, the resistivity exhibits a semiconducting behavior with two different regions separated by a plateau at \( T \approx 20 \) K and is similar to what is observed in Refs. 4 and 21. The activation energy above 50 K obtained by fitting the Arrhenius equation \( \rho(T) = \rho_0 \exp(E_a/2k_B T) \) is estimated to \( E_a \sim 350–450 \) K in agreement with experimental measurements.\(^{40}\) Furthermore, it appears from our calculations that the low-temperature resistivity is highly anisotropic. Along the \( c \) axis, we observe a large increase of \( \rho(T) \) values, in agreement with experimental measurements.\(^{17}\)

The efficiency of thermoelectric materials is determined by the value of the figure of merit \( ZT \) defined above. The thermodynamic properties, such as the velocity of sound and the Debye temperature in these compounds, are different, as the Fe modes here are in a larger energy range. The experimental Sb phonons are very well reproduced by the calculation. In contrast, for the Fe phonons, although the shape is very well reproduced, the experimental modes are somewhat softer than calculated ones, which might be indicative of some significant softening due to electron-phonon interactions already at 50 K. The Debye level can be obtained from the low-energy range in the reduced DPS, \( \lim_{E \to 0}[g(E)/E^2] \) (see inset to Fig. 6). From the Debye level, the velocity of sound \( v_s \) can be obtained by \( \lim_{E \to 0}[g(E)/E^2] = M_R/(2\pi^2 \hbar^3 v_s^3) \), with the mass of the resonant atom \( M_R \) and the density of the material \( \rho \). The mean values of \( v_s \) obtained from the calculated and experimental Fe DPS are 3300(50) and 3050(50) m/s, respectively, in good agreement with the \( v_s \) obtained by the measured Sb DPS (see Table I). The difference between the calculated and experimental velocity of sound can be partly explained by differences in temperature because the experimental DPS was obtained at a finite temperature of 50 K. The partial velocity of sound \( v_s \) along the longitudinal and transversal directions is obtained from the calculated phonon dispersion curves using the acoustic approximation (see Table II). The DPS also gives reliable and direct access to the element-specific mean force constants both for Fe (Ref. 35) and for Sb (Ref. 42) by the relation \( F^m = M_R/\hbar \int_0^\infty g(E)E^2dE \). The obtained \( F^m \) values for Fe and Sb from calculation and experiment are in good agreement (see Table I). The element-specific Debye temperatures can be obtained from the DPS with the relation \( \theta^D = 3/(2\pi^2 \hbar^3 \int_0^\infty g(E)dE/E^3) \), obtained in the high-temperature limit (see Ref. 35). The obtained calculated and experimental values are in agreement (see Table I).

2. Elastic constants

The polycrystalline average of the \( c_{44} \) shear modulus exhibits first softening with increasing temperature up to 120 K, which is the typical behavior observed in most materials (see Fig. 7). However, above 120 K, hardening with increasing temperature is observed, which correlates with the onset of the gradual transition from diamagnetic to paramagnetic behavior and the activation of charge carriers (see experimental data in Fig. 2). This anomalous hardening is a further indication of strong coupling between lattice and electronic degrees of freedom. The qualitative temperature dependence reported herein is not affected by the modeling problems related...
TABLE III. Elastic constants of FeSb₂ with and without contribution from spin-orbit interaction (SOI) in Voigt notation and in units of GPa.

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3. Specific heat

Using the calculated phonon frequencies and density of states, the temperature dependence of specific heat $C_v$ was computed and compared with measurements. The theoretical and experimental results are illustrated in Fig. 8 and are in excellent agreement. The curves evolve smoothly at low temperatures following the Debye model $C_v \sim \left( \frac{T}{\Theta_D} \right)^3$ and approaches the Dulong-Petit limit $C_v \sim 75 \text{ J mol}^{-1} \text{ K}^{-1}$ at high temperatures.

E. Electron-phonon coupling

Our results for the phonon dispersion along several symmetry directions together with the corresponding linewidth broadening due to electron-phonon coupling are displayed in Fig. 9. The energy range of phonon modes extending from 0 up to 35 meV as well as the trend of the electron-phonon coupling along the different branches are consistent with a recent direct experimental measurement using inelastic neutron scattering.44 Among the various areas explored in the first Brillouin zone, we note that the stronger electron-phonon couplings are observed in the vicinity of the $Z[0,0,0.5]$, $X[0.5,0,0]$, and $S[0.5,0.5,0]$ points for the optical modes. As shown in Fig. 6, the optical modes are largely dominated by contribution from Fe ions. The maximum value of the linewidth broadening is achieved at $X$.

The Eliashberg spectral function $\alpha^2 F(\omega)$ is shown in Fig. 10. The value of the dimensionless electron-phonon coupling strength defined through $\lambda = 2 \int \alpha^2 F(\omega) \omega d\omega / \omega$ is 0.24; this value is fairly low compared to calculated values for simple metals.45

FIG. 7. (Color online) Temperature dependence of the shear modulus $c_{44}$ in a sintered polycrystalline sample of FeSb₂.

FIG. 8. (Color online) Plot of the specific heat of FeSb₂ (full line) as a function of temperature. Results are in very good agreement with measurements (dotted line).
the electrical resistivity was highlighted. It can be seen in the
position, but not the large amplitude of the \( T \sim 12 \) K peak in
S, suggesting the possible presence of phonon drag effects\(^{20}\)
as it was already found in pure semiconductors silicon\(^{27,48}\) and
germanium.\(^{49,50}\)

The correlation between the magnetic susceptibility and
the electrical resistivity was highlighted. It can be seen in the
behavior of the resistivity when raising temperature. In the
low-spin-state diamagnetic phase, the value of the resistivity
is four orders of magnitude larger than the value in the para-
magnetic high-spin state at room temperature. This significant
increase of the electrical conductivity in the paramagnetic
region agrees with Goodenough’s hypothesis\(^{51}\) that thermal
excitation favors the population of the less localized orbitals
rather than more localized orbitals. Therefore, the magnetic
susceptibility can be explained either by a low- to high-spin
transition or, as mentioned above, by a thermally activated
Pauli susceptibility. Further band-structure calculations and
experiments are, however, necessary to clarify this issue. For
instance, in the band-structure calculation, the orientation of
localized and nonlocalized orbitals should be known.

The combined experimental and theoretical study has
enabled a direct comparison of measurements and calculations.
Although differences were observed between the measured
and calculated sound velocities, a very good agreement
was obtained for the partial density of phonon states and the tem-
perature dependence of heat capacity. The correlation between
the temperature evolution of the shear moduli and the transition
from diamagnetic to paramagnetic behavior is emphasized. We
have also studied the effects due to electron-phonon coupling.
The linewidth broadening along high-symmetry directions
was analyzed. The stronger electron-phonon interaction was
observed for optical modes of Fe around the \( Z, X, \) and
\( S \) points. This suggests significant mode asymmetry. Since
there are no experimental data available yet, our \textit{ab initio}
estimations of the linewidth broadening due to electron-
phonon interaction presented here may have potential practical
implications. They can provide a basis for addressing a detailed
experimental study of the effects of electron-phonon coupling
in the thermolectric properties of FeSb\(_2\). Inelastic scattering
experiments could more deeply investigate this domain, and
work is ongoing in this direction. Studies led until now do not
reveal a credible and conclusive explanation of the fascinating
low-temperature behavior of FeSb\(_2\).

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