Correlation of power handling capability and intermodulation distortion in YBa$_2$Cu$_3$O$_{7-\delta}$ thin films

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The nonlinearity of the microwave properties of coplanar thin-film YBa$_2$Cu$_3$O$_{7-\delta}$ resonators are examined by measurements of the degradation of the quality factor $Q_L$ and the increase of the two-tone third-order intermodulation distortion signal with increasing microwave power. A linear correlation between the data of the characteristic microwave powers, which are obtained for the degradation of $Q_L$ and the intermodulation signal, is experimentally observed and explained in terms of a theoretical model based solely on well-known expressions for the nonlinear surface resistance. Due to these experimental observations and the theoretical model, we conclude that the degradation of the resonator $Q$ factor and the generation of intermodulation distortion are determined by the same physical mechanism and that thermal effects can most likely be excluded.

The use of superconducting films in microwave devices allows a dramatic reduction of the device dimension at comparable or higher performance with respect to conventional nonsuperconducting devices. As a consequence, larger power densities are encountered in these devices. Therefore, it is important to investigate the power handling capability and the generation of intermodulation distortion (IMD) signals, which are consequences of the nonlinear microwave properties of the superconductors. Especially, the characterization of the third-order IMD is of significant interest because it can lead to the largest parasitic signals with frequencies close to the fundamental frequency.\(^1\) In this work, the microwave power handling capability and IMD of epitaxial YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) films on CeO$_2$-buffered $r$-cut sapphire and LaAlO$_3$ substrates are examined. A linear correlation between the two characteristic microwave powers $P_e$ and $P_\gamma$ that describe the degradation of the loaded quality factor $Q_L$ and the increase of the IMD signal with increasing input power respectively, is experimentally observed. Finally, this correlation is explained in terms of a simple model.

YBCO films with optimized microwave properties are prepared on LaAlO$_3$ and sapphire substrates by a magnetron sputtering technique.\(^2,3\) For the examination of the nonlinearity of the microwave properties via measurements of the degradation of the $Q$ value and increase of the IMD signals, coplanar resonators were patterned using photolithography and by ion beam etching. They consist of a 100 $\mu$m wide and 45 mm long $\lambda/2$ stripline resonator (film thickness 300 to 400 nm) which is coupled via 20 $\mu$m slits to the central conductor. The gap between resonator strip and ground electrodes is 200 $\mu$m and 41 $\mu$m for LaAlO$_3$ and sapphire, respectively. The resulting resonant frequencies $f_0$ are 0.95 GHz for LaAlO$_3$ and 1.4 GHz for sapphire. Typical values for the loaded quality factor are $Q_L(70\,\text{K})=10\,000–14\,000$ and $Q_L(4.2\,\text{K})=15\,000–20\,000$.

The power handling capability was derived from the dependence of $Q_L$ measured in the frequency domain as function of microwave input power $P_{\text{in}}$ and temperature (Fig. 1). It is defined by the input power $P_e$ at which the $Q_L$ is reduced to a fraction $\epsilon$ of its low-power value $Q_L(P_{\text{in}}=0)$. Since large degradations of the $Q$ factor are accompanied by an increasing degree of distortion of the Lorentz-shaped resonance curve, the determination of $Q_L$ according to $Q_L=f_0/\Delta f_{3\,\text{dB}}$ ($\Delta f_{3\,\text{dB}}$ represents the full width at half maximum of the resonance curve) becomes invalid for high input powers. Therefore, on the one hand, large enough $\epsilon$ values have to be taken for the criterion, on the other hand, a clearly measurable decrease of $Q_L$ demands $\epsilon$ values which are not too close to 1. Therefore, criteria ranging between $\epsilon=0.8$ and 0.9 have been used for the definition of $P_e$.

A typical result of IMD measurements is shown in Fig. 2. The two fundamental signals are positioned in the center

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FIG. 1. Loaded quality factor of a coplanar resonator on LaAlO$_3$ as a function of the microwave input power for different temperatures is shown. Data points for which $Q_L$ is reduced $\epsilon=0.9$ and 0.8 are marked. The determination of $P_e$ is indicated for one temperature and criterion.
of the resonance at frequencies $\omega_1$ and $\omega_2$ with $\delta=(\omega_1 - \omega_2)/2\pi=5$ KHz. Thus, the fundamental and IMD signals lie within a frequency range of 30 KHz. This is smaller than half of the resonator bandwidth for all experiments and guarantees that the reduction of the intermodulation signal due to the characteristic of the resonator is smaller than 1 dB.

The output powers $P_1(\omega_1)$ and $P_2(\omega_2)$ scale linearly with $P_{in}$ of the fundamental signals in the linear regime of $R_s$, i.e., for low input powers. At higher input power the increasing deviation from the linear scaling marks the transition to the nonlinear regime. In contrast to the behavior of the fundamental signals, the IMD signals turn out to be far more sensitive. Even at extremely low input power, an IMD signal can be detected (noise level for the particular measurement in Fig. 2 is about $-97$ dBm). It increases with increasing input power according to the theoretical expectation $P_3 \propto P_{in}^\beta$ with an exponent $\beta$, which ranges between 2.2 and 3.1 for all measurements. Values well below the theoretically expected value of $\beta=3$ are also reported for high quality YBCO films in the literature.

Since the IMD signal is a consequence of the nonlinear behavior of the microwave properties, it is questionable as to whether it can be used for a characterization of the power handling capability, too. Such a characterization could be achieved by (i) the determination of the third-order intercept (TOI) point $P_{TOI}$ that is defined by the intercept between the linear extrapolations of the output power of fundamental and IMD signals or (ii) a characteristic power $P_\gamma$ at which the ratio between fundamental and IMD signals is $\gamma^5$. Both methods are indicated in Fig. 2. Although both methods in principle yield the same temperature dependencies of the parameters $P_{TOI}$ and $P_\gamma$, respectively, as long as the scaling of the fundamental and IMD signal is not changed, it turned out to be more accurate to avoid extrapolations of the experimental data and rather use $P_\gamma$ for the characterization of the IMD. Furthermore, $\gamma$ should be large enough in order to ensure that the relevant data is recorded in the linear regime of $R_s$. Therefore, we choose values of $\gamma=10^{-50}$, $10^{-40}$, i.e., $\gamma$ ranging between $-50$ and $-40$ dB.

Figure 3 shows a plot of the correlation between the characteristic data $P_\varepsilon$ and $P_\gamma$, which are obtained from measurements of the degradation of the Q value and the increase of the IMD signal, respectively. It clearly shows a linear correlation between both sets of experimental data independently of the chosen criteria $\varepsilon$ and $\gamma$. Therefore, there seems to exist a linear relation between the two characteristic powers $P_\varepsilon$ and $P_\gamma$, and, thus, the degradation of the Q value and the increase of the IMD signal are most likely caused by the same physical mechanism. Furthermore, the fact that the linearity is observed independent of the criteria $\varepsilon$ and $\gamma$, and that the values of $P_\gamma$ are much lower ($\sim 10^{-1}$ dBm) than $P_\varepsilon$, seems to rule out thermal effects to be responsible for the onset of nonlinear behavior in these experiments.

In order to analyze this experimental observation, we evaluate the correlation of the power handling capability and the IMD product on the basis of standard theoretical expressions, and demonstrate that the experimental observation can be explained in terms of a simple model.

At linear response, the electric field $E_s$ induced parallel to the surface of a superconductor is proportional to the magnetic field $H_s$ according to $E_s/\gamma=Z_s \gamma$, (Z_s denotes the bulk surface impedance). The component of the electric field that is in phase with the magnetic field is therefore defined by the surface resistance, i.e., $E_s/\gamma=H_s/\gamma$. In contrast, a nonlinear response can be expressed in most cases by a power series of $E_s/\gamma$ in terms of $H_s$:

$$E_s/\gamma(t) = \sum_{i=0}^{\infty} b_{2i+1}[H_s(t)]^{2i+1}.$$  (1)

Equation (1) implies that the electric field is defined by the magnetic field at a given time. Thus, it is valid only for nondynamic systems, i.e., without dispersive or dissipative relaxation. In this case, the coefficients $b_i$ are real constants. In case of a superposition of two harmonic microwave signals at frequencies $\omega_1$ and $\omega_2$ for simplicity, identical amplitude $H_0$ the resulting magnetic field is

$$H(t)=H_0[\cos \omega_1 t + \cos \omega_2 t] = 2H_0 \cos \delta t \cos \omega_0 t.$$  (2)

Here, $\omega_0=(\omega_1 + \omega_2)/2$ and $\delta=(\omega_1 - \omega_2)/2\pi$ define center frequency and frequency difference between center frequency and fundamental signals, respectively. The harmonic components of the electric field can be obtained by inserting Eq. (2) into Eq. (1). In the case of symmetric excitations, a symmetric spectrum of $E(\omega)$ with harmonic signals and intermodulation signals at $\omega = (n \omega_1 + m \omega_2)$ with $n$ and $m$...
positive integers is generated. The dominating nonfundamental signals are the third-order IMD at \( \omega_{3\text{MD}} = 2 \omega_1 - \omega_2 \) and \( \omega_{3\text{MD}} = 2 \omega_2 - \omega_1 \), which are usually also closest to the fundamental frequencies. Their amplitudes are

\[
E_3 = \frac{3}{4} b_3^3 H_0^3 + \frac{25}{8} b_3^2 H_0^5 + \frac{735}{64} b_3^1 H_0^7 + \cdots. \tag{3}
\]

Using the expression \( R_s(H) = E/H \) and neglecting the smaller terms of higher order in Eq. (3), we obtain the field dependence of the nonlinear surface resistance:

\[
R_s \approx R_s(0) (1 + b_3 H_0^3), \tag{4}
\]

\( R_s \) represents the surface resistance obtained for small rf power when \( H(0) \) and the correlation of the magnetic field components \( H_3 \) (at intermodulation frequencies) and \( H_0 \) (fundamental frequency), respectively:

\[
H_3 = E_3 \approx \frac{3}{4} b_3 H_0^3. \tag{5}
\]

By introducing a field-to-power conversion factor \( \eta \) with \( H = \eta \), the corresponding correlation of the microwave power \( P_3 \) at the intermodulation frequencies and \( P_0 \) of the fundamental frequencies is given by

\[
P_3 = \left( \frac{3}{4} b_3 \eta^2 \right)^2 P_0^3. \tag{6}
\]

Finally, the power of the fundamental signals can be related to the input power \( P_{\text{in}} \) according to

\[
P_0 = \alpha P_{\text{in}} \quad \text{with} \quad \alpha = \frac{4 Q_L}{Q_{C_{\text{in}}}}. \tag{7}
\]

where \( Q_L \) is the loaded quality factor, and \( Q_{C_{\text{in}}} \) and \( Q_{C_{\text{out}}} \) are the quality factors of the coupling at input and output, respectively. Using Eqs. (6) and (7), we can evaluate the third-order power intercept \( P_{\text{TOI}} \), which is defined by the power at which fundamental and IMD signals are of identical amplitude, i.e., \( P_3 = P_0 \):

\[
P_{\text{TOI}} = \frac{4}{3 b_3 \eta^2 \alpha}. \tag{8}
\]

In a similar way, the characteristic power \( P_\gamma \) of the IMD experiment can be derived. It is given by the power at which the input power and IMD signal are related according to \( P_3 = \gamma P_0 \) (in our experiments \( \gamma = 10^{-4} \), i.e., -40 dB), thus:

\[
P_\gamma = \frac{4 \sqrt{\gamma}}{3 b_3 \eta^2 \alpha}. \tag{9}
\]

On the other hand, the experimentally determined nonlinearity of the loaded quality factor \( Q_L \) is given by standard expressions and using Eqs. (4) and (7):

\[
\frac{1}{Q_L} = \frac{1}{Q_{L0}} + \frac{R_s(0)}{G} (1 + b_3 \eta^2 \alpha P_{\text{in}}). \tag{10}
\]

\( Q_{L0} \) summarizes all contributions to the quality factor except for the conductor loss in the resonator and \( G \) is the geometry factor of the resonator. The determination of the power handling via degradation of the loaded quality factor is defined by the input power \( P_e \) for which the quality factor is given by \( Q_L = e Q_{L0} (P_{\text{in}} = 0) \) (in our measurement \( e = 0.8 \)). Using Eqs. (9) and (10) this, finally, yields a linear correlation between the two quantities that characterize the power handling capability and the IMD signal, respectively:

\[
P_e = \left( 1 - \frac{e}{\epsilon} \right) \left( 1 + \frac{G}{R_s Q_{L0}} \right) \frac{3}{4} P_\gamma. \tag{11}
\]

Thus, the experimental observed linear correlation \( P_e \propto P_\gamma \) (see Fig. 3) can be explained by this simple model, which is based on standard expressions for the nonlinear microwave properties of superconductors. Moreover, even a modification of the power-law dependence of \( P_3 \) on the input power, which is observed for a number of experiments and is reported in the literature, does not affect this correlation. Using the same expressions, but substituting the exponent in Eq. (5) by using \( H_3 = 0.75 b_3 H_0^3 \), leads to:

\[
P_e \propto \frac{1}{\alpha \eta^2} \quad \text{and} \quad P_\gamma \propto \frac{1}{\alpha \eta^{2\mu-1}}. \tag{12}
\]

Thus, also in this case, a linear correlation \( P_e \propto P_\gamma \) should be expected.

In this work, the nonlinear microwave properties of coplanar thin film resonators of YBCO on CeO\(_2\)-buffered r-cut sapphire and LaAlO\(_3\) substrates are examined. The degradation of the quality factor \( Q_L \) and the increase of the two-tone third-order intermodulation distortion signal with increasing microwave power are characterized. A linear relationship between the characteristic microwave powers \( P_e \) and \( P_\gamma \) for the degradation of \( Q_L \) and the increase of the IMD signal, respectively, is observed independently of the chosen criteria \( \epsilon \) and \( \gamma \). Furthermore, the characteristic power \( P_\gamma \) of the IMD signal is much smaller compared to the onset of the degradation of the \( Q \) value at \( P_e \). These facts indicate that (i) both phenomena are most likely caused by the same physical mechanism and (ii) this mechanism seems not to be dominated by thermal effects. The linear correlation is confirmed by a theoretical model which is based on well-known expressions for the nonlinear surface resistance. The IMD measurement turns out to be far more sensitive than the determination of the degradation of \( Q_L \), which represents the standard method of characterizing the power handling capability of superconducting microwave devices and films. Moreover, considering the small microwave power that causes IMD, thermal effects can be excluded to play a role in the IMD measurements of the power handling capability. This fact represents an advantage of IMD experiments compared to the standard method for the determination of the nonlinear properties of high temperature superconductor thin films.