Stochastic optimal velocity model for pedestrian flow

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Objectives

♦ Modelling of pedestrian trajectories

♦ Continuous stochastic *optimal velocity* microscopic model

♦ Statistical calibration of the model parameters

♦ Fundamental diagram and stop-and-go wave phenomena
Overview

1. Description of the data
2. Stochastic optimal velocity model
3. Calibration of the parameters
4. Simulation results
5. Concluding remarks
Overview

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Pedestrian trajectories

- Trajectories of soldiers in laboratory conditions (in Düsseldorf, 2006)
- Uni-dimensional closed system of length 27 m (lane width 0.8 m)
- Uniform initial configurations
- Several tested density levels (from 11 to 70 pedestrians)
- Data collection from video analysis by using free software PeTrack

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Available Online
$N = 25$ pedestrians
$N = 45$ pedestrians
$N = 62$ pedestrians
Fundamental diagram
Global sample

![Graph of Speed vs Spacing](image)

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Stochastic OV model for pedestrian flow
Description of the data
Slide 10
Main features

♦ Stop-and-go waves for sufficiently high density levels

♦ Piecewise linear fundamental diagram
  → Free state: Speed constant (desired speed)
  → Congested state: Speed correlated to the spacing

♦ Noise in the dynamics
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Notations

\[ \Delta x_n = x_{n+1} - x_n \]

\[ \Delta x_n - \ell \]

\[ \ell \]

Notations \( x_n \) is the position, \( \Delta x_n \) the spacing and \( v_n \) the speed of agent \( n \)
Deterministic optimal velocity (OV) model

- Introduced by Masako Bando et al. for road traffic flow\(^6\)

\[
\begin{align*}
\frac{dx_n(t)}{dt} &= v_n(t) \, dt \\
\frac{dv_n(t)}{dt} &= \frac{1}{b} \left( V(\Delta x_n(t)) - v_n(t) \right) \, dt
\end{align*}
\]  

(1)

with \(V(\cdot)\) the OV function and \(b\) relaxation time

- Uniform solution\(^7\) unstable if \(b > 2/V'(d)\)

- Convergence to non-uniform solutions with stop-and-go for particular non-linear OV functions / Instability hard to control (collision)

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\(^7\) where \(\Delta x_n = d\) and \(v_n = V(d)\) for all \(n\)
Stochastic OV model

♦ Introduction of the relaxation in a stochastic noise

\[
\begin{align*}
\text{d}x_n(t) &= V(\Delta x_n(t)) \, \text{d}t + \varepsilon_n(t) \, \text{d}t \\
\text{d}\varepsilon_n(t) &= -\frac{1}{b} \varepsilon_n(t) \, \text{d}t + a \, \text{d}W_n(t)
\end{align*}
\]

with \( W(t) \) the Wiener process and \( a \) the noise amplitude

♦ First order OV model with inertia at the second order through the noise (Langevin equation)

♦ Uniform solution *always* stable in the deterministic case \( (a = 0) \)
White noise and the Langevin process

White noise

Ornstein-Uhlenbeck process
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Calibration of the parameters

♦ Piecewise linear OV function

\[ V(d) = \min\{v_0, \max\{0, (d - \ell)/T\}\} \]  \hspace{1cm} (3)

with 3 parameters: pedestrian size \( \ell \), time gap \( T \) and desired speed \( v_0 \)

♦ \((s_k, d_k)\) sample of observations of speed\(^8\) and distance spacing

♦ Estimation of the parameters \( p = (\ell, T, v_0) \) by least squares

\[ \tilde{p} = \arg \min_p \sum_k (v_k - V_p(d_k))^2 \]  \hspace{1cm} (4)

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\(^8\) The speed is calculated by differencing the position over \( \delta t = 0.8 \text{ s} \)
Parameters of the OV function

\[ \tilde{v}_0 = 0.92 \text{ m/s} \]
\[ \tilde{T} = 1.04 \text{ s} \]
\[ \tilde{\ell} = 0.34 \text{ m} \]
Histogram of the residuals

\[ \tilde{\sigma}_R = 0.14 \text{ m/s} \]
Estimation of the parameters for the noise

- The stationary variance and autocorrelation of the noise are $a^2 b/2$ and $e^{-\delta t/b}$ (Langevin process)

\[ \tilde{b} = -\delta t / \log(\tilde{c}_t) \quad \text{and} \quad \tilde{a} = \tilde{\sigma}_R \sqrt{2/\tilde{b}} \] (5)

with $\tilde{\sigma}_R$ the empirical std-dev of the residuals and $\tilde{c}_t$ their autocorrelation

- Estimations for all the data: $\tilde{a} \approx 0.09 \text{ms}^{-3/2}$ and $\tilde{b} \approx 4.38 \text{s}$

- Estimations depend on the spacing
Parameters for the noise by class of spacing

\[ \tilde{a} \quad (m/s^{3/2}) \]

\[ \tilde{b} \quad (s) \]

Spacing (m)

\[ 0.5 \quad 1.0 \quad 1.5 \]

\[ 0.07 \quad 0.09 \quad 0.11 \]

\[ 2 \quad 6 \quad 10 \quad 14 \]
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Simulation of the model

♦ Numerical simulation using explicit Euler-Maruyama scheme

\[
\begin{align*}
    x_n(t + dt) &= x_n(t) + dt \cdot V_p(\Delta x_n(t)) + dt \cdot \varepsilon_n(t) \\
    \varepsilon_n(t + dt) &= (1 - dt/\tilde{b}) \varepsilon_n(t) + \sqrt{dt} \tilde{a} \xi_n(t)
\end{align*}
\]

with \((\xi_n(t), n, t)\) ind. normal random variables and time step \(dt = 1e-3\) s

♦ Same settings as in the data (ring of 27 m, from 11 to 70 pedestrians, uniform initial configuration)
$N = 25$ pedestrians

Real data

Simulation

Space (m)

Time (s)

0.0 0.5 1.0 1.5 2.0
0 20 40 60 80 100 120

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Simulation results
$N = 45 \text{ pedestrians}$
$N = 62$ pedestrians

Real data

Simulation

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Simulation results
Mean value, standard deviation and correlation
Global sample

<table>
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<th>5251 obs.</th>
<th>(d)</th>
<th>(v)</th>
<th>(d_1)</th>
<th>(v_1)</th>
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<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
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<tr>
<td>Mean</td>
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<td>1</td>
<td>0.87</td>
<td>0.87</td>
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<tr>
<td>(v)</td>
<td>–</td>
<td>–</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Legends: \(d\) distance spacing, \(v\) speed (calculated over 0.8 s), \((d_1, v_1)\) predecessor spacing and speed
Speed density by class of spacing

Global sample

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Stochastic OV model for pedestrian flow

Simulation results
Spacing and speed autocorrelation functions

- $N = 25$
- $N = 45$
- $N = 62$

**Spacing**

**Speed**

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Conclusion

♦ Minimalist model for description of pedestrian trajectories and stop-and-go wave phenomena

♦ Stochastic inertia / Linear (or piecewise linear) model
  → No deterministic instability of uniform solution
  → No requirement of specific non-linear OV functions
  → No generic collision and backward motion problems

♦ Relaxation time of stochastic approach \( b \approx 5 \text{ s} \) while \( b \approx 0.5 \text{ s} \) in general with classical OV models
  → New relaxation mechanism for the modeling of stop-and-go