Isospin violation, light quark masses, and all that

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Isospin violation, light quark masses, and all that *

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Abstract Isospin violation is driven through the light quark mass difference and electromagnetic effects. I review recent progress in extracting the light quark mass difference and tests of the chiral dynamics of Quantum Chromodynamics in various reactions involving light as well as heavy quarks.

Key words quantum chromodynamics, isospin violation, quark masses, chiral Lagrangians

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1 Introduction and motivation

In the Standard Model (SM), isospin violation has two sources, namely the differences of the light quark (u, d) masses (QCD) and their charges (QED),

$$\mathcal{H}_{\rm QCD}(x) = \frac{1}{2}(m_{\rm d}-m_{\rm u})(\bar{\rm d}{\rm d}-\bar{\rm u}{\rm u})(x), \quad (1)$$

$$\mathcal{H}_{\rm QED}(x) = -\frac{{\rm i}e}{2}\left(\bar{u}\gamma_\mu A^\mu u-\bar{d}\gamma_\mu A^\mu d\right)(x) \,. \quad (2)$$

Isospin violation (IV) thus offers a unique window to access the light quark mass difference (ratio) in light or heavy-light quark systems as I will show in a variety of examples in this talk. In most cases these strong (str) and electromagnetic (em) effects are of the same size, cf. the neutron-proton mass difference discussed below, and must be treated consistently. Furthermore, in many cases the small IV effects sit on top of a large isospin-conserving "background" and thus an accurate theoretical machinery is mandatory to be able to extract the information encoded in the isospin-violating contributions. This machinery is given by chiral perturbation theory (CHPT) coupled to virtual photons. This will constitute the theoretical framework underlying the topics presented here.

2 Isospin-breaking in the pion-nucleon scattering lengths

Already decades ago, Weinberg pointed out that systems of (neutral) pions and nucleons are particularly sensitive to IV [1]. This problem was readdressed in the framework of heavy baryon CHPT about a decade ago, see e.g. Ref. [2]. Interest was rekindled due to the precision measurements of pionic hydrogen and deuterium at PSI [3]. In the framework of covariant baryon CHPT, [4] the elastic π^-p amplitude was calculated at threshold to third (leading one-loop) order in the chiral expansion. In Ref. [5], the extension to all physical channels was given based on the Feynman graphs shown in Fig. 1.

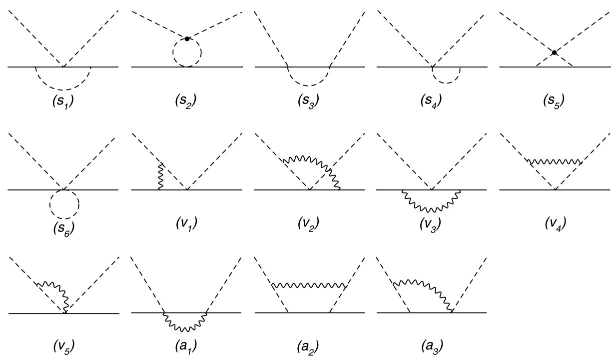

Fig. 1. IV in πN scattering: loop diagrams at third order. Solid, dashed and wiggly lines denote nucleons, pions and photons, in order.

In particular, analytical formulae to leading oder in the isospin breaking parameter $\delta = \{m_{\rm d}-m_{\rm u}, e^2\}$ are given, e.g. the IV shift for the isospin-symmetric amplitude a^+ reads:

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$$\Delta a^+ = \frac{m_p}{4\pi(m_p+M_\pi)}\left\{\frac{4\Delta_\pi}{F_\pi^2}c_1 - \frac{e^2}{2}(4f_1+f_2) - \frac{g_A^2 M_\pi}{32\pi F_\pi^2}\left(\frac{33\Delta_\pi}{4F_\pi^2}+e^2\right)\right\}, \quad (3)$$

where $f_1, f_2\,(c_1)$ are dimension two em (strong) low-energy constants (LECs), $\Delta_\pi = M_\pi^2 - M_{\pi^0}^2$ (with M_π the charged pion mass) is the charged-to-neutral pion mass difference, F_π the pion decay constant and m_p the proton mass. Note in particular the sizable prefactor 33/4 that signals the large contribution from the triangle diagram (s_5) in Fig. 1. The numerical results for all pion-proton threshold amplitudes are collected in Table 1. The imaginary part in $\pi^-p \to \pi^-p$ is due to the π^0n intermediate state in the rescattering graph (s_3). In the charge-exchange amplitude, the contribution from the triangle graph is absent and thus the corrections are smaller. Note also that the uncertainty is largely given by our poor knowledge of the dimension two and three em LECs. In particular, the IV corrections to the triangle ratio (that vanishes in the isospin limit) turn out to be (1.5±1.1)%, consistent with earlier findings in heavy baryon CHPT [6]. An remarkably large shift due to the cusp effect in the π^0p amplitude is predicted, that might be measured in third generation photoproduction experiments at HIγS or MAMI. An extension of this work above threshold is given in Ref. [7].

Table 1. Predictions for the IV shifts to all pion-proton scattering lengths in units of $10^{-3}/M_\pi$.

isospin limit	channel	shift
$a^+ + a^-$	$\pi^-p\to\pi^-p$	$-3.4^{+4.3}_{-6.5}+5.0i$
$a^+ - a^-$	$\pi^+p\to\pi^+p$	$-5.3^{+4.3}_{-6.5}$
$-\sqrt{2}a^-$	$\pi^-p\to\pi^0n$	0.4 ± 0.9
a^+	$\pi^0p\to\pi^0p$	-5.2 ± 0.2

3 The strong neutron-proton mass difference from $np\to d\pi^0$

Amongst the IV effects in hadronic reactions the ones that are charge-symmetry-breaking (CSB), i.e. that emerge from an interchange of up and down quarks, are of particular interest. Their importance is due to the fact that the neutral-to-charged pion mass difference, which is almost entirely of em origin and usually dominates IV hadronic observables, does not contribute here. Therefore, the sensitivity to the quark mass difference m_d-m_u is enhanced in observables related to CSB. Recently, experimental evidence for CSB was found in reactions involving the production of neutral pions. At IUCF non-zero values for the $dd \to \alpha\pi^0$ cross section were established [8]. At TRIUMF a forward-backward (fb) asymmetry of the differential cross section for $pn \to d\pi^0$ was reported [9],

$$A_{fb} = \frac{\int_0^{\pi/2}\left[\frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi-\theta)\right]d\cos\theta}{\int_0^{\pi/2}\left[\frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(\pi-\theta)\right]d\cos\theta} = (17.2\pm8(\text{stat})\pm5.5(\text{sys}))\cdot10^{-4}\ . \quad (4)$$

Theoretically, A_{fb} is directly related to the interference of an isospin-conserving and an IV amplitude,

$$\frac{d\sigma}{d\Omega} = A_0 + A_1 P_1(\cos\theta_\pi) + \ldots \to A_{fb} \simeq \frac{A_1}{2A_0}\ . \quad (5)$$

The first calculation utilizing chiral EFT was performed in Ref. [10]. This was improved recently in Ref. [11]. It was shown that in the rescattering graph not only the IV Weinberg-Tomozawa term but also the neutron-proton mass difference on the nucleon lines plays a role, as shown in Fig. 2. These two LO contributions combine in such a way, that A_{fb} at LO in the chiral expansion is entirely given by the strong neutron-proton mass difference.

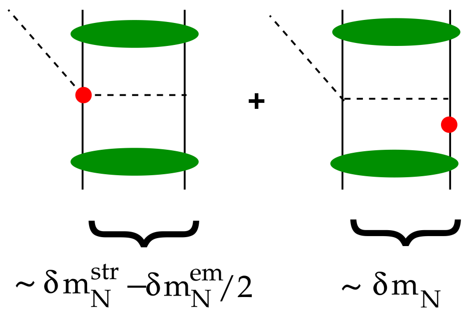

Fig. 2. (color online). LO (rescattering) diagrams for IV S-waves in $pn \to d\pi^0$. Left: IV in the πN vertex. Right: IV contribution due to the neutron-proton mass difference.

A crucial ingredient for a precise calculation of A_{fb} is to take A_0 from the precision PSI experiment [12] using isospin invariance. Putting pieces together, one can express the fb-asymmetry in terms of δm_N^{str} as

$$A_{fb}^{LO} = (11.5\pm3.5)\cdot10^{-4}(\delta m_N^{str}/\text{MeV})\ . \quad (6)$$

From the measured asymmetry Eq. (4) one can thus deduce

$$\delta m_N^{str} = 1.5\pm0.8\,(\text{exp})\pm0.5\,(\text{th})\ \text{MeV}\ . \quad (7)$$

This is nicely consistent with the extraction based on the Cottingham sum rule, $\delta m_N^{str} = 2.05\pm0.30$ MeV [13] and a recent lattice determination, [14] $\delta m_N^{str} = 2.26\pm$

0.57±0.42±0.10 MeV. This calculation is a particular beautiful example how very different quantities and processes are related through the same LECs – here the dimension two LEC c_5 that drives the strong IV in the nucleon sector [15]. It also shows that a refined NLO calculation of $A_{\rm fb}$ will offer a viable alternative to extract this fundamental quantity (the corresponding isospin-symmetric calculations of pion production where recently performed [16]).

4 EM corrections to $\eta \to 3\pi$ decays

The charged ($\eta \to \pi^+\pi^-\pi^0$) and neutral ($\eta \to 3\pi^0$) three-pion decays of the η meson are driven by isospin violation,

$$\Gamma(\eta \to 3\pi) \sim Q^{-4}, \quad Q^{-1} = \frac{m_{\rm u} - m_{\rm d}}{m_{\rm s} - \hat{m}} \; . \qquad (8)$$

The amplitude for $\eta \to 3\pi$ has been calculated to two-loop accuracy in CHPT [17] and em corrections at $\mathcal{O}(e^2\, m_{\rm quark})$ were analyzed in Ref. [18] (BKW). Since these turned out to be very small, in Ref. [19] (DKM) the formally subleading em corrections of $\mathcal{O}(e^2(m_{\rm u}-m_{\rm d}))$ have been calculated. There are various reasons why one should consider these subleading IV corrections: by restricting oneself to terms of the form $e^2\hat{m}$ (where $\hat{m} = (m_{\rm u}+m_{\rm d})/2$) and $e^2 m_{\rm s}$, one excludes some of the most obvious electromagnetic effects: real and virtual photon contributions, as well as effects due to the charged-to-neutral pion mass difference (which is predominantly of electromagnetic origin), both of which scale as $e^2(m_{\rm d}-m_{\rm u})$. These mechanisms fundamentally affect the analytic structure of the amplitudes in question: in the charged decay channel $\eta \to \pi^+\pi^-\pi^0$, there is a Coulomb pole at the boundary of the physical region (at the $\pi^+\pi^-$ threshold), while in the neutral decay channel $\eta \to 3\pi^0$, the pion mass difference induces a cusp behavior at the $\pi^+\pi^-$ thresholds (see next section). At this order, the neutral and charged amplitudes must be calculated separately.

It turns out that the em corrections are in general small, but need to be accounted for if one wants e.g. to determine the Dalitz slope parameters to a high accuracy. Also, corrections of order $e^2(m_{\rm d}-m_{\rm u})$ (DKM) turn out to be as big (or even bigger) as the ones of order $e^2 m_{\rm quark}$ (BKW). In addition, cusps in the neutral amplitude due to rescattering $\pi^0\pi^0 \to \pi^+\pi^- \to \pi^0\pi^0$ are clearly visible, see Fig. 3 where the em corrections are shown in comparison to the strong one-loop result of Ref. [20] (GL). For more details, I refer to Ref. [19] (see also [21]).

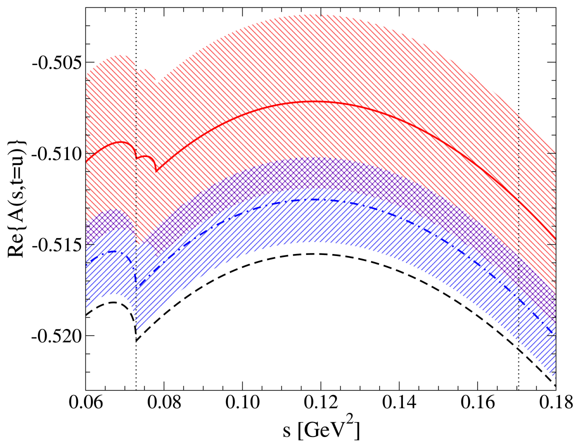

Fig. 3. (color online). Real part of the neutral amplitudes corresponding to GL (dashed), BKW (dot-dashed), and DKM (full) for $t = u$. The hatched regions denote the error bands due to a variation of electromagnetic low-energy constants. The vertical lines show the limits of the physical region.

5 The cusp in $\eta' \to \eta\pi^0\pi^0$

The discovery of the cusp in $K^+ \to \pi^+\pi^0\pi^0$ by the NA 48/2 collaboration [22] has renewed interest in such effects (for early works see e.g. Refs. [23, 24]) since it allows for a precise extraction of the pion-pion scattering lengths [25–27]. The cusp effect is driven by the rescattering from two neutral pions through an intermediate charged pion pair and should thus also be visible in other processes like $\eta \to 3\pi^0$ (as discussed before) or $\eta' \to \eta\pi^0\pi^0$. While the former reaction has been measured and the data are not yet precise enough to analyze the cusp (see e.g. Ref. [28]), it was pointed out in Ref. [29] that in the isospin limit $BR(\eta' \to \eta\pi^+\pi^-) = 2BR(\eta' \to \eta\pi^0\pi^0)$ so that the cusp should be strongly visible in $\eta' \to \eta\pi\pi$ decays. In that paper, a detailed analysis of the cusp based on non-relativistic EFT, which is the most appropriate tool to investigate such phenomena, was performed. The central results in that paper are the various decay amplitudes, calculated to two loop accuracy. Invoking theoretical information on the coupling constants involved, the size of the cusp effect can be predicted. It reduces the decay spectrum below the charged-pion threshold by more than 8%, see Fig. 4. This is a much more sizeable effect than e.g. in $\eta \to 3\pi^0$ decays. Approximate isospin symmetry dictates that the cusp of $\mathcal{O}(a^2)$ above threshold is strongly suppressed, and three-loop effects can be estimated to yield a correction below 1%. Therefore the threshold singularity in $\eta' \to \eta\pi^0\pi^0$ is determined to very high precision by the leading $\mathcal{O}(a)$ rescattering effect. Experimental verifications of these predictions at various laboratories are eagerly awaited. It was also shown in [30] that the πη threshold parameters can not be deduced from the measured decay spectrum. This can be seen as follows: Since the opening of the πη channel is at the border of the Dalitz plot there is no one-loop cusp in the physical region as in ππ scattering. Further, there is an exact cancellation between the product of two-loop and tree graph and the product of two one-loop graphs. Thus, without exact theoretical knowledge of the tree-level couplings an extraction of πη threshold parameters in this framework is not possible.

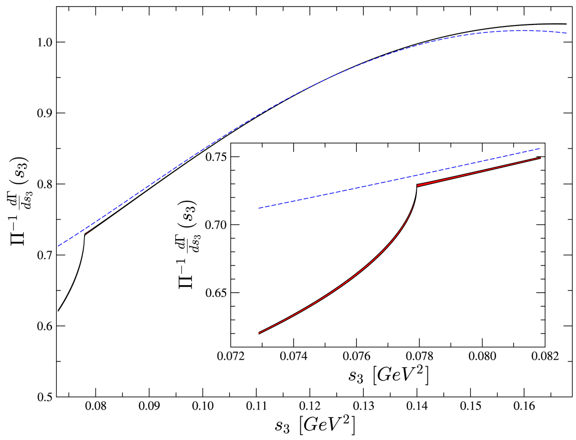

Fig. 4. (color online). The differential decay rate $\mathrm{d}\Gamma/\mathrm{d}s_3$ divided by phase space for the tree (dashed) and full (band) amplitude that clearly shows the cusp effect.

6 The light quark mass ratio m_u/m_d from ψ′ decays

The decays of the ψ′ into J/ψπ^0 and J/ψη were suggested to be a reliable source for extracting the light quark mass ratio m_u/m_d, see e.g. Refs. [31, 32]. Based on the QCD multipole expansion and the axial anomaly, the relation between the quark mass ratio and the ratio of the decay widths of these two decays was worked out up to next-to-leading order in the chiral expansion. Including leading $SU(3)$ breaking effects, one obtains

$$R_{\pi^0/\eta} = \frac{Br(\psi'\to J/\psi\pi^0)}{Br(\psi'\to J/\psi\eta)} = 3\left(\frac{m_d-m_u}{m_d+m_u}\right)^2 \frac{F_\pi^2}{F_\eta^2}\frac{M_\pi^4}{M_\eta^4}\left|\frac{\vec{q}_\pi}{\vec{q}_\eta}\right|^3 . \quad (9)$$

Using the most recent measurement of the decay-width ratio [33], one obtains $m_u/m_d = 0.40\pm0.01$ which differs significantly from the LO analysis of the Goldstone boson masses, which leads to $m_u/m_d = 0.56$. Most recent determinations of this ratio including also lattice data give $m_u/m_d = 0.47\pm0.08$ [34], which is barely compatible with the value extracted from Eq. (9). It is therefore of fundamental interest to understand theoretically the discrepancy between the values of the up-down quark mass ratio determined from different sources. In Ref. [35] it was shown that the theoretical framework to relate the ψ′ decays into J/ψπ^0(η) is not yet accurate enough for extracting the quark mass ratio to the desired precision. In that paper, the contribution from intermediate charmed meson loops (see also the talk by Zhao at this workshop) was worked out to leading order in HQEFT. Because the ψ′ and J/ψ are $SU(3)$ singlets, it is obvious that the decay ψ′ → J/ψπ^0 violates isospin symmetry, and the decay ψ′ → J/ψη violates $SU(3)$ flavor symmetry. Accordingly, the decay amplitudes reflect the flavor symmetry breaking. Here, all the charmed mesons in a flavor multiplet can contribute, and it is the mass differences within the multiplet that generates the isospin or the $SU(3)$ breaking. In fact, for heavy quarks, the velocity is the appropriate expansion parameter, here $v\sim\sqrt{(2M_{\hat{D}}-M_{\hat{\psi}})/M_{\hat{D}}}\simeq 0.5$, where $M_{\hat{D}}$ is the averaged charmed-meson mass, and $M_{\hat{\psi}}=(M_{J/\psi}+M_{\psi'})/2$. As it is well-known for charm systems, this expansion parameter is not very small but for our purposes of discussing the influence of the so far neglected loops, a leading order analysis in v suffices. The loops with virtual intermediate charmed mesons themselves scale as $v^3\cdot v^2/(v^2)^2=v$, whereas the remaining terms proportional to m_q (that lead to the prediction Eq. (9)) are proportional to an energy scale of $\mathcal{O}(v^2)$. Consequently

$$\mathcal{M}(\psi'\to J/\psi\pi^0)_{\rm direct} \sim (m_d-m_u)\,|\vec{q}_\pi| ,$$
$$\mathcal{M}(\psi'\to J/\psi\pi^0)_{\rm D-loops} \sim (m_d-m_u)\frac{|\vec{q}_\pi|}{v}. \quad (10)$$

Therefore, the charmed meson loop contribution is potentially enhanced compared to the direct term. This power counting estimate was confirmed by explicit calculations of the various one-loop graphs in Ref. [35]. One can take an extreme position and assume that the intermediate charmed-meson loop mechanism saturates the decay widths of the ψ′ → J/ψπ^0(η). This leads to $R_{\pi^0/\eta} = 11\%\ldots14\%$ (with an uncertainty of about 50% from higher orders), which is larger than the experimental number [33], $R_{\pi^0/\eta}=(3.88\pm0.23\pm0.05)\%$. This findings are similar to the ones of Ref. [36], where it was shown that meson loops invalidate the direct relation between the ratio $\Gamma(\eta'\to\pi^0\pi^+\pi^-)/\Gamma(\eta'\to\eta\pi^+\pi^-)$ and the quark mass ratio $(m_d-m_u)/(m_s-\hat{m})$. A NLO calculation of the charmed meson loop contributions to $\psi' \to J/\psi\pi^0(\eta)$ should be performed to find out how accurately the ratio m_u/m_d can indeed be inferred from these decays.

7 Mass splittings in heavy baryon multiplets

Mass splittings within isospin multiplets of hadrons appear due to both the mass difference between u and d quarks and electromagnetic effects. Since the d quark is heavier than the u, usually the hadron with more d quarks is heavier within one isospin multiplet. For instance, the neutron (udd) is heavier than the proton (uud), and the $K^0(d\bar{s})$ is heavier than the $K^+(u\bar{s})$. There is only one exception to this pattern, the Σ_c iso-triplet (Σ_c^{++}(cuu), Σ_c^+(cud) and Σ_c^0(cdd)). The mass splittings within the Σ_c iso-triplet are measured

$$\Delta_{1c} \equiv m_{\Sigma_c^+} - m_{\Sigma_c^0} = -0.9 \pm 0.4 \text{ MeV},$$
$$\Delta_{2c} \equiv m_{\Sigma_c^{++}} - m_{\Sigma_c^0} = 0.27 \pm 0.11 \text{ MeV}. \quad (11)$$

Remarkably, the state with two u quarks has the largest and the one with a u and a d quark has the smallest mass. Only recently some of the bottom cousins of the Σ_c, $\Sigma_b^\pm$, were observed by the CDF Collaboration [37]. Their masses are for the buu state $m_{\Sigma_b^+} = 5807.8 \pm 2.7$ MeV and for the bdd state $m_{\Sigma_b^-} = 5815.2 \pm 2.0$ MeV, respectively — their neutral partner Σ_b^0 has not been observed yet. Thus, here the natural ordering of the states seems to be restored. On the other hand, heavy quark symmetry relates baryons containing a b quark to those with a c quark.

In Ref. [38], we have calculated the mass splittings within the heavy baryon isospin multiplets $\Sigma_{c(b)}$ and $\Xi'_{c(b)}$ to $\mathcal{O}(p^3)$ in the chiral expansion. For doing that, we constructed both the strong and the em Lagrangians at $\mathcal{O}(p^2)$ which are responsible for the mass corrections. In contrast to mass splittings in light quark baryon multiplets, there is an additional operator that describes the hard virtual photons exchanged between the heavy quark and light quarks accompanied by a LEC β_{1h}. Remarkably, this term has a different sign for the charm baryons and the bottom baryons. This is due to the fact that the sign of the electric charge of the charm quark is different from that of the bottom quark. It is the different interference between this term and the other terms that drives the mass splittings within the Σ_c iso-triplet to have a different pattern compared to any other known isospin multiplet. This leads one to expect that the isospin mass splittings in the charm hadrons are always different from those in the bottom hadrons even if the heavy quark symmetry were exact. Besides the heavy baryons considered in Ref. [38], the D and B-meson mass splittings, $m_{D^\pm} - m_{D^0} = 4.78 \pm 0.10$ MeV and $m_{B^0} - m_{B^\pm} = 0.37 \pm 0.24$ MeV are a nice example for the effect, although the ordering does not get changed here.

There is no loop contribution to the mass splitting between the two Ξ'_c baryons, and we predict $m_{\Xi_c'^+} - m_{\Xi_c'^0} = -0.2 \pm 0.6$ MeV. The present data for the masses of the Ξ'_c baryons are not accurate enough yet to test the prediction. For the Σ_b states, the β_{1h} term interferes constructively with the other terms and hence the loop corrections are less important. The mass of the Σ_b^0 and the mass difference $m_{\Xi_b'^0} - m_{\Xi_b'^-}$ are predicted to be 5810.3±1.9 MeV and -4.0 ± 1.9 MeV, respectively, which can be tested in future experiments. For an alternative view, see Ref. [39].

8 Summary and outlook

Precision calculations in the light quark sector lead to important tests of the flavor and the symmetry structure of QCD. I have discussed the intricate interplay of strong and electromagnetic isospin violation for four very different processes, namely elastic pion-nucleon scattering, neutron-proton fusion to a deuteron and a neutral pion, higher order electromagnetic corrections in $\eta \to 3\pi$ decays and the cusp in $\eta' \to \eta\pi\pi$. There are also intriguing isospin-violating effects in heavy-light systems. He, I focused on the extraction of the light quark mass ratio from ψ' decays and the generation of mass splittings in baryon multiplets with one heavy (c,b) quark. Clearly, refined theoretical calculations are called for. I outlined a few directions that deserve more attention. I have pointed out that this field also offers various great experimental challenges that should not be missed.

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