

Nonuniversality Aspects of Nonlinear $k_\perp$ Factorization for Hard Dijets

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The origin of breaking of conventional linear $k_\perp$ factorization for hard processes in a nuclear environment is by now well established. The realization of the nonlinear nuclear $k_\perp$ factorization which emerges instead was found to change from one jet observable to another. Here we demonstrate how the pattern of nonlinear $k_\perp$ factorization, and especially the role of diffractive interactions, in the production of dijets off nuclei depends on the color properties of the underlying pQCD subprocess.

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Factorization theorems are part and parcel of the theory of hard processes. In the perturbative quantum chromodynamics (pQCD) description of high energy hard reactions one uses the so-called $k_\perp$ factorization [1] in terms of the unintegrated gluon densities [2] in the target and beam hadrons [for a review see [3]]. A concise discussion of the inadequacy of the collinear approximation and the necessity of fully unintegrated parton densities with explicit allowance for transverse momenta is found in [4]. Fundamentally, the familiar pQCD factorization for free-nucleon interactions implies that hard cross sections are linear functionals of parton densities in the projectile and target.

An extension of factorization theorems to nuclear targets is of burning urgency—the notion of nuclear gluon densities can be made meaningful only if they furnish a unified description of various nuclear hard processes. Analyzing forward dijets in deep inelastic scattering (DIS) off nuclei we found a striking breaking of $k_\perp$ factorization [5], confirmed in a related analysis of single-jet spectra in hadron-nucleus (hA) collisions [6]. Based on the pQCD treatment of diffractive dijets [7] one can define the collective nuclear unintegrated gluon density such that in terms of it the linear $k_\perp$ factorization would hold for the nuclear structure function $F_{2A}(x, Q^2)$ and the forward single jets in DIS off nuclei because of their special Abelian features. Dijets in DIS, and dijet and single-jet spectra in hA collisions, however, prove to be highly nonlinear functionals of the collective nuclear glue. For a criticism of unwarranted applications of linear $k_\perp$ factorization to nuclear targets, see, e.g., [6]; our conclusions on the breaking of linear $k_\perp$ factorization for hard scattering off nuclei have recently been corroborated in [8,9].

In this Letter, we relate the diverse $k_\perp$ factorization properties of dijet production to the color structure of the pQCD subprocess: $\gamma^*g \rightarrow Q\bar{Q}$ for leading quark-antiquark dijets in DIS, $q^*g \rightarrow qg$ for the dominant dijets in the proton (deuteron) hemisphere of hA collisions ($h = p, d$) at the Relativistic Heavy Ion Collider (RHIC), and $g^*g \rightarrow Q\bar{Q}$ for open charm production. The nonlinear $k_\perp$ factorization derives from the interaction cross section for a

color-singlet system of n partons being a (matrix) superposition of elementary dipole cross sections, and their nuclear S matrices being a (matrix) product of nuclear S matrices for elementary dipoles. We establish universality classes for nonlinear $k_\perp$ factorization. Our results shed certain light onto which extent hard processes in a nuclear environment can be described entirely in terms of the classical gluon field of a whole nucleus [10].

We require coherency over the whole nucleus, $x = [(Q^*)^2 + M_{JJ}^2]/2m_p E_a \lesssim x_A = (2R_A m_p)^{-1} \approx 0.1A^{-1/3}$, where R_A is the radius of the target nucleus of mass number A , $(Q^*)^2$ and E_a are the virtuality and energy of the beam parton a in the target rest frame, M_{JJ} is the dijet mass, and m_p is the proton mass [11]. At RHIC this corresponds precisely to qg dijets in the largest rapidity bins in the proton hemisphere of pA collisions.

To the lowest order in pQCD, all the above processes, in the laboratory frame, can be viewed as an excitation of the perturbative $|bc\rangle$ Fock state of the physical projectile $|a\rangle$ by one-gluon exchange with the target nucleon. For nuclear targets one has to deal with the non-Abelian intranuclear evolution due to multiple gluon exchanges which are enhanced by a large target thickness. The kinematical constraints in pA collisions at RHIC justify the focus on lowest Fock states only; understanding the rich pattern of nonlinear $k_\perp$ factorization found in this regime is a must for the further studies of small- x evolution.

The master formula for the dijet spectrum is derived in [6] based on [5,12,13]; here we only reproduce the main result:

$$\begin{aligned} \frac{d\sigma(a^* \rightarrow bc)}{dz_b d^2 p_b d^2 p_c} &= \frac{1}{(2\pi)^4} \int d^2 b_b d^2 b_c d^2 b'_b d^2 b'_c \\ &\times \exp[-i\mathbf{p}_b \cdot (\mathbf{b}_b - \mathbf{b}'_b) - i\mathbf{p}_c \cdot (\mathbf{b}_c - \mathbf{b}'_c)] \\ &\times \Psi(z_b, \mathbf{b}_b - \mathbf{b}_c) \Psi^*(z_b, \mathbf{b}'_b - \mathbf{b}'_c) \\ &\times \{S_{\bar{b}cb}^{(4)}(\mathbf{b}'_b, \mathbf{b}'_c, \mathbf{b}_b, \mathbf{b}_c) + S_{\bar{a}a}^{(2)}(\mathbf{b}', \mathbf{b}) \\ &- S_{\bar{b}ca}^{(3)}(\mathbf{b}, \mathbf{b}'_b, \mathbf{b}'_c) - S_{\bar{a}bc}^{(3)}(\mathbf{b}', \mathbf{b}_b, \mathbf{b}_c)\}. \quad (1) \end{aligned}$$

If $\mathbf{b}_a = \mathbf{b}$ is the projectile's impact parameter, then $\mathbf{b}_b =$

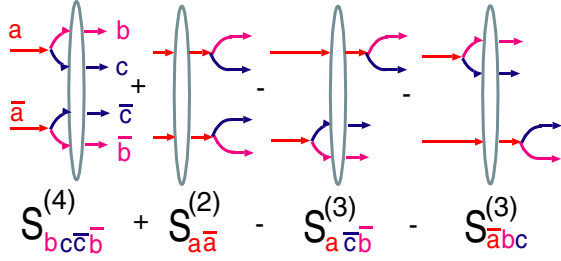


FIG. 1 (color online). The S -matrix structure of the two-body density matrix for the excitation $a \rightarrow bc$.

$\mathbf{b} + z_c \mathbf{r}$, $\mathbf{b}_c = \mathbf{b} - z_b \mathbf{r}$, where $z_{b,c}$ stand for the fraction of the light cone momentum of the projectile a carried by partons b and c . For all reactions, $\Psi(z, \mathbf{r})$ and its momentum space version $\Psi(z, \mathbf{p})$ stands for the light cone wave function of the $|bc\rangle$ Fock state of the projectile; its connection to the parton-splitting functions is found in [6]. All $S^{(n)}$ describe a scattering of color-singlet systems of n partons, as indicated in Fig. 1. $S^{(2)}$ and $S^{(3)}$ are readily calculated in terms of the 2-parton and 3-parton dipole cross sections [13,14]; general rules for the multiple scattering theory calculation of the coupled-channel $S^{(4)}$ are found in [5].

The unintegrated gluon density in the target nucleon, $\mathcal{F}(x, \kappa^2) = \partial G(x, \kappa^2) / \partial \log \kappa^2$, furnishes a universal description of the proton structure function $F_{2p}(x, Q^2)$ and of the final states. For instance, the linear $k_\perp$ factorization for forward dijets reads [for applications, see [15] and references therein]

$$\frac{2(2\pi)^2 d\sigma_N(\gamma^* \rightarrow Q\bar{Q})}{dz d^2 p d^2 \Delta} = f(x, \Delta) |\Psi(z, \mathbf{p}) - \Psi(z, \mathbf{p} - \Delta)|^2, \quad (2)$$

where $\Delta = \mathbf{p} + \mathbf{p}_{\bar{Q}}$ is the jet-jet decorrelation momentum, and $\mathbf{p} \equiv \mathbf{p}_{\bar{Q}}$, $z \equiv z_{\bar{Q}}$ refer to the $\bar{Q}$ jet.

We turn to nuclear targets. We define the collective nuclear gluon density, $\phi(\mathbf{b}, x, \kappa)$, per unit area in the impact parameter plane, in terms of the $q\bar{q}$ nuclear profile function [5,7]:

$$\Gamma_{2A}[\mathbf{b}, \sigma(x, \mathbf{r})] \equiv 1 - \exp\left[-\frac{1}{2}\sigma(x, \mathbf{r})T(\mathbf{b})\right] = \int d^2 \kappa \phi(\mathbf{b}, x, \kappa) [1 - \exp(i\kappa \cdot \mathbf{r})]. \quad (3)$$

Here $T(\mathbf{b})$ is the optical thickness of a nucleus,

$$\sigma(x, \mathbf{r}) = \int d^2 \kappa f(x, \kappa) [1 - \exp(i\kappa \cdot \mathbf{r})] \quad (4)$$

is the $q\bar{q}$ dipole cross section [14], and

$$f(x, \kappa) = \frac{4\pi\alpha_S(r)}{N_c} \frac{1}{\kappa^4} \mathcal{F}(x, \kappa^2). \quad (5)$$

The nuclear glue $\phi(\mathbf{b}, x, \kappa)$ is a highly nonlinear functional of the free-nucleon glue, its expansion in terms of the collective glue for j overlapping nucleons in the Lorentz-contracted nucleus [11], its saturation properties at small κ , and the saturation scale $Q_A(\mathbf{b}, x)$ are found in [5,7]. The inclusive spectrum of leading quarks from the excitation $\gamma^* \rightarrow Q\bar{Q}$ off nuclei is linear $k_\perp$ factorizable in terms of $\phi(\mathbf{b}, x, \kappa)$ as its counterpart for the free-nucleon target in terms of $f(x, \kappa)$, see Eq. (2)—such an Abelianization only holds for the color-singlet projectile. We shall also make use of $\Phi(\mathbf{b}, x, \kappa) = S_{abs}(\mathbf{b})\delta^{(2)}(\kappa) + \phi(\mathbf{b}, x, \kappa)$, where $S_{abs}(\mathbf{b}) = \exp[-\frac{1}{2}\sigma_0(x)T(\mathbf{b})]$ and $\sigma_0(x) = \int d^2 \kappa f(x, \kappa)$ is the dipole cross section for large dipoles.

The t -channel pQCD gluon exchange leaves the target nucleon debris in a color excited state. In the case of nuclear targets one must distinguish truly inelastic processes and coherent diffraction $aA \rightarrow (bc)A$ with retention of the target nucleus in the ground state.

Here we report closed-form analytic results for nuclear dijet spectra to the leading order in $1/N_c$; higher orders can be derived following Ref. [5]. In DIS off nuclei, dipoles propagate as color singlets until at depth β from the front face of a nucleus they excite into the octet state; the non-Abelian evolution in the slice $[\beta, 1]$ consists of color rotations within the octet state [16].

Combining the truly inelastic and diffractive components of DIS,

$$\begin{aligned} \frac{(2\pi)^2 d\sigma_A(\gamma^* \rightarrow Q\bar{Q})}{d^2 \mathbf{b} dz d^2 \mathbf{p} d^2 \Delta} &= \frac{1}{2} T(\mathbf{b}) \int_0^1 d\beta \int d^2 \kappa_1 d^2 \kappa f(x, \kappa) \Phi(1 - \beta, \mathbf{b}, x, \Delta - \kappa_1 - \kappa) \Phi(1 - \beta, \mathbf{b}, x, \kappa_1) \\ &\times |\Psi(\beta; z, \mathbf{p} - \kappa_1) - \Psi(\beta; z, \mathbf{p} - \kappa_1 - \kappa)|^2 + \delta^{(2)}(\Delta) |\Psi(1; z, \mathbf{p}) - \Psi(z, \mathbf{p})|^2, \end{aligned} \quad (6)$$

where $\Psi(\beta; z, \mathbf{p}) = \int d^2 \kappa \Phi(\beta, \mathbf{b}, x, \kappa) \Psi(z, \mathbf{p} + \kappa)$ is the wave function of the incident dipole distorted by the coherent initial state interaction (ISI) in the slice $[0, \beta]$ of a nucleus, and the collective nuclear glue for the slice β of a nucleus, $\Phi(\beta, \mathbf{b}, x, \kappa)$, is defined by [5]

$$\exp\left[-\frac{1}{2}\beta\sigma(x, \mathbf{r})T(\mathbf{b})\right] = \int d^2 \kappa \Phi(\beta, \mathbf{b}, x, \kappa) \exp(i\kappa \cdot \mathbf{r}). \quad (7)$$

The diffractive component of Eq. (6), $\propto \delta^{(2)}(\Delta)$, gives exactly back-to-back dijets [for the Δ dependence for finite-size nuclei, see Ref. [7]]. It is a quadratic functional of the collective nuclear glue and makes $\sim 50\%$ of the total DIS off heavy absorbing nuclei [17]. The first component in (6)—truly inelastic DIS—is of fifth order in gluon densities: a quartic functional of the collective nuclear glue for the two slices of a nucleus which describe the coherent ISI's of color-singlet dipoles in the slice $[0, \beta]$

and incoherent FSI's of color-octet dipoles in the slice $[\beta, 1]$, and a linear one of the free-nucleon glue $f(x, \Delta)$ which describes the hard singlet-to-octet transition. The striking distinction between ISI and FSI requires the collective nuclear glue (7) for slices of the nucleus and not the classical gluon field of the whole nucleus. The nonlinear $k_\perp$ factorization result (6) must be contrasted to the free-nucleon spectrum (2); it entails a nuclear enhancement of the decorrelation of dijets from truly inelastic DIS, while semihard dijets, $|p_{Q\bar{Q}}|^2 \lesssim Q_A^2(b, x)$, are completely decorrelated.

The $k_\perp$ factorizable free-nucleon $q^* \rightarrow qg$ dijet cross section is the differential form of the single-jet spectrum,

$$\begin{aligned} \frac{(2\pi)^2 d\sigma_A(q^* \rightarrow qg)}{d^2b dz d^2p d^2\Delta} &= \frac{1}{2} T(b) \int_0^1 d\beta \int d^2\mathbf{k}_2 d^2\mathbf{k}_1 d^2\mathbf{k} \Phi(\beta, b, x, \mathbf{k}_2) \Phi(1 - \beta, b, x, \Delta - \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}) \\ &\times \Phi\left(\frac{C_A}{C_F}(1 - \beta), b, x, \mathbf{k}_1\right) f(x, \mathbf{k}) |\Psi(\beta; z, \mathbf{p} - \mathbf{k}_1 - \mathbf{k}_2) - \Psi(\beta; z, \mathbf{p} - \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k})|^2 \\ &+ \phi(b, x, \Delta) |\Psi(1; z, \mathbf{p} - \Delta) - \Psi(z, \mathbf{p} - z\Delta)|^2 + \delta^{(2)}(\Delta) S_{abs}(b) |\Psi(1; z, \mathbf{p}) - \Psi(z, \mathbf{p})|^2. \end{aligned} \quad (9)$$

Coherent diffractive excitation of color-triplet qg dipoles, $\propto \delta^{(2)}(\Delta)$, is suppressed by the nuclear attenuation because the initial parton q^* is colored. Note that $\Psi(1; z, \mathbf{p})$ is coherently distorted over the whole thickness of the nucleus; consequently, the second term in (9)—the excitation of the color-triplet qg states—is a cubic functional of the nuclear glue. The first component of the nuclear spectrum (9)—excitation of the color sextet and 15-plet qg states—resembles strongly the truly inelastic dijet spectrum (6) for DIS: the free-nucleon glue $f(x, \mathbf{k})$ describes the hard excitation of the qg color dipole from the lower (triplet) to higher (sextet and 15-plet) states; the coherent ISI distortions in the slice $[0, \beta]$ are similar, too. The incoherent distortions are somewhat different, however. In DIS we have an equal distortion of both outgoing parton waves by pure FSI. In the qg case, the uncorrelated FSI distortion of the outgoing quark and gluon waves in the slice $[\beta, 1]$ are complemented by incoherent ISI distortions of the incident quark wave in the slice $[0, \beta]$. The ratio of Casimir operators $C_A/C_F \simeq 2$ in $\Phi((1 - \beta)\frac{C_A}{C_F}, b, x, \mathbf{k}_1)$ for FSI of gluons is a reminder that the collective nuclear glue derives

Eq. (82) of Ref. [6] (here $\mathbf{p} \equiv \mathbf{p}_g$, $z \equiv z_g$):

$$\begin{aligned} \frac{2(2\pi)^2 d\sigma_N(q^* \rightarrow gq)}{dz d^2p d^2\Delta} &= f(x, \Delta) [|\Psi(z, \mathbf{p}) - \Psi(z, \mathbf{p} - \Delta)|^2 \\ &+ |\Psi(z, \mathbf{p} - \Delta) - \Psi(z, \mathbf{p} - z\Delta)|^2]. \end{aligned} \quad (8)$$

We proceed to nuclear targets. The set of color-singlet 4-parton states $qg\bar{q}'g'$ of interest consists of $|3\bar{3}\rangle$, $|6\bar{6}\rangle$, and $|15\bar{15}\rangle$ states. The nuclear dijet spectrum takes the form

from a density matrix in color space [5,6]. As such, excitation of color sextet and 15-plet final states exhibits a sixth order nonlinearity: it is linear in the free-nucleon glue and fifth order in the collective nuclear glue.

Heavy flavor excitation, $g^* \rightarrow Q\bar{Q}$, off free nucleons is the differential form of the single-jet spectrum derived in [6] (here $\mathbf{p} \equiv \mathbf{p}$, $z \equiv z_{\bar{Q}}$):

$$\begin{aligned} \frac{2(2\pi)^2 d\sigma_N(g^* \rightarrow Q\bar{Q})}{dz d^2p d^2\Delta} &= f(x, \Delta) [|\Psi(z, \mathbf{p}) - \Psi(z, \mathbf{p} - z\Delta)|^2 \\ &+ |\Psi(z, \mathbf{p} - \Delta) - \Psi(z, \mathbf{p} - z\Delta)|^2]. \end{aligned} \quad (10)$$

Here one starts with the color-octet $Q\bar{Q}$ dipole; intranuclear interactions are color rotations in the space of the octet states. Transitions to the color-singlet $Q\bar{Q}$ dipoles and coherent diffraction are $1/N_c$ suppressed [5]. The non-Abelian evolution of the $Q\bar{Q}Q'\bar{Q}'$ system becomes a single-channel problem and the resulting nuclear dijet cross section equals

$$\begin{aligned} \frac{(2\pi)^2 d\sigma_A(g^* \rightarrow Q\bar{Q})}{dz d^2p d^2b d^2\Delta} &= S_{abs}(b) \phi(b, x, \Delta) \{ |\Psi(z, \mathbf{p}) - \Psi(z, \mathbf{p} - z\Delta)|^2 + |\Psi(z, \mathbf{p} - \Delta) - \Psi(z, \mathbf{p} - z\Delta)|^2 \} \\ &+ \int d^2\mathbf{k} \phi(b, x, \mathbf{k}) \phi(b, x, \Delta - \mathbf{k}) \times |\Psi(z, \mathbf{p} - \mathbf{k}) - \Psi(z, \mathbf{p} - z\Delta)|^2. \end{aligned} \quad (11)$$

Interestingly, this is precisely the differential version of the single-quark spectrum, Eq. (31) of Ref. [6], if in the nonlinear term one makes a proper identification of momenta. It satisfies the quadratic-nonlinear $k_\perp$ factorization in terms of the collective glue defined for the whole nucleus.

We summarize: we established how the color properties of the relevant pQCD subprocess define the pattern of

nonlinearity: (a) excitation of dijets in higher color multiplets from partons in a lower multiplet gives rise to the universality class with the fifth, or even sixth, order nonlinearity of the dijet spectrum in gluon densities. This includes the free-nucleon glue which describes hard excitation of higher color multiplets [see also the discussion of the $1/(N_c^2 - 1)$ expansion in Ref. [5]]; (b) in heavy flavor

excitation starting from already higher-multiplet partons (gluons) inelastic interactions can be viewed as color rotations within the same multiplet and the nonlinearity will be a quadratic one; (c) coherent diffraction is a universality class of its own, unsuppressed by large N_c and color suppressed for incident partons in lower and higher color multiplets, respectively; this point, and the very existence of coherent diffraction for incident colored partons, are also new observations; (d) the contribution from dijets in the same lower color multiplet as the incident parton forms a fourth universality class. Nonlinear $k_\perp$ factorization furnishes a unified pQCD description of dijet production in a nuclear environment in terms of one and the same collective nuclear glue of Eqs. (3) and (7). Such explicit quadratures for the dijet spectra are not contained in previous works on the subject [8,9].

Because of the important physics distinction of the initial and final state interactions in class-A final states this collective glue must be evaluated for different slices of a nucleus and nonlinear nuclear $k_\perp$ factorization cannot be described in terms of the collective glue defined for the whole nucleus. The latter remains useful for other universality classes, though. By the coherency condition, $x \lesssim x_A \approx 0.1A^{-1/3}$, our formalism applies to the largest rapidity bins of the proton hemisphere of pA collisions at RHIC; it does not hold for the midrapidity dijets studied so far at RHIC [18].

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