ELECTROSTATIC DRIFT INSTABILITIES, TURBULENCE AND ANOMALOUS TRANSPORT: INTRODUCTION AND BASIC THEORY

André L. Rogister

Institut für Plasmaphysik, Forschungszentrum Jülich GmbH, Trilateral Euregio Cluster, EURATOM Association, , D-52425 Jülich, Germany; a.rogister@fz-juelich.de

Experts consider that the enhanced transport observed in fusion devices is the consequence of low frequency instabilities which are elongated along the magnetic field lines and whose free energy sources are the gradients inherent to confinement. The cases $k_{||}qR >> 1$ and $k_{||}qR \sim 1$ will be considered, where $k_{||}$ is the parallel wave number and qR the connection length. The first limit is relevant to plasmas with large parallel flow velocity gradients ($|\partial_r U_{\varphi,i}| \sim c_i |\partial_r \ln N_i|$, where $\vec{U}_i$ and c_i are the ion bulk and thermal velocities; N_i is the density); here, a local dispersion relation leads to exact stability criteria with the help of the Nyquist diagram technique. The ordering $k_{||}qR \sim 1$ applies otherwise; it requires a non-local analysis leading to a second order differential equation whose complex eigenvalues provide the wave frequencies and growth / decay rates; two sub-cases must be considered according to whether the eigenfunctions are radially or poloidally localised. After reviewing some of the most important micro-instabilities, the non-linear saturation mechanism and ultimate turbulence level will be discussed, as well as the various aspects of anomalous transport theories.

I. INTRODUCTION

In a liquid in contact with a heat source, the temperature gradient generates ascending Bénard cells which carry the energy to the surface in contact with the cooling atmosphere. Similarly, in a magnetically confined plasma, the inherent temperature, density and parallel flow velocity gradients are free energy sources which sustain micro-instabilities leading to turbulence and anomalous transport. The interplay between macroscopic and microscopic processes leads to very complex self regulation mechanisms which have been under study, both numerically and theoretically, for decades.

The macroscopic length scales are the temperature, density and parallel flow velocity gradient scales [L, e.g., $L_N = (\partial_r \ln N)^{-1}$], the minor and major radii (r and R) and the connection length qR (the latter is the distance measured along a field line between its intersection points with a given cross section; the safety factor q is the average over a flux surface of the field line pitch angle $d\varphi / d\theta$; φ and θ are the poloidal and toroidal angles). Microscopic scales are the ion and electron gyro-radii $a_j = c_j / \Omega_j$, where $c_j = \sqrt{T_j / m_j}$ and $\Omega_j = e_j B / m_j$ are the thermal velocities and gyro-frequencies, the collisionless skin depth c / ω_{pe}, where c is the speed of light and $\omega_{pe} = \sqrt{N_e e^2 / m_e \varepsilon_0}$ the electron plasma frequency, the ion and electron Debye lengths $\lambda_{D,j} = c_j / \omega_{pj}$, etc. The stable operation regime of tokamaks is such that $\lambda_{D,i} \approx \lambda_{D,e} \approx a_e << a_i$. In this lecture, we consider oscillations with perpendicular length scales of the order of, or larger than the ion gyro-radius: $\vec{k}_\perp a_i \leq \vec{1}$.

We define the unit vectors parallel to the magnetic field lines, the pressure gradient and the binormal: $\hat{n} = \vec{B} / B$, $\hat{p} = \vec{\nabla} P / |\vec{\nabla} P|$ ($\hat{p}$ is orthogonal to the nested toroidal flux surfaces on which P is constant[1]) and $\hat{b} = \hat{n} x \hat{p}$. In the laboratory frame of reference, the phase velocity of drift waves in the direction of the binormal, $v_{ph,\beta}$, is a linear combination of the density and temperature gradient drift velocities, $T_J \hat{p} \cdot \vec{\nabla} N_J / e_J N_J B$ and $\hat{p} \cdot \vec{\nabla} T_J / e_J B$, of the $\vec{E} x \vec{B}$ drift velocity, $-\hat{p}.\vec{E} / B$, and of the parallel flow velocity $U_{||,i}$. Their angular frequency, which is the product of the binormal wave vector component $k_\beta = \vec{k}.\hat{b}$ with the phase velocity $v_{ph,\beta}$, is thus of order

$$\omega_j^* = k_\beta T_j \partial_r \ln N_j / e_j B; \quad (1)$$

ω_j^* is the diamagnetic frequency for species j and ∂_r a short-hand notation for $\hat{p}.\vec{\nabla}$ (∂_r and $\hat{p}.\vec{\nabla}$ are rigorously identical in cylindrical geometry; in more complex systems, general coordinates and Christoffel symbols must be introduced[2]). It follows that drift frequencies are small compared to the ion cyclotron frequency (thus the name *low frequency waves*):

$$\omega / \Omega_i \sim k_\beta a_i^2 / L << 1$$

It is generally assumed that perturbations with the largest parallel wavelengths are most readily unstable, on the ground that the time required for electrons to neutralise charge separation is then longest. We note that the argument is well founded if the instability drive is related to electron inertia or collisions, but its validity is less obvious otherwise. If the driving mechanism is a parallel flow shear, in particular, the unstable spectrum is not expected to be symmetric with respect to the parallel component of the wave vector $k_{||} = \vec{k}.\hat{n}$ and the most unstable modes are expected to have finite $k_{||}$'s.

In summary, we shall consider waves with frequencies and wave number components in the ranges

$$\omega \sim \omega_j^*, \tag{2}$$

$$\vec{k}_\perp a_i \leq \vec{1}, \tag{3}$$

and, respectively, either

$$k_{||} qR >> 1, \tag{4a}$$

or

$$k_{||} qR \sim 1. \tag{4b}$$

(It will be shown that the lowest characteristic value of $k_{||}$ within a radial eigenmode is typically $\sim 1/qR$).

II. $k_{||}$qR >> 1: ELECTRON AND ION DRIFT BRANCHES

II.A. Local Dispersion Relation

The electron and ion distribution functions can be split into their equilibrium and perturbed components. As the collision time scales are much shorter than the transport time scales, the equilibrium distributions are Maxwellian in leading order:

$$F_j^{(0)} = \frac{N_j}{(2\pi c_j^2)^{3/2}} exp[-\frac{v_\perp{}^2 + (v_{//} - U_{//,j})^2}{2c_j^2}] \tag{5}$$

where $v_{||} = \vec{v}.\hat{n}$ is the parallel component of the particle velocity and $v_\perp^2 = v^2 - v_{||}^2$. There is no constraint on $T_e - T_i$ if the temperature relaxation time scale ($\tau_{equi} \sim m_i / m_e \nu_e$) is larger than, or of order of the transport time scale. The difference $U_{//,e} - U_{//,i}$ is proportional to the plasma current density along the magnetic field lines; it is usually smaller than the ion thermal speed.

It is convenient to split the perturbed distribution functions into $f_j = \bar{f}_j + \tilde{f}_j$, where $\bar{f}_j$ is the gyro-phase average of f_j; the assumption $\vec{k}_\perp a_i < \vec{1}$ leads to $\tilde{f}_j < \bar{f}_j$. A straightforward expansion of Vlasov's equation shows that

$$\bar{f}_i^{(0)} = \frac{k_{//}(v_{//} - U_{//,i})F_i^{(0)} - \frac{k_\beta T_i}{eB}\partial_r F_i^{(0)}}{(\omega - \omega_E) - k_{//}v_{//}} \frac{e\phi}{T_i} \tag{6}$$

where ϕ is the electrostatic potential of the wave and

$$\omega_E = -k_\beta E_r / B \tag{7}$$

the $\vec{E} \times \vec{B}$ Doppler frequency (ω_j^* and ω_E are usually of comparable magnitudes). We note that

$$\frac{k_\beta T_i}{eB}\partial_r F_i^{(0)} = \omega_i^* \{1 + \eta_i [\frac{v_\perp{}^2 + (v_{//} - U_{//,i})^2}{2c_i^2} - \frac{3}{2}] + \frac{v_{//} - U_{//,i}}{c_i} \frac{\partial_r U_{//,i}}{c_i \partial_r \ln N_i}\} F_i^{(0)} \tag{8}$$

where $\eta_i = \partial_r lnT_i / \partial_r ln N_i = L_N / L_T$. The most general ion response corresponds to $\omega' \equiv \omega - \omega_E - k_{||}U_{||,i} \sim k_{||}c_i$, which yields $U_{||,i} \sim c_i$ and $k_{||}L \sim k_\beta a_i$. It follows that $U_{||,e} \cong U_{||,i}$ and $\omega - \omega_E - k_{||}U_{||,e} << k_{||}c_e$ which leads to the *adiabatic* electron response

$$f_e^{(0)} = \frac{e\phi}{T_e} F_e^{(0)} \tag{9}$$

The expressions of the electron and ion densities are readily obtained at leading order. Inserting them into the charge neutrality condition provides the dispersion relation

$$1 + \tau_i = D(\omega') \tag{10}$$

where $\tau_i = T_i / T_e$ and

$$D(\omega') = N_i^{-1} \int_{-\infty}^{\infty} dv'_{\parallel} (\omega' - k_{\parallel} v'_{\parallel})^{-1}$$
$$\left\{ \omega' - \omega_i^* \left[1 + \eta_i \left(\frac{v'^2_{\parallel}}{2c_i^2} - \frac{1}{2} \right) + \frac{v'_{\parallel}}{c_i} \frac{\partial_r U_{\parallel,i}}{c_i \partial_r \ln N_i} \right] \right\} F_i^{(0)} \tag{11}$$
$$= \frac{1}{\sqrt{2\pi\tau_i}} \int_{-\infty}^{\infty} du (z - \xi u)^{-1}$$
$$\left[z + \tau_i \left(1 - \frac{\eta_i}{2} \right) + \beta u + \frac{\eta_i}{2} u^2 \right] exp(-u^2 / 2\tau_i)$$

We have defined $z = \omega' / \omega_e^*$, $\xi = k_{\parallel} c_s / \omega_e^*$, $\beta = \partial_r U_{\parallel,i} / c_s \partial_r \ln N_i$ and $v'_{\parallel} = v_{\parallel} - U_{\parallel,i}$ [so that $F_i^{(0)} \propto exp(v_{\perp}^2 + v'^2_{\parallel}) / 2c_i^2$]. If $|Im\omega'| << |Re\omega'|$, the function $D(\omega')$ can be rewritten as

$$D(\omega') = (2\pi\tau_i)^{-1/2} P \int_{-\infty}^{\infty} du (z - \xi u)^{-1}$$
$$\left[z + \tau_i \left(1 - \frac{\eta_i}{2} \right) + \beta u + \frac{\eta_i}{2} u^2 \right] exp(-u^2 / 2\tau_i) \tag{12}$$
$$- i \sqrt{\frac{\pi}{2\tau_i}} \frac{1}{|\xi|} \left[z + \tau_i \left(1 - \frac{\eta_i}{2} \right) + \beta \frac{z}{\xi} + \frac{\eta_i}{2} \frac{z^2}{\xi^2} \right]$$
$$exp(-z^2 / 2\tau_i \xi^2)$$

where P is the Cauchy principal part integral and the imaginary part corresponds to the residue following the causal prescription of Landau[3]. Worth noting is that

(i) the dispersion relation involves only the two dimensionless parameters $z \propto \omega' / k_{\beta}$ and $\xi \propto \omega' / k_{\parallel}$;

(ii) the radial derivative of the fluctuation does not appear; the dispersion relation is thus scalar (i.e. local);

(iii) unstable oscillations will slowly disintegrate if $\partial_r \omega \neq 0$; their radial extent and amplitude will thus be controlled by the equilibrium inhomogeneities and the fluxes.

II.B. Nyquist Stability Analysis

The dispersion function $D(\omega')$ can be written in a closed analytical form only if $\tau_i \to 0$. Exact instability criteria must otherwise be obtained by means of the Nyquist diagram technique[4,5]. Instability occurs if $\Im m\omega' > 0$ [as $\phi \propto exp(-i\omega t)$]; assuming $\omega_e^* > 0$ without loss of generality, that requests $\Im m z > 0$. Let z trace out a closed contour in the complex plane, going from $-\infty$ to $+\infty$ on the real axis and closing anti-clockwise on a semicircle at infinity in the upper half-plane. The function $D(z)$ will correspondingly trace a closed contour in the complex D-plane (the Nyquist contour). If the point $D(z) = 1 + \tau_i$ [cf. Eq. (10)] falls in a region encircled by, and lying to the left of this contour, then the dispersion relation admits a root with $\Im m z > 0$, i.e. the plasma is unstable.

A detailed analysis[6] of the dispersion equation (11) shows that instability occurs if

$$|\beta| \geq |\beta_{thr}| = \sqrt{\eta_i (2 - \eta_i)(1 + \tau_i)} \tag{13}$$

At threshold, the modes are damped for all values of ξ, except

$$\xi = \xi_{thr} = -\beta_{thr} / 2(1 + \tau_i) \tag{13'}$$

which correspond to marginal stability. The frequency of the marginally unstable mode is

$$z_{thr} = 1 - 0.5\eta_i \tag{13''}$$

Noting that $\beta / \xi \equiv -k_{\beta} \partial_r U_{\parallel,i} / k_{\parallel} \Omega_i$, we find

$$(k_{\beta} \partial_r U_{\parallel,i} / k_{\parallel} \Omega_i)_{thr} = 2(1 + \tau_i) \tag{14}$$

Inequality (13) requests that either

$$\eta_i \leq 1 - \sqrt{1 - \beta^2 / (1 + \tau_i)} \tag{15}$$

or

$$\eta_i \geq 1 + \sqrt{1 - \beta^2 / (1 + \tau_i)} \tag{16}$$

If $\beta = 0$, (16) reduces to the standard criterion $\eta_i > 2$ for the ion temperature gradient (ITG) instability; (13'') then yields $z_{thr} = 0$; hence $\omega'_{thr} = 0$ which is the ITG frequency in slab geometry without finite Larmor radius (FLR) effects ($k_{\perp}^2 a_i^2 \to 0$). Correspondingly, (15) and (13'') yield $\eta_i < 0$ and $z_{thr} = 1$; thus $(\omega')_{thr} = \omega_e^*$, the frequency of electron drift waves without FLR.

II.C. Discussion

We note that

i) The growth / decay rate can easily be calculated near marginal stability; indeed, the numerator in the integrand of (12) divides exactly by the denominator upon introducing $z = z_{thr} + \delta z$, $\xi = \xi_{thr} + \delta\xi$ and expanding[6].

ii) The dispersion function $D(z)$ can readily be evaluated in the limit $\tau_i \to 0$, leading to the dispersion equation

$$z^2 - z - \xi(\beta + \xi) = 0 \tag{17}$$

from which the marginal instability relations $|\beta_{thr}| = 1$, $\xi_{thr} = -\beta_{thr}/2$ and $z_{thr} = 1/2$ are readily obtained. Those agree with (13), (13') and (13'') if $\eta_i = 1$ [Note that η_i does not appear in (17) and that the right-hand side of (13) maximises for $\eta_i = 1$]. Restricting their calculations to the case $\eta_i = 0$, D' Angelo[7] and Catto *et al*[8]. obtained marginal instability relations which differ qualitatively from (13), (13') and (13").

iii) The threshold condition (14) can be rewritten as

$$k_{\|}qR = (k_\beta a_i)(qR\partial_r U_{\|,i}/c_i)/2(1+\tau_i) \tag{18}$$

Marginally unstable modes with $k_{\|}qR >> 1$ thus occur if $(qR\partial_r U_{\|,i}/c_i)/2(1+\tau_i) >> 1$; those are not linked to any particular rational surface (see next Section).

III. k$_{\|}$qR ~ 1: ELECTRON AND ION DRIFT EIGENVALUE EQUATIONS

Under most experimental conditions, neither $(qR\partial_r U_{\|,i}/c_i)/2(1+\tau_i)$ nor $k_{\|}qR$ are large numbers [Counter examples may occur in the pedestal of the high (H) confinement mode[9]]. In such cases, rational surfaces play a most important role. To simplify the discussion, we ignore the torus curvature at first.

III.A. Cylindrical Geometry

The parallel wave vector can be written as

$$k_{\|} = \vec{k}.\hat{n} = k_\varphi \frac{B_\varphi}{B} + k_\theta \frac{B_\theta}{B} = [m + nq(r)]\frac{B_\theta}{rB} \tag{19}$$

where $m = k_\theta r$ and $n = k_\varphi R$ are the poloidal and toroidal mode numbers. In a plasma with finite magnetic shear, i.e. $\hat{s} \equiv r\partial_r \ln q \neq 0$, $k_{\|}$ is a function of position which vanishes at the *rational* surface $r = r_{m,n}$ defined by

$$q(r_{m,n}) = -m/n \tag{20}$$

Expanding the right-hand side of (19) in the vicinity of $r = r_{m,n}$ yields

$$k_{\|} = n(r - r_{m,n})(\partial_r q)(B_\theta/rB) = -\frac{k_\beta \hat{s}}{qR}(r - r_{m,n}) \tag{21}$$

where $k_\beta = k_\theta B_\varphi / B$. It may be verified that $1/|k_\theta \hat{s}| \cong 1/|k_\beta \hat{s}|$ is the distance between the neighbouring rational surfaces $r_{m,n}$ and $r_{m\pm1,n}$. Moderate values of $k_{\|}qR$ therefore request that the mode (m,n) be centred on the rational surface defined in (20) and overlaps a few neighbouring rational surfaces $r_{m\pm p,n}$ ($p << m$) at most. The short scale radial dependence introduced in the dispersion equation by $k_{\|} = k_{\|}'(r - r_{m,n})$ must be balanced by another short scale mechanism; in cylindrical geometry, the latter is finite Larmor radius effect:

$$k_r a_i \to a_i \partial_r$$

The technique of the Nyquist diagram has not been extended to that situation. Expansions of the dispersion function are therefore necessary; for that purpose, it is usually assumed that

$$\lambda \equiv L_{N(T)}/R << 1 \tag{22}$$

and $L_{N(T)} \sim r$. The perturbations are then described by second order ordinary differential equations in which the complex frequency plays the role of eigenvalue:

$$[a_s^2(\partial_x^2 - k_\beta^2) + \frac{k_\beta^2 c_s^2}{\omega'^2}\frac{x^2}{L_s^2} + c(\omega')]f(x,\omega') = 0 \tag{23}$$

where $x = r - r_{m,n}$ and $k_{\|} = -k_\beta x / L_s$; $L_s = qR/\hat{s}$ is the magnetic shear length. The orthogonal eigenmodes of Eq. (23) are of the form

$$f(x,\omega') = H_l(\sqrt{i\sigma}x/a_s)exp(-i\sigma x^2/2a_s^2) \tag{24}$$

where the H_l's are Hermite polynomials[10] of order l and σ and ω' solutions of the equations

$$\sigma = \pm k_\beta c_s a_s / \omega' L_s \tag{25}$$

$$i\sigma = [c(\omega') - k_\beta^2 a_s^2]/(2l+1) \tag{26}$$

Two classes of oscillations can be identified according to the frequency range: the electron and the ion drift branches.

III.A.1 The electron drift branch

The frequencies of the electron drift modes are

$$\omega - \omega_E = \omega' \sim \omega_e^*$$

and their parallel phase velocities satisfy the inequalities

$$c_i << \omega' / k_\| \sim k_\beta a_s (qR / L_N) c_s << c_e .$$

Electrons behave adiabatically under those conditions, for they average out the wave structure; hence $f_e = -(e_e\phi / T_e) F_e$, $n_e = (e\phi / T_e) N_e$ and $t_e = 0$. Inspection of the fundamental equations shows that the complex frequencies can be expanded as

$$\omega' = \omega_e^* + \omega^{(1)} \tag{27}$$

where $\omega^{(1)} \sim \lambda\omega_e^*$; moreover

$$t_i / T_i \sim n_i / N_i = n_e / N_e$$

and

$$c(\omega') = -\alpha_i^{-1} \omega^{(1)} / \omega_e^* \tag{28}$$

where $\alpha_i = [1 + \tau_i(1 + \eta_i))]$. Inserting (27) and (28) into (25) and (26) yields

$$\sigma = \mp(1 - \omega^{(1)} / \omega_e^*) L_N / L_S \tag{29a}$$

$$\frac{\omega^{(1)}}{\omega_e^*} = \alpha_i [-k_\beta^2 a_s^2 \pm i(2l+1)(\alpha_i k_\beta^2 a_s^2 + 1)\frac{L_N}{L_S}] \tag{29b}$$

We require that unstable solutions ($\Im m\omega^{(1)} > 0$) be spatially bounded; thus $\Im m\sigma = \pm \Im m\omega^{(1)} L_N / \omega_e^* L_S < 0$ which, in turn, leads to $\pm L_N / \omega_e^* L_S < 0$. Introducing this constraint into (29b) yields

$$\Im m\omega^{(1)} = -\alpha_i (2l+1)(\alpha_i k_\beta^2 a_s^2 + 1) | \omega_e^* L_N / L_S | ; \tag{29c}$$

thus the modes are actually damped. The inequality $\Im m\sigma = \pm \Im m\omega^{(1)} L_N / \omega_e^* L_S < 0$ stems from a causality requirement, as does the Landau contour deformation[3] in velocity space.

The right-hand side of (29c) is referred to as "magnetic shear damping"; it vanishes in the limit $\hat{s} \to 0$. The usual interpretation is that shear damping is the consequence of the wave energy being radiated far away from the reference rational surface[11]. The argument leading to (29c), the expression of $\Im m\omega^{(1)}$ and its interpretation must however be taken with caution: the contribution $\Im m\sigma = \pm \Im m\omega^{(1)} L_N / \omega_e^* L_S$ to σ is indeed of order $\lambda \equiv L_{N(T)} / R << 1$; thus it may not, rigorously, be retained at this order of the expansion and it has to be checked whether other corrections at next order do not invalidate the above prescription.

III.A.2 The ion drift branch

The frequencies of ion drift modes are in the range

$$\omega - \omega_E = \omega' \sim \lambda\omega^*$$

and their parallel phase velocities such that

$$c_i \sim \omega' / k_{||} << c_e .$$

The electrons are again considered as adiabatic. Inspection of the ion equations shows that

$$t_i / T_i \sim \lambda^{-1} n_i / N_i \tag{30}$$

There is a pair of growing and damped bounded solutions if $\eta_i > 2/3$ with

$$\gamma = \Im m\omega' = \pm(\eta_i - 2/3)^{1/2} |\hat{s}| c_i / qR \tag{31}$$

(Slightly different results were obtained in Ref. [12]). It must be emphasised that the theory of the ion drift branch is on a not as firm ground than that of the electron branch, owing to the smaller value of $\omega' / k_{||} c_i$; that may explain some of the discrepancy between the threshold values given here and in Section II.B.

III.B. Toroidal Geometry

The description of drift waves in toroidal geometry differs from that in cylindrical geometry in many respects.

i) The frequencies associated to the $\vec{\nabla} B$ and curvature drifts both generate new Fourier components (side-bands) and, if their width is larger than the distance

$$r_{m\pm1,n} - r_{m,n} = \mp 1 / n\partial_r q = \pm 1 / k_\theta \hat{s} , \tag{32}$$

couple modes centred on neighbouring rational surfaces $r_{m\pm p,n} .....r_{m,n}$, $p << m$; thereby they introduce novel radial derivatives associated to the departure of the ion orbits from magnetic surfaces:

$$\omega_{B,i} = -\Omega_i^{-1}(\frac{v_\perp^2}{2} + v_\parallel^2)\frac{(\hat{n}x\vec{k}).\vec{\nabla}B}{B} \\ \rightarrow (\Omega_i R)^{-1}(\frac{v_\perp^2}{2} + v_\parallel^2)(i\frac{B_\varphi}{BR}\sin\theta\,\partial_r - \cos\theta\, k_\beta) \quad (33)$$

[We have defined $B = B_0(1-\varepsilon\cos\theta)$, where $\theta = 0$ in the outboard equatorial plane].

ii) The requirement of long parallel wavelengths cannot be fulfilled with a mere Fourier decomposition in θ. To avoid that difficulty, J.W. Connor *et al.*[13] introduced the *ballooning formalism*, a transformation of the original two-dimensional problem into a one-dimensional one in an infinite domain without periodicity constraint. A less abstract representation has been introduced in Ref. [14], relying on invariance properties that the mode representation must satisfy with respect to translations as the distance between neighbouring rational surfaces ...$r_{m-1,n}$ $r_{m,n}$, $r_{m+1,n}$, ... is constant (Similar arguments are introduced in the theory of crystal lattices[15]).

Two limiting cases must be envisaged according to whether the cylindrical modes m,n and $m\pm 1,n$ do or do not overlap.

III.B.1. Toroidal geometry without overlapping

If the width of the cylindrical eigenmodes is small compared to the distance $r_{m\pm1,n} - r_{m,n} = \mp 1/n\partial_r q$, then the modes centred on neighbouring rational surfaces are obviously independent. However, a *primary* oscillation with poloidal and toroidal mode numbers m and n drives *side bands* $m\pm 1,n$ via the θ dependence of $\omega_{B,i}$. Those modify the dynamics of the primary mode.

In the case of the electron branch, the coefficient of the second order derivative in Eq. (23) is, as a result, multiplied by the factor[16,17] $1+2q^2$

$$a_s^2(\partial_x^2 - k_\theta^2) \rightarrow (1+2q^2)a_s^2\partial_x^2 - a_s^2 k_\theta^2 \quad (34)$$

It can be verified that this leads to an increase of the shear damping rate (29c) by $(1+2q^2)^{1/2}$ (It is interesting to note that similar factors appear in the neoclassical theory of transport, see, e.g., Ref. [18]).

In the case of the ion drift branch, the multiplication factor of $a_s^2\partial_x^2$ is[19]:

$$\{1+12.5(1+\tau_i)[L_N/R(1.5\eta_i - 1)k_\theta a_i]^2\} \quad (35)$$

The growth rate remains essentially unchanged. Results obtained in the framework of this model permit to explain the formation of internal transport barriers in negative central shear discharges[20].

III.B.2. Toroidal geometry with strong overlapping

The last case to consider is when the distance between neighbouring rational surfaces is small compared to the characteristic radial scale of the cylindrical modes. We seek for solutions satisfying the condition

$$\phi_{m+p,n}(x - x_{m+p,n}) = \exp(ip\theta_0)\phi_{m,n}(x - x_{m,n})$$

with $p << m$ (Those form a complete set, as the phase angle θ_0 is arbitrary) and expand

$$\phi_{m\pm1,n}(x - x_{m,n}) = [1 \pm \Delta\partial_x + \frac{\Delta^2}{2}\partial_x^2]\phi_{m\pm1,n}(x - x_{m\pm1,n}) \\ = \exp(\pm i\theta_0)[1 \pm \Delta\partial_x + \frac{\Delta^2}{2}\partial_x^2]\phi_{m,n}(x - x_{m,n})$$

where $\Delta = x_{m+1,n} - x_{m,n}$.

For the electron branch, two successive substitutions in $\omega_{B,i}\phi_{m\pm1,n}$ lead to the multiplying factor[21,22]

$$1+(1-2\hat{s})L_N\Delta^2\cos\theta_0/Ra_s^2. \quad (36)$$

of the second order derivative in (23). L_N being generally negative, this coefficient may change sign with respect to the cylindrical geometry if $\hat{s} < 1/2$ and $\cos\theta_0 > 0$ or $\hat{s} > 1/2$ and $\cos\theta_0 < 0$. (We recall that $\theta = 0$ refers to the outer equatorial plane). When this occurs, the electron drift branch is no longer radiating, i.e. behaving asymptotically as $\exp(-i\sigma x^2/2a_s^2)$ with σ real [Eqs. (24) and (25)], but evanescent; magnetic shear damping disappears under those circumstances[23]. However, magnetic shear damping is enhanced if $(1-2\hat{s})L_N\cos\theta_0 > 0$!

With the help of Poisson's summation formula[24], it may finally be shown that the superposition of the individual modes localised on the rational surfaces $r_{m\pm p,n}$ with phase factors $p\theta_0$ peaks at $\theta = \theta_0$ (Hence the appellation *ballooning mode*).

IV. SUMMARY AND FURTHER COMMENTS ON DRIFT MICROINSTABILITIES

We have seen that oscillations are adequately described by a *local* dispersion relation if $k_{||}qR >> 1$; that condition is verified for the most readily unstable modes when $(qR\partial_r U_{||,i} / c_i) / 2(1+\tau_i) >> 1$. Here, $k_{||}$ can be considered as nearly constant and the oscillations are localised to a region in which their distortion and disintegration resulting from the frequency gradient is balanced by the linear excitation. Exact stability criteria are obtained by means of the Nyquist diagram.

If, on the contrary, $k_{||}qR$ should be finite for instability, then the mode must be localised near a *rational* surface defined by $q(r_{m,n}) = -m/n$ so that $k_{||} = k'_{||}x$, where $x = r - r_{m,n}$ and $k'_{||} = -k_\beta / L_S$. The strong space dependence that is introduced by $k_{||}$ must be balanced by other processes. In cylindrical geometry, localisation of the mode is provided by finite gyro-radius effects; those lead to a second order differential eigenvalue problem whose solutions yield the radial wave functions and the complex frequencies. In toroidal geometry, the departure of the ion orbits from rational surfaces owing to the $\vec{\nabla}B$ and curvature drifts combines with finite gyro-radius effects; if, furthermore, the modes centred on neighbouring rational surfaces do overlap, then they couple via the geometry. Toroidicity has a stabilising role in the former case and may be either stabilising or destabilising in the latter, according to the value of the magnetic shear parameter and the ballooning angle θ_0.

In what precedes, the electrons were considered in the adiabatic approximation. That is clearly not justified for those being trapped, in which case averaging between the bouncing angles leads to a bounced averaged drift kinetic equation[25] where inertia plays, as for ions, an important role. As they are localised in a velocity sub-space $v_{||} / v_\perp \sim \varepsilon^{1/2}$, the collision rate of trapped electrons is enhanced by a factor ε^{-1}, see Ref. [26]. Collisions of trapped electrons are found to destabilise the electron drift branch if $\omega_e^* \le 2.3\nu_e / \varepsilon$ and to stabilise it otherwise[27]. The trapped electrons population being largest in the outboard equatorial plane, we consider modes which are ballooning around $\theta_0 = 0$. Equation (36) shows that shear damping may then be suppressed for $\hat{s} < 1/2$ but is enhanced for $\hat{s} > 1/2$ (which is the case at the plasma edge).

We have concentrated on the Trapped Electron (TE) and Ion Temperature Gradient (ITG) instabilities. Other mechanisms may however lead to enhanced fluctuation levels. In high collisionality plasmas, collisions experienced by the bulk of electrons may be responsible for the rippling and the drift resistive ballooning (DRBM) instabilities. The former[28,29,30] is driven by the resistivity gradient $\partial_r \eta \propto \partial_r (Z_{eff} T_e^{-3/2})$, the latter by curvature[31]. The DRBM has been considered as a candidate to explain the origin of the density limit in limiter tokamaks[32]; here, the stability theory was however developed for a uniform temperature plasma: it remains to be seen whether the temperature gradient is stabilising or destabilising.

The above instabilities are essentially electrostatic and their characteristic perpendicular length scale is the ion gyro-radius. The electromagnetic Electron Temperature Gradient (ETG) instability whose characteristic length scale is the collisionless skin depth c / ω_{pe} has also been proposed[33] to explain anomalous electron heat transport.

V. INTRODUCTION TO NONLINEAR ASPECTS AND TRANSPORT.

Electron energy transport in fusion devices is commonly observed to be up to three orders of magnitude larger than neo-classical predictions. Owing to their $\sqrt{m_i / m_e}$ larger neo-classical transport rate, the ion energy anomalous factor is smaller, typically up to 10.

Schematically, attempts to explain the observed energy transport can be classified along two main lines:

i) Enhanced losses are proportional to the fluctuation induced particle and energy fluxes across the equilibrium magnetic surfaces, averaged over many fluctuation periods[34-36], namely

$$\Gamma_j = < n_j u_{r,j} > \qquad \text{and} \qquad Q_j = < p_j u_{r,j} > \qquad (37)$$

where n_j, p_j and $u_{r,j} = -ik_\beta \phi / B$ are the density, pressure and radial velocity fluctuations; the relations between Γ_j and Q_j, on the one hand, and the fluctuation spectrum $S(\vec{k})$, on the other hand, are provided by quasi-linear theory.

ii) Enhanced losses result from magnetic field lines braiding and destruction of magnetic surfaces owing to self sustained magnetic islands[37-39,33].

Quenching of linear instabilities occurs in two ways:

α) via wave energy transfer through $\vec{k}$-space to damped modes, as a result of non-linear ion scattering (i.e., resonant interaction of ions with the beat of two linear waves) and three wave resonant interaction;

β) through relaxation of the equilibrium profiles towards marginal stability, owing to enhanced particle and energy transport.

Since the linear growth rate is usually mode number dependent, marginal linear stability cannot be achieved at all mode numbers simultaneously and a certain amount of nonlinear energy transfer and damping must always take place in the relaxed state [The spectrum should otherwise be singular: $S(\vec{k}) \propto \delta(\vec{k})$]. It is important to note that narrow spectra and relaxation to nearly stable profiles do hint at a strongly non-linear relation between transport and linear growth rate, on the one hand, and to profile *stiffness* or *resiliency*[40], characterised by *critical gradients*, on the other hand; in that case, the concept of local transport may have to be replaced by that of ballistic transport.

In any case, the linear marginal stability condition provides a useful first approximate relation between the various equilibrium profiles. If, for example, the density profile is known (e.g., taken from experiment) and the assumption $T_i(r) \propto T_e(r)$ is made, the marginal stability relation yields $T_e(r)$; a first estimate of the fluctuation level through the discharge then obtains by balancing the known power deposition profile with the divergence of the quasi-linear expression of the anomalous electron energy flux[33,41,16].

The turbulence level and spectrum obtain from a non-linear wave kinetic equation[42-44] in which the equilibrium profiles enter as parameters. The evolution and relaxation to a stationary state of the latter is provided by the energy, momentum and particle balance equations where the anomalous flows are related to the turbulence spectrum via quasi-linear theory. That approach was followed in [34], where a strong non-linear increase of the fluxes above threshold led to stiff profiles, close to marginal instability. It is however a very complicated one, since requires solving for the turbulent spectrum. For that reason, a simple order of magnitude argument, known as mixing length estimate and attributed to Kadomtsev[45,46] is used in most instances.

The argument assumes that instability saturation occurs when the amplitude of the oscillating electric and diamagnetic radial velocities associated with the perturbations - those are of order $k_\beta (T_j / eB_\varphi)(n_j / N_j)$- becomes comparable to the equilibrium diamagnetic drift velocity $(T_j / eB_\varphi)/L_{N(T)}$ (The dominant non-linear terms in the equations describing the perturbations are then indeed comparable to the linear terms; whether they are stabilising or destabilising however requires a detailed non-linear analysis!). The mixing length hypothesis thus yields the relation

$$n_j / N_j \sim e\phi / T_j \sim t_j / T_j \sim 1 / k_\beta L_{N(T)} \tag{38}$$

Identifying the anomalous heat flux $Q_j \sim < p_j i k_\beta \phi / B_\varphi >$ with the usual definition $Q_j = -\chi_j N_j \partial_r T_j$ leads to

$$\chi_j = \chi_{Bohm} L_T < i k_\beta e^{i\varsigma_j} \left(\frac{e\phi}{T_j} \right)^2 > \sim \chi_{Bohm} \frac{\varsigma_j}{k_\beta L_{N(T)}} \tag{39}$$

where we let $p_j / P_j = e^{i\varsigma_j} e\phi / T_j$ and

$$\chi_{Bohm} = T / eB \tag{40}$$

is Bohm' s diffusion coefficient. Since, typically, $k_\beta \sim a_i^{-1}$ for drift waves, $\chi_j \sim \chi_{Bohm} \varsigma_j a_i / L_{N(T)}$ is known as gyro-Bohm diffusion; note that it vanishes if their is no phase lag between the pressure and potential oscillation ($\varsigma_j = 0$). The ad hoc expression often encountered in the literature

$$\chi \sim \gamma / k_\perp^2 \tag{41}$$

follows from (39) if the characteristic linear growth rate γ is of the order of the diamagnetic frequency (Which mode's growth rate to consider can however only be specified by genuine non-linear analysis).

Drift waves will be distorted and break up into shorter eddies in the presence of strong $\vec{E}x\vec{B}$ velocity shear. That is equivalent to stabilisation of the primary drift turbulence[47] via spectral energy transfer to small eddies which are damped. Rapidly varying radial electric fields may be triggered by external causes, such as orbit losses at the plasma edge, or by the drift wave itself, as observed in numerical simulations[48]. The origin of the self-generated radial electric field and associated $\vec{E}x\vec{B}$ *zonal* flows is as follows. The electron and ion averaged radial flows induced by an isolated drift mode are not intrinsically ambipolar[49,50], although the oscillations are charge free ($n_e = n_i$). According to Eqs. (23a) and (23b) of Ref. [50], the quasi-linear radial current associated to a particular electron drift mode is:

$$J_r / e = i(2\Omega_i B^2)^{-1} \partial_r [\alpha_i N_i k_\beta (\phi \, \partial_r \phi^* - \phi^* \partial_r \phi)] \tag{42}$$

where $\alpha_i = 1 + (1+\eta_i) T_i / T_e$. In order to avoid charge separation, the above "primary" current must be balanced by a "secondary" polarisation current according to

$$\vec{\nabla} \cdot (\vec{J} + \partial \vec{D} / \partial t) = o \ . \tag{43}$$

This polarisation current is responsible for the zonal flow[51] which is thus, effectively, a poloidally and toroidally symmetric ($m = n = 0$) sheared $\vec{E}x\vec{B}$ flow; here $\vec{E} = \vec{D} / \varepsilon_0$ where ε_0 is the permitivity. It is interesting to note that the above ion current is proportional to the Reynold stresses

$$u_r^* \partial_r u_\beta + u_r \partial_r u_\beta^* = i k_\beta B^{-2} (\phi \, \partial_r^2 \phi^* - \phi^* \partial_r^2 \phi) \tag{44}$$

which arise from the $\vec{u}.\vec{\nabla}\vec{u}$ term in the ion momentum equation and to which zonal flows are therefore linked.

The complexity of the linear and non-linear theories of instabilities and transport has led to the development of genuinely first principle codes, namely the particle in cells (PIC) code and the non-linear gyro-Landau-fluid and gyrokinetic codes; those, however, can follow the plasma evolution only for a very limited time, of the order of the millisecond[52-54] (to be compared to the particle and energy confinement times, of the order of 100 ms in present devices). Other codes, based on the ad hoc mixing length argument, allow comparison of experiment and theory on more reasonable time-scales[55] but are, of course, somewhat empirical. Those questions are discussed in the next lecture[56].

REFERENCES

1. M.D. KRUSKAL and R.D. KULSRUD, “Equilibrium of a Magnetically Confined Plasma in a Toroid”, PHYS. FLUIDS **1**, 265 (1958).
2. A. ROGISTER and D. LI, “Kinetic and Transport Theories of Turbulent, Axisymmetric, Low Collisionality Plasmas and Turbulence Constraints”, PHYS. FLUIDS B **4** 804 (1992) (In particular Appendix A).
3. L.D. LANDAU, “On the Vibrations of the Electronic Plasma”, J. PHYS. (U.S.S.R.) **10**, 25 (1946).
4. T.H. STIX, *Waves in Plasmas*, pp. 193-197 (American Institute of Physics, New York, 1992).
5. R. J. GOLDSTON and P. H. RUTHERFORD, *Introduction to Plasma Physics*, pp. 468-475 (Institute of Physics Publishing, Bristol, 1995).
6. A.L. ROGISTER, R. SINGH, and P.K. KAW, “On Ion Temperature Gradient and Parallel Velocity Shear Instabilities”, PHYS. PLASMAS **11**, 2106 (2004).
7. N. D’ ANGELO, “kelvin-Helmholtz Instability in a Fully Ionised Plasma in a Magnetic Field”, PHYS. FLUIDS **8** 1748 (1965).
8. P.J. CATTO, M.N. ROSENBLUTH, and C.S. LIU, “Parallel Velocity Shear Instabilities in an Inhomogeneous Plasma with a Sheared Magnetic Field”, PHYS. FLUIDS **16** 1719 (1973).
9. A.L. ROGISTER, “Enhanced Dα Confinement Mode: a Theoretical Model”, NUCL. FUSION **44**, 869 (2004).
10. M. ABRAMOWITZ and I. STEGUN, *Handbook of Mathematical Functions*, Ch. 22 (Dover Publications, Inc., New York, 1965).
11. L.D. PEARLSTEIN and H.L. BERK, “Universal Eigenmode in a Strongly Sheared Magnetic Field”, PHYS. REV. LETT. **23** 220 (1969).
12. B. COPPI, M.N. ROSENBLUTH, and R.Z. SAGDEEV, “Instabvilities due to Temperature Gradients in Complex Magnetic Field Configurations”, PHYS. FLUIDS **10**, 582 (1967).
13. J.W. CONNOR, R.J. HASTIE and J.B. TAYLOR, “Shear, Periodicity, and Plasma Ballooning Modes”, PHYS. REV. LETT. **40**, 396 (1978), and “High Mode Number Stability of an Axisymmetric Toroidal Plasma”, PROC. R. SOC. LOND. A. **365**, 1 (1979).
14. A. ROGISTER and G. HASSELBERG , Nonlinear Gyrokinetics of High β Axisymmetric Plasmas”, PLASMA PHYS. CONTROL. FUSION **27**, 193 (1985).
15. C. KITTEL, *Introduction to solid state physics*, Ch. 7 (John Wiley, New York, 1976).
16. A. ROGISTER, “Enhanced Shear Damping and Ion Transit Resonance Damping of Electron Drift Waves in Toroidal Plasmas”, PHYS. PLASMAS **2**, 2729 (1995).
17. J.W. CONNOR and R.J. HASTIE, “Microinstability in Tokamaks with Low Magnetic Shear”, PLASMA PHYS. CONTROL. FUSION, **46**, 1501 (2004).
18. P.H. RUTHERFORD, “Collisional Diffusion in an Axisymmetric Torus”, PHYS. FLUIDS **13**, 482 (1970).
19. A.L. ROGISTER, “Theory of Non-overlapping Low Frequency Modes in Axisymmetric Toroidal Plasmas”, PHYS. PLASMAS **7** (2000) 5070.
20. A.L. ROGISTER, “First Principle Model of Internal Transport Barriers in Negative Central Shear Discharges”, NUCL. FUSION **41** (2001)1101.
21. W.M. TANG, “Microinstability Theory in Tokamaks”, NUCL. FUSION **18**, 1089 (1978).
22. G. HASSELBERG and A. ROGISTER, “Ballooning Limit of Collisional Drift Waves Driven by Temperature Gradients”, PLASMA PHYS. **22**, 805 (1980).
23. J.B. TAYLOR, “Does Magnetic Shear Stabilise Drift Waves”, in *Plasma Physics and Controlled Nuclear Fusion Research 1976*, Vol.2, p. 323 (Proceedings of 6th International Conference, IAEA, Vienna, 1977).
24. M.J. LIGHTHILL, *Fourier Analysis and Generalised Functions* p. 67 and 68 (University Press, Cambridge, 1962).
25. A.L. ROGISTER, “Gyro and Drift Kinetic Equations in Toroidal Plasmas”, in *Proceedings of the Second Carolus Magnus Summer School on Plasma Physics*, TRANS. FUSION TECHNOLOGY, **29**, 2T, 81 (1996).
26. L. SPITZER, *Physics of Fully Ionised Gases*, Sections 1.2 and 2.5. (Interscience Publishers, New York, 1962).
27. W.M. MANHEIMER and C.N. LASHMORE-DAVIES, *MHD Microinstabilities in Confined Plasmas*, Chapt. 18 and 19 (Institute of Physics, Bristol, 1989).
28. B.B. KADOMTSEV and A.V. NEDOSPASOV, “Instability of the Positive Column in a Magnetic Field and the ‘Anomalous’ Diffusion Effect”, J. NUCL. ENERGY **C1**, 230 (1960).
29. H.P. FURTH, J. KILLEEN, and M.N. ROSENBLUTH, “Finite-Resistivity Instabilities of a Sheet Pinch”, PHYS. FLUIDS **6**, 459 (1963).
30. A. ROGISTER, “Theory of the Rippling Instability in Toroidal Devices”, PLASMA PHYS. CONTROL. FUSION **28**, 547 (1986).

31. S.V. NOVAKOVSKII, P.N. GUZDAR, J.F. DRAKE, C.S. LIU, and F.L. WAELBROECK, "New Unstable Branch of Drift Resistive Ballooning Modes in Tokamaks, PHYS. PLASMAS **2**, 781 (1995).
32. M.Z. TOKAR, "Synergy of Anomalous Transport and Radiation in the Density Limit", PHYS. REV. LETT. **91**, 095001 (2003).
33. P.N. GUZDAR, C.S. LIU, J.Q. DONG, and Y.C. LEE, "Model for Thermal Transport in Tokamaks", PHYS. REV. LETT. **57**, 2818 (1986).
34. A.L. ROGISTER, G. HASSELBERG, F.G. WAELBROECK, and J.WEILAND, "Transport through Dissipative Trapped Electron Mode and Toroidal Ion Temperature Gradient Mode in TEXTOR", NUCL. FUSION **28**, 1053 (1988), and A.L. ROGISTER, G. HASSELBERG, A. KALECK, A. BOILEAU, H.W.H. VAN ANDEL, and M. VON HELLERMANN, "Enhanced Small-Scale Turbulence Oscillations Correlated with Sawtooth Relaxations in the TEXTOR Tokamak", NUCL. FUSION **26**, 797 (1986).
35. R.E. WALTZ, R.R. DOMINGUEZ, F.W. PERKINS, "Drift Wave Model Tokamak Ignition Projections with a Zero-Dimensional Transport Code", NUCL. FUSION **29**, 351 (1989).
36. J. SHEFFIELD, "Tokamak Transport in the Presence of Multiple Mechanisms", NUCL. FUSION **29**, 1347 (1989).
37. A.B. RECHESTER and M.N. ROSENBLUTH, "Electron Heat Transport in a Tokamak with Destroyed Magnetic Surfaces", PHYS. REV. LETT. **40**, 38 (1978).
38. T.H. STIX, "Plasma Transport Across a Braided Magnetic Field", NUCL. FUSION **18**, 353 (1978).
39. P.H. REBUT, P.P. LALLIA, M.L. WATKINS, "The Critical Temperature Gradient Model of Plasma Transport: Applications to JET and Future Tokamaks", in *Plasma Physics and Controlled Nuclear Fusion Research 1988* Vol. **2**, p. 191 (Proceedings of the 12th International Conference, IAEA, Vienna, 1989).
40. B. COPPI, COMMENTS PLASMA PHYS. CONTR. FUSION **5**, 261 (1980).
41. W.M. MANHEIMER, K.R. CHU, E.OTT, and J.P. BORIS, "Marginal-Stability Calculation of Electron Temperature Profiles in Tokamaks", PHYS. REV. LETT. **37**, 286 (1976).
42. R.Z. SAGDEEV and A.A. GALEEV, *Nonlinear Plasma Theory*, Ch. III (Benjamin, New York, 1969).
43. A. ROGISTER and G. HASSELBERG, "Drift-Wave Spectra Obtained from the Theory of Nonlinear Ion-Landau Damping in Sheared Magnetic Fields", PHYS. REV. LETT. **48**, 249 (1982).
44. F.Y. GANG, P.H. DIAMOND, and M.N. ROSENBLUTH, "A Kinetic Theory of Trapped-Electron-Driven Drift Wave Turbulence in a Sheared Magnetic Field", PHYS. FLUIDS B **3**, 68 (1990).
45. B.B. KADOMTSEV and O.P. POGUTSE, "Turbulence in Toroidal Systems", in *Reviews of Plasma Physics*, Vol. 5, p. 249 (Edited by M.A. Leontovich, Consultants Bureau, New York, 1970).
46. B.B. KADOMTSEV, *Tokamak Plasma: a Complex Physical System* (Institute of Physics Publishing, Bristol & Philadelphia, 1992).
47. T.S. HAHM and K.H. BURREL, "Flow Shear Induced Fluctuation Suppression in Finite Aspect Ratio Shaped Tokamak Plasma", PHYS. PLASMAS **2**, 1648 (1995).
48. Z. LIN, T. HAHM, W. LEE, W. TANG and R. WHITE, " Turbulent Transport Reduction by Zonal Flows: Massively Parallel Simulations", SCIENCE 281, 1835 (1998).
49. P.H. DIAMOND and Y.-B. KIM, "Theory of Mean Poloidal Flow Generation by Turbulence", PHYS. FLUIDS B 3, 1626 (1991).
50. A.L. ROGISTER and D. LI, "Theoretical Model of the Plasma Edge, Part I", NUCL. FUSION **33**, 1799 (1993).
51. P.H. DIAMOND, M.N. ROSENBLUTH, F.L. HINTON, M. MALKOV, J. FLEISHER, AND A. SMOLYAKOV, "Dynamics of Zonal Flows and Self-Regulating Drift-Wave Turbulence", in *Fusion Energy 1998*, Vol.**4**, p. 1421 (Proceedings of the 17th International Conference, IAEA, Vienna, 1999).
52. R.D. SYDORA, V.K. DECYK, J.W. DAWSON, "Fluctuation-Induced Heat Transport Results from a Large Global 3D Toroidal Particle Simulation Model", PLASMA PHYS. CONTROL. FUSION **38**, A281 (1996).
53. D.W. ROSS, R.V. BRAVENEC, W. DORLAND, *et al.*, "Comparing Simulation of Plasma Turbulence with Experiment", PHYS. PLASMAS **9**, 177 (2002).
54. D.W. ROSS and W. DORLAND, "Comparing Simulation of Plasma Turbulence with Experiment. II. Gyrokinetic Simulations", PHYS. PLASMAS **9**, 5031 (2002).
55. X. GARBET, P. MANTICA, C. ANGIONI, *et al.*, "Physics of Transport in Plasmas", PLASMA PHYS. CONTROL. FUSION **46**, B557 (2004).
56. X. GARBET, "Introduction to Drift Wave Turbulence Modelling", those proceedings.