Consideration of multifaceted asymmetric radiation from the edge (MARFE) as a dissipative structure

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(Received 29 November 2001; accepted 18 February 2002)

Multifaceted asymmetric radiation from the edge (MARFE) is considered as an example of dissipative structures which develop under critical conditions in different physical and technical systems. The model proposed results in a system of algebraic equation including a relation similar to Maxwell's Rule that determines such characteristic parameters as the plasma temperature in MARFE, its extent in poloidal and radial directions. Predictions of this approximate approach are compared with the results of one- and two-dimensional numerical simulations. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1468233]

I. INTRODUCTION

Multifaceted asymmetric radiation from the edge (MARFE) is a region of high density and low temperature being a source of strong radiation from hydrogen neutrals and impurities.[1–3] This region arises at the plasma edge in tokamaks when the electron density exceeds some critical level and looks like a luminous toroidal loop at the high field side in limiter discharges or in the vicinity of the X-point in divertor configuration.

MARFE development results not only in a significant change of local plasma parameters but leads to a noticeable deterioration of the global discharge performance. Thus in plasma states with good energy confinement, e.g., in the H- or radiative improved mode, MARFE often causes a back transition to the low confinement mode,[4,5] which happens, most probably, owing to increasing anomalous transport of energy and particles caused by electromagnetic fluctuations. This is confirmed by the observations of significant enhancement in the level of fluctuations during MARFE formation.[6] The characteristics of anomalous transport, in particular, particle and heat diffusivities are in turn essentially involved in MARFE models and determine such an important parameter as the density threshold, etc.[7–15]

A firm model for the effect of MARFE on global plasma behavior should take into account the described complex nonlinear interrelations and that can not be done in practice without any constraint. On the one hand a reliable numerical realization, i.e., finding of converged solutions for the equations involved, cannot be achieved within a reasonable computation time if the constituents of the model are treated in a too detailed way. On the other hand the treatment should be sophisticated enough to allow a sufficiently accurate determination of the most important characteristics. In the MARFE case these are the temperature and density in the MARFE and surrounding hot plasma, the dimensions of MARFE in the poloidal and radial directions.

Methods, which satisfy these requirements, have been developed in investigations of the so-called dissipative structures, which arise under certain critical conditions in different physical and technical systems having nonlinear properties,[16] e.g., a nonmonotonous temperature dependence of energy sources and sinks.[17] MARFE reveals many characteristic features of dissipative structures. For example, it develops spontaneously when the mean plasma density exceed a critical level; MARFE is sharply separated from surrounding regions with ''normal'' plasma parameters so one can apply the concepts of ''phases,'' ''boundaries,'' etc.

It is well known that characteristics of a medium should satisfy some requirements to allow formation of dissipative structures.[16] In particular, homogeneous stationary states should be unstable in a certain parameter range. This takes place, e.g., if the energy loss from the system, Q_{loss}, increases with decreasing temperature T (see Fig. 1). It is normally believed that for MARFE development the so-called thermal condensation instability is of importance. This instability was proposed initially as a possible mechanism for the formation of stars of relatively small mass.[18,19] If the local plasma temperature and, consequently, pressure reduce spontaneously a mass flow arises and the plasma density n increases at the location in question. This leads to growing radiation losses being usually proportional to the electron density, and if the energy sink exceeds the heat conduction from hotter regions of the plasma the temperature drops further and an instability develops.

For sufficiently low and high temperature the variation of losses with T should change its character and they should increase with increasing T. This results in a N-like temperature dependence of Q_{loss} and for the same level of the energy input there can be three stationary homogeneous states in which the energy source P is balanced by the sink. Two of them, namely, those of the highest and lowest temperature, are stable and the one of the intermediate temperature is unstable. At the plasma edge in fusion devices such a behavior is ensured by the fact that the most significant contribution to the energy losses comes from the radiation of impurities released from the machine walls. On the one hand these particles can be excited and radiate effectively only if the electron temperature exceeds a certain level T_{min}. For typical

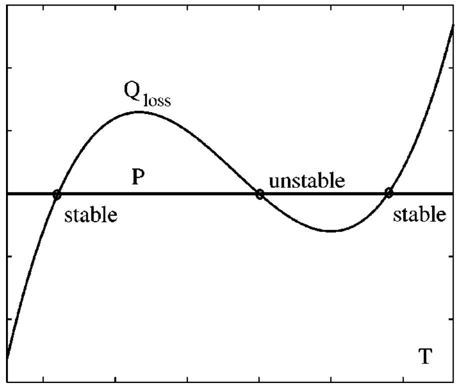

FIG. 1. The temperature dependence of energy losses from a homogeneous medium where dissipative structure can arise.

impurities like carbon and oxygen $T_{\min}$ is of several electronvolts.[20] On the other hand if T exceeds a value $T_{\max}$ close to the ionization potential of Li-like ions, impurities are mostly in ''dim'' He-like state and their radiation losses are strongly reduced. Out of the temperature range in-between $T_{\min}$ and $T_{\max}$ the energy losses are mainly governed by the conduction and convection in the plasma and Q_{loss} grows up with increasing T. Thus if the impurity concentration is high enough there could be a strong maximum in the total energy loss for $T_{\min} \leqslant T \leqslant T_{\max}$.

The N-like temperature variation of Q_{loss} is an important precondition for the coexistence of nearly homogeneous regions where the plasma parameters are roughly given by two stable solutions of the heat balance equation. These regions, which will be called ''phases'' henceforth, are separated by a thin ''border,'' where the parameters vary very sharply. The precondition mentioned above, allows to satisfy an integral relation analogous to Maxwell's Rule of ''equal areas,'' however, only for certain values of control parameters.[16,17] If the control parameters are constant over the system a stationary position of the border is unstable and one of the phases forces another one out. The situation changes, however, if there are spatial inhomogeneities in control parameters which can stabilize the position of the border. In the case of MARFE such inhomogeneities are predetermined by the toroidal magnetic geometry of tokamak which leads to the poloidal variation of the distance between neighboring magnetic surfaces.

Techniques for a qualitative description of dissipative structures provide simple rules which can easy be applied in order to estimate important characteristics of MARFE, e.g., the plasma temperature, its poloidal and radial extent. This will be done in the present paper and the results will be compared with numerical solution of one- and two-dimensional transport equations. These equations and boundary conditions to them, the assumptions done for the densities of the main and impurity particles, the radiative characteristics of impurities will be discussed in the next section. In third section a ''phase'' model of the plasma edge with MARFE will be presented beginning with the derivation of an approximate one-dimensional equation for the plasma temperature at the last closed magnetic surface (LCMS). Algebraic equations for the temperatures in the cold MARFE and hot surrounding ''phase,'' for the poloidal position of the interface between ''phases,'' which is an analog to Maxwell's Rule, and an equation for the radial width of the radiative layer will be also derived there. In Sec. IV the results of the ''phase'' model will be checked by comparison with the numerical integration of one- and two-dimensional heat transport equations.

II. BASIC ASSUMPTIONS AND EQUATIONS

It is well-known that MARFE physics is essentially predetermined by the variation of the energy losses from plasma with electron temperature. In many MARFE models the radiation from impurities released from tokamak walls, e.g., carbon and oxygen is considered as the main energy loss.[7–11] Other approaches presume that hydrogen neutrals, which enter the plasma due to surface and volume recombination of charged particles, are more important.[21,22] It is out of the scope of the present paper to analyze the actual nature of the energy losses. In both cases they vary with the plasma temperature T in the same specific way, i.e., increase significantly when T exceeds a certain minimum level $T_{\min}$ and decay fast when the temperature becomes larger than some maximum value $T_{\max}$.

Henceforth it will be assumed for certainty that namely impurity radiation governs the energy losses. Their power density can be written as $Q_{\text{rad}} = n n_r L_c$, where n and n_r are the densities of electrons and radiating impurity ions and L_c is the so-called ''cooling rate.'' In the literature one can find different approximations for the temperature dependence of the cooling rate. The most simple ''box'' model assumes $L_c = L_c^{\max}$ for $0 \leqslant T \leqslant T_{\max}$ and $L_c = 0$ for $T > T_{\max}$.[9] We approximate L_c by a more adequate temperature dependence, which allows, nevertheless, to progress far enough in an analytical consideration and get physically transparent results, $L_c = L_c^{\max} \times \psi(T)$ with

$$\psi(T) = 4\,\frac{(T - T_{\min})(T_{\max} - T)}{(T_{\max} - T_{\min})^2}. \tag{1}$$

For example $L_c^{\max} \approx 10^{-6}\ \text{eV}^{-1}\,\text{cm}^3\,\text{s}^{-1}$, $T_{\min} \approx 2$–3 eV, and $T_{\max} \approx 30$–40 eV for carbon impurity.[20]

The temperature dependence of the energy losses, Q_{loss}, is determined not only by the cooling rate but also by the densities of exciting and radiating particles. These are governed by continuity and motion equations where only the variation along the magnetic field (direction l) will be taken into account. (Particle transport perpendicular to the magnetic surfaces may play an important role in MARFE physics[12,15,22] but for the main aim of this paper an inclusion of this effect is not of principal importance.) Thus for the background plasma particles one has

$$\frac{\partial n}{\partial t} + \frac{\partial nV}{\partial l} = 0, \quad m_i \frac{\partial nV}{\partial t} + 2\,\frac{\partial nT}{\partial l} = 0,$$

where V and m_i are the parallel velocity and mass of the background ions. Combining these equations we get

$$m_i \frac{\partial^2 n}{\partial t^2} = 2 \frac{\partial^2 nT}{\partial l^2}.$$

According to observations the characteristic time of MARFE formation is larger than the transit time for the ion sound and the plasma pressure has time to come to equilibrium. Thus,

$$n(l) \sim 1/T(l) \tag{2}$$

is maintained by MARFE development.

The motion of impurity ions parallel to the magnetic field is strongly influenced by the forces due to interaction with background plasma particles. For a slow subsonic motion the so called thermal force caused by collisions with the main ions and being proportional to the temperature gradient and the force from electric field are of importance. From continuity and force balance equations one finds for the density of impurity ions,[23]

$$n_r(l) \sim T^{\xi_I}(l) \tag{3}$$

with the exponent ξ_I determined by the masses and charges of the main and impurity ions. In the present study we adopt ξ_I as a given parameter.

Considering the heat transfer in the plasma edge we keep in mind the experimentally observed toroidal symmetry of MARFE. Moreover a linear stability analysis shows[13] that toroidally symmetric perturbations have the largest growth rate among those, which can potentially lead to MARFE development. In this case the length of the line of force is determined by the poloidal angle ϑ, $\partial l = qR\partial\vartheta$, where q is the safety vector and R is the torus major radius. For definiteness $\vartheta = 0$ is assumed at the high field side where MARFE is located. By taking the continuity and momentum equations into account the heat transport at the edge is modeled by the equation,[13]

$$5n\frac{\partial T}{\partial t} - g^{rr}\frac{\partial}{\partial x}\left(\kappa_\perp \frac{\partial T}{\partial x}\right) - \frac{1}{(qR)^2}\frac{\partial}{\partial \vartheta}\left(\kappa_\parallel \frac{\partial T}{\partial \vartheta}\right) = -Q_{\mathrm{rad}}. \tag{4}$$

Here $x = r - a \ll a$ with a and r being the minor radii of the LCMS and the magnetic surface in question; $\kappa_\perp$ and $\kappa_\parallel$ are the perpendicular and parallel components of the plasma heat conductivity. The metric coefficient g^{rr} accounts for the geometry of magnetic surfaces. We will consider only the case of circular surfaces being non-concentric due to a small Shafranov shift Δ and $g^{rr} \approx 1 - 2(d\Delta/dx)\cos\vartheta$.[13] Henceforth the variation of the parameters q, $\kappa_\perp$, and $d\Delta/dx$ will be neglected in the thin edge region under consideration.

Equation (4) implies that even for a temperature homogeneous on magnetic surfaces the heat flux from the plasma core varies with the poloidal angle. Transport models provide often radial fluxes which are not determined by real space gradients, but by gradients in magnetic flux space. Formally that would lead to no poloidal inhomogeneity in the equation above. This inconsistency disappears if we take into account that the magnetic flux representation is a result of averaging over the magnetic surface. Equation (4) includes, however, poloidally local not yet averaged radial fluxes. The latter are ϑ-dependent both in neoclassical theory and in numerous models for anomalous transport in toroidal devices.[14] Here only the part of this dependence due to metric coefficients is taken into account. A more consistent consideration will be done elsewhere.

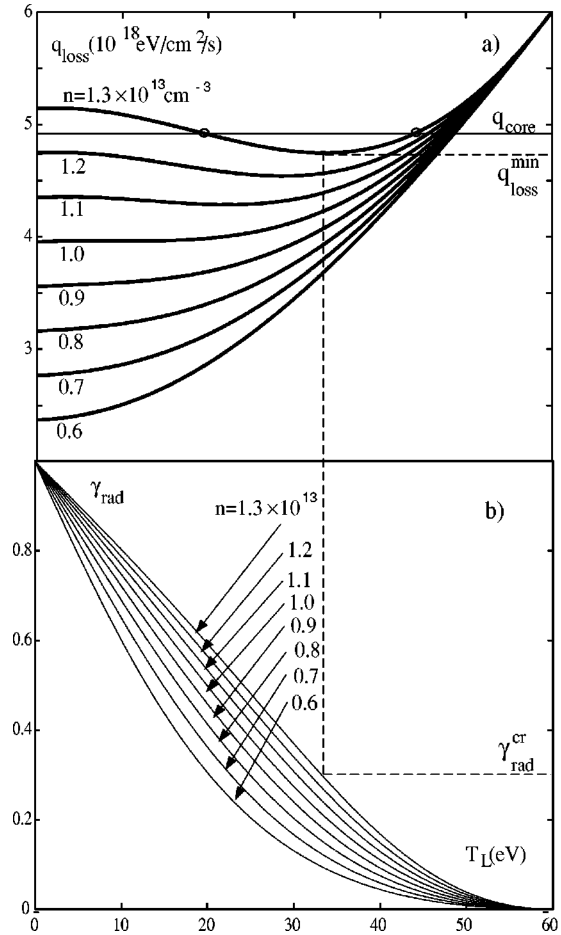

FIG. 2. Characteristics of a poloidally homogeneous radiative layer at the plasma edge: energy losses with conduction and radiation (a) and the radiation level (b) vs the temperature at the LCMS for different edge density.

The boundary condition for Eq. (4) at the LCMS is given by the temperature e-folding length δ_T, which is determined by the transport in the scrape-off layer (SOL), $\partial T/\partial x = T/\delta_T$. At the interface between the edge and core plasma regions where the temperature exceeds $T_{\max}$ at any poloidal position the density of the heat flux to the edge, $q_{\mathrm{core}} = \int_0^\pi \kappa_\perp (\partial T/\partial x)(d\vartheta/\pi)$, is equal to P/S_0 with P being the power launched into the plasma and S_0 the interface area.

III. "PHASE" MODEL FOR MARFE

In this section it will be demonstrated that the plasma edge region described by heat transport equation (4) with the loss term in the form given by Eq. (1) is a physical system where a structure of a cold dense "phase" can develop. For this purpose an approximate one-dimensional equation, which describes the heat transport at the LCMS, will be derived from Eq. (4). Then it will be demonstrated that solutions of this equation describe sharp transitions between regions of relatively smoothly distributed parameters, i.e., "phases." A relation similar to Maxwell's Rule will be derived for determination of the plasma parameters in phases and the position of the border between them.

A. Thermal instability of poloidally symmetric radiative layer

An important feature of a system, in which dissipative structures can develop, is the existence of a range of parameters where homogeneous stationary states are unstable. In the case of a radiative layer at the plasma edge one can presume only a homogeneity in the poloidal direction. In the radial direction this layer is always inhomogeneous because the heat flow from the plasma core is transported through the layer to the SOL by the plasma heat conduction and a radial temperature gradient exists. It was shown in Ref. 24 that for a wide class of the temperature dependencies of the cooling rate L_c a poloidally homogeneous radiative layer can be unstable if the temperature at the LCMS, T_L, is smaller than a critical level. For unstable states the losses through the LCMS with the plasma conduction and radiation computed as a function of T_L increase when T_L drops.

For stationary poloidally homogeneous states the heat transfer equation reduces to

$$\frac{d^2T}{dx^2}=\psi(T)\times\frac{Q_{\max}}{\kappa_\perp},$$

with $Q_{\max}=nn_rL_c^{\max}$. For a moment we assume that T_L is given as a boundary condition instead of the heat influx into the edge region from the plasma core. The equation above is multiplied by $2(dT/dx)$ and integrated over the edge radiative layer, i.e., between $x=0$ where $T=T_L$ and $x=x_{\rm rad}$, where $T=T_{\max}$. By taking into account the given e-folding length for the temperature at the LCMS we compute the heat influx from the core, $\kappa_\perp\,(dT/dx)\,(x_{\rm rad})$. In stationary states this is equal to the surface density of the energy loss from the plasma edge with radiation and conduction, $q_{\rm loss}$, and thus

$$q_{\rm loss}=\sqrt{\left(\frac{\kappa_\perp}{\delta_T}T_L\right)^2+\kappa_\perp Q_{\max}\Delta_T(T_{\max})},$$

where

$$\Delta_T(T)=\frac{4(T_{\max}+T_{\min})(T^2-T_*^2)-8(T^3-T_*^3)/3-8T_{\min}T_{\max}(T-T_*)}{(T_{\max}-T_{\min})^2}$$

and $T_*=\max(T_L,T_{\min})$. One can see that the first term under the square root in $q_{\rm loss}$, which stems from the conductive losses, increases with T_L. Conversely, the second one, which arises owing to radiation, becomes larger when T_L drops. The latter occurs because the radiative layer widens with decreasing T_L. Therefore there could be a minimum in $q_{\rm loss}$ as a function of T_L. This is demonstrated in Fig. 2(a) by $q_{\rm loss}(T_L)$ computed for $\kappa_\perp=10^{18}\ \mathrm{cm}^{-1}\,\mathrm{s}^{-1}$, $\delta_T=2$ cm, 1% of carbon and several values of the plasma density. These parameters are typical for the plasma edge in TEXTOR (Ref. 25) and other limiter tokamaks[6] under conditions when MARFE formation has been observed. With increasing density the contribution from radiation losses becomes larger and a minimum appears for $n\gtrsim10^{13}\ \mathrm{cm}^{-3}$. It is straightforward to find an analytical expression for this critical density using the condition that $dq_{\rm loss}/dT_L$ and $d^2q_{\rm loss}/dT_L^2$ reduce to zero simultaneously.

The instability of states with $dq_{\rm loss}/dT_L<0$ can be easy understood. If T_L is spontaneously reduced the conductive loss through LCMS drops. However for the states in question this drop is smaller than the rise of the radiated power. The latter occurs because the radiation layer widens with diminishing T_L. Thus the total energy loss from the radiative layer grows and the initial perturbation of the temperature develops further. As it has been demonstrated in Ref. 24 such an ‘‘energetic’’ approach predicts instability of stationary states in agreement with a conventional analysis based on a solution of the eigenvalue problem for small temperature perturbations.

The critical value of the radiation level $\gamma_{\rm rad}=[q_{\rm loss}-(\kappa_\perp/\delta_T)\,T_L]/q_{\rm loss}$ at which the radiative layer becomes unstable, $\gamma_{\rm rad}^{\rm cr}$, is an important parameter for a check of theoretical models for MARFE. For the parameters used above Fig. 2(b) shows $\gamma_{\rm rad}$ vs T_L and one can see that $\gamma_{\rm rad}^{\rm cr}$ can be noticeably less than 1. This is in agreement with observations in ohmic plasmas in Tokamak Experiment for Technology Oriented Research (TEXTOR) (Ref. 25) and Divertor Injection Tokamak Experiment (DITE),[26] where a radial detachment occurred instead of MARFE when the plasma density was pushed to the critical level. This contradicts however to the conclusion drawn in Refs. 9, 27 that radial detachment is impossible for a radiation level <1. This contradiction stems from the boundary condition of zero T_L assumed in Refs. 9, 27, which implies $\delta_T=0$. Physically this situation corresponds to an instant loss of the energy at the LCMS which stabilizes the radiative layer at any radiation level. In reality, however, δ_T is finite and the radiative layer becomes unstable at a $\gamma_{\rm rad}^{\rm cr}<1$.[24]

In a real plasma T_L is not in our hands but is determined by the balance between the power lost from the radiative layer, $q_{\rm loss}$, and that supplied by the plasma heat conduction from the plasma core, $q_{\rm core}$. As one can see from Fig. 1(a) stationary states exist only for $q_{\rm core}>q_{\rm loss}^{\min}$. The level of $q_{\rm loss}^{\min}$ increases when the plasma density is ramped up since the radiation losses grow with n. When $q_{\rm loss}^{\min}$ becomes larger than $q_{\rm core}$ a stationary radiative layer cannot exist: T_L drops and the layer widens; when T_L becomes lower $T_{\min}$ the layer detaches from the LCMS. The dynamic process of such a radial ‘‘detachment’’ and formation of ‘‘detached’’ plasma has been modeled numerically in Ref. 28.

B. "Phases" in the plasma edge with poloidally inhomogeneous heat flux from the core

1. Equation for poloidal temperature profile at LCMS

The picture given above presumes that the density of the heat flux from the plasma core is poloidally homogeneous. However, there is always an inhomogeneity due to the specifics of magnetic geometry in toroidal fusion systems. Even in a limiter configuration without any X-point there is a Shafranov shift of magnetic surfaces. Therefore the distance between two neighboring surfaces varies with the poloidal angle and is larger at the high field side than at the low field side. Formally, this is characterized by the poloidal dependence of the metric coefficient g^{rr} with a minimum at the inner edge, $\vartheta=0$.

If $g^{rr}(\vartheta=0)<q_{\rm loss}^{\rm min}/q_{\rm core}<g^{rr}(\vartheta=\pi)$, a part of the radiative layer at the high field side (HFS) should undergo a detachment while that at the low field side (LHS) remains presumably in the attached state. However, different poloidal positions at the edge are connected by spiral magnetic field lines. Thus when a detachment begins at the HFS the parallel electron heat conduction supplies additional energy to this part of the edge due to increasing difference in temperatures at the outer and inner board. As a result the process of detachment can be terminated and a new equilibrium arises, in which a zone of cold intensively radiative plasma, MARFE, is separated by a thin border from the "attached" region of a relatively high temperature. This situation is similar to coexistence of different phases and a constraint like Maxwell's Rule can be of interest for analysis.[16,17]

The plasma temperature is assumed to be roughly homogeneous inside MARFE and "attached" region and close to its values at the symmetry planes of these phases, $T_m=T(\vartheta=0)$ and $T_a=T(\vartheta=\pi)$, respectively. With given values of the averaged over the LCMS densities of electrons and radiative particles, $\overline{n}$ and $\overline{n_r}$, Eqs. (2) and (3) allow us to connect their poloidal variation with the temperature profile,

$$n(\vartheta)\approx\overline{n}\frac{T_*}{T_L(\vartheta)},\quad n_r(\vartheta)\approx\overline{n_r}\left[\frac{T_L(\vartheta)}{T_{*I}}\right]^{\xi_I}, \tag{5}$$

where $T_*=[T_a^{-1}(1-\vartheta_m/\pi)+T_m^{-1}\vartheta_m/\pi]^{-1}$ and $T_{*I}=[T_a^{\xi_I}(1-\vartheta_m/\pi)+T_m^{\xi_I}(\vartheta_m/\pi)]^{1/\xi_I}$, and ϑ_m is the position of the interface between the MARFE and attached "phases."

In order to apply concepts of a qualitative theory for dissipative structures[16,17] we derive from the two-dimensional heat transfer equation in the radiative layer a one-dimensional approximate equation for the poloidal variation of the temperature at the LCMS, $T_L(\vartheta)$. For this purpose the divergence of the perpendicular heat flux is approximated as follows:

$$g^{rr}\frac{\partial}{\partial x}\left(\kappa_\perp\frac{\partial T}{\partial x}\right)\approx g^{rr}\frac{q_{\rm core}-\dfrac{\kappa_\perp}{\delta_T}T_L}{x_{\rm rad}}.$$

Here $x_{\rm rad}(\vartheta)$ is the local width of the radiative layer determined as the distance from the LCMS at which $T=T_{\max}$. On the one hand $x_{\rm rad}$ should approach its maximum, $x_{\rm rad}^{\max}$, at the MARFE symmetry plane, $\vartheta=0$. On the other hand at the magnetic surface $x=x_{\rm rad}^{\max}$ the plasma temperature and, thus, the parallel heat conductivity, being proportional to $T^{2.5}$, are sufficiently high. Therefore the temperature profile should be relatively flat here, i.e., $T(x_{\rm rad}^{\max},\vartheta)\approx T_{\max}$. As a result $x_{\rm rad}$ also does not change too strongly with ϑ and $x_{\rm rad}^{\max}$ will be assumed for $x_{\rm rad}$ henceforth. Finally the stationary $T_L(\vartheta)$ profile is governed by the following approximate equation:

$$\frac{1}{(qR)^2}\frac{d}{d\vartheta}\left(\kappa_\parallel\frac{dT_L}{d\vartheta}\right)\approx W(\vartheta,T_L)$$
$$\equiv\overline{Q}_{\max}\frac{T_*}{T_L}\left(\frac{T_L}{T_{*I}}\right)^{\xi_I}\psi(T_L)+\frac{\dfrac{\kappa_\perp}{\delta_T}T_L-q_{\rm core}}{x_{\rm rad}^{\max}}g^{rr}(\vartheta) \tag{6}$$

with $\overline{Q}_{\max}=\overline{nn_r}L_c^{\max}$.

An analytical approach for the determination of the parameters T_a, T_m, ϑ_m, and $x_{\rm rad}^{\max}$, which are the most principal characteristics of a state with MARFE, is outlined in the next section.

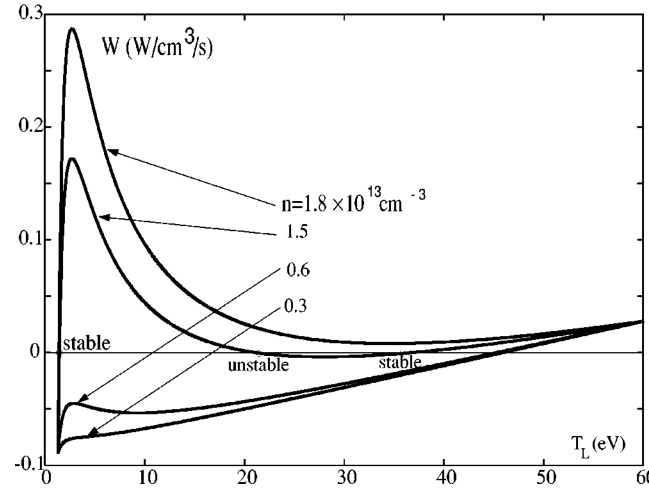

FIG. 3. Function W on the RHS of Eq. (5), vs T_L for a fixed poloidal position and for different values of $\overline{n}$.

2. Equations for "phase" characteristics

The function $\psi(T_L)$ reduces to zero beyond the temperature range $T_{\min}\leqslant T_L\leqslant T_{\max}$. It is therefore easy to comprehend that the right-hand side (RHS) W of Eq. (6) computed as a function of T_L for a fixed poloidal angle ϑ can have a N-like form (see Fig. 3). This takes place if the maximum density of radiation losses exceeds a critical level, which can be found from the requirements that both $\partial W/\partial T_L$ and $\partial^2 W/\partial T_L^2$ reduce to zero,

$$\overline{Q}_{\max}^{\mathrm{cr}}=\frac{\kappa_\perp g^{rr}(\vartheta)}{4\delta_T x_{\mathrm{rad}}^{\max}}\frac{T_{\mathrm{cr}}(T_{\max}-T_{\min})^2}{[(1+\xi_I)T_{\mathrm{cr}}^2-\xi_I(T_{\min}+T_{\max})T_{\mathrm{cr}}-(1-\xi_I)T_{\min}T_{\max}]},$$

where the critical temperature at LCMS, T_{cr}, at which the minimum in W is approached is given by the relation,

$$T_{\mathrm{cr}}=\frac{T_{\min}+T_{\max}}{2}\frac{1-\xi_I}{1+\xi_I}\times\left[\sqrt{1-\left(\frac{2}{\xi_I}-1\right)\frac{1+\xi_I}{1-\xi_I}\frac{4T_{\min}T_{\max}}{(T_{\min}+T_{\max})^2}}-1\right].$$

One can see if $W(T_{\mathrm{cr}},\vartheta)<0$ for $\overline{Q}_{\max}=\overline{Q}_{\max}^{\mathrm{cr}}$ there is a range of $\overline{Q}_{\max}$ in which W reduces to zero for three values of T_L. This situation is typical for many nonlinear systems and has been exhaustively analyzed, e.g., in Ref. 17 by dealing with structures in superconductors. With a ϑ-independent g^{rr} the system could exist in three homogeneous states of different temperatures. The state with an intermediate T_L is unstable and the initial conditions predetermine to which of two other states the system will evolve in time. For a poloidally varying g^{rr} a homogeneous state cannot be realized and one should expect that the system will be separated into two "phases" where parameters correspond to their values in stable states.

A qualitative theory of Ref. 17 allows us to determine the temperatures in both phases if we assume that the border region, where T_L varies between T_m and T_a, is very thin. The results of one- and two-dimensional numerical modeling presented in next section confirm this presumption. In this case the distance between magnetic surfaces and metric coefficient g^{rr} vary very little through the border extent and $g^{rr}\approx g^{rr}(\vartheta_m)$ here. According to Eq. (5) the approximate homogeneity of the temperature profile in both phases bounded to ϑ_m requires

$$W(T_m,\vartheta_m)\approx 0,\quad W(T_a,\vartheta_m)\approx 0. \tag{7}$$

By multiplying Eq. (6) with $\kappa_\parallel(dT_L/d\vartheta)\,d\vartheta$ and integrating between T_m and T_a in the temperature space, we will get

$$\int_{T_m}^{T_a}W(T_L,\vartheta_m)\kappa_\parallel dT_L=\left(\kappa_\parallel\frac{dT_L}{d\vartheta}\right)^2_{T_a}-\left(\kappa_\parallel\frac{dT_L}{d\vartheta}\right)^2_{T_m}\approx 0. \tag{8}$$

For $\kappa_\parallel(T)\sim T^{2.5}$ the integral can be calculated analytically.

In order to find an equation for $x_{\mathrm{rad}}^{\max}$ we again make use of the flatness of poloidal temperature profile in the phases. This allows us to neglect the last term in the RHS of Eq. (4) by determination of the radial temperature variation at $\vartheta=0$, $\Theta=T(x,\vartheta=0)$. The particle densities at $x>0$ are determined by Eq. (2) with T_m being replaced by Θ when $\Theta<T_a$; for $\Theta\geqslant T_a$ we assume $n=\overline{n},n_r=\overline{n}_r$ in order to avoid a situation with a density in the MARFE region smaller than on the same magnetic surfaces in the "attached" phase. Under these assumptions one gets $d^2\Theta/dx^2\approx(\overline{Q}_{\mathrm{rad}}/\kappa_\perp g^{rr}_{\vartheta=0})\,\Lambda(\Theta)$, where

$$\Lambda(\Theta)=\psi(\Theta)\frac{\left[\left(\frac{T_a}{\Theta_*}\right)^{\xi_I}(\pi-\vartheta_m)+\vartheta_m\right]^{1/\xi_I}}{\frac{\Theta_*}{T_a}(\pi-\vartheta_m)+\vartheta_m}$$

and $\Theta_*=\min(\Theta,T_a)$. With the adopted e-folding length for the temperature at the LCMS a double integration of the equation for Θ multiplied with $2(d\Theta/dx)$ results in

$$x_{\mathrm{rad}}^{\max}\approx\int_{T_m}^{T_{\max}}\frac{d\Theta}{\sqrt{\left(\frac{T_m}{\delta_T}\right)^2+2\frac{\overline{Q}_{\mathrm{rad}}}{\kappa_\perp g^{rr}_{\vartheta=0}}\int_{T_m}^{\Theta}\Lambda(\Theta_*)d\Theta}}. \tag{9}$$

Relations (7)–(9) give a set of equations, which allow to determine the temperatures in phases, T_m and T_a, and the characteristic dimensions of MARFE in the poloidal and radial direction, ϑ_m and $x_{\mathrm{rad}}^{\max}$.

IV. COMPARISON OF "PHASE MODEL" PREDICTIONS WITH ONE- AND TWO-DIMENSIONAL NUMERICAL CALCULATIONS

In order to check the predictions of the "phase model" for the characteristic parameters in states with MARFE they will be compared with numerical solutions of one- and two-dimensional heat transport equations (4) and (6). The presumption that the plasma edge is broken up into two phases, corresponding to cold dense MARFE region and hot attached zone, separated by a relatively thin border is validated in Fig. 4. This shows poloidal profiles of the temperature at the

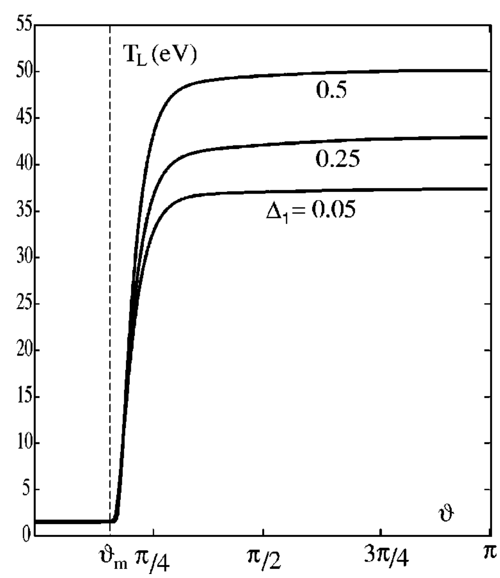

FIG. 4. Poloidal profiles of the plasma temperature at the LCMS computed by numerical solution of Eq. (5) for different values of Δ_1 and the position of the border between MARFE and attached region, ϑ_m, from the "phase model."

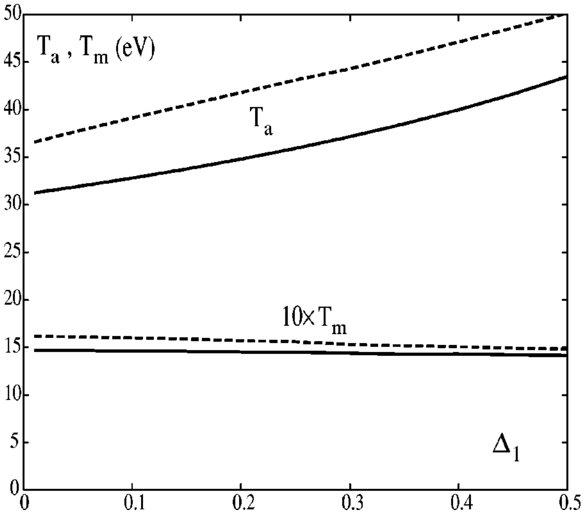

FIG. 5. The plasma temperature in MARFE, T_m, and in the ''attached'' region, T_a, computed from the ''phase'' model (solid curves) and found by numerical solution of Eq. (5) (broken curves).

LCMS computed in one-dimensional modeling for different magnitudes of the parameter $\Delta_1 = -2(d\Delta/dr)(a)$, which prescribes the level of poloidal inhomogeneity in the heat flux from the core. The range of Δ_1 magnitudes has been chosen by taking into account that the $\Delta(r) \approx \Delta_0(1 - r^2/a^2)$ and $\Delta_1 \approx 4\Delta_0/a$ with Δ_0 and a being the shift of the plasma axis and the plasma minor radius, respectively. For typical TEXTOR values, $a \approx 46$ cm and $\Delta_0 \approx 6$ cm, this gives $\Delta_1 \approx 0.5$. Other parameters were such that the ''phase model'' predicts the same position of the phase interface, $\vartheta_m = 0.5$. The numerically found profiles show the separation of phases at roughly the same poloidal angle even for a very low level of g^{rr} inhomogeneity.

It is of interest to note that there is not any phase separation and MARFE does not appear if Shafranov shift is absent at all and $\Delta_1 = 0$, $g^{rr}(\vartheta) \equiv 1$. This is in contradiction with the results of Refs. 7, 29, where stationary poloidally inhomogeneous solutions of Eq. (4) were constructed under similar assumptions on the temperature dependence of the cooling rate, absence of plasma flows, etc., but without any ϑ-dependence in the problem parameters. However, the stability of these solutions was not examined. Our analysis based on numerical integration of nonstationary heat transport equation shows that such solutions do not realize. This is in agreement with the linear stability analysis,[10,13] which predicts that poloidally symmetric modes leading not to MARFE but to radial detachment, become first unstable in the case without external inhomogeneity.

Figure 5 presents the plasma temperature in MARFE and ''detached plasma,'' T_m and T_a, vs Δ_1 found from the ''phase model'' and one-dimensional computations. The difference between the results is mainly due to the following approximations in the ''phase model:'' (i) the finite width of the border between phases is disregarded by the determination of the poloidal density profiles and (ii) the plasma temperatures in phases are identified with their values at the symmetry planes. Figure 6 demonstrates two-dimensional temperature profiles computed for $\Delta_1 = 0$ and 0.5 and the

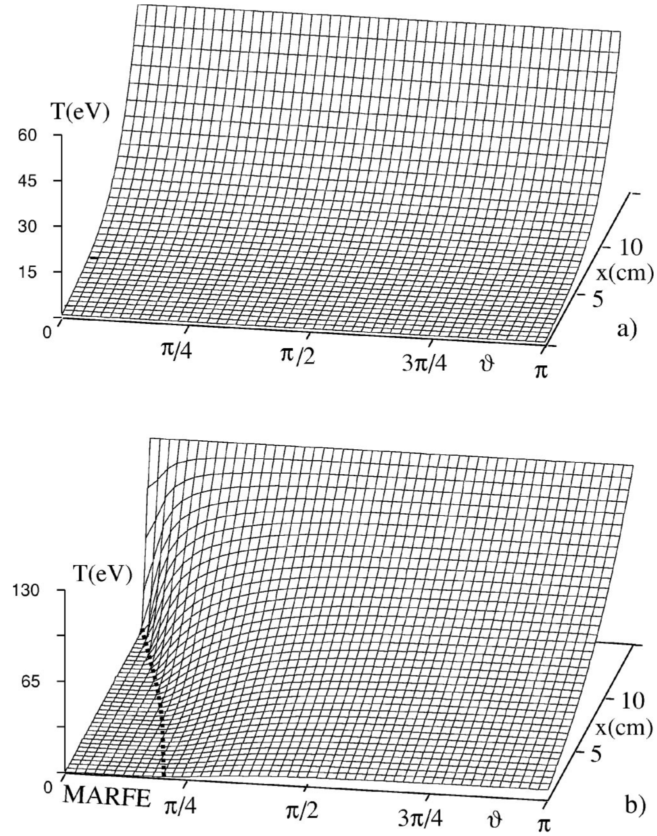

FIG. 6. Two-dimensional temperature profiles at the plasma edge found from numerical integration of Eq. (4) for $\Delta_1 = 0$ and $\Delta_1 = 0.5$.

latter one reproduces well the structure of MARFE. The radial width of the radiative layer at the MARFE symmetry plane agrees with that found from Eq. (9) with an accuracy of 30%.

V. CONCLUSION

The ''phase'' model for the plasma edge in limiter tokamaks based on concepts developed for consideration of dissipative structures predicts the main edge parameters in states with MARFE in a reasonable agreement with numerical solutions of one- and two-dimensional heat transport equations. This model, whose realization requires significantly less computer time than direct computations, could be useful for a self-consistent numerical modeling of the influence of MARFE on global plasma performance.

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