A mathematical model is presented that incorporates the mass transport by diffusion in the porous structure of thick substrate type solid oxide fuel cells (SOFCs). On the basis of the mean transport pore model a multidimensional study allows for an optimization of the structural parameters of the substrates with respect to cell performance. Next to the mass transport in the porous substrates the electrochemical kinetics, methane/steam reforming and shift reaction, and energy equations are integrated in the model and boundary as well as operation conditions can be varied. Two-dimensional simulations for both anode as well as cathode substrate type SOFC operating on partially prereformed methane are presented and discussed.

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Modeling of Mass and Heat Transport in Planar Substrate Type SOFCs

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A mathematical model is presented that incorporates the mass transport by diffusion in the porous structure of thick substrate type solid oxide fuel cells (SOFCs). On the basis of the mean transport pore model a multidimensional study allows for an optimization of the structural parameters of the substrates with respect to cell performance. Next to the mass transport in the porous substrates the electrochemical kinetics, methane/steam reforming and shift reaction, and energy equations are integrated in the model and boundary as well as operation conditions can be varied. Two-dimensional simulations for both anode as well as cathode substrate type SOFC operating on partially prereformed methane are presented and discussed.

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Mathematical modeling of solid oxide fuel cells (SOFCs) is a helpful tool in examining various design and operation parameters such as geometry, dimensions, fuels, temperatures, and pressures. With the appropriate models and parameter values influences on gas concentration, temperature, current density, and stress distributions can be determined. Results can be used to optimize designs and select optimal operation conditions. During recent years, a number of mathematical models have been published on mass and heat transport in SOFC stacks.1,5 Most models considered the direction of fuel gas flow only, to determine the concentration and temperature profiles in either a tubular or planar type SOFC. The actual cell (anode/electrolyte/cathode) is treated as one solid component with averaged properties. Until recently, such an approximation was sufficient for the standard planar concept with a relatively thick supporting electrolyte, because only the main balances of a cell or stack were analyzed. However, more recently, the planar concept with a thin electrolyte layer supported on a thick substrate formed by one of the electrodes has gained much more interest.6-9 This development of the so-called substrate concept is aimed at a reduction of the operation temperature of the SOFC to below 800°C. Thus, the anode/electrolyte/cathode assembly can no longer be treated as one solid component. Mass transport in the thick substrate, which can be either the anode or the cathode, must be considered. The diffusion path length for the gases from the gas channel to the three-phase boundary (TPB) can be as long as 1.5 mm which as a consequence can result in diffusion limitations. Furthermore, an anode substrate leads to a spatial separation of the reforming reaction and the electrochemical reactions, the latter being restricted to only a very thin (<20 μm) region at the electrode/electrolyte interface.

Yakabe et al.10,11 performed three-dimensional simulations for a complete anode substrate cell using the parallel pore model to describe mass transport by diffusion in the thick porous anode. In our group a one-dimensional isothermal model was developed previously, based on the mean transport pore model12 (MTPM) to describe mass transport of gases in porous electrodes, and including the reforming reaction and the electrochemistry for an anode substrate concept.13 Energy balances were implemented to consider nonisothermal behavior.14 The major difference between our calculations and the aforementioned simulations of Yakabe et al. is the model used to describe the mass transport in the porous electrodes. The MTPM has an advantage over the parallel pore model, that both the permeability coefficient and the diffusion coefficient are calculated from structural parameters of the porous material, which can be determined directly for real components by permeation and diffusion experiments in a Wicke-Kallenbach cell. In this paper we describe the extension of the existing model to two dimensions. The analysis emphasizes mass transport in porous electrodes, especially in the problematical zone below the interconnect ribs. Furthermore, the cathode substrate concept is considered as well, which has not been done before.

Model

The model was developed for a SOFC based on a planar substrate type concept. The fuel cell is considered to operate on either hydrogen or a (prereformed) methane/steam mixture as fuel and oxygen from air as oxidant. Next to the description of the electrochemical reactions, the methane/steam reforming and shift reactions, the heat-transfer and the mass-transfer processes are included in the model. Distinct from previous models, the fuel and oxidant flow, i.e., multicomponent transport, were developed for a realistic fuel cell comprising porous elements, based on a finite-volume-based computational fluid dynamics (CFD) approach. For this purpose the model calculations were performed using the commercial CFD-package FLUENT.

Model assumptions.—Ideal gas mixtures, incompressible and laminar flow due to small gas velocities, and pressure gradients are assumed. The electrodes have a homogeneous, isotropic structure and no gradients within mechanical properties. The electrolyte is regarded as an infinite thin layer between anode and cathode. In the model limitations, further sources according to the electrochemical conversion arise in the boundary cells of the anode next to the electrolyte. The thickness of these boundary cells depends on the truncation grade of the anode.

Fuel cell geometry.—Figure 1 is a schematic diagram for a planar SOFC concept using either a cathode or an anode substrate. Air diffuses from the air channels into the cathode and reacts electrochemically at the TPB. Gas channels are realized in the interconnect. The width of the gas channels and the interconnect ribs for current passage are equivalent. The dimensions of the components used in the calculations are given in Table I. In the calculations only half of the anode substrate shown in Fig. 1a consists of a thick supporting anode and a thin cathode. The fuel gas that flows through the fuel gas channel, diffuses into the porous anode and, in hydrocarbon fuel, is reformed to a mixture containing the electrochemically active gases CO and H2. At the same time inside the anode the shift reaction takes place. The product gases diffuse deeper into the anode. At the TPB, which is replaced in the model by the last layer of grid nodes in the anode, H2...
and CO are oxidized. The electrolyte is simplified and integrated as a surface. In both cases the interconnect is not truncated.

The coordinate system is fixed for all simulations, where x is the direction of the gas flow in the channels, y is the direction in the cell plane perpendicular to the gas flow channel, and z the direction normal to the cell plane. The origin is fixed at the intersection point of the anode and the middle of the gas channel entrance. All simulations for two-dimensional cases are performed at the entrance of the gas/air channel, i.e., at x = 0.

**Mass transport.—**The MTPM\(^{12}\) is used to describe the mass transport of the gases in the porous substrate. This model is based on the assumption that the structure of the porous medium is isotropic. The porous structure can be described by three structural parameters: (i) \( \Psi \): the ratio of porosity (\( e \)) to tortuosity (\( \tau \)); (ii) \( \bar{r} \): the mean value of pore radii; and (iii) \( \bar{r}^2 \): the mean value of squared pore radii.

Table 1. Physical parameters.

<table>
<thead>
<tr>
<th>Fuel cell geometry</th>
<th>Anode substrate</th>
<th>Cathode substrate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anode thickness, ( d_A ) [m]</td>
<td>( 1.5 \times 10^{-3} )</td>
<td>( 60 \times 10^{-6} )</td>
</tr>
<tr>
<td>Cathode thickness, ( d_K ) [m]</td>
<td>( 40 \times 10^{-6} )</td>
<td>( 0.6 \times 10^{-3} )</td>
</tr>
<tr>
<td>Electrolyte thickness, ( d_E ) [m]</td>
<td>( 0.17 \times 10^{-6} )</td>
<td>( 0.17 \times 10^{-6} )</td>
</tr>
<tr>
<td>Fuel channel width, [m]</td>
<td>( 1.5 \times 10^{-3} )</td>
<td>( 1.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>Oxidant channel width, [m]</td>
<td>( 1.5 \times 10^{-3} )</td>
<td>( 1.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>Rib width fuel side, [m]</td>
<td>( 1.5 \times 10^{-3} )</td>
<td>( 1.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>Rib width oxidant side, [m]</td>
<td>( 1.5 \times 10^{-3} )</td>
<td>( 1.5 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

**Fuel cell operation**

<table>
<thead>
<tr>
<th>Fuel gas composition, ( y_i ) [mol/mol]</th>
<th>CH(_4)</th>
<th>CO</th>
<th>H(_2)</th>
<th>H(_2)O</th>
<th>CO(_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.171</td>
<td>0.025</td>
<td>0.473</td>
<td>0.283</td>
<td>0.050</td>
</tr>
<tr>
<td>Oxidant gas composition, ( y_i ) [mol/mol]</td>
<td>O(_2)</td>
<td>N(_2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.21</td>
<td>0.79</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel gas inlet temperature, [K]</td>
<td>973</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oxidant gas inlet temperature, [K]</td>
<td>973</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interconnect wall temperature, [K]</td>
<td>973</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat conductivity coefficient, ( \lambda ) [W/mK]</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cell voltage, ( U ) [V]</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where \( y_i \) is the mole fraction of component i, \( p_i \) is the total pressure, \( R \) is the gas constant, and \( T \) the absolute temperature. The porous nature of the medium is represented by the factor \( \Psi \) in the first term.

Within the MTPM the mass transport is described as the sum of a diffusion flux and a permeation flux

\[
N_i = N_i^d + N_i^p
\]

**Diffusion effects** can be either molecular diffusion or Knudsen diffusion, depending on the size of the pores of the porous medium. In the transition region both types exist

\[
N_i^d + \sum_{j=1}^{0} y_j N_i^d - y_i N_i^d = -c_i \frac{dy_i}{dz}
\]

The first term of Eq. 3 describes the Knudsen diffusion. The second term describes the molecular diffusion and is derived from the Stefan-Maxwell equations.

The effective Knudsen and binary diffusion coefficients are calculated according to

\[
D_i^k = \psi(\bar{r}^2)^2 \frac{8RT}{\pi M_i}
\]

and

\[
D_{ij}^m = \psi \cdot \frac{1.013 \times 10^{-2} \cdot T^{1.75} \cdot \frac{1}{M_i} + \frac{1}{M_j}}{p[ \sum U^{1.73} + (\sum U)2.73]^2}
\]

respectively. Explanation of the symbols in the Eq. 4 and 5 and following can be found in the List of Symbols.

The permeation flux density is described by Darcy’s Law

\[
N_i^p = \frac{B_i dp}{RT \frac{dz}{\xi}}
\]

where the permeability coefficient \( B_i \) is determined by the Weber-equation\(^{12}\)

\[
B_i = D_i^k \frac{K_n}{1 + K_n} + D_{ij}^m \frac{m_{rel,i}}{1 + K_n} + \frac{(r^2) \Psi p}{8n_g}
\]

where \( m_{rel,i} \) is the relative molar mass

\[
m_{rel,i} = \sqrt{ \frac{M_i}{\sum \gamma_i M_j} }
\]

The MTPM summarizes all mass-transport phenomena in a concentration-dependent matrix \( G \). The overall mass transport can be calculated by
one-dimensional simulation, comparing exact calculations and approximated 
Figure 2. Methane concentration profile in a thick anode substrate for a 
D simulation model\textsuperscript{13,14} uses the exact MTPM Eq. 9 and 10. In the 
model according to Faraday’s law 
for all components effected by the shift reaction. \textsuperscript{16} The electrochemical reactions have an exo-

thermic character and are producing heat.

\textbf{Heat transport.}—The basic energy equation\textsuperscript{17} for porous media 
describes different phenomena such as heat convection by the fluid and heat conduction in the solid. 

\[
e_{\Psi} \left( \frac{\partial \delta T}{\partial t} + u \frac{\partial \delta T}{\partial x} + v \frac{\partial \delta T}{\partial y} + w \frac{\partial \delta T}{\partial z} \right) = \lambda_{\text{eff}} \left( \frac{\partial^2 \delta T}{\partial x^2} + \frac{\partial^2 \delta T}{\partial y^2} + \frac{\partial^2 \delta T}{\partial z^2} \right) + q + \phi 
\]

where \( q \) is an energy source term reflecting both the exothermic and endothermic reactions and \( \phi \) is a term accounting for energy dissipation.

Because only steady-state simulations are performed, the time-
dependent first term can be set to zero. The effective thermal conductivity coefficient \( \lambda_{\text{eff}} \) consists of a solid and a fluid conduction part 

\[
\lambda_{\text{eff}} = \varepsilon \lambda_s + (1 - \varepsilon) \lambda_f
\]

In a first estimation the Peclet number was calculated inside the 
anode and results showed that heat transport by convection is neg-
ligible in comparison with the predominating thermal conduction, 
where experimentally determined values for the effective thermal conductivity coefficient of the anode material\textsuperscript{18} were used.

\textbf{Parameter values and boundary conditions.}—The parameter values 
used in the following simulations are summarized in Table I. A 
cell voltage of 0.7 V is used as boundary condition. The fuel gas 
composition used represents a partially \( 30\% \) prereformed methane/ 
steam (33/67 vol \%)/vol \%) mixture.

The structural parameters for porous substrates used in the 
MTPM were determined by diffusion and permeation 

experiments.\textsuperscript{19,20} Anode substrates (Ni/yttrium stabilized zirconia 
(YSZ) cerments) manufactured at the Forschungszentrum Jülich were 
characterized previously.\textsuperscript{13,21} Structural parameters for a standard 
anode substrate NZ40 are listed in Table II. Cathode substrates (Sr-
doped LaMnO\textsubscript{3}) were kindly supplied by the University of 
Karlsruhe. The cathode samples were around 400 \( \mu \)m thick and 
were sintered between 1200 and 1300°C. Higher sintering tempera-
tures lead to an increased density of the samples. Figure 3 shows 
that with this increasing density the average pore size decreases, 
Figure 4 shows the values of the factor \( \Psi \) (porosity/tortuosity) in dependence of the density and sintering temperature. T1200 is 
sintered at 1200°C and shows the highest factor \( \Psi \) as well as the 
highest pore radius of this production series. T1275, sintered at 
1275°C, is a probe with an average \( \Psi \) and pore radius. Because of 
the substantial differences depending on the production series, struc-
tural parameters from two different cathode substrate series, as listed 
in Table II, were selected for the simulations.

Because these structural parameters could be determined only for 

thick substrates, the same values were used for the thin anode and 
cathode layers in the cathode substrate and anode substrate, 
respectively.

For each of these substrates the one-dimensional mass transport 
according to the exact MTPM Eq. 9 and 10 was simulated using the 
software tool MATLAB. The reflected diffusion coefficients were 
calculated for the boundary conditions chosen with Eq. 11. The values 
for the reflected diffusion coefficients, listed in Table II, are used in 
the two-dimensional simulation in FLUENT.
one-dimensional simulations have shown similar gradients but for temperature gradient of around 2-3 K/mm can be calculated. Recent center of the channel. With the assumption of a linear distribution, a temperature determined for several cathode substrates.

Figure 3. Mean value of pore radii as function of density and sintering temperature determined for several cathode substrates.

For the cathode substrates the variation in the structural parameters shows that the reflected diffusion coefficient decreases with higher sintering temperatures. The main influence on the reflected diffusion coefficient was the structural parameter \( \Psi \). Sample A65a refers to another production series sintered at 1200°C with a thickness of about 600 \( \mu \)m. The property of T1200 is a higher average pore size radius and a higher average square compared to A65a, but the lower factor \( \Psi \) results in a lower reflected diffusion coefficient.

Results and Discussion

Two-dimensional cases were simulated for both types of anode and cathode substrate concept SOFC operating on prereformed methane. Figure 5 shows the resulting concentration profiles for methane, hydrogen, and air as well as the temperature distribution in both cases. Particularly in Fig. 5a the fast reforming reaction is reflected by the methane distribution in the porous medium of the anode substrate. A large amount of methane already reacts in the center of the gas channel. This depth is even less below the interconnect rib. There is hardly any methane reaching the anode/electrolyte interface. However, the anode/electrolyte interface is sufficiently supplied with hydrogen resulting from the methane/steam reforming reaction, as can be seen in Fig. 5b. Depending on the degree of prereforming, different temperature gradients arise in the anode substrate. The temperature distribution inside the anode (Fig. 5d) reveals a maximum temperature of around 973 K and a minimum temperature of around 971.5 K, which is situated around 0.5 mm in the \( z \) direction from the gas channel at the center of the channel. With the assumption of a linear distribution, a temperature gradient of around 2-3 K/mm can be calculated. Recent one-dimensional simulations have shown similar gradients but for higher boundary temperatures using a slower reforming kinetic. The distribution and steepness of the temperature curve depend not only on the boundary conditions and reaction kinetics, but also on the properties of the electrodes, that is, the effective heat conductivity, as was shown in a previous study. At the interface region anode/fuel channel the gradient can be higher due to truncation.

The boundary conditions are valid for coflow operation with a homogeneous temperature \( (T = 973.15 \text{ K}) \) at the interfaces of both electrodes with the channels and with the interconnect. Because channel and interconnect are cut in the middle, adiabatic conditions exist at the edges of this balance room.

In the cathode substrate concept the temperature profile in the solid is more homogeneous (Fig. 5h), so that only very small temperature gradients arise. More problematic is the oxygen distribution in the cathode substrate. The oxygen molar fraction decreases from 0.21 at the air channel side to 0.07 under the interconnect (Fig. 5g). For this case the better structural parameters of sample A65a have been used in the calculation of the reflected diffusion coefficient. For a higher porosity and a higher sintering temperature the depletion of oxygen results in an oxygen molar fraction close to zero at the cathode/electrolyte interface.

In Fig. 6 the local current density profile at the cathode/electrolyte interface is shown for the simulation case with type A65a cathode substrate. The current density profile shows a maximum above the center of the air channel \( (y = 0) \) and drops to a minimum below the center of the interconnect rib. The diffusion of oxygen in a thin cathode layer for which the same structural parameters are assumed in the anode substrate case is even more hampered below the interconnect ribs, as can be seen from the corresponding local current density profile shown in Fig. 7. Part of the anode/electrolyte interface is electrochemically inactive due to the depletion of oxygen below the interconnect rib. Here the question must be put forward, whether it is allowed to use the structural parameters determined for relatively thick substrates for these thin \( (50 \mu \text{m}) \) electrode

![Figure 4](image4.png)  
**Figure 4.** Value of the factor \( \Psi \) [ratio of porosity \( (\varepsilon) \) to tortuosity \( (r) \)] as function of density and sintering temperature determined for several cathode substrates.
Figure 5. Simulated two-dimensional concentration profiles of (a, e) methane, (b, f) hydrogen, and (c, g) oxygen and (d, h) temperature profiles in (a-d) an anode substrate concept and (e-h) a cathode substrate concept SOFC.
layers as well. Also one could place a question mark at the structural parameters for the cathode substrates determined in this study. There is some difference between the parameters obtained for the cathode substrates and the anode substrate. In particular the values for the porosity-to-tortuosity ratio of porosity and/or conditions. In a next step an extension to the third dimension is planned.

Acknowledgments

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List of symbols

- \( B \): permeability coefficient, \( \text{m}^2/\text{s} \)
- \( c \): molar concentration, \( \text{mol/m}^3 \)
- \( c_p \): molar heat capacity, \( \text{J/mol/K} \)
- \( d_A \): thickness anode, \( \text{m} \)
- \( d_E \): thickness electrolyte, \( \text{m} \)
- \( d_K \): thickness cathode, \( \text{m} \)
- \( d/dx \): differential coefficient
- \( D \): diffusion coefficient, \( \text{m}^2/\text{s} \)
- \( D^k_i \): effective Knudsen diffusion coefficient, \( \text{m}^2/\text{s} \)
- \( D_m^e \): effective molar diffusion coefficient, \( \text{m}^2/\text{s} \)
- \( F \): Faraday constant, \( \text{A s/mol} \)
- \( k \): reaction rate constant, \( \text{kmol CH}_4/\text{mol Ni} \cdot \text{s} \)
- \( K_a \): adsorption equilibrium constant, \( 1000/\text{Pa} \)
- \( m_{rel} \): relative molar weight
- \( M \): molar mass, \( \text{kg/kmol} \)
- \( N \): molar permeation flux density, \( \text{mol/m}^2 \cdot \text{s} \)
- \( N_d \): molar diffusion flux density, \( \text{mol/m}^2 \cdot \text{s} \)
- \( p \): pressure, \( \text{Pa} \)
- \( \bar{r} \): mean value of pore radii, \( \text{m} \)
- \( r^2 \): mean value of squared pore radii, \( \text{m}^2 \)
- \( R \): gas constant, \( 8.314 \text{J/mol/K} \)
- \( R_t \): reaction rate, \( \text{mol/m}^3 \cdot \text{s} \)
- \( r \): time, \( \text{s} \)
- \( T \): temperature, \( \text{K} \)
- \( u \): velocity in \( x, y, z \) direction, \( \text{m/s} \)
- \( dx \): length, \( \text{m} \)
- \( y \): mole fraction, \( \text{mol/mol} \)
- \( z_a \): number of electrons

Greek

- \( \epsilon \): porosity
- \( \Phi \): energy dissipation, \( \text{W/m}^3 \)
- \( \eta \): viscosity, \( \text{kg/m} \cdot \text{s} \)
- \( \lambda \): thermal conductivity, \( \text{W/m/K} \)
- \( \tau \): tortuosity
- \( \nu \): diffusion volume
- \( \omega \): reaction rate constant, \( \text{kmol CH}_4/\text{mol Ni} \cdot \text{s} \)
- \( \Psi \): effective molar diffusion coefficient, \( \text{m}^2/\text{s} \)

Subscripts

- \( \epsilon_l \): electrochemical
- \( f \): fluid phase
- \( i \): species
- \( j \): species II
- \( g \): gas
- \( \text{ref} \): reflected
- \( s \): solid phase

References

15. I. Drescher, Juli-3699, Berichte des Forschungszentrums Jülich, D82 (Diss. RWTH Aachen, 1999).