The nucleon axial–vector coupling beyond one loop[#1]

Véronique Bernard[⋆#2] and Ulf-G. Meißner[‡∗#3]

⋆*Université Louis Pasteur, Laboratoire de Physique Théorique*
3-5, rue de l'Université, F–67084 Strasbourg, France

‡*Universität Bonn, Helmholtz–Institut für Strahlen– und Kernphysik (Theorie)*
Nußallee 14-16, D-53115 Bonn, Germany

∗*Forschungszentrum Jülich, Institut für Kernphysik (Theorie)*
D-52425 Jülich, Germany

Abstract

We analyze the nucleon axial-vector coupling to two loops in chiral perturbation theory. We show that chiral extrapolations based on this representation require lattice data with pion masses below 300 MeV.

[#2]email: bernard@lpt6.u-strasbg.fr
[#3]email: meissner@itkp.uni-bonn.de

arXiv:hep-lat/0605010v1 14 May 2006

1. The axial–vector coupling constant g_A is a fundamental property of the nucleon that can e.g. be determined in neutron β–decay (for a review on the nucleons axial properties, see [1]). It is directly related to the fundamental pion–nucleon coupling constant by the Goldberger-Treiman relation and thus of great importance for the problem of nuclear binding. In the last few years, first attempts to calculate g_A using various approximations to QCD on a discretized space–time (lattice QCD) have been published, see e.g. [2–5]. These results are obtained for quark masses considerably larger than the physical ones, the lowest quark masses considered e.g. in the most recent study [5] correspond to a pion mass of about 350 MeV (which should already be close to the so–called chiral regime). It is therefore necessary to perform a chiral extrapolation to connect these lattice results with the physical values of the quark masses.[#4] Long before the advent of these lattice data it was noted that the chiral expansion of the axial–vector coupling does not show the expected convergence behaviour for an SU(2) quantity — the correction of order M_π^3 is of the order of 30% at the physical pion mass although it is two orders down compared to the leading term [8]. One therefore can not expect the one–loop representation to be very accurate for increasing pion mass. In fact, the complete (fourth order) one–loop result is steeply rising with growing M_π while the lattice data show essentially no pion mass dependence [#5]. A possible solution to this problem was offered in Ref. [9] where an effective field theory with explicit delta degrees of freedom at leading one–loop order could lead to a flat pion mass dependence of g_A, requiring, however, a fine tuning of certain low–energy constants. For a recent update, see [10]. Two remarks on that result are in order: First, it should also be noted that most lattice data available at that time are far outside the regime of applicability of the effective field theory. Second, to judge upon the usefulness of such an approach requires a systematic analysis of many other observables which has not been done so far[#6] (for further discussions on this issue, see e.g. [11,12]). To obtain a deeper understanding of the chiral expansion of g_A it appears therefore timely to go beyond the one–loop approximation in nucleon chiral perturbation theory. This is precisely the issue of this letter. Using renormalizations group methods, we will determine the coefficient of the double log in g_A – that arises first at two–loop order – from the existing one–loop results (note that the spectral function of the axial form factor to two loops was already worked out in Ref. [13]). We also give the general structure of the two–loop representation of g_A, and determine the numerically leading contributions to the single logarithm and polynomial terms at order M_π^4 and M_π^5, generated by graphs with insertion proportional to the large dimension two low–energy constants c_3 and c_4 and the dimension three LEC $\bar{d}_{16}$. We thus achieve more detailed information on the quark mass expansion of the nucleon axial–vector coupling constant.

2. Our calculation is based on the effective Lagrangian of pions and nucleons coupled to external sources. The various contributions to S-matrix elements and transition currents are organized in powers of the small parameter q, where q collectively denotes small pion four-momenta, the pion mass and baryon three-momenta. The effective SU(2) Lagrangian is given as a string of terms with increasing chiral dimension,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \mathcal{L}_{\pi N}^{(5)} + \mathcal{L}_{\pi N}^{(6)} + \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \dots \tag{1}$$

where the ellipsis denotes terms not needed in what follows. The local operators at the various orders are accompanied by low-energy constants (LECs), these are denoted as $c_i, d_i, e_i, \dots$ for the

---

[#4]In addition, one has to correct for finite volume and finite size effects, which we do not consider in what follows. For recent studies, see [6,7].

[#5]One should be somewhat cautious to draw too strong conclusions from such observations because most of the available lattice QCD results are far outside the range of applicability of chiral perturbation theory or any model-independent scheme.

[#6]This was to some extent attempted in [9] where one LEC was constrained by matching to the pion-nucleon theory, in which this LEC had been determined earlier.

dimension two, three, four, ... pion-nucleon Lagrangian and l_i for the mesonic LECs of dimension four. According to the power counting, the tree approximation is given by tree graphs with insertions from $\mathcal{L}_{\pi N}^{(1,2)}$. The one–loop approximation contains further tree graphs with insertions from $\mathcal{L}_{\pi N}^{(3,4)}$ and one–loop graphs with insertions from $\mathcal{L}_{\pi N}^{(1)}$ and at most one insertion from $\mathcal{L}_{\pi N}^{(2)}$. At two–loop order, we have two–loop graphs with insertions from $\mathcal{L}_{\pi N}^{(1)}$ and at most one insertion from $\mathcal{L}_{\pi N}^{(2)}$, one–loop graphs with insertions from $\mathcal{L}_{\pi N}^{(3,4)}$ and further tree graphs related to $\mathcal{L}_{\pi N}^{(5,6)}$ (and the corresponding mesonic contributions). Since we are interested in the quark mass expansion of the axial-vector coupling g_A, it is most convenient to work in the heavy baryon framework (for a review, see [14]). In two–flavor chiral perturbation theory, the quark mass expansion is mapped onto an expansion in the pion mass, whose physical value is denoted by M_π. Consequently, the chiral expansion of g_A takes the form

$$
\begin{aligned}
g_A &= g_0 \left\{ 1 + \left(\frac{\alpha_2}{(4\pi F)^2} \ln \frac{M_\pi}{\lambda} + \beta_2 \right) M_\pi^2 + \alpha_3 M_\pi^3 \right. \\
&\quad \left. + \left(\frac{\alpha_4}{(4\pi F)^4} \ln^2 \frac{M_\pi}{\lambda} + \frac{\gamma_4}{(4\pi F)^2} \ln \frac{M_\pi}{\lambda} + \beta_4 \right) M_\pi^4 + \alpha_5 M_\pi^5 \right\} + \mathcal{O}(M_\pi^6) , \\
&= g_0 \left\{ 1 + \Delta^{(2)} + \Delta^{(3)} + \Delta^{(4)} + \Delta^{(5)} \right\} + \mathcal{O}(M_\pi^6) ,
\end{aligned}
\tag{2}
$$

with g_0 the chiral limit value of g_A, $g_A = g_0[1 + \mathcal{O}(M_\pi^2)]$, λ is the scale of dimensional regularization, and $\Delta^{(n)}$ denotes the relative correction at order M_π^n. Further, F denotes the pion decay constant in the chiral limit, $F_\pi = F[1 + \mathcal{O}(M_\pi^2)]$. To the order we are working, we require the quark mass expansion of F_π,

$$
F_\pi = F \left[1 + \frac{M_\pi^2}{16\pi^2 F^2} \bar{\ell}_4 + \mathcal{O}(M_\pi^4) \right] ,
\tag{3}
$$

in terms of the scale–independent LEC

$$
\bar{\ell}_4 = 16\pi^2 \ell_4^r(\lambda) - 2\ln(M_\pi/\lambda) ,
\tag{4}
$$

where $\ell_4^r(\lambda)$ is the corresponding scale–dependent renormalized LEC. Note that this explicit quark mass dependence of F_π has to be accounted for when one studies the axial coupling as a function of the pion mass. Note also that when we generate the numerical value of $\ell_4^r(\lambda)$ from $\bar{\ell}_4$, we have of course to use the physical value of the pion mass in Eq. (4). Furthermore, the chiral expansion of the pion decay constant generates contributions to α_4, β_4, γ_4 and α_5. This can be seen from Eq. (2) which is expressed in terms of the chiral limit value F instead of the physical value, as it is commonly done. At a fixed pion mass, these two representations are of course equivalent. The explicit expressions of these additional quark mass dependent terms are given below. The third order one-loop coefficients α_2 and β_2 in Eq. (2) were first given in [15] and the one–loop fourth order calculation was completed in [8] with (we use the by now standard notation of Refs. [14, 16])

$$
\begin{aligned}
\alpha_2 &= -2 - 4g_0^2 , \\
\beta_2 &= \frac{4}{g_0} \left(d_{16}^r(\lambda) - 2g_0 d_{28}^r(\lambda) \right) - \frac{g_0^2}{(4\pi F)^2} , \\
\alpha_3 &= \frac{1}{24\pi F^2 m_0} \left(3 + 3g_0^2 - 4m_0 c_3 + 8m_0 c_4 \right) ,
\end{aligned}
\tag{5}
$$

with

$$
\begin{aligned}
d_{16}^r(\lambda) &= \bar{d}_{16} + \frac{g_0(4 - g_0^2)}{8(4\pi F)^2} \ln \frac{M_\pi}{\lambda} , \\
d_{28}^r(\lambda) &= \bar{d}_{28} - \frac{9g_0}{16(4\pi F)^2} \ln \frac{M_\pi}{\lambda} .
\end{aligned}
\tag{6}
$$

As in [16], we set $\bar{d}_{28} = 0$ in what follows. Again, note that these relations for $d_i^r(\lambda)$ ($i = 16, 28$) have to be taken at the physical value of M_π when it comes to pin down their numerical values. The dimension two LECs c_3, c_4 can be determined e.g. from the analysis of elastic pion–nucleon scattering and the dimension three LEC d_{16} from the reaction $\pi N \to \pi\pi N$ (for a detailed discussion see e.g. Ref. [11] and references therein). In Eq. (5), m_0 denotes the nucleon mass in the chiral limit. To the order we are working, it is related to the physical nucleon mass m_N via

$$m_N = m_0 - 4c_1 M_\pi^2 + \mathcal{O}(M_\pi^3) \ , \tag{7}$$

with c_1 another dimension two LEC that can be determined e.g. from low energy pion-nucleon scattering data or the pion–nucleon sigma term. This quark mass dependence of the nucleon mass induces corrections at fifth order from the third order coefficient $\alpha_3 \sim 1/m_N$. In what follows, we always work with m_0 and absorb this induced contribution in the combination of LECs contributing to α_5.

In this letter, we are going to evaluate the coefficient α_4 of the double logarithm which arises at two–loop order. This requires only parameters from the one–loop calculation, as already stressed by Weinberg in his seminal paper [17]. The coefficients β_4, γ_4 and α_5 contain combinations of LECs from $\mathcal{L}_{\pi N}^{(2,3)}$ and unknown LECs from $\mathcal{L}_{\pi N}^{(4,5,6)}$, we will estimate these using naturalness arguments and also from the description of the available lattice data (at small enough pion masses). Note that we can, in addition, work out the numerically large contributions to these coefficients $\sim c_i/m_0$ and $\sim c_i/m_0^2$ from the expansion of the corresponding relativistic one-loop graphs (together with the induced contributions from the quark mass expansion of F_π).

3. The application of renormalization group (RG) methods to chiral effective Lagrangians was pioneered by Weinberg [17]. He showed that the coefficient of the double log $\sim \ln^2 M_\pi$ can be entirely expressed in terms of coupling constants of the one–loop generating functional. For recent applications of such RG methods in chiral perturbation theory for mesons, see e.g. [18–20], a nice discussion of this and related issues is given in [21]. Here, we wish to apply the same arguments to the effective pion–nucleon Lagrangian. According to the power counting, the double logs are generated from two–loop graphs at $\mathcal{O}(q^5)$. Employing a mass–independent renormalization scheme (here: dimensional regularization), the two–loop divergences take the generic form

$$k(d) \frac{\lambda^{2\epsilon}}{(4\pi)^4} \left[\frac{1}{\epsilon^2} + \frac{2}{\epsilon} \ln \frac{M_\pi}{\lambda} + \ln^2 \frac{M_\pi}{\lambda} + \dots \right] \ , \tag{8}$$

with d the number of space–time dimensions, $\epsilon = d - 4$ and $k(d)$ is a function of d that depends on the specific diagram under consideration. This function can also be expanded around $d = 4$, $k(d) = k_0 + k_1\epsilon + \mathcal{O}(\epsilon^2)$. The leading term in this expansion generates the non-local divergence $\sim k_0 \ln M/\epsilon$ that must be cancelled by one–loop graphs with insertions from the dimension three effective pion–nucleon Lagrangian (parameterized by the unrenormalized LECs d_i). Such graphs give the generic contribution

$$-\frac{h_i(d)}{2} \frac{\lambda^{2\epsilon}}{(4\pi)^4} \left[\frac{\kappa_i}{\epsilon^2} + \frac{\kappa_i}{\epsilon} \ln \frac{M_\pi}{\lambda} + \frac{(4\pi)^2 d_i^r(\lambda)}{\epsilon} + (4\pi)^2 d_i^r(\lambda) \ln \frac{M_\pi}{\lambda} + \dots \right] \ , \tag{9}$$

where the d_i are the dimension three LECs that have the form [16]

$$d_i(d) = \lambda^\epsilon \left[\frac{\kappa_i}{(4\pi)^2 \epsilon} + d_i^r(\lambda) + \dots \right] \ , \tag{10}$$

where we use the basis of operators enumerated in [16] with the corresponding β–functions κ_i listed there. Here, $h_i(d)$ is a function specific for the coefficient under consideration, that itself depends on
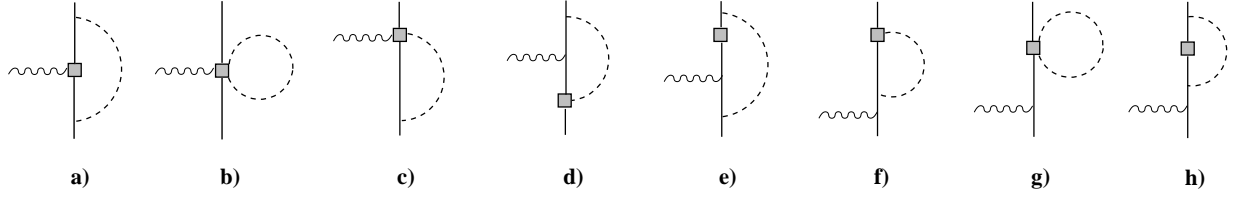
4

Figure 1: Topologies of the one–loop graphs that generate the coefficient of the double log at two–loop order. The hatched square denotes a dimension three insertion proportional to some of the LECs d_i.

d via $h_i(d) = h_{i0} + h_{i1}\epsilon + \mathcal{O}(\epsilon^2)$. As shown by Weinberg, the elimination of the non–local divergence is guaranteed by the RG condition

$$k_0 = \frac{1}{4} h_{i0} \kappa_i . \tag{11}$$

Using this equation, we can now calculate the coefficient of the double log. There are two types of diagrams contributing, namely irreducible and reducible ones, the latter being related to wave function renormalization. It is important to note that the notion of reducibility here refers to the two-loop graphs. The non-vanishing irreducible two-loop contribution is generated from the graphs shown in Fig. 1. The following operators (given in terms of their LECs) contribute to the various graphs: a) d_{16}, d_{25} b) d_{10}, d_{11}, d_{12}, d_{13}, and d_{16}, c), d_1, d_2, d_{14}, d_{26} and d_{30} d) d_{16} and d_{29}, e) d_{24} and d_{28}, f) d_{16}, d_{25} and d_{29}, g) d_{26}, d_{27} and d_{28}, and h) d_{24} and d_{28}. Using the β-functions from [16] gives the contribution to the double log generated by these diagrams. It reads

$$\alpha_4^{\text{irr}} = 2 \left(\frac{4}{3} + \frac{5}{3}g_0^2 - g_0^4 \right) . \tag{12}$$

Furthermore, there are reducible graphs generated from wave function renormalization. These are given by $g_A^{1-\text{loop}} \cdot Z^{1-\text{loop}}$ and their contribution to the double log is

$$\alpha_4^{\text{red}} = 9 \left(g_0^2 + 2g_0^4 \right) . \tag{13}$$

Putting pieces together (i.e. the contributions from the irreducible diagrams, Eq. (12), the reducible graphs, Eq. (13), and the induced term $\tilde{\alpha}_4$, see Eq. (17) below), we have thus for the coefficient $k_0 \equiv \alpha_4$ of the double log

$$\alpha_4 = \alpha_4^{\text{irr}} + \alpha_4^{\text{red}} + \tilde{\alpha}_4 = -\frac{16}{3} - \frac{11}{3}g_0^2 + 16g_0^4 . \tag{14}$$

This is the central result of this paper and allows us to analyze the leading two–loop correction to the axial–vector coupling constant. In the formulation using the Lagrangian given in [16], one has to deal with a large number of equation of motion terms. These can be, however, eliminated from the effective Lagrangian as done in [22]. We have therefore also performed the calculation in the basis given in that paper and employing the pertinent β-functions. We find the same result as in Eq.(14), which serves as an excellent check on our calculation[#7]. Note that this procedure generates also part of the single log coefficient γ_4 in Eq. (2). We only give the contribution generated from the operator proportional to the axial LEC d_{16} (note that some of the other LECs d_i are also known, see Ref. [24], but only d_{16} plays a prominent role in the chiral expansion of g_A)

$$\gamma_4^{d_{16}} = -12 \, d_{16}^r(\lambda) \left(\frac{5}{3g_0} + g_0 \right) . \tag{15}$$

[#7]Provided one corrects for the typographical error in β_{11} in that paper, see also [23].

Note further that using a relativistic formulation (see e.g. Ref. [25]), it is easy to work out the $1/m_0$ and $1/m_0^2$ corrections to the large contribution $\alpha_3 \simeq 94$ (for the parameters given below, see also the discussion in [11]) that will give a sizeable contribution to the coefficients β_4, γ_4 and α_5, respectively. These terms are given by

$$
\begin{aligned}
\gamma_4^{c_i} &= \frac{4(c_4 - c_3)}{m_0} \ , \\
\beta_4^{c_i} &= \frac{c_4}{m_0} \frac{1}{4\pi^2 F^2} \ , \\
\alpha_5^{c_i} &= \frac{c_3}{m_0^2} \frac{1}{16\pi F^2} \ .
\end{aligned}
\tag{16}
$$

The numerical values of these contributions will be given below, but we remark already that in particular $\gamma_4^{c_i}$ will contribute sizeably. Finally, we collect here the induced terms form the quark mass expansion of the pion decay constant, cf. Eq. (3). Denoting these by a tilde, they read

$$
\begin{aligned}
\tilde{\alpha}_4 &= 4\,\alpha_2 \ , \\
\tilde{\gamma}_4 &= -\frac{2}{F^2}\,\alpha_2\,l_4^r(\lambda) - \frac{4g_0^2}{(4\pi F)^2} \ , \\
\tilde{\beta}_4 &= \frac{2g_0^2}{(4\pi F)^2 F^2} l_4^r(\lambda) \ , \\
\tilde{\alpha}_5 &= -\frac{2\alpha_3}{F^2}\left(l_4^r(\lambda) - \frac{1}{8\pi^2}\ln\frac{M_\pi}{\lambda}\right) \ .
\end{aligned}
\tag{17}
$$

We end this section with a brief comment of the one-loop chiral EFT representation given in [9]. Including an explicit delta to leading one–loop order generates some of the terms $\sim M_\pi^3$ and part of the coefficients γ_4 and β_4 (plus some other higher order terms $\sim M_\pi^{2n}/\Delta^{2m}$ with $2n - 2m = 2$ and Δ is the delta-nucleon mass splitting). However, already at third order in the pion mass, this neglects other resonance contributions to the LECs c_3 and c_4 (for a detailed discussion, see e.g. [26]) and therefore that representation can not be considered as accurate as the one developed here (for pion masses in the chiral regime).

4. We are now in the position to put pieces together. First, we consider the contribution of the various (incomplete) terms at fourth and fifth order in the pion mass for the physical values of the quark masses. For that, we express all parameters in terms of their chiral limit values. We take $\bar{\ell}_4 = 4.33$ corresponding to $F = 87\,\mathrm{MeV}$ and $m_0 = 880\,\mathrm{MeV}$. If not stated otherwise, we use $\bar{d}_{16} = -1.76\,\mathrm{GeV}^{-2}$, $c_3 = 3.5\,\mathrm{GeV}^{-1}$ and $c_4 = -4.7\,\mathrm{GeV}^{-1}$. We also work at $\lambda = m_0$. Note that we have varied these LECs within their allowed ranges, but this did not lead to any sizeable changes to the results given below. Furthermore, we vary g_0 between 1.0 and 1.2. To be definite, let us set $g_0 = 1$. Collecting pieces, we obtain

$$
\begin{aligned}
\alpha_4 &= \alpha_4^{\mathrm{irr+red}} + \tilde{\alpha}_4 = 31 - 24 = 7 \ , \\
\gamma_4 &= \gamma_4^{c_i} + \tilde{\gamma}_4 + \gamma_4^{d_{16}} = (37.3 + 3.2 + 74.8)\ \mathrm{GeV}^{-2} = 115.3\ \mathrm{GeV}^{-2} \ , \\
\beta_4 &= \beta_4^{c_i} + \tilde{\beta}_4 = (13.3 + 0.9)\ \mathrm{GeV}^{-4} = 14.2\ \mathrm{GeV}^{-4} \ , \\
\alpha_5 &= \alpha_5^{c_i} + \tilde{\alpha}_5 = (-16.0 - 4.3)\ \mathrm{GeV}^{-5} = -20.3\ \mathrm{GeV}^{-5} \ .
\end{aligned}
\tag{18}
$$

Of course, the coefficients γ_4, β_4 and α_5 receive further corrections from LECs that have to be determined e.g. from an analysis of lattice data or estimated assuming naturalness. We denote these
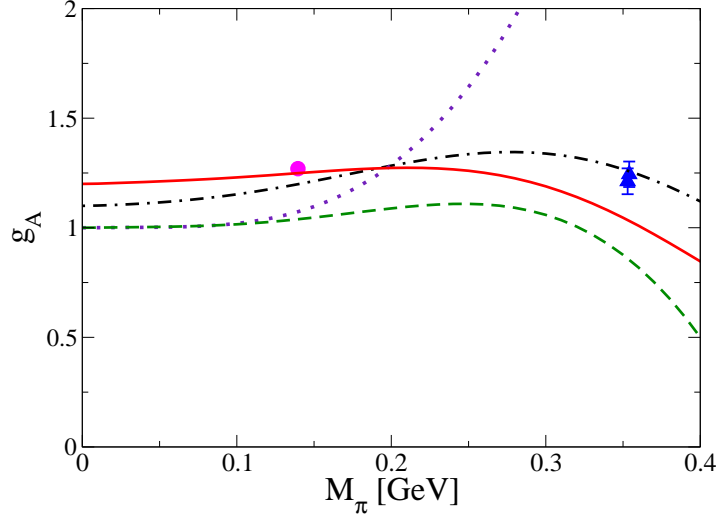
Figure 2: The axial-vector coupling as a function of the pion mass. Solid (red) line: $g_0 = 1.2, \bar{d}_{16} = -1.76\,\text{GeV}^{-2}$, $\gamma_4^f = 50\,\text{GeV}^{-2}$, $\beta_4^f = 60\,\text{GeV}^{-4}$, $\alpha_5^f = 20\,\text{GeV}^{-5}$; Dot-dashed (black) line: $g_0 = 1.1, \bar{d}_{16} = -0.92\,\text{GeV}^{-2}$, $\gamma_4^f = 40\,\text{GeV}^{-2}$, $\beta_4^f = 20\,\text{GeV}^{-4}$, $\alpha_5^f = 50\,\text{GeV}^{-5}$; Dashed (green) line: $g_0 = 1.0, \bar{d}_{16} = -1.76\,\text{GeV}^{-2}$, $\gamma_4^f = -50\,\text{GeV}^{-2}$, $\beta_4^f = \alpha_5^f = 0$. The dotted (violet) line is the complete one–loop result with $g_0 = 1, \bar{d}_{16} = -1.76\,\text{GeV}^{-2}$ and using the physical values of the nucleon mass and the pion decay constant. The (magenta) circle denotes the physical value of g_A at the physical pion mass and the triangles are the lowest mass data from Ref. [5].

additional contributions by γ_4^f, β_4^f and α_5^f. For the moment, we set $\gamma_4^f = \beta_4^f = \alpha_5^f = 0$. In the notation of Eq. (2), these results translate into

$$
\begin{aligned}
\Delta^{(2)} &= -15.3\ \%\ , \\
\Delta^{(3)} &= 25.6\ \%\ , \\
\Delta^{(4)} &= \Delta_\alpha^{(4)} + \Delta_\gamma^{(4)} + \Delta_\beta^{(4)} = (0.6 - 6.3 + 0.5)\ \% = -5.6\ \%\ , \\
\Delta^{(5)} &= -0.1\ \%\ .
\end{aligned}
\tag{19}
$$

The sizeable fourth order contribution is entirely due to the large coefficient γ_4, largely due to the insertion of the operator $\sim d_{16}$, cf. Eq. (15). Note also that the fifth order term is very small at the physical point. Ignoring the higher order corrections, one can calculate the chiral limit value of g_A from Eq. (2) using the values collected in Eq. (19),

$$
g_0 = 1.21\ [1.12]\ ,
\tag{20}
$$

where the number in the brackets refers to the choice $\bar{d}_{16} = -0.92\,\text{GeV}^{-2}$ and we use $g_A = 1.267$. Of course, these numbers will be affected by the unknown LEC contributions γ_4^f, β_4^f and α_5^f. These values are consistent with the findings in [7]. We also remark that the chiral expansion of m_N is much better behaved and one thus can successfully apply one–loop extrapolation functions to pion masses below 450 MeV (for detailed discussions, see e.g. [11, 27, 28]).

We show in Fig. 2 some typical examples for the pion mass dependence of g_A for values of $\gamma_4^f, \beta_4^f, \alpha_5^f$ that lead to an approximately flat behaviour for not too high pion masses. These values are of natural size as a comparison with the induced pieces collected in Eq. (18) reveals. This is very different from the one–loop representation, which fails to generate a flat quark mass dependence for values above

the physical pion mass, see e.g. [9, 11]. We have studied many more combinations of the LECs and found that for this representation to be useful (that is leading to a moderate theoretical uncertainty), the pion mass should be less then 300 MeV. This can also been seen if one compares to the complete one-loop result as depicted by the dotted line in Fig. 2. The existing lattice results are at still too high pion masses for a model-independent extrapolation to the physical values of the quark masses. For a particular choice of the LECs, we can describe the trend of the lattice data up to $M_\pi \simeq 600\,\text{MeV}$, but the theoretical uncertainty is simply too large for such values of the pion mass.

5. In this letter, we have studied the pion mass dependence of the nucleon axial-vector coupling constant g_A. This is a fundamental observable for our understanding of the nucleon structure in the regime of strong QCD. First lattice simulations have appeared and so far, chiral extrapolation functions appearing in the literature are either based on (leading) one-loop chiral effective field theory results with explicit deltas (for a critical discussion, see e.g. [12]) or are very model-dependent. We have provided the two–loop representation in baryon chiral perturbation theory, see Eq. (2), and determined the coefficient of the double log term $\sim M_\pi^4 \ln^2 M_\pi$ based on renormalization group arguments. We have also determined some numerically important contributions to the terms $\sim M_\pi^4 \ln M_\pi, M_\pi^4$ and M_π^5. We have shown that with LECs of natural size one can indeed obtain a flat pion mass dependence of g_A for pion masses below 400 MeV. We conclude that lattice data for pion masses below 300 MeV are required to use this representation with a moderate theoretical uncertainty. Such data should be available in the near future.

Acknowledgements

References

[1] V. Bernard, L. Elouadrhiri and U.-G. Meißner, J. Phys. G **28** (2002) R1 [arXiv:hep-ph/0107088].

[2] S. Ohta [RBC Collaboration], Nucl. Phys. Proc. Suppl. **119** (2003) 389 [arXiv:hep-lat/0210006].

[3] S. Sasaki, K. Orginos, S. Ohta and T. Blum [the RIKEN-BNL-Columbia-KEK Collaboration], Phys. Rev. D **68** (2003) 054509 [arXiv:hep-lat/0306007].

[4] A. A. Khan *et al.*, Nucl. Phys. Proc. Suppl. **140** (2005) 408 [arXiv:hep-lat/0409161].

[5] R. G. Edwards *et al.* [LHPC Collaboration], Phys. Rev. Lett. **96** (2006) 052001 [arXiv:hep-lat/0510062].

[6] S. R. Beane and M. J. Savage, Phys. Rev. D **70** (2004) 074029 [arXiv:hep-ph/0404131].

[7] A. A. Khan *et al.*, arXiv:hep-lat/0603028.

[8] J. Kambor and M. Mojžiš, JHEP **9904** (1999) 031 [arXiv:hep-ph/9901235].

[9] T. R. Hemmert, M. Procura and W. Weise, Phys. Rev. D **68** (2003) 075009 [arXiv:hep-lat/0303002].

[10] M. Procura, B. U. Musch, T. R. Hemmert and W. Weise, Nucl. Phys. Proc. Suppl. **153** (2006) 229 [arXiv:hep-lat/0512026].

[11] U.-G. Meißner, PoS **LATT2005** (2005) 009 [arXiv:hep-lat/0509029].

[12] G. Colangelo, A. Fuhrer and C. Haefeli, Nucl. Phys. Proc. Suppl. **153** (2006) 41 [arXiv:hep-lat/0512002].

[13] V. Bernard, N. Kaiser and U.-G. Meißner, Nucl. Phys. A **611** (1996) 429 [arXiv:hep-ph/9607428].

[14] V. Bernard, N. Kaiser and U.-G. Meißner, Int. J. Mod. Phys. E **4** (1995) 193 [arXiv:hep-ph/9501384].

[15] V. Bernard, N. Kaiser, J. Kambor and U.-G. Meißner, Nucl. Phys. B **388** (1992) 315.

[16] N. Fettes, U.-G. Meißner and S. Steininger, Nucl. Phys. A **640** (1998) 199 [arXiv:hep-ph/9803266].

[17] S. Weinberg, PhysicaA **96** (1979) 327.

[18] G. Colangelo, Phys. Lett. B **350** (1995) 85 [Erratum-ibid. B **361** (1995) 234] [arXiv:hep-ph/9502285].

[19] J. Bijnens, G. Colangelo and G. Ecker, Phys. Lett. B **441** (1998) 437 [arXiv:hep-ph/9808421].

[20] M. Büchler, arXiv:hep-ph/0511087.

[21] M. Büchler and G. Colangelo, Eur. Phys. J. C **32** (2003) 427 [arXiv:hep-ph/0309049].

[22] G. Ecker and M. Mojžiš, Phys. Lett. B **365** (1996) 312 [arXiv:hep-ph/9508204].

[23] J. Gasser, M. A. Ivanov, E. Lipartia, M. Mojžiš and A. Rusetsky, Eur. Phys. J. C **26** (2002) 13 [arXiv:hep-ph/0206068].

[24] N. Fettes, V. Bernard and U.-G. Meißner, Nucl. Phys. A **669** (2000) 269 [arXiv:hep-ph/9907276].

[25] J. Schweizer, "Low Energy Representation for the Axial Form Factor of the Nucleon", Master Thesis, University of Bern, 2000.

[26] V. Bernard, N. Kaiser and U.-G. Meißner, Nucl. Phys. A **615** (1997) 483 [arXiv:hep-ph/9611253].

[27] V. Bernard, T. R. Hemmert and U.-G. Meißner, Nucl. Phys. A **732** (2004) 149 [arXiv:hep-ph/0307115].

[28] M. Frink, U.-G. Meißner and I. Scheller, Eur. Phys. J. A **24** (2005) 395 [arXiv:hep-lat/0501024].