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Abstract

The ambipolarity constraint and the parallel momentum balance equation of neoclassical theory, accounting for finite Larmor radius effects and inertia, allow to describe the radial electric field and the related spin up in collision dominated edge - plamas with steep gradients. Thus they may contribute significantly to the understanding of the L-H transition.

The variation of the toroidal velocity from the last closed magnetic surface up to a position r within the plasma is predicted to be proportional to the integral of the product $u_\theta \frac{\partial lnT}{\partial r}$ i.e., to $\sum_d [\frac{T_i'^{\,2}}{T_i}] L_d$, if the interaction with the neutral gas can be neglected. The summation is over different radial domains, such as the edge pedestal. L_d is the radial extension of the respective domain. The dimensionless parameter $\Lambda = \frac{\nu_i q^2 R^2}{\Omega_i r L_{T_i}} = \frac{\hat{\nu}_i a_{i_p}}{L_{T_i}}$ [where $\hat{\nu}_i = \frac{qR\nu_i}{c_i} > 0.22$ is the relevant collision parameter and a_{i_p} the poloidal ion Larmor radius] characterizes the ratio of the diamagnetic rotation frequency to the heat diffusion rate along magnetic field lines. Conventional neoclassical theory assumes $\Lambda \to 0$. However, e. g. in ALCATOR C-MOD ohmic H-mode pedestals, Λ is sufficiently large that conventional neoclassical results are invalid: it follows from the neoclassical theory that the poloidal velocity decreases below the standard prediction $v_{neo} = \frac{-1.83 T_i'}{eB}$ as Λ^2 increases and changes sign for $\Lambda^2 = \Lambda_0^2$ (typically $\approx$ 1-2).

The equations are treated analytically using a linear interpolation for the poloidal velocity, $v_\theta(\Lambda^2)$, based on $v_\theta(\Lambda_0^2) = 0$ and on the neoclassical value v_{neo} for small Λ. This allows to account for finite Λ effects in the just mentioned integration.

The equations are also solved numerically (1) to benchmark with a simplified analytical theory with Λ=0 and vanishing neutral gas density; (2) to compare with the analytical theory accounting for finite Λ effects and (3) to explore the parameter space in regions where the analytical theory is not valid, in particular in the cases where the neutral gas density is larger than $10^{14}m^{-3}$.
The method resorts to an ODE - solver for the classical momentum balance which is combined with a solver for transcendental equations yielding v_Θ.

The results concern the comparison with the analytical solution and the experimental results of the ohmically heated ALCATOR plasma. For $\Lambda = 0$ the numerical solution and the analytical one agree exactly. For finite $\Lambda \approx 1$ the deviations are surprisingly small. The toroidal spin up of the ALCATOR plasma, characterized by a very short decay length $L_\psi = 0.76$ cm, is $\approx 40 \, \frac{km}{sec}$. This compares well the measured value of 35 $\frac{km}{sec}$. The radial electric field profile assumes the characteristic shape and absolute values reported by the DIII-D Group.

Contents

1 Introduction

Anomalous plasma transport and the concomitant deterioration of the confinement times of tokamaks much below the neoclassical prediction, in particular if auxiliary heating is applied, are key issues in fusion research.

Therefore the surprising experimental discovery of the transition to a high confinement mode [1] above a certain operating power has evoked considerable interest in improved confinement regimes.

Thus e. g. the H (high) ([1] - [5]), the VH (very high) [6], the RI (radiation improved) [7] and the ERS (enhanced reversed shear) ([8] - [11]) modes were investigated. The importance of these modes is underlined by the fact that ignition of the International Tokamak Experimental Reactor (ITER) [12] requires an improved confinement regime.

The theoretical models for these transitions and in particular for the L (low) to H transition, ([13] - [17]) are generally speaking not fully selfconsistent. In fact, a model accounting for the interaction of turbulence, MHD - activity and neoclassical effects selfconsistently is out of range.

Here we concentrate on the revisited neoclassical theory ([22] - [24]) which, in contrast to the conventional neoclassical theory, allows within the framework of a rigorous analytical approach to calculate the velocity field parallel to the flux surfaces and the ambipolar electric field. This theory is valid in the collision dominated regime which is sometimes, e.g. in [4], encountered in the edge region.

In standard neoclassical theory it is assumed that all macroscopic lengths (connection length qR, minor radius a, and the absolute values of the decay lengths of the ion Temperature T and ion density n, $L_T = \{\frac{\partial ln\, T}{\partial r}\}^{-1} = T(\frac{\partial T}{\partial r})^{-1}$ and $L_n = \{\frac{\partial ln\, n}{\partial r}\}^{-1} = n(\frac{\partial n}{\partial r})^{-1}$, respectively, are much larger than the ion Larmor radius.

If, however, the decay lengths of the plasma parameters are intermediate between the ion Larmor radius and the connection length, finite Larmor radius (F. L. R.) effects and inertia become important. Therefore standard neoclassical transport theory (predicting e. g. 'automatic' ambipolarity) does not apply to the collision dominated edge region.

Within the framework of the revisited neoclassical theory ([22]-[24],[27]), the analysis of poloidal or toroidal rotation in a collision dominated toroidal plasma with steep gradients is based on the fluid equations with mass and momentum sources. The revised theory has introduced important modifications into the parallel momentum equation, when the parameter

$$\Lambda_1 = \frac{\nu_{ii}}{\Omega_i}\frac{q^2 R^2}{L_T 4r}$$

exceeds $\frac{1}{3}$. (Ω_i, ν_{ii} are the ion gyro- and the ion - ion collision frequency, respectively.)

Since the poloidal velocity is determined in this theory by means of a cubic equation, it has also been speculated that the poloidal velocity as derived by the revisited neoclassical theory would not be unique and under certain conditions allow for bifurcated equilibria.

Poloidal plasma rotation in toroidal systems is related to various instability mechanisms [32]. For example, plasma rotation is always accompanied by a radial electric field, whose origin appears to be complicated due to various competing effects, e. g. the friction with the neutral gas and steepening of the temperature profile. It is usually believed that the neoclassical transport should be ambipolar and independent of the radial electric field. However, this requirement is strictly valid only in a plasma without steep gradients as source of toroidal momentum.

Stringer [19] was the first to notice that the resistive diffusion rate in a toroidal plasma can not only be non-ambipolar, but can also be negative for some values of the poloidal rotation velocity.

Rosenbluth and Taylor [20], considered the stability of toroidal diffusion using a fluid model and proved that if the resistivity were the only dissipative mechanism, then even if all plasma deformations were excluded, there could be no stable poloidal rotation velocity.

The rapidly growing poloidal or toroidal rotations, can also be observed as spin-up phenomena, sometimes of unknown origin, in tokamaks.

An interaction between a poloidal or toroidal spin-up and the turbulence driven anomalous transport is also believed to be a likely reason for the aforementioned L-H mode transition in tokamaks.

A further consequence of an unstable rotation is that a poloidally asymmetric particle transport may also render the radial electric field unstable [21].

In the here envisaged high collisionality regime with steep gradients only stationary solutions are investigated. The F. L. R. effects enter the ion fluid equations via the gyro - stress tensor [22]. Since ambipolarity is no longer automatically ensured in these equations, an ambipolarity constraint can be derived allowing to determine the selfconsistent radial electric field E_r. This constraint together with the momentum balance provide the possibility to compute the parallel ion speed $v_{||}$ and the radial electric field E_r or, equivalently, the poloidal velocity component $u_\Theta = v_E + \frac{B_\theta}{B} v_{||} + v_n + \tilde{v}_T$ selfconsistently. As mentioned, both equations are nonlinear in u_Θ and $v_{||}$. ([22]-[24],[27]). (The electric drift speed and the density or temperature related velocities v_E, v_n, $\tilde{v}_T$ are given by $v_E = -\frac{E_r}{B_\phi}$, $v_n = \frac{T}{eB_\phi}\frac{\partial ln\, n}{\partial r}$ $\tilde{v}_T = \frac{T}{eB_\phi}\frac{\partial ln\, T}{\partial r}$)

The momentum source or sinks due to the recycled neutrals and NBI are included in these equations.

The crucial parameter Λ_1 measures the ratio of the contributions arising from perpendicular viscosity to those from the parallel viscosity. To compare with the results of the standard neoclassical theory $|\Lambda_1|$ is chosen to approach zero.

By increasing $|\Lambda_1|$ until 0.5 (as indicated by ASDEX - parameters) the modifications due to the full theory can be investigated.

2 Ambipolarity and momentum balance in the revisited neoclassical theory

The revisited neoclassical theory provides an ambipolarity constraint, which together with the parallel and radial momentum equations allows to account e.g. for the radial electric field, the plasma rotation and the formation of transport barriers [24].

Equations for a two-component plasma, describing the continuity of species j with sources $S_j(\vec{x},t)$, the momentum balance with friction $\vec{R}_j(x,t)$ and momentum input $\vec{S}_j^M(\vec{x},t)$, and the energy balance with analogous terms and energy input are

$$\frac{\partial n_j}{\partial t} + \nabla(n_j \vec{u}_j) = S_j \tag{1}$$

$$m_j n_j \frac{d\vec{u}_j}{dt} = -\nabla \vec{P}_j - \nabla \vec{\vec{\Pi}}_j - eZ_j n_j (\vec{E}_j + \frac{1}{c}\vec{u}_j \times \vec{B}) - \vec{R}_j + \vec{S}_j^M \tag{2}$$

and

$$\frac{3}{2} n_j \frac{dT_j}{dt} + P_j \nabla u_j = -\nabla \vec{q}_j - \vec{\vec{\Pi}}_j : \nabla \vec{u}_j + R_j^E + S_j^E \tag{3}$$

The viscous tensor is given by

$$\vec{\vec{\Pi}} = -\eta_0 \vec{\vec{W}}_0 - \eta_1 (\vec{\vec{W}}_1 + 4\vec{\vec{W}}_2) + \eta_3 (\vec{\vec{W}}_3 + 2\vec{\vec{W}}_4) = \tag{4}$$

The tensors $\vec{\vec{W}}_0$, $\vec{\vec{W}}_1$ $\vec{\vec{W}}_2$, $\vec{\vec{W}}_3$, $\vec{\vec{W}}_4$, accounting for the parallel, perpendicular and gyro stress are given in [18] (recently extended and completed by Mikhailovsky and Tsypin [26]). The terms S_j^E on the R.H.S. of equation (3) denotes energy sources and losses, whereas R_j^E denotes terms relating to collisional energy transfer and frictional heating. The correct forms of these

terms require a kinetic and atomic approach. However, here we shall assume them, as given functions.
The radial electric field satisfies the radial momentum balance

$$E_r + (\vec{U} \times \vec{B}) = \frac{1}{eN}\frac{\partial P_i}{\partial r}$$

.
In the revisited neoclassical theory [1], a plausible ordering inside the separatrix is introduced by means of a small parameter $\mu \approx 0.1$ as

$$\frac{c_j}{qRv_j} \approx \frac{L_\psi}{r} \approx \frac{r}{qR} \approx \frac{B_\theta}{B_\phi} \approx \mu$$

and

$$\frac{e_j V}{T_j} \approx \sqrt{\frac{m_e}{m_i}} \approx \frac{a_i}{L_\psi} \approx \mu^2$$

where L_ψ is the radial scalelength of the temperature, c_j and a_j are the thermal speed and the gyro radius of the species j, respectively; V is the loop voltage and ν_j the collision frequency between like particles. Using the magnetic field aligned orthonormal unit vectors $(\hat{p}, \hat{b}, \hat{n})$ in radial, binormal and parallel directions, and the small parameter μ, the velocity of species j can be assumed as,

$$\hat{p}\vec{U}_j = \mu^6 U^{(6)}_{\psi,j}(\psi,\chi) + ...$$

$$\hat{b}\vec{U}_j = \mu^2 U^{(2)}_{\beta,j}(\psi) + \mu^3 U^{(3)}_{\beta,j}(\psi,\chi)...$$

$$\hat{n}\vec{U}_j = \mu U^{(1)}_{||,j}(\psi) + \mu^2 U^{(2)}_{||,j}(\psi,\chi)...$$

Assuming that the magnetic field, density, temperature, potential, etc., are independent of the poloidal angle in dominant order, these are also expanded in perturbation series. For example, the density and the magnetic field are written as,

$$N(\psi, \chi) = N^{(0)}(\psi)[1 + \mu N^{(1)}(\psi, \chi)...]$$

and

$$B(\psi, \chi) = B^{(0)}(\psi)[1 + \mu B^{(1)}(\psi, \chi)...]$$

respectively. For a tokamak plasma with circular cross section, also use is made of the toroidal unit vectors $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\phi)$. Taking toroidal and parallel projections of the momentum equation and averaging them over the magnetic flux surfaces, and imposing the ambipolarity condition, one obtains a pair of coupled non-linear equations for the toroidal and poloidal ion velocities in terms of other plasma variables, such as temperature, density, and electric field [1].

3 Classical Toroidal Momentum Transport

The main result of the revisited neoclassical theory [27] is an equation describing the radial transport of toroidal momentum in a collisional subsonic plasma with steep gradients. For large aspect ratio and circular cross - section we get the time - dependent u_ϕ - equation

$$\frac{\partial}{\partial r}[\eta_2(\frac{\partial u_\phi}{\partial r} - \frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial r}\frac{B_\phi}{B_\theta}u_\theta)]$$

$$= m_i n_i(\frac{\partial}{\partial t} + \nu_{cx})u_\phi - m_i n_i(\dot{M}_{\phi,i} + j_r B_\theta) \tag{5}$$

Q and S are given in the appendices F and E. m_i, n_i are the ion mass and ion density, respectively.

The poloidal rotation driven by the temperature gradient seen inside the paranthesis on the right hand side of (5) results from the gyro stress tensor and acts like another source term, i.e., as a toroidal momentum source or sink, depending on the sign of its radial gradient. External momentum sources can be direct, such as fast ions provided by the neutral beam injection (NBI), collisions by alpha particles, or indirect and due to particle sources such as aforementioned charge exchange reactions with cold recycled neutrals. The poloidal rotation velocity u_θ, the charge exchange reactions with the recycled cold neutrals, the momentum injection $\dot{M}_{\phi,i}$ by NBI and by a probe generating the radial current density j_r determine the toroidal flow velocity u_ϕ [25]. The charge exchange reactions are characterized by the charge exchange frequency

$$\nu_{cx} =< \sigma\ v >_{cx} \quad n_0 \tag{6}$$

$< \sigma \; v >_{cx}$ is the rate coefficient for charge exchange, and n_0 the neutral gas density. We define

$$g = \frac{u_\phi}{v_T}$$

$$h = \frac{u_\theta}{v_T}$$

and

$$x = \frac{r - r_{inf}}{L_\psi}$$

Here we used the (constant, positive) velocity

$$v_T = \frac{1}{eB_\phi}\frac{T_{inf}}{L_\psi}$$

T_{inf} is the temperature at the inflection point P_{inf} of the temperature profile (Fig. 1). The inflection point has the radius r_{inf} and is defined as the locus of vanishing curvature of the temperature profile. L_ψ is the temperature scale length at P_{inf} in absolute value (appendix A). We get

$$\frac{v_T}{L_\psi}\frac{\partial}{\partial x}[\eta_2(\frac{\partial g}{\partial x} - \frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\; T}{\partial x}\frac{B_\phi}{B_\theta}h)]$$

$$= m_i n_{inf}\hat{n} v_T(\frac{\partial}{\partial t} + \nu_{cx})g - m_i n_{inf}\hat{n}(\dot{M}_{\phi,i} + j_r B_\theta) \qquad (7)$$

n_{inf} is the density at the inflection point and $\hat{n}$ is defined by $n_i = \hat{n} n_{inf}$. The characteristic time t_c is given by

$$t_c^{-1} = \frac{1}{m_i n_{inf} L_\psi^2}\eta_{2,inf} \qquad (8)$$

We introduce this time by multiplying with $\frac{1}{m_i n_{inf} L_\psi^2}$. With the decomposition

$$\eta_2 = \eta_{2,inf}\hat{\eta}_2$$

we can write

$$\frac{\eta_{2,inf}}{m_i n_{inf} L_\psi^2}(\frac{\partial}{\partial x}[\hat{\eta}_2(\frac{\partial g}{\partial x} - \frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial x}\frac{B_\phi}{B_\theta}h)]$$

$$= \hat{n}(\frac{\partial}{\partial t} + \nu_{cx})g - \frac{\hat{n}\dot{M}_{\phi,i}}{v_T}] + \frac{J_r B_\theta}{m_i v_T n_{inf}} \qquad (9)$$

We define

$$G = \frac{\partial g}{\partial x} - \frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial x}\frac{B_\phi}{B_\theta}h \qquad (10)$$

and get

$$\frac{\eta_{2,inf}}{m_i n_{inf} L_\psi^2}\frac{\partial}{\partial x}(\hat{\eta}_2 G)$$

$$= \hat{n}(\frac{\partial}{\partial t} + \nu_{cx})g - \frac{\hat{n}\dot{M}_{\phi,i}}{v_T} + \frac{j_r B_\theta}{m_i v_T n_{inf}} \qquad (11)$$

By differentiating we obtain

$$\frac{\eta_{2,inf}}{m_i n_{inf} L_\psi^2}(G\frac{\partial}{\partial x}\hat{\eta}_2 + \hat{\eta}_2\frac{\partial}{\partial x}G)$$

$$= \hat{n}_i(\frac{\partial}{\partial t} + \nu_{cx})g - \frac{\hat{n}\dot{M}_{\phi,i}}{v_T} + \frac{J_r B_\theta}{m_i v_T n_{inf}} \qquad (12)$$

or

$$\frac{\eta_{2,inf}}{m_i n_{inf} L_\psi^2}\hat{\eta}_2(G\frac{\partial}{\partial x}ln(\hat{\eta}_2) + \frac{\partial}{\partial x}G)$$

$$= \hat{n}(\frac{\partial}{\partial t} + \nu_{cx})g - \frac{\hat{n}\dot{M}_{\phi,i}}{v_T} + \frac{J_r B_\theta}{m_i v_T n_{inf}} \qquad (13)$$

Furthermore we get

$$\frac{\eta_{2,inf}}{m_i n_{inf} L_\psi^2}\hat{\eta}_2 \frac{\partial}{\partial x} G = \hat{n}(\frac{\partial}{\partial t} + \nu_{cx})g - \frac{\hat{n}\dot{M}_{\phi,i}}{v_T} +$$
$$\frac{J_r B_\theta}{m_i v_T n_{inf}} - \frac{1}{m_i n_{inf} L_\psi^2}\eta_2 G \frac{\partial}{\partial x} ln(\eta_2) \tag{14}$$

and

$$\frac{\partial}{\partial x} G = \frac{m_i n_{inf} L_\psi^2}{\eta_{2,inf}\hat{\eta}_2} = [\hat{n}(\frac{\partial}{\partial t} + \nu_{cx})g - \frac{\hat{n}\dot{M}_{\phi,i}}{v_T} +$$
$$\frac{J_r B_\theta}{m_i v_T n_{inf}} - \frac{\eta_{2,inf}}{m_i n_{inf} L_\psi^2}\hat{\eta}_2 G \frac{\partial}{\partial x} ln(\eta_2)] \tag{15}$$

The coefficient $\frac{1}{m_i n_{inf} L_\psi^2}\eta_{2,inf} = \eta'$ containing the viscosity can be rewritten as

$$\eta' = \frac{\frac{6}{5} n_{inf} k T_{inf} \tau_{ii}}{\Omega_i^2 \tau_{ii}^2 m_i n_{inf} L_\psi^2} \tag{16}$$

We use $kT_{inf} = m_i c_{inf}^2 \hat{T}$, where c_{inf} is the thermal speed at the inflection point and get

$$\eta' = \frac{\frac{6}{5}\hat{T} c_{inf}^2}{\Omega_i^2 \tau_{ii} L_\psi^2} \tag{17}$$

or (appendix D)

$$\eta' = 8.57 \ \ 10^{-7} \frac{a_{i,inf}^2}{L_\psi^2} \frac{n_{inf} Z_{eff_{inf}}}{T_{inf}^{\frac{3}{2}}} \sqrt{\frac{1}{A}} \tag{18}$$

$a_{i,inf}$ is the ion Larmorradius at the inflection point.
We assume

$$Z_{eff_i} = Z_{eff_{inf}} \hat{T}^Z$$

where Z is a constant. Then the radial dependence of η_2 is given by (one $\hat{T}$ is cancelled)

$$\hat{\eta}_2 = \hat{n}\hat{T}^{(-\frac{1}{2}+Z)} \tag{19}$$

We get the derivative

$$\frac{\partial}{\partial x}(\hat{\eta}_2) = \hat{T}^{-\frac{1}{2}+Z}\frac{\partial \hat{n}}{\partial x} + n(-\frac{1}{2} + Z)\hat{T}^{-\frac{3}{2}+Z}\frac{\partial \hat{T}}{\partial x} \tag{20}$$

The logarithmic derivative is

$$\frac{\partial}{\partial x}ln(\eta_2) = \frac{\partial}{\partial x}ln(\hat{\eta}_2) = \frac{\hat{T}^{-\frac{1}{2}+Z}\frac{\partial \hat{n}}{\partial x} + \hat{n}(-\frac{1}{2} + Z)\hat{T}^{-\frac{3}{2}+Z}\frac{\partial \hat{T}}{\partial x}}{\hat{n}\hat{T}^{-\frac{1}{2}+Z}} \tag{21}$$

or

$$\frac{\partial}{\partial x}ln(\eta_2) = \frac{1}{\hat{n}}\frac{\partial \hat{n}}{\partial x} + \frac{(-\frac{1}{2} + Z)}{\hat{T}}\frac{\partial \hat{T}}{\partial x} \tag{22}$$

4 Poloidal rotation

Using the ambipolarity condition and the extended forms of the stress tensors in the parallel momentum equation, one can cancel the time derivative and the source terms. The result - in the lowest order - is a nonlinear equation between the radial derivatives of the poloidal and toroidal plasma velocities. For large aspect ratio and circular cross - section it can be written as [25]

$$u_\theta + \frac{1.833}{eB_\phi}\frac{\partial T}{\partial r} = 0.36\frac{\eta_1}{\eta_0(1+\frac{Q^2}{S^2})}q^2R^2\frac{eB_\phi}{T_i}\frac{\partial ln\ T}{\partial r}$$
$$\{\frac{T}{eB_\theta}\frac{\partial u_\phi}{\partial r} + \frac{1}{2}u_\phi^2$$
$$-u_\phi\frac{B_\phi}{B_\theta}(u_\theta - \frac{T}{eB_\phi}\frac{\partial ln\ (Tn_i^2)}{\partial r})+$$
$$1.9\frac{B_\phi^2}{B_\theta^2}[u_\theta - 0.8\frac{T}{eB_\phi}\frac{\partial ln\ (Tn_i^{1.6})}{\partial r}]^2\} \qquad (23)$$

This may be rewritten as

$$u_\theta + \frac{1.833}{eB_\phi}\frac{\partial T}{\partial r} =$$
$$T_{36}\{\frac{T_i}{eB_\theta}\frac{\partial u_\phi}{\partial r} + \frac{1}{2}u_\phi^2$$
$$-u_\phi\frac{B_\phi}{B_\theta}(u_\theta - \frac{T_i}{eB_\phi}\frac{\partial ln\ (Tn_i^2)}{\partial r})+$$
$$1.9\frac{B_\phi^2}{B_\theta^2}[u_\theta - 0.8\frac{T}{eB_\phi}\frac{\partial ln\ (Tn_i^{1.6})}{\partial r}]^2\} \qquad (24)$$

T_{36} is given by

$$T_{36} = 0.36\frac{\eta_1}{\eta_0(1+\frac{Q^2}{S^2})}q^2R^2\frac{eB_\phi}{T}\frac{\partial ln\ T}{\partial r}$$

We use $v_T = \frac{1}{eB_\phi}\frac{T_{inf}}{L_\psi}$ and get the following expression for the LHS of the preceding equation

$$u_\theta + \frac{1.833}{eB_\phi}\frac{\partial T}{\partial r} = v_T(h + 1.833\frac{\partial \hat{T}}{\partial x}) \tag{25}$$

T_{36} may be rewritten as

$$T_{36} = 0.36\frac{\eta_1}{\eta_0(1+\frac{Q^2}{S^2})}q^2R^2\frac{eB_\phi}{T}\frac{\partial ln\ T}{\partial r} =$$

$$0.36\frac{\eta_1}{\eta_0(1+\frac{Q^2}{S^2})}q^2\frac{eB_\phi L_\psi}{T}\frac{R}{L_\psi}\frac{R}{L_\psi}L_\psi\frac{\partial ln\ T}{\partial r}$$

$$= 0.36\frac{\eta_1}{\eta_0(1+\frac{Q^2}{S^2})}q^2\frac{1}{v_T\hat{T}}\frac{R^2}{L_\psi^2}\frac{\partial ln\ T}{\partial x} \tag{26}$$

The viscosities η_1, η_0 can be written as [18]

$$\eta_1 = \frac{\frac{6}{5}n_iT\tau_{ii}}{\Omega_i^2\tau_{ii}^2} \tag{27}$$

$$\eta_0 = \frac{0.96n_iT\nu_{ii}}{\nu_{ii}^2} \tag{28}$$

For the ratio $\frac{\eta_1}{\eta_0}$ we get

$$\frac{\eta_1}{\eta_0} = \frac{\frac{\frac{6}{5}n_iT\nu_{ii}}{\Omega_i^2}}{\frac{0.96P_i\nu_{ii}}{\nu_{ii}^2}} = \frac{6}{5\cdot 0.96}\frac{\nu_{ii}^2}{\Omega_i^2} \tag{29}$$

Therefore the factor T_{36} in the preceding term becomes

$$T_{36} = 0.36 \frac{\eta_1}{\eta_0 (1 + \frac{Q^2}{S^2})} q^2 \frac{1}{v_T \hat{T}} \frac{R^2}{L_\psi^2} \frac{\partial ln\ T}{\partial x}$$

$$= 0.36 \frac{6}{5\ 0.96} \frac{\nu_{ii}^2}{\Omega_i^2} \frac{1}{(1 + \frac{Q^2}{S^2})} q^2 \frac{1}{v_T \hat{T}} \frac{R^2}{L_\psi^2} \frac{\partial ln\ T}{\partial x}$$

$$= 0.45 \frac{\nu_{ii}^2}{\Omega_i^2} \frac{1}{(1 + \frac{Q^2}{S^2})} q^2 \frac{1}{v_T \hat{T}} \frac{R^2}{L_\psi^2} \frac{\partial ln\ T}{\partial x} \qquad (30)$$

The other terms of equation (24) are

$$\frac{T}{eB_\theta} \frac{\partial u_\phi}{\partial r} = \frac{B_\phi}{B_\theta} \frac{T}{eB_\phi L_\psi} \frac{\partial u_\phi}{\partial x} = \frac{B_\phi}{B_\theta} v_T^2 \hat{T} \frac{\partial g}{\partial x}$$

$$-u_\phi \frac{B_\phi}{B_\theta} (u_\theta - \frac{T}{eB_\phi} \frac{\partial ln\ (T n_i^2)}{\partial r}) =$$

$$-v_T^2 g \frac{B_\phi}{B_\theta} (h - \hat{T} \frac{\partial ln\ (T n_i^2)}{\partial x}) \qquad (31)$$

and

$$1.9 \frac{B_\phi^2}{B_\theta^2} [u_\theta - 0.8 \frac{T_i}{eB_\phi} \frac{\partial ln\ (T n_i^{1.6})}{\partial r}]^2 =$$

$$1.9 \frac{B_\phi^2}{B_\theta^2} v_T^2 [h - 0.8 \hat{T} \frac{\partial ln\ (T n_i^{1.6})}{\partial x}]^2 \qquad (32)$$

Thus equation (24) can be written as

$$v_T (h + 1.833 \frac{\partial \hat{T}}{\partial x}) =$$

$$T_{36} \{ \frac{B_\phi}{B_\theta} v_T^2 \hat{T} \frac{\partial g}{\partial x} + \frac{1}{2} v_T^2 g^2$$

$$-v_T^2 g \frac{B_\phi}{B_\theta}(h - \hat{T}\frac{\partial ln\ (Tn_i^2)}{\partial x})$$

$$+1.9\frac{B_\phi^2}{B_\theta^2}v_T^2[h - 0.8\hat{T}\frac{\partial ln\ (Tn_i^{1.6})}{\partial x}]^2\} \qquad (33)$$

We multiply with $\hat{T}v_T$ and cancel v_T^2

$$(h + 1.833\frac{\partial\hat{T}}{\partial x})\hat{T} =$$

$$C_\eta\{\frac{B_\phi}{B_\theta}\hat{T}\frac{\partial g}{\partial x} + \frac{1}{2}g^2$$

$$-g\frac{B_\phi}{B_\theta}(h - \hat{T}\frac{\partial ln\ (Tn_i^2)}{\partial x})$$

$$+1.9\frac{B_\phi^2}{B_\theta^2}[h - 0.8\hat{T}\frac{\partial ln\ (Tn_i^{1.6})}{\partial x}]^2\} \qquad (34)$$

C_η is defined by

$$C_\eta = v_T\hat{T}T_{36} =$$

$$0.45q^2\frac{R^2}{L_\psi^2}\frac{\nu_{ii}(n_{inf}, T_{inf})^2}{\Omega_i^2}(\hat{T}^{-\frac{3}{2}}T^Z\hat{n})^2\frac{1}{(1+\frac{Q^2}{S^2})}\frac{\partial ln\ T}{\partial x} \qquad (35)$$

5 Description of the plasma rotation by a 2nd order equation and a transcendental equation

We start with equation (7) for g

$$\frac{v_T}{L_\psi^2}\frac{\partial}{\partial x}[\eta_2(\frac{\partial g}{\partial x} - \frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial x}\frac{B_\phi}{B_\theta}h)]$$

$$= m_i n_{inf}\hat{n} v_T(\frac{\partial}{\partial t} + \nu_{cx})g - m_i n_{inf}\hat{n}\dot{M}_{\phi,i} + J_r B_\theta \qquad (36)$$

n_{inf} is the density at the inflection point and $\hat{n}$ is defined by $n_i = \hat{n} n_{inf}$. To get the characteristic time we multiply - as before - by $\frac{1}{m_i n_{inf} v_T}$ and get using $\eta_2 = \eta_{2,inf}\hat{\eta}_2$

$$\frac{\eta_{2,inf}}{m_i n_{inf} L_\psi^2}(\frac{\partial}{\partial x}[\hat{\eta}_2(\frac{\partial g}{\partial x} - \frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial x}\frac{B_\phi}{B_\theta}h)]$$

$$= \hat{n}(\frac{\partial}{\partial t} + \nu_{cx})g - \frac{\hat{n}\dot{M}_{\phi,i}}{v_T}] + \frac{J_r B_\theta}{m_i v_T n_{inf}} \qquad (37)$$

The characteristic time t_c is given by

$$t_c^{-1} = \frac{1}{m_i n_{inf} L_\psi^2}\eta_{2,inf}$$

We define

$$G = \frac{\partial g}{\partial x} - \frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial x}\frac{B_\phi}{B_\theta}h \qquad (38)$$

and get the first order equation

$$\frac{\eta_{2,inf}}{m_i n_{inf} L_\psi^2}\frac{\partial}{\partial x}(\hat{\eta}_2 G)$$

$$= \hat{n}(\frac{\partial}{\partial t} + \nu_{cx})g - \frac{\hat{n}_i \dot{M}_{\phi,i}}{v_T} + \frac{J_r B_\theta}{m_i v_T n_{inf}} \tag{39}$$

By differentiating we obtain

$$\frac{\eta_{2,inf}}{m_i n_{inf} L_\psi^2}(G\frac{\partial}{\partial x}\hat{\eta}_2 + \hat{\eta}_2 \frac{\partial}{\partial x} G)$$

$$= \hat{n}(\frac{\partial}{\partial t} + \nu_{cx})g - \frac{\hat{n}\dot{M}_{\phi,i}}{v_T} + \frac{J_r B_\theta}{m_i v_T n_{inf}} \tag{40}$$

or, resolving with respect to $\frac{\partial}{\partial x}G$ we get

$$\frac{\partial}{\partial x}G = \frac{m_i n_{inf} L_\psi^2}{\eta_{2,inf}\hat{\eta}_2}[\hat{n}(\frac{\partial}{\partial t} + \nu_{cx})g - \frac{\hat{n}\dot{M}_{\phi,i}}{v_T} +$$

$$\frac{J_r B_\theta}{m_i v_T n_{inf}} - \frac{\eta_{2,inf}}{m_i n_{inf} L_\psi^2}\hat{\eta}_2 G \frac{\partial}{\partial x} ln(\hat{\eta}_2)] \tag{41}$$

In the last term the factor $\hat{\eta}_2$ appears because the 'ln' was introduced. The equation can be written in the form (omitting $\frac{\partial g}{\partial t}$)

$$\frac{\partial}{\partial x}G = T_{CX} + T_{NB} + T_{Pr} - G\frac{\partial}{\partial x} ln(\hat{\eta}_2) \tag{42}$$

Here

$$T_{CX} = t_c \frac{\hat{n}}{\hat{\eta}_2}\nu_{cx} g = t_c \hat{T}^{0.5-Z}\nu_{cx} g$$

$$T_{NB} = t_c \frac{\hat{n}}{\hat{\eta}_2 v_T}\dot{M}_{\phi,i}$$

and

$$T_{Pr} = t_c \frac{J_r B_\theta}{m_i v_T n_{inf}\hat{\eta}_2}$$

account for the friction caused by the neutral gas, the momentum source due to neutral beam injection and the momentum source due to a probe, respectively.

6 Boundary values

The solution of the preceding equations is strongly influenced by the the boundary values which are therefore discussed in the following.

Firstly we assume a symmetric streaming of the scrape-off plasma into the divertor. This yields as boundary value h($r = r_s$) = 0. We note that this assumption determines the value of $\Lambda(r = r_s)$ at the boundary. Via the temperature profile we then get the radius r_s. Thus the condition h($r = r_s$) = 0 is **not** a boundary condition for the differential equations, but determines the radial extension.

Secondly we assume $g(r = r_s) = 0$ because of the absence of momentum sources such as neutral beam injection (NBI) and because neutrals may be reducing $g(r = r_s)$ to a low value. (We note, however, that at JET [33] and JT60, i. e. devices with strong NBI, a considerable spin up at the separatrix had been observed.)

The analytical or numerical integration can start at the inside (r=0) or at the outside ($r = r_s$).

For integrating from the outside to the inside - what may be considered as the 'natural' procedure - g and $\frac{\partial g}{\partial x}$ are prescribed and G follows from equation (38). If the neutral gas can be neglected, NBI and the probe are switched of, the boundary condition is G = 0.

If the neutral gas density is finite as in chapter 13, the derivative $\frac{\partial g}{\partial x}|_{r=r_s}$ must be adjusted such that $\frac{\partial g}{\partial x}|_{r=0} = 0$ ('shooting method').

For integrating from the inside to the outside g and G are prescribed. From equation (38) follows $G(x = -\infty) = 0$. Now g$(r = 0)$ must be adjusted to hit a prescribed boundary value g$(r = r_s)$ at the outside.

7 Halfanalytical solution

We envisage the equation

$$\frac{v_T}{L_\psi}\frac{\partial}{\partial x}[\eta_2(\frac{\partial g}{\partial x} - \frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial x}\frac{B_\phi}{B_\theta}h)]$$

$$= m_i N_{inf}\hat{n}v_T(\frac{\partial}{\partial t} + \nu_{cx})g - m_i N_{inf}\hat{n}\dot{M}_{\phi,i} \qquad (43)$$

Considering stationarity and neglecting injection, probe current and the neutral gas yields

$$\frac{\partial}{\partial x}[\eta_2(\frac{\partial g}{\partial x} - \frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial x}\frac{B_\phi}{B_\theta}h)] = 0 \qquad (44)$$

Integrating radially we get

$$\eta_2(\frac{\partial g}{\partial x} - \frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial x}\frac{B_\phi}{B_\theta}h) = Const \qquad (45)$$

In the interior the term $\frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial x}\frac{B_\phi}{B_\theta}h$ vanishes, therefore

$$\eta_2\frac{\partial g}{\partial x}|_{x=-\infty} = Const \qquad (46)$$

In an infinitesimally small volume around the magnetic axis the thermodynamic quantities are constant because of the nesting of the flux surfaces around this axis. Therefore we get for the preceding equation Const=0. We integrate radially again and get the general solution

$$g - g_0 = \int_{x=-\infty}^{x} dx'[\frac{0.107q^2}{(1+\frac{Q^2}{S^2})}\frac{\partial ln\ T}{\partial x}\frac{B_\phi}{B_\theta}h] \qquad (47)$$

g_0 is the boundary value of g at r=0 or $x = -\infty$ (concerning the boundary values also see chapter 6).

We consider the neoclassical approximation of equation (24) for $C_\eta = 0$

$$h = -1.833\frac{\partial \hat{T}}{\partial x} \tag{48}$$

With this equation we get

$$g - g_0 = \int_{x=-\infty}^{x} dx'[\frac{0.107q^2}{(1+\frac{Q^2}{S^2})}\frac{\partial ln\ T}{\partial x}\frac{B_\phi}{B_\theta}(-1.833\frac{\partial T}{\partial x})] \tag{49}$$

or

$$g - g_0 = \int_{x=-\infty}^{x} dx'\frac{0.107q^2}{(1+\frac{Q^2}{S^2})}\frac{B_\phi}{B_\theta}(\frac{-1.833}{\hat{T}}[\frac{\partial \hat{T}}{\partial x}]^2)] \tag{50}$$

8 Analytical integration resorting to a special temperature profile

We evaluate equation (50) in the case of the profile

$$T = T_{inf}(1 - tanh\frac{r - r_{inf}}{\Delta_T}) = T_{inf}(1 - tanh(x)) \tag{51}$$

which is normalized to T_{inf}, the temperature at the inflection point $r = r_{inf}$ (Fig. 1). The profile (51) stands for the pedestal at the boundary and thus contains the main property leading to the spin up of the plasma. With the derivatives

$$\frac{\partial ln\ T}{\partial x} = -\frac{1}{(1 - tanh(x))csh^2(x)} \tag{52}$$

and

$$\frac{\partial T}{\partial x} = -\frac{1}{csh^2(x)} \tag{53}$$

we get

$$g(x') - g_0 = \int_{-\infty}^{x'} dx \frac{0.107q^2}{(1 + \frac{Q^2}{S^2})} \frac{1}{(1 - tanh(x))csh^2(x)} \frac{B_\phi}{B_\theta} \times$$

$$(1.833\ \frac{1}{csh^2(x)})] \tag{54}$$

We use the identities

$$\frac{1}{(1 - tanh(x))csh^4(x)} = \frac{(1 - tanh(x)^2)^2}{(1 - tanh(x))} =$$

$$\frac{(1 - tanh(x))^2(1 + tanh(x))^2}{(1 - tanh(x))} =$$

$$(1 - tanh(x))(1 + tanh(x))^2 \tag{55}$$

and

$$\frac{1}{(1 - tanh(x))csh^4(x)} =$$

$$= (1 - tanh^2(x))(1 + tanh(x)) = \frac{1 + tanh(x)}{csh^2(x)} \tag{56}$$

This yields

$$g(x') - g_0 = \frac{0.107q^2}{(1 + \frac{Q^2}{S^2})} \frac{B_\phi}{B_\theta} \times$$

$$(1.833\ (tanh(x) + 0.5tanh^2(x))|_{-\infty}^{x'} \tag{57}$$

9 Analytical treatment in the case of finite Λ_r

The u_ϕ - equation is in the case of vanishing neutral gas density given by

$$\frac{\partial g}{\partial x} = -\frac{0.107q^2}{1+\frac{Q^2}{S^2}} \frac{\partial ln\ T}{\partial x} \frac{B_\phi}{B_\theta} h \tag{58}$$

With $h = h' \frac{\partial \hat{T}}{\partial x}$ and $g = g' \frac{B_\phi}{B_\theta}$ we obtain

$$\frac{\partial g'}{\partial x} = -\frac{0.107q^2}{1+\frac{Q^2}{S^2}} \frac{\partial ln\ T}{\partial x} h' \frac{\partial \hat{T}}{\partial x} \tag{59}$$

The u_θ - equation can be written as

$$\begin{aligned}
&\frac{\tilde{v}_T}{\frac{\partial \hat{T}}{\partial x}}(h + 1.833\frac{\partial \hat{T}}{\partial x}) = \tilde{v}_T(h' + 1.833) = \\
&T_{36}\{\frac{B_\phi}{B_\theta}\frac{\tilde{v}_T^2}{[\frac{\partial \hat{T}}{\partial x}]^2}\hat{T}\frac{\partial g}{\partial x} + \frac{1}{2}\frac{\tilde{v}_T^2}{[\frac{\partial \hat{T}}{\partial x}]^2}g^2 \\
&-\frac{\tilde{v}_T^2}{[\frac{\partial \hat{T}}{\partial x}]^2}g\frac{B_\phi}{B_\theta}(h - \hat{T}\frac{\partial ln\ (Tn_i^2)}{\partial x}) \\
&+1.9\frac{B_\phi^2}{B_\theta^2}\frac{\tilde{v}_T^2}{[\frac{\partial \hat{T}}{\partial x}]^2}[h - 0.8\hat{T}\frac{\partial ln\ (Tn_i^{1.6})}{\partial x}]^2\}
\end{aligned} \tag{60}$$

or

$$\begin{aligned}
&\tilde{v}_T(h' + 1.833) = \\
&T_{36}[\frac{B_\phi}{B_\theta}]^2\{\frac{\tilde{v}_T^2}{[\frac{\partial \hat{T}}{\partial x}]^2}\hat{T}\frac{B_\theta}{B_\phi}\frac{\partial g}{\partial x} + \frac{1}{2}\frac{\tilde{v}_T^2}{[\frac{\partial \hat{T}}{\partial x}]^2}[\frac{B_\theta}{B_\phi}]^2 g^2 \\
&-\frac{\tilde{v}_T^2}{[\frac{\partial \hat{T}}{\partial x}]^2}g\frac{B_\theta}{B_\phi}(h - \hat{T}\frac{\partial ln\ (Tn_i^2)}{\partial x})
\end{aligned}$$

$$+1.9\frac{\tilde{v}_T^2}{[\frac{\partial \hat{T}}{\partial x}]^2}[h - 0.8\hat{T}\frac{\partial ln\ (Tn_i^{1.6})}{\partial x}]^2\} \tag{61}$$

We replace $\frac{\partial g}{\partial x}$ according to equation (58) cancel one $\tilde{v}_T$, and employ $h = h'\frac{\partial \hat{T}}{\partial x}$:

$$(h' + 1.833) =$$

$$T_{36}\tilde{v}_T[-\frac{B_\phi}{B_\theta}]^2\{\frac{1}{[\frac{\partial \hat{T}}{\partial x}]}\hat{T}\frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial x}h' + \frac{1}{2}\frac{1}{[\frac{\partial \hat{T}}{\partial x}]^2}[\frac{B_\theta}{B_\phi}]^2 g^2$$

$$-\frac{1}{[\frac{\partial \hat{T}}{\partial x}]^2}g\frac{B_\theta}{B_\phi}(h - \hat{T}\frac{\partial ln\ (Tn_i^2)}{\partial x})$$

$$+1.9\frac{1}{[\frac{\partial \hat{T}}{\partial x}]^2}[h - 0.8\hat{T}\frac{\partial ln\ (Tn_i^{1.6})}{\partial x}]^2\} \tag{62}$$

Using $v_T = \tilde{v}_T\frac{L_T}{L_\psi}$ and

$$\Lambda_r = \frac{\nu_{ii}}{\Omega_i}\frac{q^2R^2}{L_T r}$$

we get for $T_{36}[\frac{B_\phi}{B_\theta}]^2$ successively the relations

$$T_{36}[\frac{B_\phi}{B_\theta}]^2 = 0.36\frac{6}{5\ 0.96}\frac{\nu_{ii}^2}{\Omega_i^2}\frac{1}{(1+\frac{Q^2}{S^2})}q^2\frac{1}{\tilde{v}_T\frac{L_T}{L_\psi}}\frac{R^2}{L_\psi^2}\frac{\partial ln\ T}{\partial x}\frac{B_\phi^2}{B_\theta^2}$$

$$T_{36}[\frac{B_\phi}{B_\theta}]^2 = 0.36\frac{6}{5\ 0.96}\frac{\nu_{ii}^2}{\Omega_i^2}\frac{1}{(1+\frac{Q^2}{S^2})}q^2\frac{1}{\tilde{v}_T\frac{L_T}{L_\psi\hat{T}}\hat{T}}\frac{R^2}{L_\psi L_T}\frac{B_\phi^2}{B_\theta^2}$$

$$T_{36}[\frac{B_\phi}{B_\theta}]^2 = 0.36\frac{6}{5\ 0.96}\frac{\nu_{ii}^2}{\Omega^2}\frac{1}{(1+\frac{Q^2}{S^2})}q^2\frac{R^2}{\tilde{v}_T L_T^2}\frac{B_\phi^2}{B_\theta^2}$$

$$T_{36}[\frac{B_\phi}{B_\theta}]^2 = 0.45\frac{\Lambda_r^2 r^2}{q^2R^2}\frac{1}{(1+\frac{Q^2}{S^2})}\frac{1}{\tilde{v}_T}\frac{B_\phi^2}{B_\theta^2}$$

$$T_{36}[\frac{B_\phi}{B_\theta}]^2 = 0.45\frac{\Lambda_r^2}{(1+\frac{Q^2}{S^2})}\frac{1}{\tilde{v}_T}$$

and finally

$$T_{36}[\frac{B_\phi}{B_\theta}]^2\tilde{v}_T = 0.45\frac{\Lambda_r^2}{(1+\frac{Q^2}{S^2})}$$

With $C_\eta = v_T\hat{T}T_{36}$ we obtain

$$T_{36}[\frac{B_\phi}{B_\theta}]^2\tilde{v}_T = \frac{C_\eta}{\hat{T}}[\frac{B_\phi}{B_\theta}]^2\frac{\partial T}{\partial x} = 0.45\frac{\Lambda_r^2}{(1+\frac{Q^2}{S^2})}$$

For h' we get the equation

$$(h' + 1.833) =$$

$$0.45\frac{\Lambda_r^2}{(1+\frac{Q^2}{S^2})}\{\frac{1}{[\frac{\partial\hat{T}}{\partial x}]}\hat{T}\frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial x}h' + \frac{1}{2}\frac{1}{[\frac{\partial\hat{T}}{\partial x}]^2}[\frac{B_\theta}{B_\phi}]^2 g^2$$

$$-\frac{1}{[\frac{\partial\hat{T}}{\partial x}]^2}g\frac{B_\theta}{B_\phi}(h - \hat{T}\frac{\partial ln\ (Tn_i^2)}{\partial x})$$

$$+1.9\frac{1}{[\frac{\partial\hat{T}}{\partial x}]^2}[h - 0.8\hat{T}\frac{\partial ln\ (Tn_i^{1.6})}{\partial x}]^2\} \qquad (63)$$

We use (appendix C)

$$\Lambda_r^2 = \Lambda_{r,inf}^2\hat{T}^{-2c}[\frac{\partial\hat{T}}{\partial x}]^2$$

with

$$2c = 5 - \frac{2}{\eta} - 2Z$$

or

$$Z = \frac{5}{2} - \frac{1}{\eta} - c$$

E. g., the following cases are envisaged in chapter (13):

1. c=1, Z=0 $\rightarrow \eta = \frac{2}{3}$

2. c=1, $\eta = \frac{3}{2} \rightarrow$ Z=$\frac{5}{6} = 0.833$

3. c=1, $\eta = 1.6 \rightarrow$ Z=0.875

The choice c=1 is important for the analytical treatment. Using the model profile (51) we have

$$\frac{\partial \hat{T}}{\partial x} = \frac{\partial (1 - tanh(x))}{\partial x} = -\frac{1}{cosh^2(x)} =$$

$$-(1 - tanh^2(x)) = -(1 - tanh(x))(1 + tanh(x)) = -\hat{T}(2 - \hat{T})$$

We rewrite equation (63) as

$$h' + 1.833 = \frac{0.45\Lambda}{1 + \frac{Q^2}{S^2}}\{\frac{0.107q^2}{1 + \frac{Q^2}{S^2}}h' + \frac{1}{2}\frac{1}{[\frac{\partial \hat{T}}{\partial x}]^2}(g')^2$$

$$-\frac{1}{\frac{\partial \hat{T}}{\partial x}}g'(h' - (1 + \frac{2}{\eta}) + 1.9[h' - 0.8(1 + \frac{1.6}{\eta}]^2\} \tag{64}$$

Now we assume $h' = 0$ and get an equation for $\Lambda = \Lambda_0$ at the zero of h'

$$1.83 = \frac{0.45\Lambda_0^2}{1 + 0.26[0.625(1 + 2\eta_i^{-1})]^2\Lambda_0^2}\{0.5\frac{(g_0')^2}{\hat{T}_0^2(2 - \hat{T}_0)^2}$$

$$+\frac{g_0'}{\hat{T}_0(2 - \hat{T}_0)}(1 + 2\eta_i^{-1}) + 1.9[0.8(1 + 1.6\eta_i^{-1})]^2\}$$

T_0, g'_0 are temperature and the normalized velocity g' at the zero of h'. We approximate the solution by

$$h' = h'_{-\infty}(1 - \frac{\Lambda^2}{\Lambda_0^2})$$

For small Λ_r we get the solution at the magnetic axis.

We use the u$_\theta$ - equation and take the limit for x=-∞. Only the term with g' remains

$$(h')_{-\infty} = -1.83 + \frac{0.45}{\hat{T}^{2c}}\Lambda_r(-\infty)[\hat{T}(2-\hat{T})]^2\frac{1}{2}(g')^2_{-\infty}\frac{1}{[\hat{T}(2-\hat{T})]^2}$$

or

$$(h')_{-\infty} = -1.83 + \frac{0.225}{2^{2c}}\Lambda_r^2(-\infty)(g')^2_{-\infty}$$

We use the equation

$$\frac{Q}{S} = 0.5(h' - 0.625(1 + \frac{2}{\eta}))\Lambda_r$$

and combine it with the Ansatz for h'

$$\frac{Q}{S} = 0.5((h')_{-\infty}(1 - \frac{\Lambda_r^2}{\Lambda_0^2}) - 0.625(1 + \frac{2}{\eta}))\Lambda_r$$

From the relation

$$f^2 = \frac{\Lambda_r^2}{\Lambda_0^2}$$

we get

$$f = \frac{\Lambda_{r,inf}}{\Lambda_0}\frac{2-\hat{T}}{\hat{T}^{c-1}}$$

At the boundary we obtain

$$1 = \frac{\Lambda_{r,inf}}{\Lambda_0}\frac{2-\hat{T}_0}{\hat{T}_0^{c-1}}$$

We get

$$\frac{Q}{S} = 0.5((h')_{-\infty} - 0.625(1+\frac{2}{\eta}) - (h')_{-\infty}f^2)\Lambda_r$$

or - if we replace Λ_r -

$$\frac{Q}{S} = 0.5((h')_{-\infty} - 0.625(1+\frac{2}{\eta}) - (h')_{-\infty}f^2)\Lambda_0 f$$

We discuss

$$F = \frac{2-\hat{T}}{\hat{T}^{c-1}}$$

as a function of $\hat{T}$. F has an extremum for

$$\hat{T} = 2\frac{c-1}{c-2}$$

We investigate $\frac{Q^2}{S^2}$ as function of f^2. It has a maximum for

$$f^2 = f_{max}^2 = \frac{1}{3}[1 - \frac{0.625(1+\frac{2}{\eta})}{(h')_{-\infty}}]$$

and a minimum for

$$f^2 = f_{min}^2 = 1 - \frac{0.625(1+\frac{2}{\eta})}{(h')_{-\infty}}$$

This value is larger than unity and outside the range of interest. The maximum value is

$$\frac{Q^2}{S^2} = 0.26\Lambda_0^2[(h')_{-\infty}]^2\frac{4}{27}[1 - \frac{0.625(1+\frac{2}{\eta})}{(h')_{+\infty}}]^3$$

We approximate $\frac{Q^2}{S^2}$ by a parabola which interpolates between f=0, f=f_m and f=1. We get

$$\frac{Q^2}{S^2} = 0.26\Lambda_0^2[(h')_{-\infty}]^2[f_m^2(9f_m^2-1) - (5f_m^2-1)f^2]f^2$$

The toroidal velocity is then given by

$$g'(\widehat{T}) = g'(\widehat{T_0}) + 0.107q^2(h')_{-\infty} \int_{\widehat{T_0}}^{\widehat{T}} \frac{(1-f^2)(2-\widehat{T})d\widehat{T}}{1+\alpha f^2+\beta f^4}$$

α and β are given by

$$\alpha = 0.26\Lambda_0^2 (h')^2_{-\infty} f_m^2 (9f_m - 1)$$

and

$$\beta = -0.26\Lambda_0^2 h'^2_{-\infty} f_m^2 (5f_m - 1)$$

One can factorize $1+\alpha f^2+\beta f^4$ and gets

$$1+\alpha f^2+\beta f^4 = \beta(f^2-a)(f^2-b)$$

where

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{-\alpha \pm \sqrt{\alpha^2-4\beta}}{2\beta}$$

In the expression

$$f = \frac{\Lambda_{r,inf}}{\Lambda_0}\frac{2-\hat{T}}{\hat{T}^{c-1}}$$

we put $\hat{T} = \hat{T}_0$ and get

$$1 = \frac{\Lambda_{r,inf}}{\Lambda_0}\frac{2-\hat{T}_0}{\hat{T}_0^{c-1}}$$

or

$$\frac{\Lambda_{r,inf}}{\Lambda_0} = \frac{\hat{T}_0^{c-1}}{2-\hat{T}_0}$$

$$\Lambda_0 = \Lambda_{r,inf}\frac{2-\hat{T}_0}{\hat{T}_0^{c-1}}$$

We consider c=1 implying

$$f = \frac{\Lambda_{r,inf}}{\Lambda_0}(2-\hat{T})$$

because in this case a simple analytical integration is possible. This integration yields the jump

$$u_\phi(\hat{T}=2)-u_\phi(\hat{T}_0) = -\frac{0.107q^2(h')_{-\infty}}{2\sqrt{\alpha^2-4\beta}}\frac{\Lambda_0}{\Lambda_{r,inf}}ln[[\frac{-a}{1-a}]^{1-a}[\frac{-b}{b-1}]^{b-1}]$$

The radial dependance is given by

$$u_\phi(\hat{T}) - u_\phi(\hat{T}_0) = -\frac{0.107q^2(h')_{-\infty}}{2\sqrt{\alpha^2-4\beta}}\frac{\Lambda_0}{\Lambda_{r,inf}}ln[f^2-a]^{1-a}[b-f^2]^{b-1}]$$

We assume

$$\eta = 1.6$$

and (neglecting the small term $\frac{0.225}{2^{2c}}\Lambda_r^2(-\infty)(g')^2_{-\infty}$) approximately)

$$(h')_{-\infty} = -1.83$$

and use the boudary value

$$(h')_{r_s} = 0$$

to compute Λ_0^2. We get the equation

$$1.83 = \frac{0.45\Lambda_0^2}{1+0.26[0.625(1+2\eta_i^{-1})]^2\Lambda_0^2}\{0.5\frac{u_{\phi,0}^2}{\hat{T}_0^2(2-\hat{T}_0)^2}$$

$$+\frac{u_{\phi,0}}{\hat{T}_0(2-\hat{T}_0)}(1+2\eta_i^{-1}) + 1.9[0.8(1+1.6\eta_i^{-1})]^2\}$$

In addition we assume $u_{\phi,0} = 0$ (concerning the boundary values also see chapter 6) and get

$$1.83 = \frac{0.45\Lambda_0^2}{1 + 0.26[0.625(1+2\eta_i^{-1})]^2\Lambda_0^2}\{$$
$$+1.9[0.8(1+1.6\eta_i^{-1})]^2\}$$

or

$$1.83{+}1.83\ 0.26[0.625(1{+}2\eta_i^{-1})]^2\Lambda_0^2) = 0.45\Lambda_0^2\{+1.9[0.8(1{+}1.6\eta_i^{-1})]^2\}$$

Therefore

$$\Lambda_0^2(1.83\ 0.26[0.625(1{+}2\eta_i^{-1})]^2{-}0.45\ 1.9[0.8(1{+}1.6\eta_i^{-1})]^2) = -1.83$$

or

$$\Lambda_0^2 = \frac{-1.83}{(1.83\ 0.26[0.625(1+2\eta_i^{-1})]^2 - 0.45\ 1.9[0.8(1+1.6\eta_i^{-1})]^2)}$$

and finally

$$\Lambda_0^2 = 1.47$$

g' and h' allow to compute the radial electric field which is given by

$$E_r = B_\theta u_\phi - B_\phi u_\theta + \frac{T_i}{e_i}\frac{\partial P_i}{\partial r}$$

This can be rewritten as

$$\frac{E_r}{B_\phi} = \frac{B_\theta}{B_\phi}u_\phi - u_\theta + \frac{T}{eB_\phi L_\psi}\frac{1}{nT}[n\frac{\partial T}{\partial x} + T\frac{\partial n}{\partial x}]$$

Introducing the ratio η entails

$$\frac{E_r}{B_\phi} = \frac{B_\theta}{B_\phi}u_\phi - u_\theta + \frac{T_i}{e_iB_\phi L_\psi}\frac{1}{\hat{T}}\frac{\partial \hat{T}}{\partial x}[1 + \frac{1}{\eta}]$$

or with the help of the velocity v_T

$$\frac{E_r}{B_\phi} = v_T\{\frac{B_\theta}{B_\phi}g - h + \frac{\partial\hat{T}}{\partial x}[1 + \frac{1}{\eta}]\}$$

We introduce g' and h' and get

$$\frac{E_r}{B_\phi} = v_T\{g' + \frac{\partial\hat{T}}{\partial x}[-h' + 1 + \frac{1}{\eta}]\}$$

The radial electric field is then

$$E_r = \frac{T_{inf}}{e_i L_\psi}[g' + (-h' + 1 + \frac{1}{\eta})\frac{\partial\hat{T}}{\partial x}]$$

10 Numerical solution method

The second order equation is replaced by two first order equations. The 1st equation is that for $\frac{\partial g}{\partial x}$ which can be written as

$$\frac{\partial g}{\partial x} = G + \frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial x}\frac{B_\phi}{B_\theta}h \tag{65}$$

The 2nd equation is that for G which is related to the first derivative of g by

$$G = \frac{\partial g}{\partial x} - \frac{0.107q^2}{1+\frac{Q^2}{S^2}}\frac{\partial ln\ T}{\partial x}\frac{B_\phi}{B_\theta}h \tag{66}$$

Thus the 2nd equation looks like

$$\frac{\partial}{\partial x}G = \frac{m_i n_{inf} L_\psi^2}{\eta_{2,inf}\hat{\eta}_2}[\hat{n}\nu_{cx}g - \frac{\hat{n}\dot{M}_{\phi,i}}{v_T} +$$
$$\frac{J_r B_\theta}{m_i v_T n_{inf}} - \frac{\eta_{2,inf}}{m_i n_{inf} L_\psi^2}\hat{\eta}_2 G\frac{\partial}{\partial x}ln(\eta_2)] \tag{67}$$

or

$$\frac{\partial}{\partial x}G = \frac{1}{\eta'\hat{\eta}_2}[\hat{n} + \nu_{cx}g - \frac{\hat{n}\dot{M}_{\phi,i}}{v_T} +$$
$$\frac{J_r B_\theta}{m_i v_T n_{inf}} - \eta'\hat{\eta}_2 G\frac{\partial}{\partial x}ln(\eta_2)] \tag{68}$$

In addition we have the equation for the poloidal rotation which can be written as a nonlinear equation for h

$$(h + 1.833\frac{\partial\hat{T}}{\partial x})\hat{T} = T_{45}[\frac{B_\phi}{B_\theta}\hat{T}\frac{\partial g}{\partial x} + \frac{1}{2}g^2$$
$$-g\frac{B_\phi}{B_\theta}(h - \hat{T}\frac{\partial ln\ (Tn_i^2)}{\partial x})$$

$$+1.9\frac{B_\phi^2}{B_\theta^2}[h-0.8\hat{T}\frac{\partial ln\ (Tn_i^{1.6})}{\partial x}]^2] \tag{69}$$

For a given temperature and density profile and the radial dependence of the normalized toroidal velocity g this equation is solved for h by means of the solver ZEROIN [28] for transcendental equations.

11 Friction due to recycled neutrals

The charge exchange frequency of a neutral gas with the density $n_0\frac{1}{m^3}$ is given by

$$\nu_{cx} = 4.7 \ \ 10^{-14}\frac{m^3}{sec} \ \ n_0\frac{1}{m^3}$$

We multiply with the characteristic time (8) and get as charge exchange term in equation (67)

$$T_{CX} = \frac{\hat{n}}{\hat{\eta}_2}t_c\nu_{cx}g = \hat{T}^{0.5-Z}t_c\nu_{cx}g \tag{70}$$

We assume that the neutral gas density decays as

$$n_0 = N_0 exp(\frac{L_\psi}{L_{neu}}x)$$

N_0 is the neutral gas density at the inflection point.
In the case of the Alcator [30] we have $t_c = 3.3\ 10^{-3}$ sec, $N_0 = 5\ \ 10^{14}$ and one gets

$$T_{cx}(r = r_{inf}) = 4.710^{-14} \ \ 510^{14} \ \ 3.3\ 10^{-3} = 8.08 \ \ 10^{-2}$$

This has an influence on the step $g - g_0$ already.

12 Neutral injection

The injected current I_b in Amp gives rise to an increase of the mean density by

$$\dot{N}_0 = \frac{I_b[A]}{e_0}$$

e_0 is the elementary charge. Resorting to the injected power P and the beam energy E_0 we get

$$\dot{N}_0 = \frac{P_{MW}}{E_{0,keV}} \frac{10^3}{e_0}$$

or

$$\dot{N}_0 = 0.624 \ \ 10^{22} \frac{P_{MW}}{E_{0,keV}}$$

At large energies the rate coefficient $< \sigma v >_{cx}$ saturates at

$$< \sigma v >_{cx} = 1.5 \ \ 10^{-7} \frac{cm^3}{sec}$$

The charge exchange frequency is

$$< \sigma v >_{cx} \dot{N}_0 = 1.5 \ \ 10^{-13} \frac{m^3}{sec} 0.624 \ \ 10^{22} \frac{P_{MW}}{E_{0,keV}}$$

or

$$< \sigma v >_{cx} \dot{N}_0 = 0.94 \ \ 10^9 \frac{m^3}{sec} \frac{P_{MW}}{E_{0,keV}}$$

Acounting for the geometry of a pencil beam and of a circular tokamak plasma [24] we get for the deposited momentum

$$\dot{M}_{||,i} = \frac{\dot{N}_0 < \sigma v >_{cx}}{\sqrt{2\epsilon}(2\pi)^2 r R}$$

This can be rewritten as

$$\dot{M}_{||,i} = 0.94 \ \ 10^7 \frac{-1}{r^{\frac{3}{2}} R^{\frac{1}{2}}} \frac{P_{MW}}{E_{0,keV}} \frac{100}{\sqrt{2} 4\pi^2}$$

or as

$$\dot{M}_{||,i} = 0.94 \ \ 10^7 \frac{-1}{r^{\frac{3}{2}} R^{\frac{1}{2}}} \frac{P_{MW}}{E_{0,keV}} 1.79$$

Finally we have

$$\dot{M}_{||,i} = 1.68 \ \ 10^7 \frac{-1}{r^{\frac{3}{2}} R^{\frac{1}{2}}} \frac{P_{MW}}{E_{0,keV}}$$

13 Results

The equations for u_ϕ and u_θ are solved numerically (1) to benchmark with a simplified analytical theory for Λ_1=0 and vanishing neutral gas density and (2) to compare with the theory accounting for finite Λ_1 effects and (3) to explore the parameter space in regions where the analytical theory is not valid, in particular in the case where the neutral gas density larger than $10^{17}m^{-3}$ and/or neutral injection is important.
The method resorts to an ODE - solver for the classical momentum balance which is combined with a solver for transcendental equations yielding v_Θ.
Several integration procedures such as the Adams - Pece -, Burlish - Stoer -, or the fifth order Cash - Carp - Runge - Kutta - method were applied showing that the results are almost independent from the special numerical method.

13.1 Comparison with the simplified analytical theory of section 8 and the halfanalytical theory of section 7

. For the case T_{inf}=165 eV, L_ψ = 0.76 cm, r_{inf}=20.8 cm, r_s = 21.5 cm (Fig. 1), we get the jump $\Delta u_\phi = u_\phi - u_{\phi_0} = 140 \frac{km}{sec}$, the analytical and the numerical solutions agree exactly (Fig. 2). This jump is much larger than the measured one ($\Delta u_\phi \approx 35 \frac{km}{sec}$) because the approximation $\frac{Q^2}{S^2} = 0$ was made, i. e. in the numerical and the analytical approach $\Lambda_1 = C_\eta = 0$ was assumed.

The halfanalytical treatment in chapter (7) resorts to the neoclassical solution (48). One gets exact agreement with the ODE

- integration (Fig. 3) if in the numerical method the solution of equation (24) is replaced by the neoclassical value ($C_\eta = 0$). The corresponding jump of the approximate solution $\Delta u_\phi = 43$ $\frac{km}{sec}$ is now close to measured one.
Since the neoclassical approximation of u_θ is rather accurate the halfanalytical solution (circles) agrees well with the exact solution (triangles). In Fig. 4 both solution are compared. We see that the 'neoclassical jump' is around 10% larger than the corresponding jump of the exact numerical solution $\Delta u_\phi = 35$ $\frac{km}{sec}$ which is now close to measured one.

13.2 Comparison with the approximate analytical theory of section 9, accounting for finite Λ_1

The input data are those of ALCATOR C-MOD [30] (partly used already before): T_{inf} = 165 eV, $n_i = 1.87\ 10^{20} m^{-3}$, L_ψ = 0.76 cm, r_{inf} = 20.8 cm, r_s = 21.5 cm, $\epsilon = 1.6$, $Z_{eff_{inf}} = 1.57$, Z = 0.875, η= 1.6, R = 67 cm, B_θ = 0.625 T and B_ϕ = 5.2 T. Here $Z_{eff_{inf}}$ and Z are defined by $Z_{eff}(r) = Z_{eff_i} \hat{T}^Z$, i. e. $Z_{eff_{inf}}$ is $Z_{eff}(r)$ at the inflection point and Z is a constant. We proceed with the comparison of numerical and analytical solutions in the case of the exact expression for Λ. The jumps Δu_ϕ and the radial dependence of u_ϕ according to the numerical integration (triangles) and according to the analytical theory (quadrangles) agree well (Fig. 5). The deviation of the maxima (jumps) is $\approx 8\%$. Both jumps, $\Delta u_\phi = 34$ $\frac{km}{sec}$ (numerical) and $\Delta u_\phi = 37$ $\frac{km}{sec}$ (analytical), agree approximately with the experimental one (≈ 35 $\frac{km}{sec}$) [30]. The deviations of the maxima of u_θ and of the minima of E_r according to the numerical integration and the analytical theory are less than $\approx 8\%$ and 10%, respectively (Fig. 6 and Fig. 7). The radial dependence shown in Fig. 7 is similar to that reported at DIII - D.

13.3 General radial profiles of the plasma parameters

In Fig. 8 the profile

$$\hat{T} = \hat{T}_0 + (1 - \hat{T}_0)exp(-\frac{x}{1 - \hat{T}_0}) \tag{71}$$

for x < 0 and

$$\hat{T} = exp[-x] \tag{72}$$

for x > 0, discussed in appendix A in more detail, is shown. This profile is continuous, has a contiuous derivative, but in contrast to the profile (51) the second derivative is discontinuous. In Fig. 9 the normalized velocity g is shown in the range [x=-12,...,x=1.4]. As in the solutions before the toroidal velocity increases within the distance L_ψ to roughly the same value as before in Fig. 5. The poloidal velocity (Fig. 10) has a sharp maximum of 10 $\frac{km}{sec}$. It becomes zero at r=22 cm. Analogously the electric field (Fig. 11) has a sharp local minimum.
We now generalize the profile (51) to

$$T = T_{inf}\{(1 - tanh(x)) + (\frac{T_0}{T_{inf}} - 2)(1 - [\frac{r}{r_{inf}}]^2)^2\}$$

T_0 is the temperature at the plasma center (Fig. 12). This profile has been measured at ALCATOR C - MOD [30]. In Fig. 13 g is shown in the same range as before in Fig. 5. The major part of the step in u_ϕ is located in the boundary region. u_ϕ increases there to around 32 $\frac{km}{sec}$. In the central part of the plasma the increase is around 10 $\frac{km}{sec}$. ($T_0 = 1020$ eV was assumed.) The poloidal velocity has a dominant peak in the boundary region (Fig. 14), and the electric field is somewhat modified in the central region (Fig. 15). However, one has to keep in mind the restricted validity of these calculation including thethe central core plasma, because there another collisionality regime may be entered where modifications of the present theory are needed.

13.4 Influence of the neutral gas

It turns out that the solution is very sensitive with respect to the neutral gas density. This may indicate the the neutral gas plays a decisive role in the L - H transition. In fact, the neutral gas physics in limiter and divertor devices are quite different.

In Fig. 16 the influence of the neutral gas is investigated. Even the very low density of $N_0 = 5 \cdot 10^{14}/m^3$ an effect on the step can be seen which is reduced in size by 20% and gets a pedestal at the outside. In the poloidal rotation velocity and the radial electric field the influence of such a small neutral gas density can be neglected.

If the neutral density N_0 is increased to $N_0 = 5 \cdot 10^{15}/m^3$ the step in g is reduced to about 20% , i. e. almost removed (Fig. 16). Therefore the system may not be able to transit into the H - mode (Fig. 10).

Since not all charge exchange neutrals leave the plasma and contribute to the momentum loss (as it is assumed in equation [15]), the aforementioned neutral gas densities are probably underestimated by a factor 2 - 3. However, the inclusion of a more detailed neutral gas model would be beyond the scope of this report.

14 Conclusions

The good agreement between the numerical and the analytical model shows that both models are consistent and the comparison with the experimental data should be reasonable. In fact, this comparison shows that in (ohmic discharges of) the ALCATOR - device neoclassical physics prevail during the H - mode.

Surprising is the strong influence of the neutral gas. A very small density $n_0 = 5\ 10^{15}\ m^{-3}$ destroys the step already.

Preliminary calculations accounting for neutral beam injection indicate that in this case the neoclassical viscosity must be corrected by an anomalous contribution.

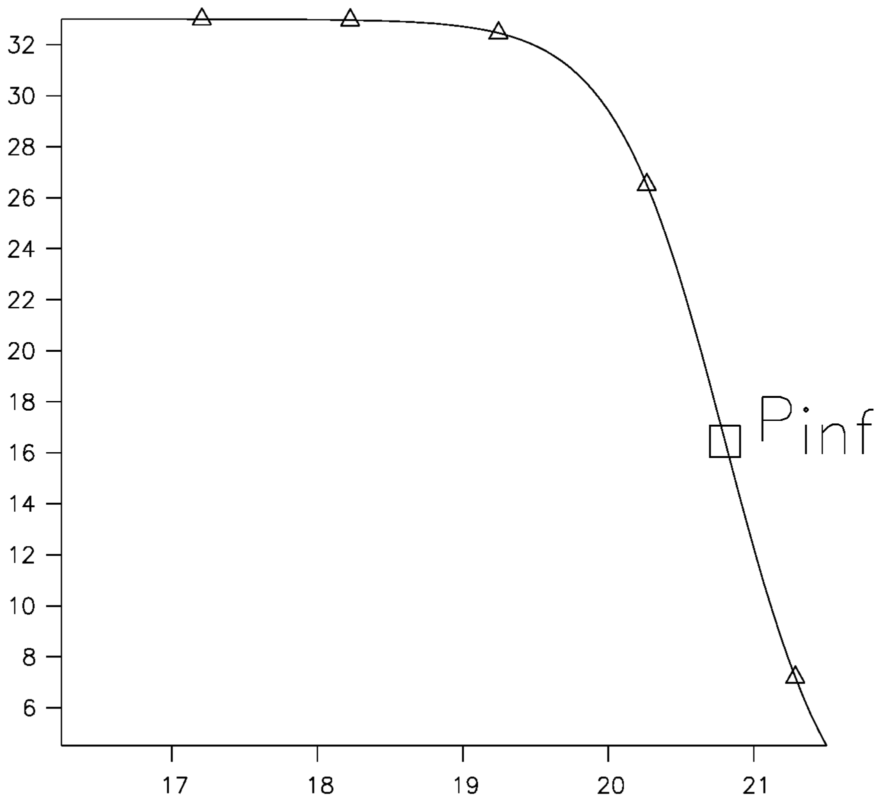

Figure 1: Ion temperature T[10eV]. The inflection point of the temperature profile, P_{inf}, is the locus of vanishing curvature. The decaylength there and the maximum at r=0 (165 eV) are adjusted to ALCATOR C-MOD data.

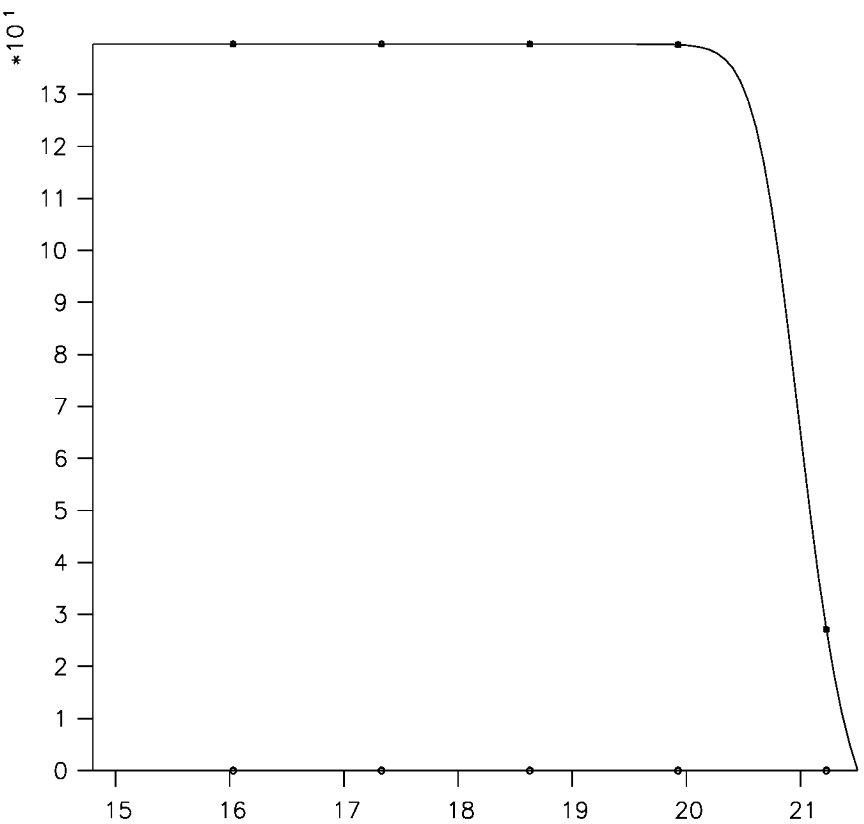

Figure 2: Toroidal velocity $U_\phi[km/sec]$. Due to the assumptions $\frac{Q}{S} = 0$ and $C_\eta = 0$ the analytical and the numerical solutions agree exactly. By the same reason the jump of the toroidal velocity $\Delta u_\phi = u_\phi - u_{\phi_0} = 140\ \frac{km}{sec}$ is much too large.

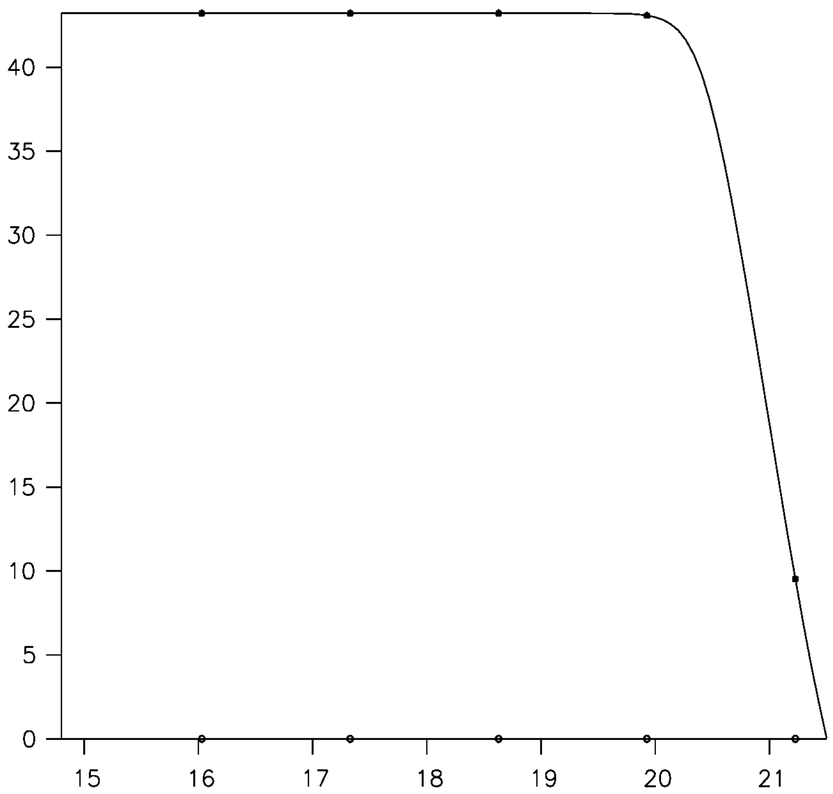

Figure 3: Toroidal velocity $U_\phi[km/sec]$. The halfanalytical treatment in chapter (7) resorts to the neoclassical solution (48). One gets exact agreement with the ODE - integration if the solution of equation (24) is replaced by the neoclassical value ($C_\eta = 0$). The corresponding jump of the approximate solution $\Delta u_\phi = 43\ \frac{km}{sec}$ is now close to measured one.

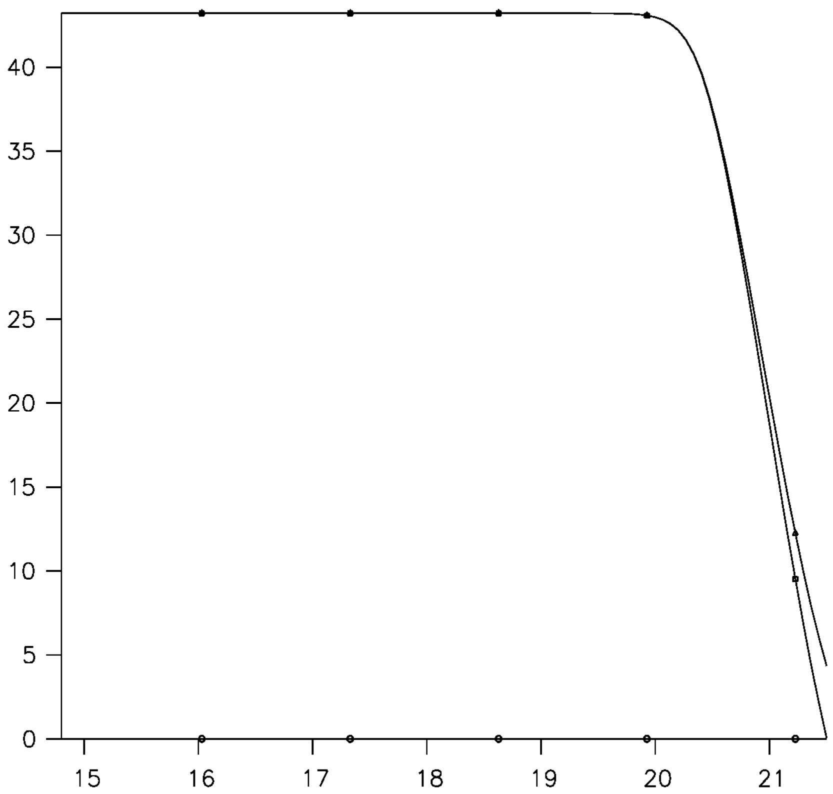

Figure 4: Toroidal velocity $U_\phi[km/sec]$. One gets approximate agreement of the halfanalytical solutions (triangles) with the exact ODE - integration. As in Fig. 3 the halfanalytical solution is obtained by replacing u_θ by the neoclassical value ($C_\eta = 0$). The corresponding jump of the approximate solution $\Delta u_\phi = 43\ \frac{km}{sec}$ is now close to measured one.

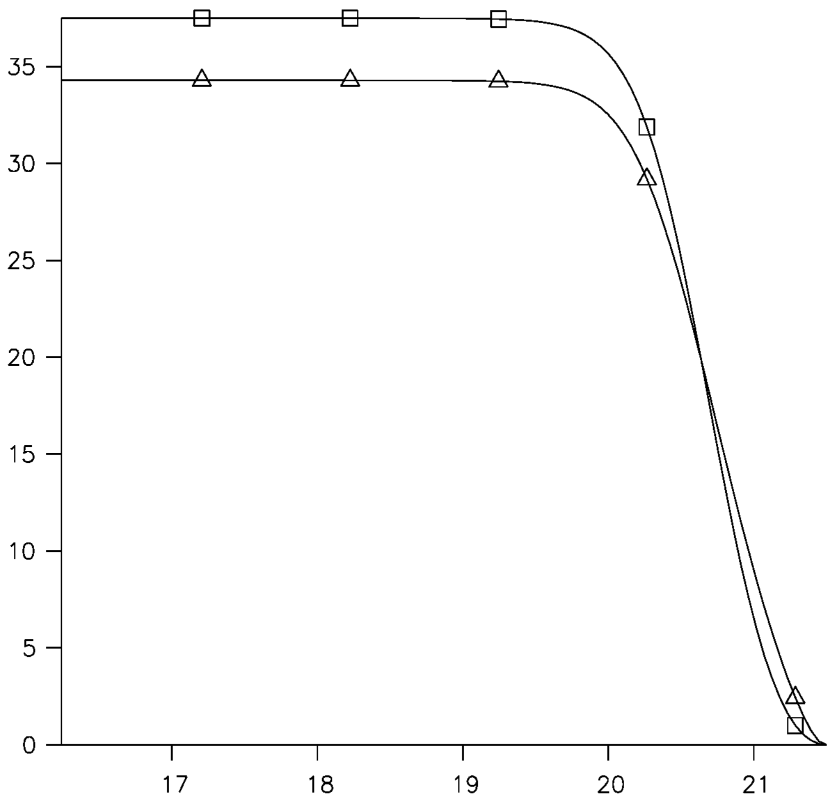

Figure 5: Toroidal velocity $U_\phi[km/sec]$. The jumps Δu_ϕ and the radial dependence of u_ϕ according to the numerical integration (triangles) and according to the analytical theory (quadrangles) of chapter (9) agree well. The deviation of the maxima (jumps) is $\approx 8\%$. Both jumps agree approximately with the experimental one ($\approx 35\ \frac{km}{sec}$) [30].

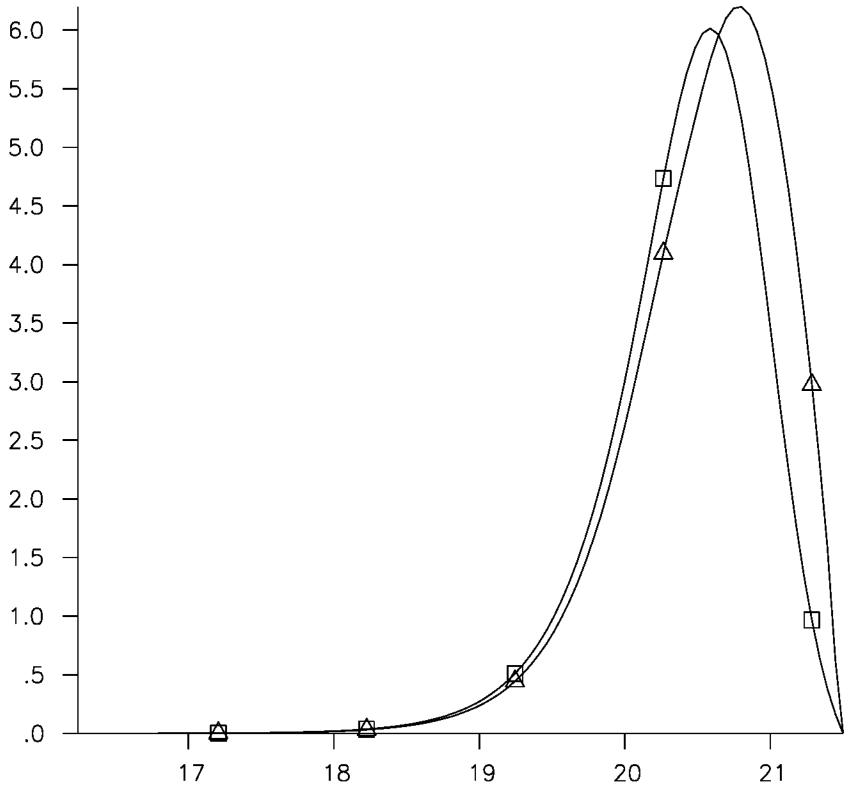

Figure 6: Poloidal velocity $U_\theta[km/sec]$. The deviations of the maxima of u_θ according to the numerical integration and the analytical theory are less than $\approx 8\%$.

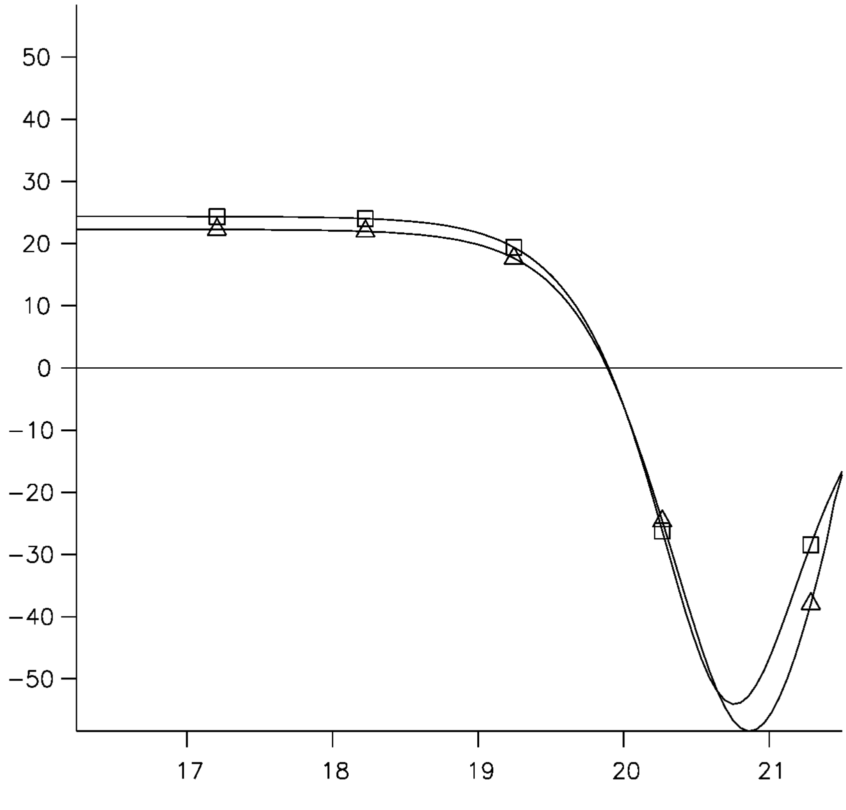

Figure 7: Radial electric field $E_r[kV/m]$. The deviations of minima of E_r according to the numerical integration and the analytical theory are less than $\approx 10\%$. The radial dependence is similar to that reported at DIII - D.

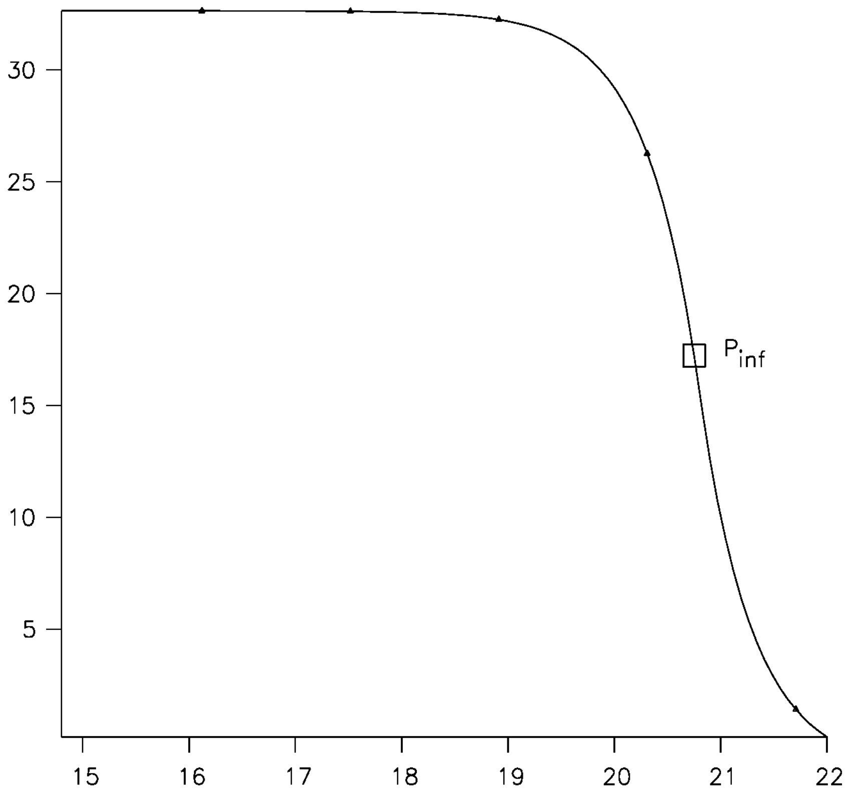

Figure 8: Ion temperature T[10eV]. The temperature profile in the appendix (A) was chosen to demonstrate the effect of different profile shapes. It has an inflection point at $r = r_{inf}$. As in Fig. 1 the central value is 2 times larger than the value at the inflection point, so that similar results can be expected as Figs. 6 and 7.

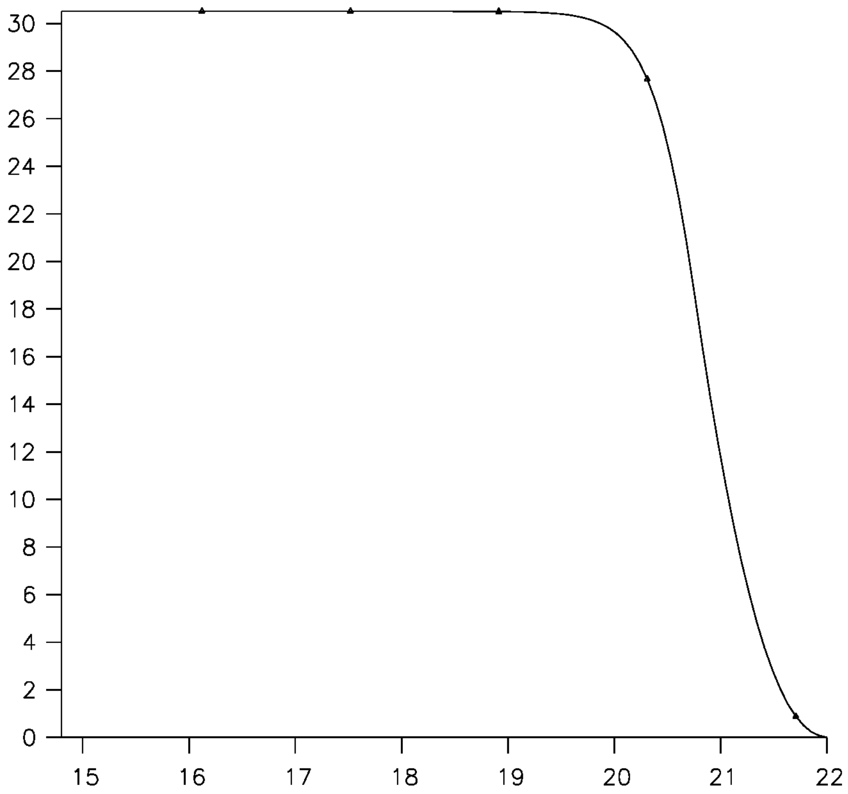

Figure 9: Toroidal velocity $U_\phi[km/sec]$. The total jump in g is three times larger than in Fig. 5.

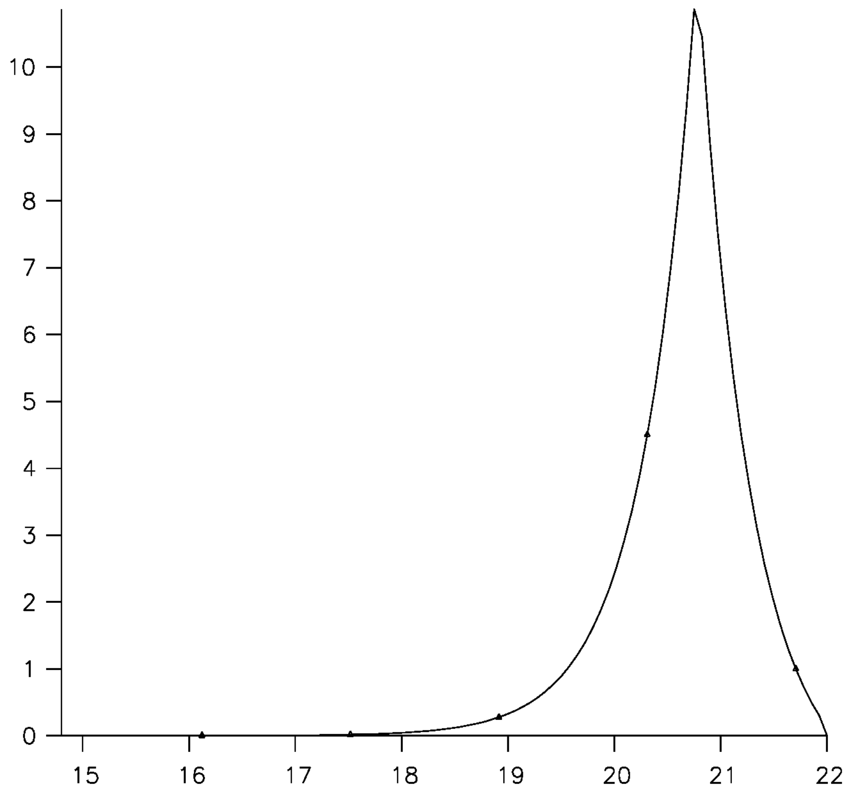

Figure 10: Poloidal velocity $U_\theta[km/sec]$. The maximum of u_θ is roughly the same as in Fig. 6 analytical theory are less than $\approx 8\%$. The different profile shape of Fig. 8 effects a rather sharply peaked maximum.

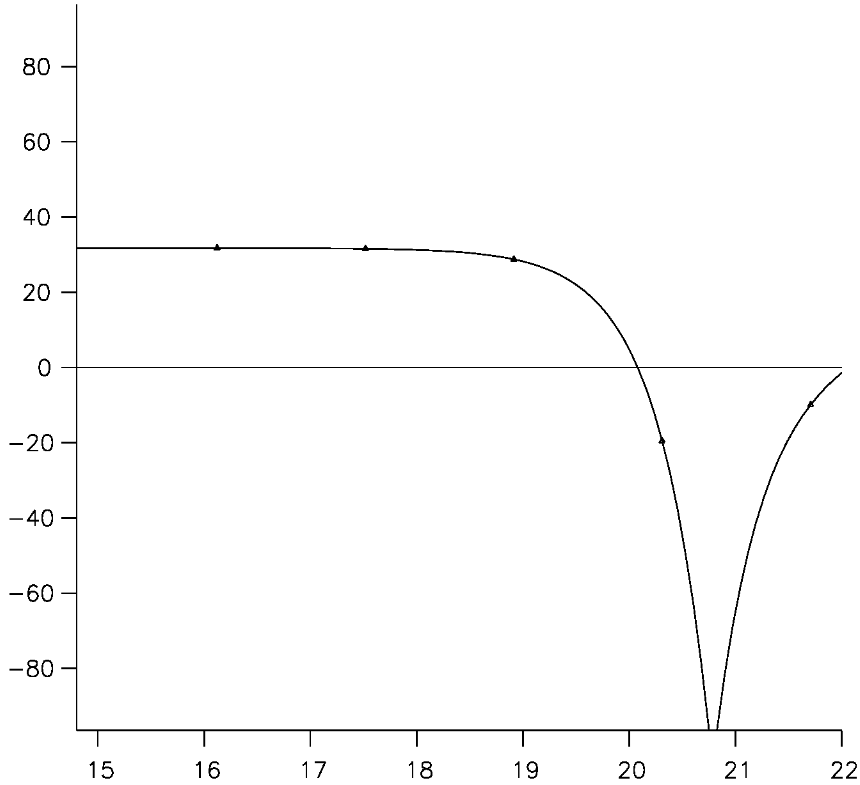

Figure 11: Radial electric field $E_r[kV/m]$. The minimum of E_r is of the same order as in Fig. 7. Even in the case of a strongly changed temperature profile the electric field is positive at r=0.

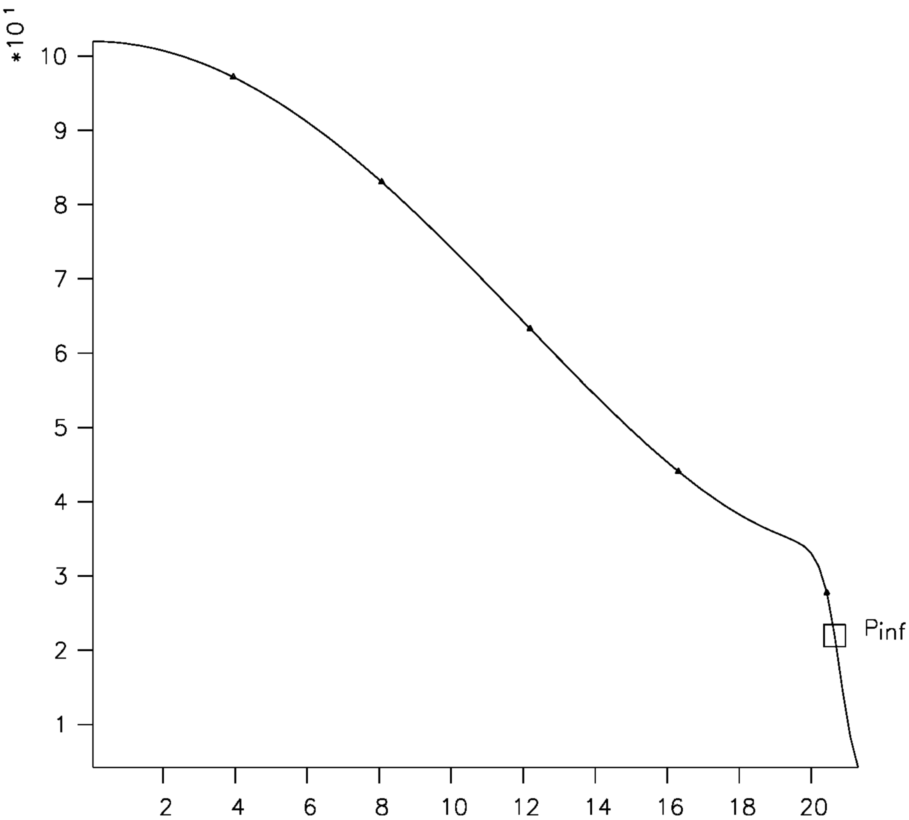

Figure 12: Ion temperature T[10eV]. The more realistic temperature profile has a pedestal like in Fig 1 and a parabolic part connecting the edge with the center.

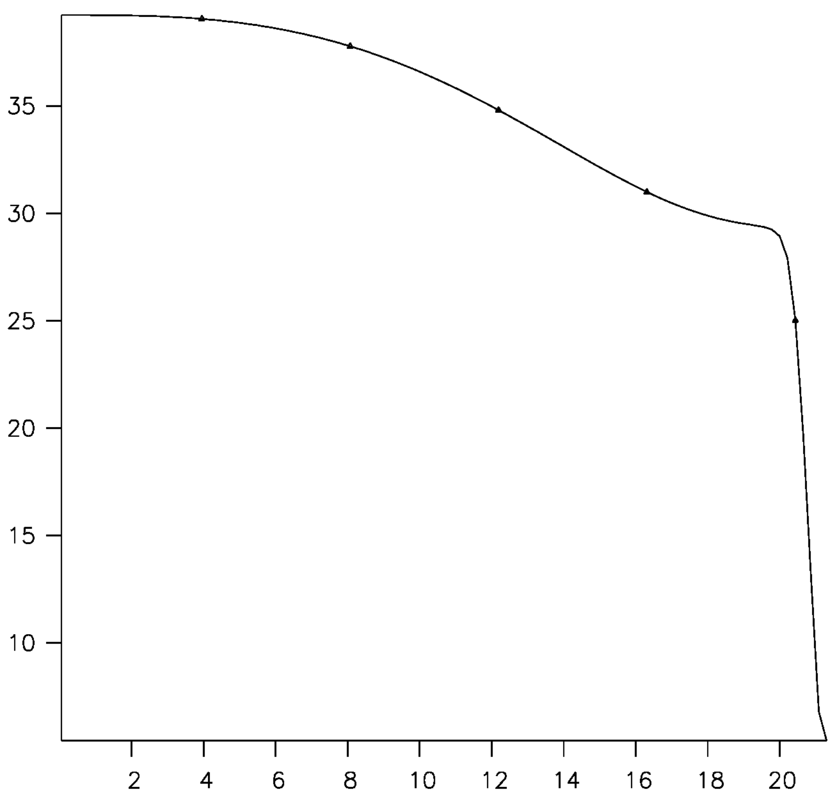

Figure 13: Toroidal velocity $U_\phi[km/sec]$. The input data are those of ALCATOR C-MOD [30]. The jump Δu_ϕ and the radial dependence of u_ϕ in the edge are the same as in the corresponding Fig. 5. In going to the center the spin - up increases further because of the temperature gradient.

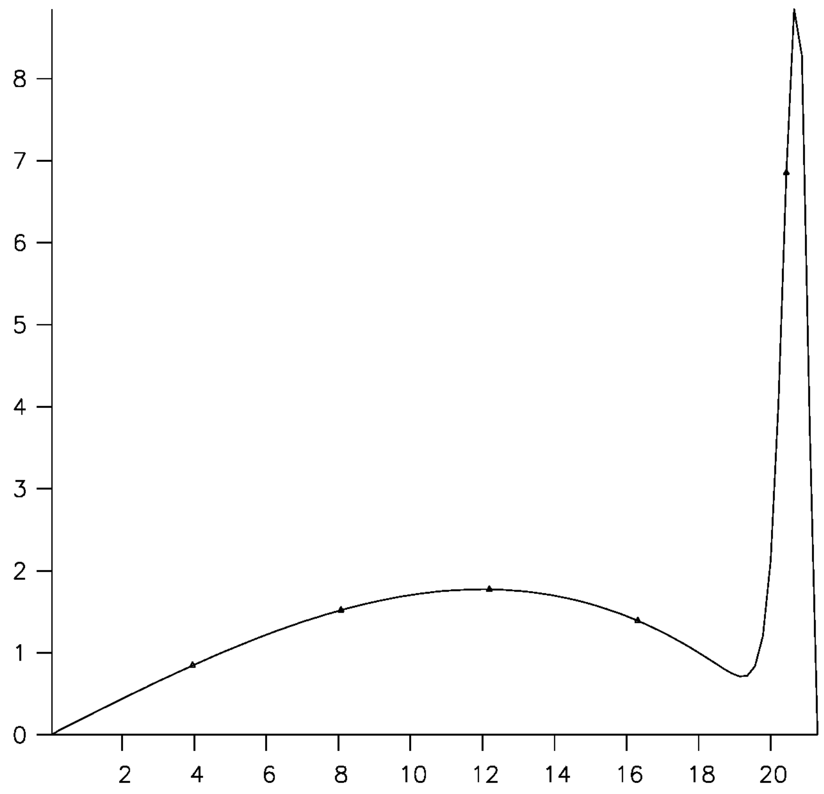

Figure 14: Poloidal velocity $U_\theta [km/sec]$. The maximum of u_θ due to the pedestal is still pronounced. The poloidal rotation velocity at smaller radii is -according to the temparature gradient - finite but small.

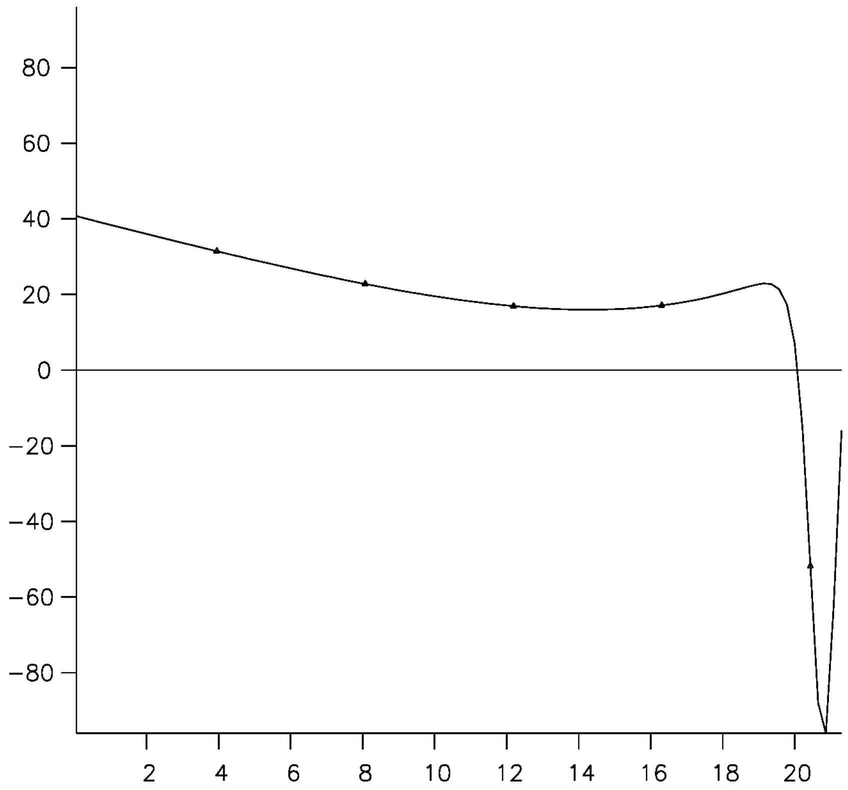

Figure 15: Radial electric field $E_r[kV/m]$. The electric field deviates at the plasma interior from that of Fig. 7 by at most 20%

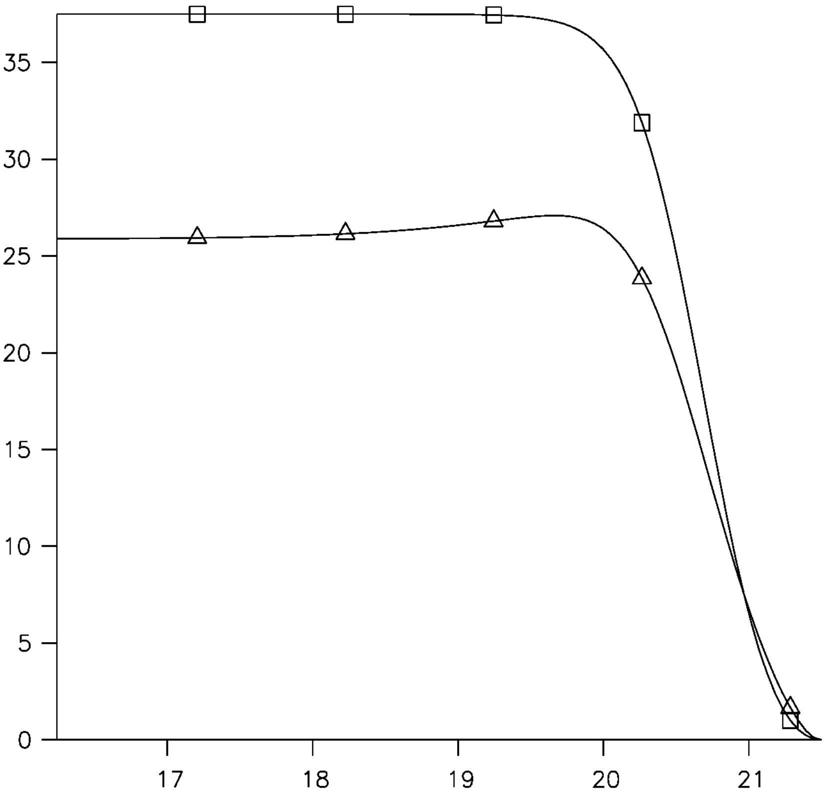

Figure 16: Toroidal velocity $U_\phi[km/sec]$. Even at the very low neutral density of $N_0 = 5 \cdot 10^{14} \frac{1}{m^3}$ an effect on the step can be seen which is reduced in size by 25%.

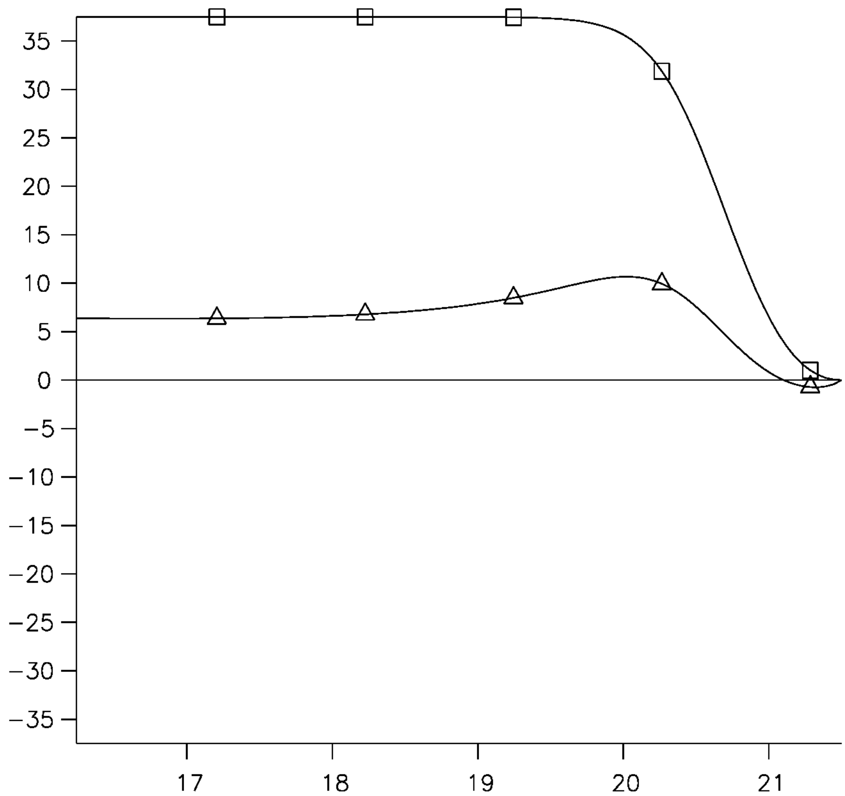

Figure 17: Toroidal velocity $U_\phi[km/sec]$. If the neutral density N_0 is increased to $N_0 = 5 \cdot 10^{15} \frac{1}{m^3}$, the step in g is reduced to about 16%, i. e. almost removed.

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A Temperature and density profiles with L_ψ as decay length

Inside the last closed flux surface with radius r_s an exponential decay hat been assumed for the temperature and density profile.

$$\hat{T} = \hat{T}_0 + (1 - \hat{T}_0) exp(-\frac{r - r_{inf}}{L_\psi (1 - \hat{T}_0)}) \tag{73}$$

or

$$\hat{T} = \hat{T}_0 + (1 - \hat{T}_0) exp(-\frac{x}{1 - \hat{T}_0}) \tag{74}$$

with $\hat{T}(r = 0) = \hat{T}_0$ and $\hat{T}(r = r_{inf}) = 1$. L_ψ is related to the temperature scale - length L_{Ts} (see below). Analogously we have for the density profile

$$\hat{n} = \hat{n}_0 + (1 - \hat{n}_0) exp(-\frac{r - r_{inf}}{L_{\psi_n} (1 - \hat{n}_0)}) \tag{75}$$

or with $r - r_{inf} = L_\psi x$

$$\hat{n} = \hat{n}_0 + (1 - \hat{n}_0) exp(-\frac{L_\psi}{L_{\psi_n} (1 - \hat{n}_0)} x) \tag{76}$$

with $\hat{n}(r = 0) = \hat{n}_0$.

$\hat{T}_0, \hat{n}_0$ are the central values and r_{inf} is the radius of the inflection point. L_{ψ_n} is related to the density scale length in an analogous way as is L_ψ to the temperature scale length.

The first and second derivative have the simple forms

$$\frac{d\hat{T}}{dx} = -exp(-\frac{x}{1 - \hat{n}_0}) \tag{77}$$

$$\frac{d\hat{n}}{dx} = -\frac{L_\psi}{L_{\psi_n}} exp(-\frac{L_\psi}{L_{\psi_n}} \frac{x}{1 - \hat{n}_0}) \tag{78}$$

and

$$\frac{d^2\hat{T}}{dx^2} = \frac{1}{1-\hat{T}_0} exp(\frac{-x}{1-\hat{T}_0}) \tag{79}$$

$$\frac{d^2\hat{n}}{dx^2} = [\frac{L_\psi}{L_{\psi_n}}]^2 \frac{1}{1-\hat{n}_0} exp(-\frac{L_\psi}{L_{\psi_n}} \frac{x}{1-\hat{n}_0}) \tag{80}$$

At the inflection point we have

$$(\frac{d\hat{T}}{dx})_{x=0} = -1 \tag{81}$$

$$(\frac{d\hat{n}}{dx})_{x=0} = -\frac{L_\psi}{L_{\psi_n}} \tag{82}$$

The temperature and density scale lengths are

$$L_T = \frac{T}{\frac{\partial T}{\partial r}} = \frac{L_\psi \hat{T}}{\frac{d\hat{T}}{dx}} = L_\psi [\frac{\hat{T}_0}{-exp(-\frac{x}{1-\hat{T}_0})} - (1-T_0)] \tag{83}$$

$$L_n = \frac{n}{\frac{\partial n}{\partial r}} = \frac{L_\psi \hat{n}}{\frac{d\hat{n}}{dx}} = \frac{L_\psi [\hat{n}_0 + (1-\hat{n}_0) exp(-\frac{r-r_{inf}}{L_{\psi_n}(1-\hat{n}_0)})}{-\frac{L_\psi}{L_{\psi_n}} exp(-\frac{L_\psi}{L_{\psi_n}} \frac{x}{1-\hat{n}_0})}$$

$$= L_{\psi_n} [\frac{\hat{n}_0}{-exp(-\frac{L_\psi}{L_{\psi_n}} \frac{x}{1-\hat{n}_0})} - (1-\hat{n}_0)] \tag{84}$$

respectively.

The temperature and density scale lengths at the inflection point are then

$$L_{T_{inf}} = (\frac{T}{\frac{\partial T}{\partial r}})_{inf} = -L_\psi \tag{85}$$

$$L_{n_{inf}} = (\frac{n}{\frac{\partial n}{\partial r}})_{inf} = -L_{\psi_n} \tag{86}$$

They are then independent from the maximum values $\hat{T}_0$ and $\hat{n}_0$.

Outside the last closed flux surface also an exponential decay hat been assumed for the temperature and density profile. For the temperature we have

$$\hat{T} = exp[\frac{r - r_{inf}}{L_\psi}] \tag{87}$$

or

$$\hat{T} = exp[-x] \tag{88}$$

with $\hat{T}(x = 0) = 1$ and $\hat{T}(r = \infty) = 0$. The first and second derivative have the simple form

$$\frac{d\hat{T}}{dx} = -exp[-x] \tag{89}$$

$$\frac{d^2\hat{T}}{dx^2} = exp[-x] \tag{90}$$

The temperature scale length is

$$\frac{T}{\frac{\partial T}{\partial r}} = \frac{L_\psi \hat{T}}{\frac{d\hat{T}}{dx}} = -L_\psi \tag{91}$$

It is a constant and equal to $(\frac{T}{\frac{\partial T}{\partial r}})_{r=r_{inf}}$ so that the radial temperature dependance is continuous and 'smooth' at r=r_{inf}. (the curvature, however, is discontinuous.)

For the density analogous formulae are valid. In the calculations we assume $L_{\psi_n} = L_\psi$. Thus we have utilized the same model behaviour for both T and n with the same scale length L_ψ.

B Ion - ion collisionality and collision frequency

In the following we collect some standard formulas for the collisionality and the collision frequency. For the collisionality $\hat{\nu}_i$ we have ($N_i = \frac{n_i}{10^{13}}$, $N_{inf} = \frac{n_{inf}}{10^{13}}$)

$$\hat{\nu}_i = \frac{qR}{c_i \tau_i} = 7.14 \ \ 10^2 \frac{qRN_i Z_{eff_i}}{T_i^2} \tag{92}$$

With the thermal speed

$$c_i = \sqrt{\frac{kT_i}{A}} = 0.99 \ 10^4 \sqrt{\frac{T_i[eV]}{m_i}} \frac{m}{sec} \tag{93}$$

we get the collision frequency

$$\nu_i = \frac{1}{\tau_i} = 7.14 \ \ 10^2 \frac{N_i Z_{eff_i}}{T_i^2} 0.99 \ 10^4 \sqrt{\frac{T_i[eV]}{A}} \tag{94}$$

or

$$\nu_i = \frac{1}{\tau_i} = 7.14 \ \ 10^6 \frac{N_i Z_{eff_i}}{T_i^{\frac{3}{2}} \sqrt{A}} \tag{95}$$

The collision time is then

$$\tau_i = \frac{1}{\nu_i} = \frac{1.414}{7.14} 10^{-6} \frac{T_i^{\frac{3}{2}} \sqrt{\frac{A}{2}}}{N_i Z_{eff_i}} \tag{96}$$

or

$$\tau_i = \frac{1}{\nu_i} = 1.98 \ 10^{-7} \frac{T_i^{\frac{3}{2}} \sqrt{\frac{A}{2}}}{N_i Z_{eff_i}} \tag{97}$$

This formula compares well with that of Braginski if a Coulomb logarithm $\lambda_{coul} = 15$ is assumed. More exactly we have

$$\tau_i = \frac{1}{\nu_i} = \frac{1.98 \ 10^6}{\left(\frac{\lambda_{coul}}{15}\right)} T_i^{\frac{3}{2}} \sqrt{\frac{A}{2}} n_i Z_{eff_i} \tag{98}$$

C Parameter Λ_1

The connection of Λ_1 with the velocity S is given by

$$\Lambda_1 = \frac{2\tilde{v}_T}{S}\frac{B_\phi}{B}$$

S is defined by

$$S = \frac{8rc_i}{\hat{\nu}_i qR} \tag{99}$$

Therefore

$$\Lambda_1 = \frac{2\tilde{v}_T}{\frac{8rc_i}{\hat{\nu}_i qR}}\frac{B_\phi}{B}$$

where $\tilde{v}_T = \frac{1}{eB_\phi}\frac{T}{L_T}$ with $L_T = \frac{T}{\frac{\partial ln\, T}{\partial r}}$. Therefore we get

$$\Lambda_1 = \frac{2\frac{1}{eB_\phi}\frac{T}{L_T}}{\frac{8rc_i}{\hat{\nu}_i qR}}\frac{B_\phi}{B} = \frac{\nu_i}{\Omega_i}\frac{q^2R^2}{L_T 4r}$$

At the inflection point we have ($L_T = L_{T,inf}$)

$$\Lambda_{1,inf} = \frac{2\frac{1}{eB_\phi}\frac{T_{inf}}{L_{T,inf}}}{\frac{8r_{inf}c_{inf}}{\frac{qR}{c_{inf}\tau_{ii,inf}}qR}}\frac{B_\phi}{B}$$

or

$$\Lambda_{1,inf} = 2\frac{1}{eB_\phi}\frac{T_{inf}^2 R^2}{L_{T,inf}8r_{inf}c_{inf}^2\tau_{ii,inf}}$$

with $\Omega_i = \frac{eB}{m_i}$ we get

$$\Lambda_{1,inf} = \frac{\nu_{ii,inf}}{\Omega_i}\frac{q^2R^2}{L_{T,inf}4r_{inf}}$$

The quantity $\Lambda'_{1,inf}$ used in appendix G is defined by

$$\Lambda'_{1,inf} = \frac{\nu_{ii,inf}}{\Omega_i}\frac{q^2R^2}{L_\psi 4r_{inf}} = \frac{L_{T,inf}}{L_\psi}\Lambda_{1,inf} = -\Lambda_{1,inf}$$

for $L_{T,inf} = -L_\psi$ as in the case of the preceding temperature profiles.

The radial dependence due to the plasma parameters is given by

$$\Lambda_1 = \Lambda_{1,inf}\hat{n}\hat{T}^{-\frac{3}{2}}Z_{eff}\frac{r_{inf}}{r}L_{T,inf}\frac{dln(T)}{dr}$$

or

$$\Lambda_1 = \Lambda'_1\frac{L_\psi}{L_T}$$

Λ'_1 is given by

$$\Lambda'_1 = \Lambda_1\frac{L_T}{L_\psi} = \frac{\nu_{ii}}{\Omega_i}\frac{q^2R^2}{L_\psi 4r}$$

we assume $\hat{n} = \hat{T}^{\frac{1}{\eta}}$ and $\hat{Z}_{eff} = \hat{T}^Z$ where η can be written as

$$\eta = \frac{\frac{dln(\hat{T})}{dx}}{\frac{dln(\hat{n})}{dx}}$$

We get (appendix B)

$$\Lambda_1 = \Lambda_{1,inf}\frac{1}{\hat{T}^{\frac{3}{2}-\frac{1}{\eta}-Z}}\frac{r_{inf}}{r}L_{T,inf}\frac{dln(T)}{dr}$$

or

$$\Lambda_1 = \Lambda_{1,inf}\frac{1}{\hat{T}^{\frac{5}{2}-\frac{1}{\eta}-Z}}\frac{r_{inf}}{r}\frac{d\hat{T}}{dx}$$

If we assume that the exponent of $\hat{T}$, $\frac{5}{2}-\frac{1}{\eta}-Z$, is equal to zero, we get

$$\Lambda_1 = \Lambda_{1,inf}\frac{r_{inf}}{r}L_{T,inf}\frac{dln(T)}{dr}$$

or

$$\Lambda_1 = \Lambda_{1,inf} \frac{r_{inf}}{r} \frac{L_{T,inf}}{L_T}$$

It is sometimes convenient to introduce the quantity Λ_r. We use the expression

$$\Lambda_1 = \frac{2\tilde{v}_T}{S} \frac{B_\phi}{B}$$

and replace S

$$\Lambda_1 = 2\tilde{v}_T \frac{\hat{\nu}_i q R}{8 r c_i} \frac{B_\phi}{B}$$

Then we replace $\tilde{v}_T$ and get

$$\Lambda_1 = 2 \frac{T}{e B_\phi L_T} \frac{\hat{\nu}_i q R}{8 r c_i} \frac{B_\phi}{B}$$

We use $q = \frac{r B_\phi}{R B_\theta}$, $T = m_i c_i^2$, $eB = m_i \Omega_i$ and obtain

$$\Lambda_1 = 2 \frac{m_i c_i^2}{m_i \Omega_i L_T} \frac{\hat{\nu}_i r R}{R B_\theta 8 r c_i} \frac{1}{B}$$

With $a_i = \frac{c_i}{\Omega_i}$ we get

$$\Lambda_1 = \frac{a_i}{L_T} \frac{\hat{\nu}_i}{B_\theta 4} B_\phi$$

We define $\Lambda_r = 4\Lambda_1$ and obtain

$$\Lambda_r = \frac{a_i}{L_T} \frac{\hat{\nu}_i}{B_\theta} B_\phi$$

With a_{p_i}, the Larmorradius in the poloidal field we get the simple expression

$$\Lambda_r = \frac{a_{p_i}}{L_T} \hat{\nu}_i$$

D Reciprocal time η'

The quantity η', is defined as

$$\eta' = \frac{1}{m_i n_{inf} L_\psi^2}\eta_2 = \frac{\frac{6}{5}N_{inf}kT\tau_{ii}}{\Omega_i^2\tau_{ii}^2 m_i n_i L_\psi^2} \tag{100}$$

With the expression for the thermal speed c_i we get

$$\eta' = \frac{\frac{6}{5}c_i^2}{\Omega_i^2\tau_{ii}L_\psi^2} \tag{101}$$

Replacing τ_{ii} by means of the collisionality yields

$$\eta' = \frac{\frac{6}{5}c_i^2}{\Omega_i^2 L_\psi^2}\frac{\hat{\nu}_i c_i}{qR} \tag{102}$$

With the expressions for the collisionality and the thermal speed we get

$$\eta' = \frac{\frac{6}{5}c_i^2}{\Omega_i^2 L_\psi^2}\frac{7.14\ \ 10^2\frac{qRN_iZ_{eff_i}}{T_i^2}c_i}{qR} \tag{103}$$

Introducing the Larmorradius $a_i = \frac{c_i}{\Omega_i}$ gives

$$\eta' = \frac{\frac{6}{5}a_i^2}{L_\psi^2}\frac{7.14\ \ 10^2\frac{qRN_iZ_{eff_i}}{T_i^2}0.99\ 10^4\sqrt{\frac{T_i[eV]}{A}}}{qR} \tag{104}$$

or

$$\eta' = \frac{\frac{6}{5}a_i^2}{L_\psi^2}714\frac{N_iZ_{eff_i}}{T_i^{\frac{3}{2}}}0.99\ 10^4\sqrt{\frac{1}{A}} \tag{105}$$

and finally

$$\eta' = 8.57\ \ 10^{-7}\frac{a_i^2}{L_\psi^2}\frac{n_iZ_{eff_i}}{T_i^{\frac{3}{2}}}\sqrt{\frac{1}{A}} \tag{106}$$

Replacing τ_{ii} by means of Λ_1 we get

$$\eta' = \frac{\frac{6}{5}c_i^2}{\Omega_i^2 L_\psi^2}|\Lambda_1|\frac{L_\psi 4r}{q^2R^2}\Omega_i \tag{107}$$

or, simplifying

$$\eta' = \frac{\frac{6}{5}a_i^2}{L_\psi^2}|\Lambda_1|\frac{L_\psi 4r}{q^2R^2}\Omega_i \tag{108}$$

a_i is the Larmorradius. Outside (inside) the inflection point we have

$$\eta' = \eta'_{inf}\hat{T}^{-\frac{1}{2}}\hat{n} \tag{109}$$

with

$$\eta'_{inf} = \frac{\frac{6}{5}a_i^2}{L_\psi^2}|\Lambda_{1,inf}|\frac{L_\psi 4r_{inf}}{q^2R^2}\Omega_i \tag{110}$$

E Velocity S

We use two expressions giving the exact result, the first resorts to $\chi_{||}$ and the second to Λ_1. The first is

$$\frac{S}{\tilde{v}_T} = \frac{2r\chi_{||}}{4nq^2R^2\tilde{v}_T} \tag{111}$$

We use

$$\chi_{||} = 4c^2n\tau_{ii} = \frac{4T_in\tau_{ii}}{m_i}$$

and get

$$\frac{S}{\tilde{v}_T} = \frac{2r\chi_{||}}{nq^2R^2\tilde{v}_T} = \frac{2r4T_i}{m_i\nu_{ii}q^2R^2\tilde{v}_T} \tag{112}$$

or with $\tilde{v}_T = \frac{T}{eB_\phi}\frac{\partial ln\,T}{\partial r}$

$$\frac{S}{\tilde{v}_T} = \frac{2r4T_in}{m_i\nu_{ii}nq^2R^2\tilde{v}_T} = \frac{\Omega_iL_T8r}{\nu_{ii}q^2R^2} = \frac{2}{\Lambda_1} \tag{113}$$

At the inflection point we have

$$\frac{S_{inf}}{\tilde{v}_{T,inf}} = \frac{2r_{inf}4T_{inf}n}{m_i\nu_{ii,inf}nq^2R^2\tilde{v}_{T,s}} = \frac{\Omega_iL_{T,inf}8r_{inf}}{\nu_{ii,inf}q^2R^2} = \frac{2}{\Lambda_{1,inf}} \tag{114}$$

or with $v_T = \frac{1}{eB_\phi}\frac{T_{inf}}{L_\psi}$

$$\frac{S_{inf}}{v_T} = \frac{2r_{inf}4T_{inf}n}{m_i\nu_{ii,inf}nq^2R^2v_T} = \frac{\Omega_iL_\psi 8r_{inf}}{\nu_{ii,inf}q^2R^2} = \frac{2}{\Lambda_{1,inf}} \tag{115}$$

Alternatively the formula

$$\frac{S_{inf}}{v_T} = \frac{2}{\Lambda'_{1,inf}} \tag{116}$$

can be employed which is analytically identical with the afore-mentioned formula. The scaling with the plasmaparameters is given by

$$\frac{S}{v_T} = \frac{S}{\tilde{v}_T}\frac{\partial \hat{T}}{\partial x} = \frac{S}{\tilde{v}_T}L_\psi\frac{\partial \hat{T}}{\partial r} = \frac{S}{\tilde{v}_T}L_\psi\frac{\frac{\partial \hat{T}}{\partial r}}{\hat{T}}\hat{T} = \tag{117}$$

$$\frac{S}{\tilde{v}_T}\frac{L_\psi}{L_T}\hat{T} = \frac{S_{inf}}{v_T}\frac{\hat{T}^{\frac{3}{2}}}{\hat{n}}\frac{L_\psi}{L_T}\hat{T} \tag{118}$$

$$\frac{S}{v_T} = \frac{S_{inf}}{v_T}\frac{\hat{T}^{\frac{3}{2}}}{\hat{n}} \tag{119}$$

$$\frac{S}{4v_T} = \frac{1}{2\Lambda_{1,inf}}\frac{\hat{T}^{\frac{3}{2}}}{\hat{n}} \tag{120}$$

We also have

$$\frac{S}{\tilde{v}_T} = \frac{2}{\Lambda_1}\frac{B_\phi}{B} = \frac{8}{\Lambda_r}\frac{B_\phi}{B} \tag{121}$$

and

$$\frac{S}{v_T} = \frac{2}{\Lambda_1}\frac{B_\phi}{B}\frac{\tilde{v}_T}{v_T} = \frac{2}{\Lambda_1}\frac{B_\phi}{B}\frac{\text{Ł}_\psi}{L_T} = \frac{2}{\Lambda_1'}\frac{B_\phi}{B} \tag{122}$$

F Velocity Q

Q is defined in [22] and is given by

$$Q = (4u_\theta - 5v_n - 2.5\tilde{v}_T)\frac{B_\phi}{B}$$

$$Q = (4u_\theta - 5v_n(1 + \frac{1}{2\eta}))\frac{B_\phi}{B}$$

with the neoclassical value $u_\theta = \kappa\frac{\partial T}{\partial x}v_T$

$$Q = (4\kappa\frac{\partial T}{\partial x}v_T - 5v_n(1 + \frac{1}{2\eta}))\frac{B_\phi}{B}$$

We get by normalizing

$$\frac{Q}{4v_T} = h - \hat{T}\frac{\partial ln(n^2T)}{\partial x}\frac{2.5}{4}$$

and

$$\frac{Q}{\tilde{v}_T} = (4h' - \frac{5}{\eta} - 2.5)\frac{B_\phi}{B}$$

h' is defined by

$$h' = \frac{u_\theta}{\tilde{v}_T}$$

Using the neoclassical expression

$$u_\theta = \frac{\kappa}{eB}\frac{\partial T}{\partial r}$$

and

$$h' = \frac{\frac{\kappa}{eB}\frac{\partial T}{\partial r}}{\tilde{v}_T} = \kappa$$

we get

$$\frac{Q}{\tilde{v}_T} = (4\kappa - \frac{5}{\eta} - 2.5)\frac{B_\phi}{B}$$

It follows the derivative $\frac{\partial Q}{\partial h} = 4v_T$. We get the ratio

$$\frac{Q}{S} = \frac{h - \hat{T}\frac{\partial ln(n^2T)}{\partial x}\frac{2.5}{4}}{\frac{1}{2\Lambda_1'}\frac{B_\phi}{B}}$$

$$\frac{Q}{S} = \frac{h - \hat{T}\frac{\partial ln(n^2T)}{\partial x}\frac{2.5}{4}}{\frac{S_{inf}}{4v_T}\frac{\hat{T}^{\frac{3}{2}}}{\hat{n}}\frac{B_\phi}{B}}$$

We also get by using Q and S normalized to $\tilde{v}_T$

$$\frac{Q}{S} = \frac{(4h' - \frac{5}{\eta} - 2.5)\frac{B_\phi}{B}}{\frac{8}{\Lambda_r}\frac{B_\phi}{B}}$$

or with $\frac{2.5}{8} = 0.3125$

$$\frac{Q}{S} = \frac{(0.5h' - 0.3125(1 + \frac{2}{\eta}))\frac{B_\phi}{B}}{\frac{1}{\Lambda_r}\frac{B_\phi}{B}}$$

$$\frac{Q}{S} = (0.5h' - 0.3125(1 + \frac{2}{\eta}))\Lambda_r$$

$$\frac{Q}{S} = 0.5(h' - 0.625(1 + \frac{2}{\eta}))\Lambda_r$$

G Evaluation of T_{36}

Two exact expressions are given, the first involves the collision-frequency and the second Λ_1.

G.1 1st expression

In the first case we have

$$T_{36} = 0.36\frac{\eta_{1s}}{\eta_{0s}(1+\frac{Q^2}{S^2})}q^2\frac{1}{\hat{T}v_T}\frac{R^2}{L_\psi^2}(\frac{\partial ln\ T}{\partial x})$$

or

$$T_{36} = 0.36\frac{6}{5\ 0.96}\frac{\nu_{ii,s}^2}{\Omega^2}\frac{1}{(1+\frac{Q^2}{S^2})}q^2\frac{1}{\hat{T}v_T}\frac{R^2}{L_\psi^2}(\frac{\partial ln\ T}{\partial x})$$

G.2 2nd expression

We introduce

$$\Lambda_1' = \frac{\nu_i}{\Omega_i}\frac{q^2R^2}{L_\psi 4r_{inf}}$$

(Λ_1' does not depent on L_T) into T_{36} and get

$$T_{36} = 0.36\frac{6}{5\ 0.96}{\Lambda_1'}^2\frac{L_\psi^2 16r^2}{q^4R^4}\frac{1}{(1+\frac{Q^2}{S^2})}q^2\frac{1}{\hat{T}v_T}\frac{R^2}{L_\psi^2}(\frac{\partial ln\ T}{\partial x})$$

or

$$T_{36} = 0.36\frac{6}{5\ 0.96}{\Lambda_1'}^2\frac{16r^2}{q^2R^2}\frac{1}{(1+\frac{Q^2}{S^2})}\frac{1}{\hat{T}v_T}(\frac{\partial ln\ T}{\partial x})$$

The dependence on the plasma parameters is given by

$$T_{36} = 0.45q^2\frac{R^2}{L_\psi^2}\frac{\nu_{ii,inf}^2}{\Omega^2}(\hat{T}^{-\frac{5}{2}}\hat{n}\hat{T}^Z)^2\frac{1}{(1+\frac{Q^2}{S^2})}\frac{1}{v_T}(\frac{\partial\hat{T}}{\partial x})$$

H Electric drift velocity

The electric drift speed is given by

$$-\frac{E_r}{B_\phi} = v_E = v_\Theta - \frac{B_\theta}{B} v_{||,i} - v_N - \tilde{v}_T$$

We use

$$g = \frac{v_{||,i}}{v_T}$$

$$h = \frac{v_\Theta}{v_T}$$

and get the normalized drift speed

$$\hat{v}_E = \frac{v_E}{v_T} = \frac{v_\Theta - \frac{B_\theta}{B} v_{||,i} - v_N - \tilde{v}_T}{v_T}$$

$$\hat{v}_E = h - \frac{B_\theta}{B} g - \frac{v_N}{v_T} - \frac{\tilde{v}_T}{v_T}$$

The normalized speeds are given by

$$\frac{\tilde{v}_T}{v_T} = \frac{\frac{T}{eB}\frac{\partial ln\ T}{\partial r}}{\frac{T_{inf}}{eBL_\psi}} = \hat{T}\frac{\partial ln\ T}{\partial r} L_\psi = \hat{T}\frac{\partial ln\ T}{\partial x} = \frac{\partial T}{\partial x}$$

$$\frac{v_N}{v_T} = \frac{\frac{T}{eBL_\psi}\frac{\partial ln\ n}{\partial r}}{\frac{T_{inf}}{eB}} = \hat{T}\frac{\partial ln\ n}{\partial r} L_\psi = \hat{T}\frac{\partial ln\ n}{\partial x} = \frac{\hat{T}}{\hat{n}}\frac{\partial n}{\partial x}$$

I Temperature profile in Alcator C - MOD

$$\frac{T_i(r)}{T_i(0)} = \frac{T_i(0) - 2T(r_{inf})}{T_i(0)}(1 - [\frac{r}{r_{inf}}]^2)^2 +$$
$$\frac{T_i(r_{inf})}{T_i(0)}[1 - tanh\frac{r - r_{inf}}{\Delta_T}] \tag{123}$$

or

$$T_i(r) = (T_i(0) - 2T(r_{inf}))(1 - [\frac{r}{r_{inf}}]^2)^2 +$$
$$T_i(r_{inf})[1 - tanh\frac{r - r_{inf}}{\Delta_T}] \tag{124}$$

The decaylength at the inversion point is Δ_T again because

$$\frac{dT_i(r)}{dr} = -(T_i(0) - 2T(r_{inf}))2(1 - [\frac{r}{r_{inf}}]^2)2\frac{r}{r_{inf}^2}$$
$$\frac{T_i(r_{inf})}{\Delta_T}\frac{1}{-cosh^2\frac{r-r_{inf}}{\Delta_T}} \tag{125}$$

at $r = r_{inf}$ we have

$$\frac{dT_i(r)}{dr} = -\frac{T_i(r_{inf})}{\Delta_T} \tag{126}$$

The derivative of the pedestal is given by

$$\frac{d}{dr}\frac{T_i(r)}{T_i(0)} = \frac{d}{dr}\frac{T_i(r_{inf})}{T_i(0)}[1 - tanh\frac{r - r_{inf}}{\Delta_T}] \tag{127}$$

and

$$\frac{d}{dr}\frac{T_i(r)}{T_i(0)} = \frac{T_i(r_{inf})}{T_i(0)}\frac{1}{\Delta_T}[-\frac{1}{cosh^2[\frac{r-r_{inf}}{\Delta_T}}]] \tag{128}$$

The logarithmic derivative of the pedestal can be written as

$$\frac{1}{T_i(r)}\frac{d}{dr}T_i(r) = T_i(r_{inf})\frac{1}{\Delta_T}[-\frac{1}{cosh^2[\frac{r-r_{inf}}{\Delta_T}]}]$$

$$\frac{1}{T_i(r_{inf})[1 - tanh\frac{r-r_{inf}}{\Delta_T}]} \tag{129}$$

or as

$$\frac{1}{T_i(r)}\frac{d}{dr}T_i(r) = \frac{1}{\Delta_T}[-\frac{1}{cosh^2[\frac{r-r_{inf}}{\Delta_T}]}]$$

$$\frac{1}{[1 - tanh\frac{r-r_{inf}}{\Delta_T}]} \tag{130}$$

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