 Proposal for a direct measurement of the nonadiabatic spin-transfer torque parameter $\beta$ and the spin-polarization rate $P$

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We propose a method to measure the nonadiabatic spin-transfer torque parameter $\beta$ and the spin-polarization rate $P$ independently. These quantities are crucial for current-driven magnetization processes, but their value could so far hardly be measured. It is shown that the motion of transverse domain walls in cylindrical nanowires, driven by a magnetic field and by an electric current, can be used to extract those values. This domain wall type propagates with a precessional motion around the wire, where the domain wall acts as a magnetic dipole rotating at well-defined frequency. The spin-polarization rate $P$ can be deduced from the current-induced axial motion of the domain wall, while the rotational frequency of the domain wall and its variation with an external field allows one to determine the nonadiabatic spin-transfer torque. This proposal for an experiment to directly measure these quantities is based on analytical calculations and micromagnetic simulations.

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I. INTRODUCTION

In spite of intense research on current-driven magnetization dynamics, the basic parameters governing the equation of motion remain largely inaccessible for direct experimental measurements. Those parameters are the spin polarization $P$ and the nonadiabatic spin-transfer torque parameter $\beta$. Theory predicts that these quantities have a direct impact on spin-transfer effects, by which spin-polarized electric currents can displace domain walls (DWs) in ferromagnets or excite radio-frequency oscillations in nanomagnets [1,2]. The current-driven DW displacement [3] is governed by two microscopic processes: the adiabatic [4] and the nonadiabatic [5,6] spin-transfer torque. While powerful computer codes can solve the equations describing these effects and hence simulate the motion of domain walls, the quantitative reliability of the simulation results often hinges on the values $P$ and $\beta$. The main problem in measuring the values consists in disentangling the nonadiabatic spin-transfer torque parameter $\beta$ from the spin polarization $P$, since their direct impact on the magnetization is usually given by a product $\beta P$, and not by the individual quantities. The dimensionless parameter $\beta$ is the ratio of the nonadiabatic to the adiabatic spin torque [5,6]. Even though $\beta$ is believed to be small (i.e., comparable to the Gilbert damping parameter $\alpha$ [7]) [5], the nonadiabatic term is nonnegligible in the current-driven motion of DWs in thin strips [5]. The DW speed, the intrinsic pinning, and the critical value for the Walker breakdown [6,8] are related to the value of the nonadiabatic spin torque [5,6]. The physical origin of this spin torque term is still under debate. While some ascribe the nonadiabatic spin transfer to a Larmor precession of the spins of the conduction electrons [9], it is more frequently related to the spatial mistracking of the spin orientation of the conduction electrons and the local magnetization [10–12]. The latter picture is usually discussed in an $s$–$d$ model, which considers the exchange coupling between itinerant and local electrons. In this theory, the strength of the nonadiabatic spin torque is a result of the exchange coupling and of spin-flip relaxation processes [5]. A connection between the Gilbert damping $\alpha$ and the nonadiabatic spin-torque parameter $\beta$ has also been discussed [13,14]. In order to probe these theories and to obtain reliable input parameters for micromagnetic simulations, a precise measurement of the magnitude of the nonadiabatic torque term is necessary.

In addition to its unclear physical origin, there is also some controversy concerning the numerical value of $\beta$. Estimates for $\beta$ have been obtained from theory and experiments [16,17]. Some studies suggest that $\beta$ is equal to $\alpha$ [18–20], while others predict the necessity of a difference between these parameters [21,22]. Experiments evidencing the current-induced transformation of DW structures imply that $\alpha$ and $\beta$ are not equal [23]. Measurements from various groups have reported significant differences for the ratio $\beta/\alpha$ as well as for the value of $\beta$, ranging from small ($\beta = \alpha = 0.02$ [24], $\beta = 2\alpha = 0.02$ [25], $\beta = 2\alpha \sim 3\alpha = 0.02$ [26]) to intermediate ($\beta = 2\alpha = 0.04$ [16], $\beta = 8\alpha = 0.04$ [27]) and large values ($\beta = 0.15$ [28], $\beta = 1.45$ [29]). The methods proposed so far to measure $\beta$ are generally based on the impact of a spin-polarized current on DWs in thin strips. Only two approaches have been reported so far which deviate from this rule: One relies on the current-induced spin wave attenuation [17,26], the other on the impact of the spin torque effect on the vortex dynamics [28]. The advantage of using domain walls is that the main dynamical properties, such as the DW velocity [24], the depinning current or field [16,29,30], the dwell time [31], or the DW resonant frequency [25,27], are relatively easily accessible in experimental measurements. However, these
studies require a comparison with micromagnetic simulation data and hence $\beta$ is obtained indirectly, by fitting the value in the micromagnetic simulations so that the best agreement with the experiment is achieved. This procedure obviously precludes a parameter-free measurement. Moreover, with these schemes it is usually not possible to separate $\beta$ from its product with the spin-polarization rate $P$ [16]. Here we propose an alternative way for a parameter-free and robust measurement of $\beta$ and $P$ [32].

II. DYNAMICS OF ONE-DIMENSIONAL HEAD-TO-HEAD WALLS

The method proposed in this paper is based on the combination of the field- and current-induced motion of transverse DWs in soft magnetic cylindrical nanowires. Fabricating such nanowires of several microns length, with a well-defined diameter of a few tens of nanometers and with homogeneous material properties is feasible, as demonstrated and discussed, e.g., in Ref. [33] and references therein. It was recently reported that transverse DWs in round wires show fundamentally different dynamical properties compared to DWs in thin-film strips [34]. Here we describe how the unique dynamic properties of these domain walls can be used to measure $\beta$ and $P$ independently. The proposal is presented using an analytical model and micromagnetic simulation studies, but the simulations here only represent virtual experiments. They are not required to evaluate $\beta$ by means of a comparison or fitting procedure.

In sufficiently thin cylindrical magnetic nanowires (a few tens of nanometers in diameter), a one-dimensional transverse DW forms between domains with opposite magnetiza-

![Fig. 1](https://example.com/fig1.png)

**FIG. 1.** (Color online) Simulation result showing the magnetic structure of a transverse DW in a cylindrical wire, including a drawing of the Cartesian and the spherical coordinate systems used in the analysis. The inset on the lower right displays a cross section through the middle of the DW. As the DW propagates along $z$, the magnetization in the center of the DW rotates in the $xy$ plane due to the torque terms $\tau_H$ (due to the external field) and $\tau_j^{\text{spin}}$ (due to spin-torque effects).

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transverse DWs in cylindrical wires for such measurements is that they neither display an intrinsic pinning nor a Walker breakdown when driven by a field and/or a current, irrespective of the value of $\beta$ [34]. This is due to the characteristic spiraling motion of the DW, which avoids the deformation of the DW configuration during motion. Simulations show that the DW continues to move smoothly even if unrealistically large values of currents [34] are applied. For our method we need reliable procedures to obtain three quantities: $\alpha$, $\xi = (\partial \theta / \partial t)$, $\zeta = (\partial \phi / \partial t)$. Let us begin with $\xi = (\partial \theta / \partial t)$.

### III. MEASUREMENT METHOD

The propagation speed $v$ of the current-driven DW is proportional to the angular velocity of $\theta$. More precisely, the domain wall velocity is $v = \xi / (\partial \theta / \partial z)_{c}$. Measuring the velocity of a current-driven domain wall is nowadays a rather straightforward procedure and can be achieved, e.g., by means of the time delay of the signal obtained from GMR sensors placed at different positions along the wire; cf. Fig. 2. Assuming that the value of $v$ is available from such a measurement, the value of $(\partial \theta / \partial z)_{c}$ is required to determine $\xi$. For this purpose it is helpful to recall that the domain wall has a simple one-dimensional structure $\theta = \theta(z)$ which can be calculated analytically with high accuracy, and that this domain wall profile remains almost invariant when the domain wall is in motion, owing to the negligible Döring wall mass [37] of this type of DW [34]. A simple analytic calculation, adjusted by comparison with numerical results, yields

$$\left( \frac{\partial \theta}{\partial z} \right)_{c} = \frac{\mu_{0} M_{s}^{2}}{6A} \tag{6}$$

as a very good approximation. The derivation of this equation is described in Ref. [38]. In Eq. (6), $A$ is the micromagnetic exchange constant and $\mu_{0} = 4\pi \times 10^{-7} \text{Vs/A}$ is the vacuum permeability. We have tested the validity of Eq. (6) for different wire thickness and materials, always obtaining a virtually perfect agreement with the simulation results. Hence, by measuring the current-driven domain wall velocity in a thin wire of soft-magnetic material with known parameters $A$, $M_{s}$, the angular velocity of $\theta$ is

$$\frac{\partial \theta}{\partial t} = v \sqrt{\frac{\mu_{0} M_{s}^{2}}{6A}}. \tag{7}$$

The sign of $\xi$ depends on the type of domain wall (head-to-head or tail-to-tail) and on the propagation direction.

Having clarified $\xi$, the next task consists in establishing the value of $\zeta = (\partial \phi / \partial t)$. The rotational motion of the DWs in round wires could be detected with a GMR sensor placed in the middle of the sample (cf. Fig. 2). The locally strong dipolar field connected with the rotating domain wall should be easily measurable with modern techniques, resulting in a high-frequency variation of the GMR signal. Hence, the measurement of the azimuthal angular frequency of the magnetization is simpler than the measurement of $\xi$, because it can be determined directly, without requiring additional information such as the domain wall profile. There is however an important practical difficulty that concerns the sign of $\zeta$. From experimental data it is almost impossible to distinguish between $\zeta$ and $-\zeta$ (same frequency, opposite sense of rotation). This ambiguity can be resolved by applying a homogeneous magnetic field along the wire, which acts on the domain wall in addition to the spin-polarized current. In a magnetic field along the $z$ axis, the DW moves in a similar
spiralizing way, according to the following equations [34]:
\[
\frac{d\theta}{dt} = -\alpha \frac{d\phi}{dt}, \quad (8a)
\]
\[
\frac{d\phi}{dt} = \frac{\gamma H}{1 + \alpha^2}, \quad (8b)
\]
where \( \gamma \) is the gyromagnetic ratio. The effect of the magnetic field on the the DW is to displace it in the field direction with a velocity proportional to \( \alpha \), while it precesses with the Larmor frequency around the wire axis. By combining field-driven and current-driven motion, the rotational frequency of the DW has two contributions. If a magnetic field parallel to the wire axis is used to generate the fundamental rotation frequency \( \omega_0 \) for the DW, an additional electric current will result in an increase or in a decrease, \( \omega = \omega_0 + \Delta \omega \), depending on the type of domain wall (head-to-head or tail-to-tail), the sign of \( \xi \) and on the current direction. By measuring the frequency shift \( \Delta \omega \), the value of \( \beta \) can thus be determined. Since the domain wall structure and the current direction can be considered as known, the direction of the frequency shift induced by the current provides the missing information on the sign of \( \xi \). For completeness, we point out that DWs in thin-film strips display an oscillatory mode at velocities above the Walker break down, which is characterized by repeated transformations between different DW configurations [6], and that also in this oscillatory mode a systematic but weak current dependence of the precessional frequency was reported [39].

The last quantity that needs to be determined in our measurement scheme is \( \alpha \), the Gilbert damping constant. For this, Eq. (8b) already provides in principle all that is needed. We assume that the Gilbert damping \( \alpha \) is a constant and that effects such as spin-motive forces and enhanced damping [40,41] are negligible, owing to the large domain wall width of about 20 nm. By measuring the precession frequency \( (d\phi /dt) \) of the DW in an external field \( H \) (as indicated by the sensor “c” in Fig. 2), the Gilbert damping coefficient can be determined according to
\[
\alpha = \sqrt{\frac{\gamma H}{d\phi /dt}} - 1. \quad (9)
\]
If the result of \( \alpha \) obtained from Eq. (9) is very small (\( \alpha \ll 1 \)), the error margin connected with the measurement of \( (d\phi /dt) \) can be important. A lack of accuracy in the measurement of \( (d\phi /dt) \) can in such cases even lead to unphysical imaginary values of \( \alpha \) resulting from equation (9). Therefore, especially if the measurements indicate that \( \alpha \) is very small, it is preferable to determine \( (d\theta /dt) \) in addition to \( (d\phi /dt) \) and to use Eq. (8a) instead to calculate \( \alpha \). Like in the previously discussed case of current-driven domain wall motion, the value of \( (d\theta /dt) \) in the field-driven case can be derived from the domain wall velocity by means of Eq. (7).

With \( \alpha, \xi, \) and \( \chi \) as measurable quantities, Eq. (5) can be used to determine \( \beta \). Once the value of \( \beta \) and \( \alpha \) is established, the spin-polarization rate \( P \) follows from Eqs. (3a) and (2):
\[
P = \left( \frac{2eM_s}{g\mu_B j} \right)\left( v \left( 1 + \alpha^2 \right) \right) \left( 1 + \alpha \beta \right), \quad (10)
\]
for a current-driven domain wall with velocity \( v \). This provides an alternative way to measure \( P \), which parallels the recently established method based on current-induced spin-wave Doppler shift [26,42].

**IV. MICROMAGNETIC SIMULATIONS**

In the following, we demonstrate the feasibility of our method by means of micromagnetic simulations. The simulations provide data that, in principle, should be equivalent to that obtained from the previously described measurements. We numerically solve Eq. (1) with a finite-element code [43–45] and consider, as an example, a 10 nm diameter and 4 \( \mu \)m long cylindrical Permalloy wire (saturation magnetization \( \mu_0 M_s = 1 \) T, zero anisotropy, exchange constant \( \lambda = 1.3 \times 10^{-11} \) J/m). The wire is discretized into 259 200 irregular tetrahedrons with cell size of about 1.25 nm \( \times 1.25 \) nm \( \times 5 \) nm. For the simulation of the field-driven dynamics we assume a 50 Oe field applied in the negative \( z \) direction, which corresponds to a Larmor frequency of 140 MHz. The damping parameter \( \alpha \) is set to 0.02.

The DW motion in the 50 Oe field is shown in Figs. 3(a) and 3(b), which display the average \( z \) and \( y \) component of the magnetization, respectively, as a function of time. Because of the small value of \( \alpha \) [cf. Eq. (8a)], the DW moves very slowly in the negative \( z \) direction (\( v_0 < 1 \) m/s) and the analytic value of the Larmor frequency of the DW rotation according to Eq. (8b) is well reproduced (because \( 1 + \alpha^2 \sim 1 \)). The modification of this field-driven DW motion induced by an additional current is also shown Figs. 3(a) and 3(b). From Fig. 3(a) it is evident that the electric current \( j = 10^{12} \) A/m\(^2\) drives the DW with a much higher speed than the magnetic field; i.e., \( v_1 \gg v_0 \). As discussed before, the rotational frequency \( \omega_0 \) of the DW is shifted either to higher or to lower values by spin-torque effects, depending on the type of the domain wall and the sign of the current. The simulation results displayed in Fig. 4 confirm that the frequency shift induced by the nonadiabatic torque should be large enough to be detected easily. We also performed systematic studies with different values of \( \beta \) and current density. In this context, a finite-size effect has proven to be important, which has to be removed in the analysis in order to calculate the frequency shifts precisely. Since the wire has a finite length (4 \( \mu \)m), the equilibrium position of the DW is in the middle of the wire. But as soon as the DW is displaced from the middle, an effective magnetostatic field develops in the DW region, which points along the axis and acts on the domain wall. The break of symmetry increases the size of one domain over the other, thereby creating this magnetostatic imbalance. This additional field is artificial in the sense that it does not exist for infinitely long wires. In our simulations it has little effect on the propagation of the DW, but it causes a frequency shift in the DW rotation. This effect can be noticed from a difference in the frequency shifts generated by opposite currents of the same value, as shown in Fig. 3(b). The additional frequency shift due to the off-centered position of the DW can be determined in the simulations by setting \( \beta = \alpha \), a case in which any frequency shift is exclusively resulting from this additional dipolar field. We find that, depending on the direction of the DW motion, the dipolar field changes the frequency by up to \( \pm 13 \) MHz.
FIG. 3. (Color online) Average magnetization components as a function of time for a 10 nm diameter Permalloy cylindrical wire with a transverse DW. The DW is driven by an axial magnetic field and an electric current flowing along the wire. The top panel (a) displays the longitudinal magnetization component $m_z$, while (b) shows one transverse component $m_y$, thereby displaying the combination of the linear and rotational motion of the DW, respectively. Symbols are simulated data and lines are (a) linear fits and (b) sinusoidal fits.

this shift does not depend on $\beta$, our data can be corrected accordingly. In experiments this artifact can also be easily corrected by measuring the frequency shift for different values and signs of the current, as displayed in Fig. 4. For sufficiently long wires, this correction becomes negligible.

The current-induced frequency shift resulting for different values $\beta$ as obtained from the simulations is summarized in Fig. 4. Two current values $j = 10^{12}$ A/m$^2$ and $2 \times 10^{12}$ A/m$^2$ are used, which are experimentally achievable [3,46,47]. The polarization rate $P$ is fixed to 0.7. The solid and dashed lines in Fig. 4 are analytical values calculated from Eq. (3b), in which $\partial \theta / \partial z_c$ is extracted from the equilibrium configuration of the DW. Clearly, the simulation results match the analytical results almost perfectly. The small asymmetry between the simulation data and the analytic result is attributed to a slightly reduced width of the moving DW compared to the static domain wall profile (i.e., the Döring domain wall mass [37] is not exactly zero). Such a compression of the domain wall increases the value of $\partial \theta / \partial z_c$. The amount by which the frequency shifts is directly proportional to the difference ($\beta - \alpha$). In the example shown in Fig. 4, if $P = 0.7$, $\delta = (\beta - \alpha) = 0.02$, and $j = 10^{12}$ A/m$^2$, the current-induced frequency shift is about 19 MHz. The frequency shift can be increased by a factor of 2 by applying a current of opposite sign and the same strength. If $\beta$ is very close to $\alpha$, the relative frequency shift $\delta f / f$ could be below the resolution limit of the experiment. But even if a change in frequency is not measurable, the information obtained from the experiment is important, as it may be used to determine an upper limit for $|\alpha - \beta|$.

V. EXPERIMENTAL FEASIBILITY

The proposed method is simple and parameter free, but it implies high requirements concerning the quality of the nanowires. These quality requirements effectively represent limits within which the measurement will be possible. Measurements on the current- and field-driven motion of domain walls in nanowires have been reported extensively in the literature over the past ten years. Also the required high-frequency signal of the rotating DW should be rather easy to measure by means of GMR sensors since similar techniques have been used in numerous studies on the characteristics of spin-torque driven nano-oscillators. The only obstacle might consist in the fabrication of sufficiently thin magnetic wires of well-defined and homogeneous circular shape.

If the wire is too thick, vortex-type domain walls can develop instead of the transverse walls discussed before. The dynamic properties of these vortex walls, which can contain a Bloch point in the center, are fundamentally different from the transverse head-to-head walls that provide the basis of our study. One must therefore make sure that the wire is thin enough to prevent the formation of any other domain wall type than transverse walls. The admissible thickness range has not yet been established rigorously. The information provided in Ref. [38] can serve as a guideline to determine the thickness below which the required one-dimensional domain wall structure develops (typical diameters are below 35 nm).
Another practical problem is the roughness of the wire. To a certain extent, the proposed method should be robust against imperfections of the wire shape because the domain wall width is large compared to other length scales of the material, such as defects in the atomic lattice or the average grain size in many nanocrystalline materials. This results in an effective spatial average of the parameters governing the domain wall dynamics. If structural defects are much smaller than the domain wall width, it can thus be expected that their impact on the structure and the dynamics of the domain walls is negligible. However, roughness can lead to difficulties, and we believe it is worth discussing these effects in more detail.

Previous studies have shown that edge roughness can largely suppress the Walker breakdown in flat strips, because imperfections act as nucleation sites for spin waves through which energy can dissipate that otherwise would result in the out-of-plane rotation of the magnetization and the subsequent oscillatory behavior [48]. Moreover, the intuitively expected property of structural defects to act as pinning sites was confirmed [49,50] and, although the geometry is different, it may be expected that similar effects also occur in cylindrical nanowires. For our purposes it is therefore necessary to drive the domain wall with fields or currents that are sufficiently strong to overcome any roughness-induced pinning. If pinning is prevented, and thickness variations are small enough to ensure that vortex walls cannot develop, variations of the thickness will merely change slightly the rotation frequency or the domain wall width. This will not alter the result but only lead to an increase of the line width in the frequency measurements.

In spite of amazing progress made in the past years concerning the fabrication of nanowires of well-defined radius and spectacular high-resolution microscopy studies on domain walls in such wires [51], it is still not obvious to prepare ferromagnetic nanowires of a quality as high as required by this method. But once this is ensured, the proposed scheme should provide access to a parameter-free and unambiguous measurement of the fundamental parameters $\beta$ and $P$. An important advantage would be that the measurement leaves no room for fitting or for interpretations: The value of $\beta$ obtained from these experiments will be unambiguous. The measurement is based on the assumption that $\beta$ is a material parameter which does not depend on the magnetic structure. We cannot rule out that the value obtained with our method could be specific to head-to-head domains. If, for instance, other measurements using the same material but a different magnetic structure (such as the current-induced displacement of magnetic vortices [52]) yield results that are not compatible with measurements obtained with our method, then it may be suspected that the role of $\beta$ is too complicated to be described by a single constant. This could reinforce theoretical studies on the functional form of $\beta$, similar to the ongoing discussion on the Gilbert damping $\alpha$ which could require corrections depending on the magnetic structure [53,54]. Further complications could result from spin-accumulation effects [55,56] which are usually not considered.

VI. CONCLUSION

We have presented in detail a method that should allow one to measure the nonadiabatic spin-transfer torque parameter $\beta$ in a parameter-free and direct way. This method is based on the dynamic properties of transverse type DWs in cylindrical nanowires. By measuring the current-induced frequency shifts from the Larmor precession of the DW and combining field-driven motion with current-driven dynamics, $\beta$ can be uniquely determined. This method has the advantage that $\beta$ can be obtained separately from the spin polarization $P$. The excellent agreement between analytic calculations and micromagnetic simulations corroborates the robustness of the method.

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