We agree with Gal [1] that lattice numbers for the mixing angle are not complete until numbers for the isospin splittings are correctly accounted for. In our paper [2], the calculation only includes QCD effects arising from quark mass differences, so a complete result, a full calculation with QED effects added is also needed.

Before addressing the main issue, we would first like to mention that it is not surprising that our results satisfy the Coleman-Glashow [3] and Dalitz–von Hippel (Eq. (3) of [4]) relations. Our fit function has all the SU(3) symmetry constraints built in, so it automatically obeys every symmetry relation to the appropriate order. In this case, violations of the Coleman-Glashow relation are at $O(\delta m^3)$, while violations of the Dalitz–von Hippel relation are at $O(\delta m^2)$.

We believe that lattice QCD gives reliable numbers for the part of the isospin splitting due to quark mass differences, and that the differences between the values in [2] and the experiment are mainly due to electromagnetic effects. Our reasons to believe that lattice numbers for the purely QCD contribution are accurate are the following: first, the lattice gives good values for the splittings between the multiplets ($N, \Lambda, \Sigma, \Xi$), and it is hard to see how there could be a systematic error that would spoil the isospin splittings without also showing up in splittings between multiplets. Second, at leading order, pure QCD relates the isospin splittings to the splittings between the multiplets through the relations

$$M_n^2 - M_p^2 \approx \frac{m_d - m_u}{m_s - m_u} (M_{\Sigma^0}^2 - M_{\Xi^0}^2)$$

$$M_{\Sigma^+}^2 - M_{\Xi^+}^2 \approx \frac{m_d - m_u}{m_s - m_u} (M_{\Xi^0}^2 - M_p^2)$$

$$M_{\Xi^-}^2 - M_{\Xi^0}^2 \approx \frac{m_d - m_u}{m_s - m_u} (M_{\Xi^+}^2 - M_n^2)$$

(1)

(see Fig. 2 of [5]). Our simulations show that this leading-order formula only has minor corrections from higher-order effects, and the above relations hold reasonably well, with a value $\approx 0.022$ for the quark mass ratio $(m_d - m_u)/(m_s - m_u)$. The main systematic uncertainty in this mass ratio is due to the difficulty of correcting for electromagnetic effects in the pseudoscalar meson sector. In Fig. 1 we show the splitting values. The squares and circles are consistent with each other, but do not reproduce the full result without adding a non-QCD force (namely QED).

The relations (1) do not hold in the real world, which we take this as evidence that the QED corrections to the isospin splittings are substantial. Simulations with QED included [6] show that QED effects on the isospin splittings are comparable with the effects of the $(m_d - m_u)$ difference, and that the combined simulation reproduces the physical numbers very closely. Unfortunately, we do not have lattice results for the QED shift in the $\Sigma^0$ mass, so we cannot estimate the QED effect in the mixing matrix element.

In the paper by Isgur [7] there are electrostatic (Coulomb) contributions to the isospin splittings, but they cancel completely for the combination $[M_{\Sigma^0}^2 - M_{\Xi^0}^2] - (M_n^2 - M_p^2)]$, which, by the Dalitz–von Hippel relation, is
are calculated for a particular Gaussian electromagnetic effects, one of which just shifts all masses mixing. In terms of these coefficients, we find (assuming small mixing angle)

\[ \langle \Sigma^0 | M^2_{\text{EM}} | \Lambda \rangle = \frac{\sqrt{3}}{12} \left( -2B^2_{\text{EM}} - 3B^3_{\text{EM}} + 3B^4_{\text{EM}} \right) e^2 \]  

for the mixing matrix element. (We fit to the squares of the masses—the form of the expansion is, of course, the same if a fit to the masses themselves is made.) It may readily be checked that these electromagnetic contributions satisfy the Coleman-Glashow and Dalitz–von Hippel relations for any value of the \( B_{\text{EM}} \) coefficients.

In the Isgur model, these coefficients are not all independent. The Coulomb terms of the Isgur model follow the pattern

\[ B^3_{\text{EM}} = B^4_{\text{EM}} = -B^1_{\text{EM}}, \quad B^2_{\text{EM}} = 0, \]  

which in turn ensures that there is no Coulomb contribution to the mixing; see Eq. (3). However, these interrelations are model dependent, and we need lattice calculations to see how well they hold.

We are currently working on a combined simulation with QED effects included. Preliminary results suggest that QED effects do account for much of the difference between the current QCD-only results and the experimental values. In the Isgur model, it is expected that although the individual splittings will be changed by QED effects, the QED contribution to the \( \Sigma-\Lambda \) mixing angle will cancel. This is, however, a model-dependent statement, with other choices for the spatial wave function the cancellation would not be complete. Our joint QED and QCD results are not yet at the point where we can comment on how much the mixing angle is changed by QED, but we are grateful to Gal for bringing the issue to our attention and will certainly include a discussion of the question when we have our final results.

**ACKNOWLEDGMENTS**

The numerical configuration generation (using the BQCD lattice QCD program [8]) and data analysis (using the Chroma software library [9]) was carried out on the IBM BlueGene/Q using DIRAC 2 resources (EPCC, Edinburgh, UK), the BlueGene/P and Q at NIC (Jülich, Germany), the SGI ICE 8200 and Cray XC30 at HLRN (The North-German Supercomputer Alliance) and on the NCI National Facility in Canberra, Australia (supported by the Australian Commonwealth Government). This investigation has been supported partly by the EU Grants No. 227431 (Hadron Physics2) and No. 283826 (Hadron Physics3). J.N. was partially supported by EU Grant No. 228398 (HPC-EUROPA2). H.P. was supported by DFG Grant No. SCHI 422/9-1. P.E.L.R. was supported in part by the STFC under Contract No. ST/G00062X/1 and J. M. Z. was supported by the Australian Research Council Grants No. FT100100005 and No. DP140103067. We thank all funding agencies.