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Flux modulation scheme for direct current SQUID readout revisited

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The flux modulation scheme (FMS) is the standard readout technique of dc SQUIDs, where a
step-up transformer links the SQUID to the preamplifier. The transformer’s primary winding shunts
the SQUID via a large capacitor while the secondary winding connects it to the preamplifier. A
modulation flux having a frequency of typically 100 kHz generates an ac voltage across
the SQUID, stepped up by the transformer. The SQUID with FMS is customarily operated in the
current bias mode, because a constant dc bias current flows only through the SQUID due to the
capacitor isolation. With FMS, however, the transformer ac shunts the SQUID so that in reality
the operating mode is neither purely current-biased nor voltage-biased but rather nominal current-
biased or “mixed biased.” Our objective is to experimentally investigate the consequences of ac
shunting of the dc SQUID in FMS and the transformer’s transfer characteristics. For different shunt
values we measure the change in the SQUID bias current due to the ac shunt using another SQUID
in the two-stage readout scheme, and simultaneously monitor the SQUID output voltage signal. We
then explain our measurements by a simplified graphic analysis of SQUID intrinsic current-voltage
(I–V) characteristics. Since the total current flowing through the SQUID is not constant due to the
shunting effect of the transformer, the amplitude of SQUID flux-to-voltage characteristics V(Φ)
is less as compared to the direct readout scheme (DRS). Furthermore, we analyze and compare V(Φ)
obtained by DRS and FMS. We show that in FMS, the transfer characteristics of the SQUID circuit
also depend on the isolation capacitance and the dynamic resistance of the SQUID. © 2016
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Usually, a dc SQUID is operated either with a constant
current source (current bias) or a constant voltage source
(voltage bias).1 The bias mode does not affect the noise
properties of the SQUID readout, neither does the SQUID
intrinsic noise δΦ nor the noise contribution δΦe from the
readout electronics.2 The matching between SQUID and
readout electronics has always been a serious problem in the
SQUID system development. During the last half century,
five solutions have been employed to improve this matching;
(1) the flux modulation scheme (FMS),3 (2) additional posi-
tive feedback (APF), i.e., a voltage feedback circuit shunting
the SQUID,4 (3) the two-stage scheme, which uses a second-
ary SQUID to measure the output of primary SQUID,5 (4) a
weakly damped SQUID in direct readout scheme (DRS),6,7
and (5) an unsheilded SQUID with a reference junction
to form the double relaxation oscillation SQUID (DROS)
readout scheme.8,9

Since 1967, FMS has been the standard readout tech-
nique for the current biased SQUID.3 A modulation flux ΦM
having typically a frequency of 100 kHz generates an ac volt-
age across the SQUID, where the ΦM = Φ0/2, half of the flux
quantum, is the magnetic ac flux constantly threading the

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SQUID under test is measured with the help of a high-impedance linear amplifier. (2) The SQUID signal, \( V(\Phi) \), obtained by square wave FMS is experimentally compared with that measured by DRS. We measure the transfer characteristics of the SQUID circuit with a transformer and find that SQUID’s dynamic resistance \( R_d \) plays an important role herein, also resulting in a sinusoidal response at the preamplifier’s input.

The general equivalent bias circuit in FMS can be represented by a constant dc bias current \( I_b \) impressed on the SQUID shunted by an element with an impedance \( Z_s \), see Figure 1(a). Here, the SQUID bias mode is determined by the ratio \( \alpha = Z_s/R_d \). The dimensionless parameter \( \alpha \) characterizes the type of bias. Two extreme cases are: (1) \( Z_s \) is absent (\( \alpha \rightarrow \infty \)) and (2) \( Z_s \) is very small, i.e., \( \alpha \rightarrow 0 \). Case (1) can be attained by connecting the SQUID to a high-impedance OP in DRS. In this case, the current \( I_s \) flowing through the SQUID, recorded by the ammeter \( A \) in Figure 1(a) and voltmeter \( V \) in Figure 1(b) represents the voltage across the SQUID, yielding the \( I-V \) and the \( V(\Phi) \) characteristics. Only this biasing scheme with \( I_s = \text{constant} \) can truly be called the “current bias mode”. In case (2) with \( \alpha \rightarrow 0 \), \( I_s \) flows predominantly through a small resistor \( R_s \) replacing \( Z_s \) to produce a quasi-constant voltage \( V_b \) across \( R_s \) and the SQUID connected in parallel. In this case, \( V_b \) remains constant and independent of \( \Phi \). At a selected \( V_b = I_b \times R_s = \text{constant} \), we obtain the \( I(\Phi) \) characteristics. This is the voltage bias mode. In either bias mode, the maximal transfer coefficient \( \partial V/\partial \Phi \) or \( \partial I/\partial \Phi \) is reached, helping to reduce the relative weight of \( \delta \Phi \) in the sum of \( \delta \Phi_s \) and \( \delta \Phi_n \), where \( \delta \Phi_s \) is the SQUID intrinsic noise. When \( 0 < \alpha < \infty \), we consider the bias mode to be a “mixed bias mode.”

In the discussion of the bias mode, the SQUID can be represented by \( R_d \). The impedance \( Z_s \) of the transformer PW is usually comparable to \( R_d \), both are of the same order of magnitude. This leads to \( 0 < \alpha < \infty \). Consequently, \( I_s \) varies periodically with \( \Phi_s \), so the device is no longer operated in a pure current bias mode, Figure 1(b). The ac current \( \Delta i \), generated by the ac \( \Phi_M \) of amplitude \( \Phi_0/2 \), rings through the closed circuit consisting of the SQUID and PW, thus leading to \( I_s = (I_b - \Delta i) \). The product of \( \Delta i \times Z_s \) is the ac voltage across PW. Consequently, in this case the amperometer \( A \) and voltmeter \( V \) of Figure 1(a) synchronously indicate \( I(\Phi) \) and \( V(\Phi) \) signals. It means that the SQUID operates in the mixed bias mode. Both signals are smaller than in the two extreme cases above.

For simplicity, to avoid phase shifts and a nonlinear frequency response from a SQUID circuit with a transformer, we replaced the shunt impedance \( Z_s \) by a variable resistor \( R_s \) in our first experiment. To synchronously measure the change in the SQUID current \( \Delta i \) and the SQUID output voltage \( V_{\text{out}} \), we designed and used the test circuit shown in Figure 2. The measured SQUID (SQ1) was impressed with a constant \( I_b \) and modulated by a sinusoidal \( \Phi_M \) of amplitude \( \Phi_0/2 \), because the slew rate of SQ2 electronics did not permit us to use square wave modulation. Therefore, an ac voltage was generated across the SQUID producing an ac current \( \Delta i \) through it and the shunting resistor \( R_s \), with the capacitor \( C \) providing a pathway for the ac signal. We used a second SQUID (SQ2) to monitor \( \Delta i \) through the first SQUID (SQ1). To accomplish this, SQ1 was connected in series with the integrated planar superconducting coil \( L_i \) inductively coupled via \( M_i \) to the SQUID (SQ2). The SQ2 operated in flux locked loop (FLL) thus served as the amperometer \( A \) of Figure 1(a). For calibration, we directly injected a known current through \( L_i \) (without SQ1) to measure the change of \( \Delta V_{\text{out}} \) of SQ2, thus determining the current-to-voltage transfer coefficient \( \Delta V_{\text{out}}/\Delta i = 530 \text{ mV/\mu A} \) at \( R_f = 5 \text{ k} \Omega \), \( M_i = 1.5 \text{ nH} \). The SQ1 ac output voltage \( V_{\text{out}} \) was monitored at the output of a linear amplifier (OP) with a gain of \( G = 1000 \). We selected different \( R_s \) to determine \( \Delta i \) from measured \( \Delta V_{\text{out}} \) and to record \( V_{\text{out}} \) at a given \( R_f \), and so obtained the dependence of both \( \Delta i \) and \( V_{\text{out}} \) on \( R_s \).

In the experiment, the SQ1 had the loop inductance \( L_s \sim 150 \text{ pH} \) and the junction shunt resistor of each junction equal \( 9 \Omega \). When \( R_s \) is absent (\( \alpha \rightarrow \infty \)), one obtains the \( I-V \) characteristics of the SQUID itself at \( \Phi = (n + 0.5) \Phi_0 \) (half-integer flux) and \( \Phi = n \Phi_0 \) (integer flux), which are the intrinsic SQUID properties shown in Figure 3. We introduce this figure to offer a simple graphic interpretation of our data resulting from the complex nonlinear behavior of the shunted SQUID. At a selected bias current \( I_b = I_b \), the SQUID voltage swing \( V_{\text{swing}} \) is denoted by the voltage difference \( V_{\text{swing}} = (V_{0.5} - V_0) \) between two intersections of the \( I \) horizontal line with the \( I-V \) curves at \( \Phi = n \Phi_0 \) (red) and \( \Phi = (n + 0.5) \Phi_0 \) (blue). Here, \( V_0 \) is the intersection voltage at \( \Phi = n \Phi_0 \) and, \( V_{0.5} \) at \( \Phi = (n + 0.5) \Phi_0 \). If \( V_0 \approx 0 \) (we assume it in this paper), then \( V_{\text{swing}} \approx V_{0.5} \approx 22 \mu \text{V} \), which is the maximally readable \( V_{\text{swing}} \) of SQ1.

![FIG. 1. (a) Equivalent circuit of bias mode of a shunted dc SQUID. The ammeter \( A \) and the voltmeter \( V \) symbolize the ability to measure current and voltage. (b) SQUID circuit in FMS with ac (square wave) \( \Phi_M \) of half a \( \Phi_0 \).](image1)

![FIG. 2. The test circuit for determining \( V_{\text{out}} \) and \( \Delta i \) of the SQUID (SQ1) with a sinusoidal flux modulation \( \Phi_M \) of \( \Phi_0/2 \) and constant \( I_b \) at different shunts \( R_s \). The current \( \Delta i \) is inferred from measured \( \Delta V_{\text{out}} \) of the SQUID (SQ2) operated in FLL, as described in the text. The ac voltage \( V_{\text{out}} \) is monitored at the output of the linear amplifier with a gain \( G = 1000 \).](image2)
The finite shunt resistor $R_s$ has two consequences; $\Delta i$ is generated and $V_{\text{swing}}$ is reduced. In terms of Figure 1(b), $\Delta i$ and $V_{\text{out}}$ are the synchronous readings of $I(\Phi)$ and $V(\Phi)$ signals by the ammeter $A$ and the voltmeter $V$. Table I gives the $\Delta i$ and $V_{\text{out}}$ of SQ1 measured at different $R_s$. The value of directly measured $V_{\text{out}}$ agrees rather well with $\Delta i \times R_s$, the product of the measured ac current $\Delta i$ and the selected $R_s$.

As I-V characteristics of SQ1 change from $\Phi = n\Phi_0$ to $\Phi = (n+0.5)\Phi_0$, $I_s = (I_b - \Delta i)$, and the output voltage $V_{\text{out}}$ change synchronously with $\Phi$. Figure 3 shows $I_s$ at $\Phi = (n+0.5)\Phi_0$ and $R_s = 5 \Omega$. The dependence $R_d^0.5(V)$ plotted at $\Phi = (n+0.5)\Phi_0$ (black curve) illustrates the nonlinearity of the I-V curve, which also changes nonlinearly with $\Phi$. At different $R_s$ we can obtain $V_{\text{Rs}}$ ($V_{\text{swing}}$) by graphical analysis. As noted above, the $V_{\text{swing}}$ obtained at the intersection of the horizontal line $I_s = (I_b - \Delta i)$ and the I-V curve at $\Phi = (n+0.5)\Phi_0$ agreed with experimental $V_{\text{out}}$ rather well, $V_{\text{Rs}} \approx V_{\text{out}}$. At any other $R_s$ a $V_{\text{swing}} < V_{\text{swing}}(0.5)$ determined as shown in Figure 3 will be on the inclined load line also plotted in Figure 3. The changing $I_s(\Phi)$ is an essential feature of FMS.

Now, we discuss the more realistic case of a square wave FMS with a transformer, as shown in Figure 1(b). Using a modulation technique with a frequency $f_M$, the original SQUID $V(\Phi)$ obtained by DRS with $f << f_M$ after demodulation should reappear without any distortion. However, the presence of the transformer results not only in the shunting effect discussed above, but also in introducing a frequency bandpass effect and a phase shift, so that the original $V(\Phi)$ are distorted at the transformer’s SW.

In our second experiment, we compare the SQUID signals, $V(\Phi)$, obtained by DRS and FMS with square wave $\Phi_{SM}$ of half a $\Phi_0$. Here, a typical transformer with enameled wire winding on a ferrite ring was employed. The SW/PW ratio, $n$, was 20. Inductances of $L_{PW} \approx 12 \mu H$ and $L_{SW} \approx 4.3 \text{ mH}$ were measured at $f = 100 \text{ kHz}$. The coupling coefficient $k$ between PW and SW was determined at 0.98–0.99. We designed the simulation circuit shown in the inset of Figure 4(a) to obtain the transfer characteristics illustrated in Figure 4. Generally, a SQUID can be represented by a flux-to-voltage converter in series with nonlinear dynamic resistor $R_d$, which changes with $\Phi$. In our simulation circuit, the voltage $V_{SQ}$ across a 1 $\Omega$ resistor generated by $V_{in}$ represents the SQUID’s voltage signal, while the resistor $R$ stands for the SQUID’s $R_s$, see Figure 4(a). The capacitor $C'$ shunting the SW represents the combined effect of the input capacitance of the preamplifier and the distributed capacitance of SW. It can be converted to the PW with a factor $n^2 = 400$. The transformer’s amplitude-frequency transfer characteristics (a) and phase-frequency characteristics (b) are plotted with $C = 1 \mu F$ and different $R$. In this simulation circuit, two resonances at $f_0 \approx 50 \text{ kHz}$ and $f_1 \approx 210 \text{ kHz}$ were observed; one ($f_0$) is that of $L_{PW}$ and $C$ and other ($f_1$) of $L_{SW}$ and $C'$. The resistor $R$ is in series connected to $L_{PW}$’s resonance circuit, while shunting the $L_{SW}$’s circuit via $L_{PW}$ and $C$. Increasing $R$ will damp $L_{PW}$’s circuit ($f_0$), but boost $L_{SW}$’s $C'$ ($f_1$). Increasing $C$ lowers $f_0$, while $f_1$ is remaining constant (not shown here). The phase values are independent of $R$ at $f_0$ and $f_1$, but they rapidly change with $f$, see Figure 4(b). We show that $R$ plays a very important role in the transfer characteristics of the SQUID circuit. Taking a typical SQUID’s $R_s$ of 10 $\Omega$, for example, the bandwidth of the SQUID circuit with the transformer is much smaller than

<table>
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<th>$R_s$ ($\Omega$)</th>
<th>$\Delta i$ ($\mu A$)</th>
<th>$V_{\text{out}}$ ($\mu V$)</th>
<th>$\Delta i \times R_s$ ($\mu V$)</th>
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<td>$\infty$</td>
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<td>22</td>
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FIG. 3. Measured I-V characteristics of a SQUID at $x \rightarrow \infty$. The $R_d^0.5$ (black curve) at $\Phi = (n+0.5)\Phi_0$ is obtained by taking $\partial V/\partial I$. Due to the shunting effect of $R_d$, the reduction of $I_s = (I_b - \Delta i)$ and the periodic I-V curve change from $\Phi = n\Phi_0$ to $\Phi = (n+0.5)\Phi_0$, leads to a decline of the load line resulting in $V_{\text{swing}} < V_{\text{out}}$.

FIG. 4. Transfer characteristics measured by a frequency response analyzer FRA5087 (NF company), connected to the $V_{\text{sw}}$ and $V_{\text{sw}}$ terminals of the simulation circuit shown in the inset. The transfer amplitude-frequency characteristics (a) and the phase-frequency characteristics (b) are plotted for different $R$ with $C = 1 \mu F$. At $f = 30 \text{ kHz}$, the curves from top to bottom correspond to $R = 0$, 1, 5, 10, 20, and 51 $\Omega$, respectively.
that at $R = 0$, which is usually assumed in characterizing a transformer. In reality, the transfer characteristics of the whole SQUID circuit with FMS depend not only on the transformer but also on the isolation capacitance $C$ and on the dynamic resistance $R_d$ of the SQUID.

For $\Phi_{SM}$ with $f_M = 100$ kHz, we focus on transfer characteristics in the frequency range between 100 and 700 kHz, i.e., from the fundamental to the 7th harmonics of the square wave. These characteristics (amplitude and phase) are not flat, for typical $R$ values. That means that a distortion of the square wave is unavoidable. This transformation distortion will be reflected in the demodulated SQUID $V(\Phi)$ discussed below.

Usually, one utilizes $\Phi$ linearly varied with a low frequency $f \ll f_M$ (quasi-static) to obtain $V(\Phi)$ which is a periodical function describing a voltage change across the SQUID vs. $\Phi$. The $V(\Phi)$ generally contains odd harmonic components of $V(3, 5, \ldots (2n + 1)\Phi_0)$. Harmonics become stronger with an increasing Steward-McCumber parameter $\beta_c$ of the SQUID. To experimentally compare the $V(\Phi)$ obtained by DRS and by FMS with $\Phi_{SM}$, we employed a SQUID with $\beta_c \approx 3$ and a large $R_d \approx 30 \Omega$ at the working point. Here, no hysteretic $V(\Phi)$ appears due to a large noise parameter $\Gamma$. In DRS, with $V_{\text{swing}} \approx 50 \mu V$, a transfer coefficient of $\partial V/\partial \Phi \approx 400 \mu V/\Phi_0$ was reached at the working point $W$, because the harmonics enhanced $\partial V/\partial \Phi$, see curve (I) in Figure 5. In FMS, the $V(\Phi)$, e.g., curve (I), where $\Phi$ varies with a low frequency $f$, will be firstly modulated, i.e., the voltage signals with $f_M = 100$ kHz are generated which contain all the information on $V(\Phi)$, which reappear after the demodulation. The demodulated $V(\Phi)_{DM}$ should be identical with the original $V(\Phi)$ measured by DRS. However, the measured $V(\Phi)_{DM}$, curve (II) in Figure 5, becomes a quasi-sinusoidal function due to the transformation discussed above. The same SQUID yields only a $\partial V/\partial \Phi \approx \pi \times 40 \mu V/\Phi_0 \approx 125 \mu V/\Phi_0$ at $W$. The value of $\partial V/\partial \Phi$ is 3.2 times lower than in the case of curve (I).

Although the two working points (note that the second working point is not marked here) in square wave FMS are nominally always set to the maximal $\partial V/\partial \Phi$ of the original $V(\Phi)$, curve (I), in the operation of FLL, the effective $\partial V/\partial \Phi$ at $W$ is determined by the $V(\Phi)_{DM}$, curve (II).

In conclusion, when the SQUID is operated in the current bias mode, $x = Z_2/R_d \rightarrow \infty$ should be fulfilled, i.e., the condition of $L_s = constant$ should remain valid. In FMS, the step-up transformer acts as an ac shunt, bringing the SQUID operation into the mixed bias mode. In this case, the SQUID signal voltage and current partially represent voltage and current bias characteristics. Both are determined by the value of $x$, which lies between 0.5 and 5 in a practical system. However, one can only use either the voltage or current for detecting the SQUID signal by the SQUID readout electronics. Actually, almost all readout electronics in FMS detect the SQUID’s voltage signal, i.e., the sensor is just treated as nominally being in the current bias mode.

In general, the FMS transformer steps up the SQUID signal by a factor equal to the turns ratio $SW/PW$, thus providing both a good impedance matching and suppression of the preamplifier noise. Nevertheless, three negative effects are consequences of using a transformer: (1) The voltage swing of the SQUID signal is reduced due to the decline of the line load (2) The SQUID circuit with a transformer filters out higher harmonic components of SQUID signal, i.e., reduces $\partial V/\partial \Phi$ at the working point (3) Additional thermal noise from the transformer (not discussed here) is added. Overall, the total noise in a SQUID system with FMS is not much or not at all lower than that in the DRS, especially for a large $\beta_c$, where all the negative effects are combined.

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