$J/\psi N$ interactions revisited and $\Lambda_b^0 \rightarrow J/\psi K^-(\pi^-)p$ decays

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Recently LHCb experiments confirmed the findings of the two P_c states in the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decays. In the present work, we investigate both the $\Lambda_b^0 \rightarrow J/\psi K^- p$ and the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decays, continuing the investigations of our former works on the interactions of $J/\psi N$ with its coupled channels by considering the s-/u-channel contributions, which are nontrivial in our results. We obtain consistent results with the previous works and the line shape of the $J/\psi N$ invariant mass distribution, which is analogous to the LHCb experiments, and favor the $P_c(4450)$ state as a $\bar{D}^* \Sigma_c$ bound state in our theoretical model.

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I. INTRODUCTION

Since $X(3872)$ was found by Belle in 2003 [1], much effort has been made to understand the properties of the “exotic” states and to search new states both in the theoretical and experimental aspects. Such exotic states cannot be understood as the normal color-singlet hadrons with the structure of quark-antiquark (mesons) or three quarks (baryons), which could be multiquark structures, for example, tetraquark and pentaquark states (more discussions can be found in the reviews [2,3]). Recently, two new P_c^+ states were found by LHCb in the $J/\psi p$ mass spectrum of the $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays [4] using a model-dependent analysis, which are considered as charmonium-pentaquark states with some uncertainties about their spin parity J^P quantum numbers and were predicted in the early works [5–10] and confirmed by a model-independent reanalysis with the previous experimental data sample [11]. These findings are mostly explained as molecular states [12–21], even though Ref. [22] questions their molecular properties. On the other hand, with different theoretical models, they also can be explained as a diquark state [23–31], a compact pentaquark state [32], a soliton- $\bar{D} - D$ bound state [33], or contrarily a kinematical effect or a cusp effect [34–36]. As suggested in Refs. [37,38], LHCb found a new evidence of the observation of these two states by a full amplitude analysis of the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decays [39]. It was remarked before in Ref. [40] that the similar triangle singularity also emerges in the $J/\psi p$ invariant mass distribution as in the $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays. On the other hand, the role of $\Lambda(1405)$ in the $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays was discussed with both the Murcia-Valencia model and Bonn model in Ref. [41] before the experiments. Thus, about the existence of these two pentaquark states, the confidence and the suspicion coexist since there are different theoretical opinions about them. Based on the former investigations, in the present work we go further to study the two P_c states theoretically.

Using the chiral unitary approach, where only the leading-order contact terms [Weinberg-Tomozawa (WT)]

from the lowest order Lagrangian are taken into account, Ref. [42] dynamically reproduces the $\Lambda(1405)$ resonance in the coupled channel $\bar{K}N$ s-wave interactions. With the same approach, but considering the s- and u-channel diagrams (direct and crossed graphs, or Born terms, as they are called), the $\bar{K}N$ interactions are revisited in Ref. [43] where the experimental data are described well. One step further, up to next-to-leading-order contributions from the Lagrangian, also including the s- and u-channel diagrams of the leading order contributions, the $\bar{K}N$ interactions and the $\Lambda(1405)$ resonance are investigated again in Refs. [44,45] with the analysis of the new experimental data. Along the same line, the improved analyses are done in Refs. [46–50] and in recent works [51–53] considering on-shell or off-shell contributions in the interaction potentials. Within a framework based on the chiral and hidden local symmetries, Ref. [54] finds a strong coupling of the vector meson-baryon (VB) channels¹ to the resonances $\Lambda(1405)$, $\Lambda(1670)$, which are dominated by the pseudo-scalar meson-baryon (PB) dynamics. Further, the contributions of the s-/u-channel diagrams, and the Kroll-Ruderman terms in the $PB \rightarrow VB$ transitions are taken into account in the later work [56]. Our former papers [10,57] only consider the coupling of the VB channels to the PB channels without taking into account the contributions from the s- and u-channel diagrams. This is the motivation of the present work. In principle, one could do up to next-to-leading-order calculations. But, since the lack of experimental data in the heavy quark sector is not like the case of the $\bar{K}N$ interactions, Ref. [58] faces such problems in determining the free parameters at next to leading order. Therefore, in the present work, we do not take into account the next-to-leading-order contributions. In the next section, we present our modified formalism. Then, we discuss the contributions of the s- and u-channel

¹The approximations made in this formalism are scrutinized in Ref. [55].

diagrams in the third section. Next, we investigate the $\Lambda_b^0 \rightarrow J/\psi K^-(\pi^-)p$ decays with the obtained $J/\psi p$ amplitudes. Finally, we finish with our conclusions.

II. $J/\psi N$ INTERACTIONS

In the early work [10], using the local hidden gauge formalism [59–61] and combining the heavy quark spin symmetry (HQSS) [62,63], we have considered seven coupled channels with $J/\psi N$: $\eta_c N$, $\bar{D}\Lambda_c$, $\bar{D}\Sigma_c$, $\bar{D}^*\Lambda_c$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Sigma_c^*$ which can be specified with spin J and isospin I in different sectors. In the present work, we only focus on two sectors as in Ref. [57] where the resonances appear. The elements of the interaction potential kernel V_{ij} are given in Tables I and II for the $J = 1/2, I = 1/2$ and $J = 3/2, I = 1/2$ sectors, respectively, where the coefficients μ_i^I , μ_{ij}^I ($i, j = 1, 2, 3$) and λ_2^I are the unknown low energy constants in the HQSS formalism, which specify the isospin sector and can be related to each other using $SU(3)$ flavor symmetry. As a consequence of the HQSS constraints, all of them just depend on the isospin (I) and are independent of the spin J (for more details see Ref. [10]). The values of them depend on the considered model, where we use the local hidden gauge formalism which is different compared with the one in Ref. [9]. Then, their values for the two considered sectors are given by

$$\begin{aligned}\mu_2 &= \frac{1}{4f^2} N_i N_j (2\sqrt{s} - M_i - M_j), \\ \mu_3 &= -\frac{1}{4f^2} N_i N_j (2\sqrt{s} - M_i - M_j), \\ \mu_{12} &= -\sqrt{6} \frac{m_\rho^2}{p_{D^*}^2 - m_{D^*}^2} \frac{1}{4f^2} N_i N_j (2\sqrt{s} - M_i - M_j), \\ \mu_1 &= 0, \quad \mu_{23} = 0, \\ \lambda_2 &= \mu_3, \quad \mu_{13} = -\mu_{12},\end{aligned}\quad (1)$$

where s is the Mandelstam variable of the meson and the baryon system, p_{D^*} and m_{D^*} are the four momentum and the mass of D^* , f is the pion decay constant, and the

TABLE I. The elements V_{ij} corresponding to the channels in the $J = 1/2, I = 1/2$ sector.

$\eta_c N$	$J/\psi N$	$\bar{D}\Lambda_c$	$\bar{D}\Sigma_c$	$\bar{D}^*\Lambda_c$	$\bar{D}^*\Sigma_c$	$\bar{D}^*\Sigma_c^*$
μ_1	0	$\frac{\mu_{12}}{2}$	$\frac{\mu_{13}}{2}$	$\frac{\sqrt{3}\mu_{12}}{2}$	$-\frac{\mu_{12}}{2\sqrt{3}}$	$\sqrt{\frac{2}{3}}\mu_{13}$
	μ_1	$\frac{\sqrt{3}\mu_{12}}{2}$	$-\frac{\mu_{13}}{2\sqrt{3}}$	$-\frac{\mu_{12}}{2}$	$\frac{5\mu_{13}}{6}$	$\frac{\sqrt{2}\mu_{13}}{3}$
	μ_2	0	0	0	$\frac{\mu_{23}}{\sqrt{3}}$	$\sqrt{\frac{2}{3}}\mu_{23}$
		$\frac{1}{3}(2\lambda_2 + \mu_3)$	$\frac{\mu_{23}}{\sqrt{3}}$	$\frac{2(\lambda_2 - \mu_3)}{3\sqrt{3}}$	$\frac{1}{3}\sqrt{\frac{2}{3}}(\mu_3 - \lambda_2)$	
			μ_2	$-\frac{2\mu_{23}}{3}$	$\frac{\sqrt{2}\mu_{23}}{3}$	
				$\frac{1}{9}(2\lambda_2 + 7\mu_3)$	$\frac{1}{9}\sqrt{2}(\mu_3 - \lambda_2)$	
					$\frac{1}{9}(\lambda_2 + 8\mu_3)$	

TABLE II. The elements V_{ij} corresponding to the channels in the $J = 3/2, I = 1/2$ sector.

$J/\psi N$	$\bar{D}^*\Lambda_c$	$\bar{D}^*\Sigma_c$	$\bar{D}\Sigma_c^*$	$\bar{D}^*\Sigma_c^*$
μ_1	μ_{12}	$\frac{\mu_{13}}{3}$	$-\frac{\mu_{13}}{\sqrt{3}}$	$\frac{\sqrt{5}\mu_{13}}{3}$
	μ_2	$\frac{\mu_{23}}{3}$	$-\frac{\mu_{23}}{\sqrt{3}}$	$\frac{\sqrt{5}\mu_{23}}{3}$
		$\frac{1}{9}(8\lambda_2 + \mu_3)$	$\frac{\lambda_2 - \mu_3}{3\sqrt{3}}$	$\frac{1}{9}\sqrt{5}(\mu_3 - \lambda_2)$
			$\frac{1}{3}(2\lambda_2 + \mu_3)$	$\frac{1}{3}\sqrt{\frac{5}{3}}(\lambda_2 - \mu_3)$
				$\frac{1}{9}(4\lambda_2 + 5\mu_3)$

normalization factor for the baryonic fields $N_i = \sqrt{\frac{M_i + E_i}{2M_i}}$ with M_i and E_i being the masses and the energies of the corresponding baryons in the i th channel. For the channels related with the vector mesons, we have ignored the factor $\vec{e}_i \cdot \vec{e}_j$. Note that Eqs. (1) are a bit different from the ones used in Refs. [10,57] where we have mainly ignored the normalization factors $N_{i(j)}$ and replaced the $k_i^0 + k_j^0$ term by $(2\sqrt{s} - M_i - M_j)$, where k_i^0, k_j^0 are the energies of incoming and outgoing mesons. Indeed, $N_{i(j)} \approx 1$ in the low energy region as in our cases, we recover these factors for the consistency of the later calculations.

With the chiral unitary approach, the scattering amplitudes are evaluated by solving the coupled channels Bethe-Salpeter equation using the on-shell factorization [42,43]

$$T = [1 - VG]^{-1}V, \quad (2)$$

where the propagator G is a diagonal matrix with the meson-baryon loop functions, with the dimensional regularization,² the elements of which are given by

$$\begin{aligned}G_{ii}(s) &= \frac{2M_i}{16\pi^2} \left\{ a_\mu + \ln \frac{M_i^2}{\mu^2} + \frac{m_i^2 - M_i^2 + s}{2s} \ln \frac{m_i^2}{M_i^2} \right. \\ &\quad + \frac{q_i^{cm}}{\sqrt{s}} [\ln(s - (M_i^2 - m_i^2) + 2q_i^{cm}\sqrt{s}) \\ &\quad + \ln(s + (M_i^2 - m_i^2) + 2q_i^{cm}\sqrt{s}) \\ &\quad - \ln(-s - (M_i^2 - m_i^2) + 2q_i^{cm}\sqrt{s}) \\ &\quad \left. - \ln(-s + (M_i^2 - m_i^2) + 2q_i^{cm}\sqrt{s}) \right\},\end{aligned}\quad (3)$$

where m_i, M_i are the masses of the meson and baryon in the i th channel, respectively, and q_i^{cm} is the three momentum in the center of mass frame. The only free parameters are a_μ and μ ,³ taking $a_\mu = -2.3$ and $\mu = 1000$ MeV as done in

²A general expression for n-dimensions can be found, e.g., in Ref. [64].

³In fact, two of them correlate with each other; see the second term $\ln \frac{M_i^2}{\mu^2}$ and more discussions in Refs. [43,55].

FIG. 1. Results of the modulus squared of the amplitudes. Left: $J = 1/2, I = 1/2$ sector. Right: $J = 3/2, I = 1/2$ sector.

Refs. [5,6], which are within the natural values [43]. The kernel matrix V contains the interaction potentials that are discussed above. One should keep in mind that in our present work we use the chiral unitary approach for the s-wave projections of the interaction potentials, but for the higher partial waves interactions the off-shell contributions should be considered as discussed in Ref. [65] where an approach is constructed for the meson photoproduction based on the chiral effective Lagrangian.

We show our results of the modulus squared of the amplitudes in Fig. 1 for the $J = 1/2, I = 1/2$ sector (left) and the $J = 3/2, I = 1/2$ sector (right), which are consistent with the ones obtained in Refs. [10,57]. For the $J = 1/2, I = 1/2$ sector, corresponding to the peaks in the modulus squared of the amplitudes, we find the poles in the second Riemann sheets as $(4260.9 - i22.1)$, $(4408.5 - i37.0)$, and $(4479.0 - i39.9)$ MeV, which are under the thresholds of the channels $\bar{D}\Sigma_c$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$, respectively. For the $J = 3/2, I = 1/2$ sector, we find the corresponding poles as $(4335.0 - i24.2)$, $(4418.0 - i5.1)$, and $(4479.2 - i21.7)$ MeV, which are slightly below the thresholds of $\bar{D}\Sigma_c$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$ channels, respectively. Compared to the results of Refs. [10,57], the masses of the poles are just a few MeV different, but the widths differ by 5–20 MeV. These results are also consistent with the findings of Refs. [5,6,20] within the uncertainties.

III. THE S-/U-CHANNEL CONTRIBUTIONS

Now we take into account the s-/u-channel contributions to our interaction potentials of Eq. (1) as done in

Refs. [43–48,53], shown in Fig. 2. In fact, the potentials of Eq. (1) are the WT type called V_{ij}^{WT} , where we only consider the t-channel diagrams with the local hidden gauge formalism. More details about the WT-type interactions have already been discussed in Ref. [66] for the presence of vector mesons, the massive spin 1 fields. Thus, for the present case we have

$$V_{ij} = V_{ij}^{WT} + V_{ij}^s + V_{ij}^u. \quad (4)$$

To evaluate the diagrams of Fig. 2, we need to calculate the interaction vertices from the corresponding Lagrangians. For the PB interactions, the lowest order ($\mathcal{O}(p)$) chiral Lagrangian is given by [67,68]

$$\begin{aligned} \mathcal{L}_{MB}^{(1)} = & \langle \bar{B} i \gamma^\mu \nabla_\mu B - m_0 \bar{B} B + \frac{D}{2} (\bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\}) \\ & + \frac{F}{2} (\bar{B} \gamma^\mu \gamma_5 [u_\mu, B]) \rangle, \end{aligned} \quad (5)$$

where u is the chiral field constructed by the meson fields P , $u = \exp[iP/(\sqrt{2}f)]$, B are the baryon fields, and m_0 denotes the common (average) mass of the octet baryons in the chiral limit. Besides, the low energy constants D and F are related to the usual nucleon axial-vector coupling constant $g_A = D + F = 1.26$ [67,68]. By expanding the covariant derivative term, the Lagrangian of Eq. (5) leads to the contact term of WT type and the Born term of Yukawa vertex,

FIG. 2. The diagrams of the s-channel (a) and u-channel (b) contributions.

$$\mathcal{L}_{PBPB}^{WT} = \frac{1}{4f^2} \langle \bar{B} i \gamma^\mu [P, \partial_\mu P], B \rangle; \quad (6)$$

$$\mathcal{L}_{BBP} = -\frac{1}{\sqrt{2}f} \langle D(\bar{B} \gamma^\mu \gamma_5 \{\partial_\mu P, B\}) + F(\bar{B} \gamma^\mu \gamma_5 [\partial_\mu P, B]) \rangle. \quad (7)$$

From the Lagrangian of Eq. (6), we can obtain the tree-level amplitude, written as⁴

$$\begin{aligned} V_{ij}^{WT}(\sqrt{s}, \Omega, \sigma_i, \sigma_j) &= -\frac{C_{ij}}{4f^2} N_i N_j (\chi^{\sigma_j})^\dagger \left[2\sqrt{s} - M_i - M_j \right. \\ &\quad \left. + (2\sqrt{s} + M_i + M_j) \frac{\vec{p}_i \cdot \vec{p}_j + i(\vec{p}_i \times \vec{p}_j) \cdot \vec{\sigma}}{(M_i + E_i)(M_j + E_j)} \right] \chi^{\sigma_i}, \quad (8) \end{aligned}$$

where $\vec{p}_i$ is the three momentum of the baryon in the corresponding channel i , C_{ij} is the coefficients, and χ^{σ_i} is the two-component Pauli spinor for the baryon in corresponding channel i . Then, we should do the s -wave projection with the solid angle Ω of the scattering and the spin summation of σ_i , using

$$V_{ij}^{WT}(\sqrt{s}) = \frac{1}{8\pi} \sum_{\sigma_i, \sigma_j} \int d\Omega V_{ij}(W, \Omega, \sigma_i, \sigma_j). \quad (9)$$

Thus, we obtain the WT interaction potential

$$V_{ij}^{WT}(\sqrt{s}) = -\frac{C_{ij}}{4f^2} N_i N_j (2\sqrt{s} - M_i - M_j), \quad (10)$$

which is indeed the type in Eq. (1) from the local hidden gauge Lagrangian for the light vector meson exchange cases (WT type; see the later discussions in this section), but not for the heavy vector exchange cases; see μ_{12} in Eq. (1).

Using the Lagrangian of Eq. (7), the vertex can be derived where one can substitute an s quark by a c quark for the corresponding hadrons in the SU(3) symmetry as done in Ref. [10], having

$$V_{ie}^{BBP} = C_{i,e}^P \bar{u}_e(p_e, M_e) \gamma_\mu \gamma_5 u_i(p_i, M_i) k^\mu, \quad (11)$$

where $u_i(p_i, M_i)$ is the corresponding baryon spinor and k is the incoming momentum of the pseudoscalar meson. We only have the nucleon N as the propagating particle in our present case; thus, the coefficients C_{ie}^P with isospin $I = 1/2$ are given by

$$C_{\Lambda_c, N}^{\bar{D}} = \frac{1}{\sqrt{3}} \frac{D + 3F}{2f}, \quad C_{\Sigma_c, N}^{\bar{D}} = \frac{3}{\sqrt{3}} \frac{D - F}{2f}. \quad (12)$$

Then, we can obtain the potentials from the contributions of the s/u channels as shown in Fig. 2, for the cases of $PB \rightarrow PB$ transitions,⁵

$$\begin{aligned} V_{ij}^s(\sqrt{s}, \Omega, \sigma_i, \sigma_j) &= C_{i,N}^P C_{j,N}^P \frac{1}{s - M_N^2} N_i N_j (\chi^{\sigma_j})^\dagger \left\{ (\sqrt{s} - M_N) [s - \sqrt{s}(M_i + M_j) + M_i M_j] \right. \\ &\quad \left. + (\sqrt{s} + M_N) [s + \sqrt{s}(M_i + M_j) + M_i M_j] \frac{\vec{p}_i \cdot \vec{p}_j + i(\vec{p}_i \times \vec{p}_j) \cdot \vec{\sigma}}{(M_i + E_i)(M_j + E_j)} \right\} \chi^{\sigma_i}, \quad (13) \end{aligned}$$

$$\begin{aligned} V_{ij}^u(\sqrt{s}, \Omega, \sigma_i, \sigma_j) &= -C_{i,N}^P C_{j,N}^P \frac{1}{u - M_N^2} N_i N_j (\chi^{\sigma_j})^\dagger \left\{ u(\sqrt{s} + M_N) + \sqrt{s} [M_i M_j + M_N(M_i + M_j)] - M_i M_j M_N \right. \\ &\quad \left. - M_i^2(M_j + M_N) - M_j^2(M_i + M_N) + \{u(\sqrt{s} - M_N) + \sqrt{s} [M_i M_j + M_N(M_i + M_j)] \right. \\ &\quad \left. + M_i M_j M_N + M_i^2(M_j + M_N) + M_j^2(M_i + M_N) \} \frac{\vec{p}_i \cdot \vec{p}_j + i(\vec{p}_i \times \vec{p}_j) \cdot \vec{\sigma}}{(M_i + E_i)(M_j + E_j)} \right\} \chi^{\sigma_i}, \quad (14) \end{aligned}$$

where $u = -s + m_i^2 + m_j^2 + 2E_i E_j - 2\vec{p}_i \cdot \vec{p}_j$. After s -wave projection, we obtain

$$V_{ij}^s(\sqrt{s}) = C_{i,N}^P C_{j,N}^P N_i N_j \frac{1}{\sqrt{s} + M_N} (\sqrt{s} - M_i)(\sqrt{s} - M_j), \quad (15)$$

⁴Note that, since the phase convention and the normalization are different from Refs. [43–53], there is a factor $-2\sqrt{M_i M_j}$ or $\sqrt{M_i M_j}$ between them, and thus in the final expressions for the scattering amplitudes T_{ij} .

⁵As a check, these expressions could also be obtained using a *Mathematica* package of FeynCalc [69].

$$\begin{aligned}
V_{ij}^u(\sqrt{s}) = & -C_{i,N}^P C_{j,N}^P N_i N_j \left\{ \sqrt{s} + M_N - \frac{(M_i + M_N)(M_j + M_N)}{2(M_i + E_i)(M_j + E_j)} (\sqrt{s} - M_N + M_i + M_j) \right. \\
& + \frac{(M_i + M_N)(M_j + M_N)}{4q_i^{cm} q_j^{cm}} \left[\sqrt{s} + M_N - M_i - M_j - \frac{s + M_N^2 - m_i^2 - m_j^2 - 2E_i E_j}{2(M_i + E_i)(M_j + E_j)} \right. \\
& \times (\sqrt{s} - M_N + M_i + M_j) \left. \right] \ln \frac{s + M_N^2 - m_i^2 - m_j^2 - 2E_i E_j - 2q_i^{cm} q_j^{cm}}{s + M_N^2 - m_i^2 - m_j^2 - 2E_i E_j + 2q_i^{cm} q_j^{cm}} \left. \right\}. \quad (16)
\end{aligned}$$

For the case of $PB \rightarrow VB$ transitions, we need to evaluate the BBV vertex. The Lagrangian for the interactions of vector mesons to the baryons is given by [70,71]

$$\mathcal{L}_{BBV} = g(\langle \bar{B} \gamma_\mu [V^\mu, B] \rangle + \langle \bar{B} \gamma_\mu B \rangle \langle V^\mu \rangle), \quad (17)$$

where the coupling $g = \frac{m_V}{2f}$, with the mass of the vector meson m_V , which leads to

$$V_{ie}^{BBV} = ig C_{i,e}^V \bar{u}_e(p_e, M_e) \gamma^\mu u_i(p_i, M_i) \epsilon_\mu^{(\lambda)}, \quad (18)$$

with the polarization vector $\epsilon^{(\lambda)}$ of the incoming vector meson and the coefficients C_{ie}^V with isospin $I = 1/2$ for our present case as

$$C_{\Lambda_c, N}^{\bar{D}^*} = -\frac{3}{\sqrt{6}}g, \quad C_{\Sigma_c, N}^{\bar{D}^*} = -\frac{3}{\sqrt{6}}g. \quad (19)$$

Thus, we can evaluate the potentials for the $PB \rightarrow VB$ transitions,

$$\begin{aligned}
V_{ij}^s(\sqrt{s}, \Omega, \sigma_i, \sigma_j) = & -C_{i,N}^P C_{j,N}^V \frac{i}{s - M_N^2} N_i N_j (\chi^{\sigma_j})^\dagger \left\{ s + M_i M_N - \sqrt{s}(M_i + M_N) \right. \\
& + [s + M_i M_N + \sqrt{s}(M_i + M_N)] \frac{\vec{p}_i \cdot \vec{p}_j + i(\vec{p}_i \times \vec{p}_j) \cdot \vec{\sigma}}{(M_i + E_i)(M_j + E_j)} \left. \right\} \sigma^k \epsilon_k^{(\lambda_j)} \chi^{\sigma_i}, \quad (20)
\end{aligned}$$

$$\begin{aligned}
V_{ij}^u(\sqrt{s}, \Omega, \sigma_i, \sigma_j) = & C_{i,N}^P C_{j,N}^V \frac{i}{u - M_N^2} N_i N_j (\chi^{\sigma_j})^\dagger \left\{ \left([\sqrt{s}(M_j + M_N) - (M_i M_N + M_j^2 \right. \right. \\
& + M_i M_j - u)] - [\sqrt{s}(M_j + M_N) + (M_i M_N + M_j^2 + M_i M_j - u)] \\
& \times \frac{\vec{p}_i \cdot \vec{p}_j + i(\vec{p}_i \times \vec{p}_j) \cdot \vec{\sigma}}{(M_i + E_i)(M_j + E_j)} \left. \right) \sigma^k \epsilon_k^{(\lambda_j)} - 2\vec{p}_i \cdot \vec{\epsilon}^{(\lambda_j)} (M_j + M_N) \sigma_k \left(\frac{p_i^k}{M_i + E_i} - \frac{p_j^k}{M_j + E_j} \right) \left. \right\} \chi^{\sigma_i}, \quad (21)
\end{aligned}$$

where we only take into account the spacelike polarization and ignore the unphysical timelike one as done in Refs. [56,72]. But, after the spin summation, we found that

$$\sum_{\sigma_i, \sigma_j} (\chi^{\sigma_j})^\dagger \sigma_k \chi^{\sigma_i} \delta_{\sigma_i, \sigma_j} = 0, \quad (22)$$

where $\delta_{\sigma_i, \sigma_j}$ is the usual Kronecker delta function, which is a consequence of σ_k having spin operator properties. Therefore, for the $PB \rightarrow VB$ transitions, the s/u-channel contributions are 0 and consistent with the results of Ref. [73] where a vanishing contribution for the $\pi N \rightarrow \rho N$ transitions was found. In principle, as discussed in Ref. [74], there are also contributions from the meson in the flight term and the contact term (Kroll-Ruderman term) for the $PB \rightarrow VB$ transitions that can be obtained by introducing the PB and VB interaction Lagrangian as done in Ref. [54]. In fact, the meson in flight terms is concerned in the $J/\psi N$ and $\eta_c N$ related channels for the heavy meson exchange. But, for the others channels, we also have checked that the contributions from these two terms are trivial as discussed in Ref. [10], and thus we ignore them.

Then, for the $VB \rightarrow VB$ transitions, the amplitudes of the s/u-channel diagrams are given by

$$V_{ij}^s(\sqrt{s}, \Omega, \sigma_i, \sigma_j) = C_{i,N}^V C_{j,N}^V \frac{1}{s - M_N^2} N_i N_j (\chi^{\sigma_j})^\dagger \left[\sqrt{s} - M_N + (\sqrt{s} + M_N) \frac{\vec{p}_i \cdot \vec{p}_j + i(\vec{p}_i \times \vec{p}_j) \cdot \vec{\sigma}}{(M_i + E_i)(M_j + E_j)} \right] \sigma^k \epsilon_k^{(\lambda_i)} \sigma^l \epsilon_l^{(\lambda_j)} \chi^{\sigma_i}, \quad (23)$$

$$\begin{aligned} V_{ij}^u(\sqrt{s}, \Omega, \sigma_i, \sigma_j) = & -C_{i,N}^V C_{j,N}^V \frac{1}{u - M_N^2} N_i N_j (\chi^{\sigma_i})^\dagger \left\{ \left[\sqrt{s} + M_N - M_i - M_j + (\sqrt{s} - M_N + M_i + M_j) \right. \right. \\ & \times \frac{\vec{p}_i \cdot \vec{p}_j + i(\vec{p}_i \times \vec{p}_j) \cdot \vec{\sigma}}{(M_i + E_i)(M_j + E_j)} \left. \right] \sigma^k \epsilon_k^{(\lambda_i)} \sigma^l \epsilon_l^{(\lambda_j)} + 2\vec{p}_i \cdot \vec{e}_j \vec{\sigma} \cdot \vec{e}_i \sigma_k \left(\frac{p_i^k}{M_i + E_i} + \frac{p_j^k}{M_j + E_j} \right) \\ & \left. + 2\vec{p}_j \cdot \vec{e}_i \vec{\sigma} \cdot \vec{e}_j \sigma_k \left(\frac{p_i^k}{M_i + E_i} + \frac{p_j^k}{M_j + E_j} \right) \right\} \chi^{\sigma_i}. \end{aligned} \quad (24)$$

Then, doing s-wave projection, we find

$$V_{ij}^s(\sqrt{s}) = C_{i,N}^V C_{j,N}^V N_i N_j \frac{1}{\sqrt{s} + M_N} \vec{\sigma} \cdot \vec{e}_i \vec{\sigma} \cdot \vec{e}_j, \quad (25)$$

$$\begin{aligned} V_{ij}^u(\sqrt{s}) = & -C_{i,N}^V C_{j,N}^V N_i N_j \left\{ -\frac{\sqrt{s} - M_N + M_i + M_j}{2(M_i + E_i)(M_j + E_j)} + \frac{1}{4q_i^{cm} q_j^{cm}} \left[\sqrt{s} + M_N - M_i - M_j - \frac{s + M_N^2 - m_i^2 - m_j^2 - 2E_i E_j}{2(M_i + E_i)(M_j + E_j)} \right. \right. \\ & \left. \left. \times (\sqrt{s} - M_N + M_i + M_j) \right] \ln \frac{s + M_N^2 - m_i^2 - m_j^2 - 2E_i E_j - 2q_i^{cm} q_j^{cm}}{s + M_N^2 - m_i^2 - m_j^2 - 2E_i E_j + 2q_i^{cm} q_j^{cm}} \right\} \vec{\sigma} \cdot \vec{e}_i \vec{\sigma} \cdot \vec{e}_j, \end{aligned} \quad (26)$$

where we also have the structure $\vec{\sigma} \cdot \vec{e}_i \vec{\sigma} \cdot \vec{e}_j$ as well as the ones obtained in Ref. [56].

We first add the contributions from the s-channel diagram and find that the modulus squared of the amplitudes is not much different from the ones in Fig. 1 and the peaks move just a bit. Thus, the corresponding poles are changed to $(4264.0 - i23.2)$, $(4411.2 - i38.0)$, and $(4474.9 - i40.9)$ MeV below the thresholds of $\bar{D}\Sigma_c$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$ channels, respectively in the $J = 1/2, I = 1/2$ sector, and $(4338.4 - i26.2)$, $(4420.8 - i5.9)$, and $(4478.9 - i24.1)$ MeV below the thresholds of $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$ channels, respectively, in the $J = 3/2, I = 1/2$ sector. These results are just a few MeV different both in

the masses and the widths of the poles compared to the ones in the last section. Indeed, for the results we have here, the contributions from the s-channel diagram are suppressed by the nucleon propagator, see Eqs. (15) and (25), and thus the results with its contributions are not affected much.

Then we add all the contributions from the s/u-channel diagrams. The results are shown in Fig. 3, where we can see a mess in the modulus squared of the amplitudes and not so clear peaks as in Fig. 1 since we have added the u-channel contributions. We find that these extra unphysical (sharp) peaks come from the contributions of the unphysical

FIG. 3. Results of the modulus squared of the amplitudes with all the contributions from s/u channels. Left: $J = 1/2, I = 1/2$ sector. Right: $J = 3/2, I = 1/2$ sector.

TABLE III. The summarized results for the predicted states (units of MeV), shown in the format (mass and width).

Sectors	Channels	Thresholds	Refs. [10,57]	WT term	WT + s	WT + s + u	Experiments
$J^P = \frac{1}{2}^-$	$\bar{D}\Sigma_c$	4320.8	(4262,35)	(4261,44)	(4264,46)	(4291,39)	
	$\bar{D}^*\Sigma_c$	4462.2	(4410,58)	(4409,74)	(4411,76)	(4427,60)	$P_c(4450)^{+?}$
	$\bar{D}^*\Sigma_c^*$	4526.7	(4481,57)	(4479,80)	(4475,82)	(4482,123)	$P_c(4450)^{+?}$
$J^P = \frac{3}{2}^-$	$\bar{D}\Sigma_c^*$	4385.3	(4334,38)	(4335,48)	(4338,52)	(4368,42)	$P_c(4380)^{+?}$
	$\bar{D}^*\Sigma_c$	4462.2	(4417,8)	(4418,10)	(4421,12)	(4432,11)	$P_c(4450)^{+?}$
	$\bar{D}^*\Sigma_c^*$	4526.7	(4481,35)	(4479,44)	(4479,48)	(4480,71)	$P_c(4450)^{+?}$

subthreshold effects in the u-channel as discussed in Ref. [44], which appear at some certain energies, e.g., 4258, 4385, and 4457 MeV in the $\bar{D}\Sigma_c$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$ channels, respectively, seen in Fig. 3 (see more discussions in the next section). Note that these unphysical subthreshold effects come from the left-hand cuts in the u channels that lead to the singularity or pole structures in Fig. 3, and some of them indeed close to the region of physical poles. As can be seen in Eqs. (15) and (25), there is no left-hand cut in s channels (the minus solution of the pole has no physical meaning and is not considered here), and thus no such unphysical subthreshold effects happen in s channels. This is why the results with only s-channel contributions are similar to Fig. 1. But, in the case of $\bar{K}N$ (PB) interactions in the isospin $I = 0$ sector, these unphysical effects are numerically small⁶ as discussed in Ref. [44], and analogously ignored contributions of these diagrams were found in Ref. [76] in the case of VB interactions for the light quark sectors. Note that, as discussed at the beginning of this section, the WT-type interaction potentials from the local hidden gauge formalism are in fact the t-channel diagrams with the explicit exchange of a vector meson; thus, in principle, there are also the unphysical left-hand cut contributions in the t channel as in the u channel. The fact is that we have taken an approximation where the transferred momentum q^2 has been neglected in comparison with the masses of the exchanged vector mesons m_V^2 . Therefore, there are no singularities appearing in the interaction potentials⁷ and this leads to the t-channel diagrams contributed as WT type for the light vector meson exchanges. Even though these unphysical contributions from the u channel do not affect the pole in the second Riemann sheets since these unphysical effects only happen in the first Riemann sheet, we can still find clean poles in the second Riemann sheets and no unphysical poles, as (4291.4 − *i*19.5), (4426.9 − *i*30.2), and (4482.4 − *i*61.3) MeV below the thresholds of $\bar{D}\Sigma_c$, $\bar{D}^*\Sigma_c$, and

$\bar{D}^*\Sigma_c^*$ channels, respectively, in the $J = 1/2, I = 1/2$ sector, and (4367.6 − *i*21.0), (4432.5 − *i*5.4), and (4479.6 − *i*35.3) MeV below the thresholds of $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$ channels, respectively, in the $J = 3/2, I = 1/2$ sector. We can see that there are up to 30 MeV differences in the masses and the widths for some states compared to the ones with only s-channel contributions. It means that the contributions from the u channel are bigger than from the s channel.

Finally, to summarize our results, we show the predicted states in Table III. From this table, we can see that the contributions from the s channel are constructive interference effects and thus the masses and the widths of the predicted states increase a few MeV. But, as seen from the minus sign in Eqs. (16) and (26), the contributions from the u channel are destructive interference effects. Therefore, the widths of the states decrease except for the $\bar{D}^*\Sigma_c^*$ state which increases twice compared to the ones obtained in Refs. [10,57]. These effects also lead to the increased masses of the states since the predicted states are bound states coming from the attractive interaction potentials. Especially, the $\bar{D}\Sigma_c$ and $\bar{D}\Sigma_c^*$ states are less bound by nearly 30 MeV. Therefore, the contributions from the s/u channels are not insignificant to our results. In the experimental results [4], the width of the $P_c(4380)$ state is about 205 MeV, and about 39 MeV for the $P_c(4450)$ state, where it is certain that they have opposite parities. Thus, in our present work, we can only explain one of them with sole parity of our model. But, for our results from the interactions in the free space, it is hard to distinguish them since there are some theoretical uncertainties discussed in Ref. [57]. Therefore, in the next section we continue to investigate the Λ_b^0 decays where the two P_c states are found in the LHCb experiments.

IV. $\Lambda_b^0 \rightarrow J/\psi K^-(\pi^-)p$ DECAYS

We have calculated the interaction amplitudes of $J/\psi N$ and its coupled channels above; thus, we can go further to study the $J/\psi N$ final state interactions in the $\Lambda_b^0 \rightarrow J/\psi K^-(\pi^-)p$ decays where the two P_c states are found in the $J/\psi p$ invariant mass distribution. Following the method used in Refs. [14,41] (more details and discussions about this method can be found in Ref. [77], and references

⁶Note that this does not hold for the $\bar{K}N$ interactions in the isospin $I = 1$ sector, where important contributions of the u channel were found in Ref. [50] and lead to a pole most likely in the near-threshold region of $\bar{K}N$. Some technical details about the u-channel contributions in the isospin $I = 1$ sector are discussed in Ref. [75].

⁷More discussions can be referred to Sec. IIB of Ref. [76].

FIG. 4. The diagrams for the $\Lambda_b^0 \rightarrow J/\psi K^-(\pi^-)p$ decays: (a) direct $J/\psi K^-(\pi^-)p$ vertex decays at tree level; (b) final state interactions of $J/\psi p$.

therein), as shown in Fig. 4, since the first diagram for the direct decays (a) only contributes a constant to the scattering amplitude (shown as a black circle), we only focus on the second diagram (b) of the $J/\psi N$ final state interactions (shown as a gray ellipse). Note that, even though there is no significant contribution to the fits for the direct decay diagram of Fig. 4(a) as claimed in Ref. [4], Ref. [78] shows that this tree-level contribution is nontrivial which is the same weight as the $K^- p$ amplitude as found in Ref. [41]. Since we do not consider the contributions from the $K^- p$ amplitude that was taken into account in Ref. [78] for the $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays and in Ref. [38] for the $\Lambda_b^0 \rightarrow J/\psi \pi^- p$ decays, we emphasize the contributions from the $J/\psi p$ amplitude and do not consider this tree-level contribution in the $J = 1/2$ case. Thus, treating K^- as a spectator, for the $J/\psi p$ invariant mass distribution in the $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays we have analogously

$$\frac{d\Gamma(M_{\text{inv}})}{dM_{\text{inv}}} = \frac{1}{4(2\pi)^3} \frac{1}{M_{\Lambda_b}} \tilde{p}_{J/\psi} p_K |T^{J/\psi p}|^2, \quad (27)$$

where M_{inv} is the invariant mass of the $J/\psi p$ system in the final states, there is no $4M_p$ factor for our normalization of the baryon spinors, the momenta in the center of mass frame are given by

$$\tilde{p}_{J/\psi}(M_{\text{inv}}) = \frac{\lambda^{1/2}(M_{\text{inv}}^2, m_{J/\psi}^2, M_p^2)}{2M_{\text{inv}}}, \quad (28)$$

$$p_K(M_{\text{inv}}) = \frac{\lambda^{1/2}(M_{\Lambda_b}^2, m_K^2, M_{\text{inv}}^2)}{2M_{\Lambda_b}}, \quad (29)$$

with $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$ as the usual Källén function, and the transition amplitude of $T^{J/\psi p}$ is given by [38]

$$T^{J/\psi p}(M_{\text{inv}}) = V_p h_{K^- p} G_{J/\psi p}(M_{\text{inv}}^2) t_{J/\psi p \rightarrow J/\psi p}(M_{\text{inv}}), \quad (30)$$

with $h_{K^- p} = 1$ [41]; the loop function $G_{J/\psi p}(M_{\text{inv}}^2)$ is given by Eq. (3), V_p is a constant that collects the CKM matrix elements and the kinematic prefactors, and we take the amplitude of $t_{J/\psi p \rightarrow J/\psi p}(M_{\text{inv}})$ as $T_{J/\psi N}^{I=1/2}$ that we have

evaluated in the former section and it is different from the one used in Refs. [14,38] by an approximation of the Breit-Wigner form. For the case of the $\Lambda_b^0 \rightarrow J/\psi \pi^- p$ decays, the formalism is analogous and just replaces m_K with m_π .

In our formalism in the former section, the scattering amplitudes for $T_{J/\psi N}^{I=1/2}$ have two cases, spins $J = 1/2$ and $J = 3/2$. But, for the case of $J = 3/2$, which is different from the one of $J = 1/2$ above, since J^P of Λ_b^0 is $1/2^-$, the kaon (or pion) should be in the p wave when the $J/\psi p$ final state system is $J^P = 3/2^-$ in the s wave [79] in the $\Lambda_b^0 \rightarrow J/\psi K^-(\pi^-)p$ decays; refer to the appendix of Refs. [78,80] for more details. Therefore, in this case, there is an extra momentum factor for the kaon (or pion) in the p wave in the amplitude of Eq. (30). Thus, we have for the $J = 3/2$ case

$$T^{J/\psi p}(M_{\text{inv}}) = V'_p [1 + h_{K^- p} G_{J/\psi p}(M_{\text{inv}}^2) t_{J/\psi p \rightarrow J/\psi p}(M_{\text{inv}})] p_K, \quad (31)$$

where now we have taken into account the tree-level contribution of Fig. 4(a), since there would be the contributions from the possible contact term of the tree level in the p wave (see the term of 1) as done in Ref. [78].

Thus, using $J = 1/2$ sector scattering amplitudes, we obtain the results as shown in Fig. 5 both for the $\Lambda_b^0 \rightarrow J/\psi K^- p$ (upper-left panel) and the $\Lambda_b^0 \rightarrow J/\psi \pi^- p$ (upper-right panel) decays which are compared with the experimental data in the lower panels and shown again as the theoretical lines (solid and red) with a scale factor, correspondingly. In Fig. 5, we have successfully produced the experimental line shape found in the LHCb [4,11,39] and do not take into account the contributions of the background, even though there are three clear resonant peaks in the scattering amplitudes of free space, seen in the left of Figs. 1 and 3. In Fig. 3 (left), we can see some strong unphysical poles coming from the left-hand cuts in certain u channels as discussed in the previous section. But most of them have no or little influence on the results of Fig. 5 since its corresponding channels couple weakly to the $J/\psi p$ channel (see the coefficients of μ_{12} and μ_{13} in Tables I and II, and the discussions in Ref. [10]); e.g., the strongest one of 4139 MeV in the left of Fig. 3 comes from the nondiagonal $\bar{D}\Lambda_c \rightarrow \bar{D}\Sigma_c$ channel, which only contributes a tiny peak in the upper-left panel of Fig. 5. From these results in Fig. 5, we can see the clear resonant peak of the

FIG. 5. Results of the $J/\psi p$ invariant mass distribution in the $J = 1/2$ sector: upper-left panel for the $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays and upper-right for the $\Lambda_b^0 \rightarrow J/\psi \pi^- p$ decays. The experimental results are shown in the lower panels accordingly [4,39] and compared with the theoretical ones (solid and red lines).

$P_c(4450)$ state. Therefore, from our formalism we can conclude that the $P_c(4450)$ state could be a $J^P = 1/2^-$ $\bar{D}^* \Sigma_c$ bound state. Besides, the structure of $P_c(4380)$ seems to appear on the left of the high peak, but it is just the unphysical subthreshold effect in the u channels that is the one in the $\bar{D}^* \Sigma_c$ channel. Obviously it is not possible that the $P_c(4380)$ state appears simultaneously for its different quantum numbers of J^P as discussed in the last section. Furthermore, in Fig. 5, the dip structure on the right of the

high peak is consistent with the experimental finding, which is a better line shape compared to the results of the Breit-Wigner form approximation in Ref. [14]. From the similar results with the $\Lambda_b^0 \rightarrow J/\psi K^- p$ and the $\Lambda_b^0 \rightarrow J/\psi \pi^- p$ decays, seen in the upper panels of Fig. 5, indeed, the two P_c states should also be seen in $\Lambda_b^0 \rightarrow J/\psi \pi^- p$ decays as suggested in the early research [37,38] and found in Ref. [39]. Note that we do not fit the experimental data as done in Refs. [38,78], since we do not take into account any

FIG. 6. Results of the $J/\psi p$ invariant mass distribution in the $J = 3/2$ sector: Left panel for the $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays, right for the $\Lambda_b^0 \rightarrow J/\psi \pi^- p$ decays. The solid (green) lines are evaluated with full Eq. (31), labeled as 1+GT, and the dashed (red) ones without the tree-level contributions, labeled as GT.

background contributions and do not change the parameters used in our former works. One should keep in mind that there are some uncertainties in our formalism as discussed above. For $J = 3/2$ sector, the results are shown in Fig. 6, which has three resonant peaks of the same magnitude and cannot match the experimental results of the $J/\psi p$ invariant mass distribution. These results also show that two missing states in Fig. 5 for the $J = 1/2$ sector couple weakly with the $J/\psi N$ channel as found in Ref. [10] and three predicted states having the same magnitude in Fig. 6 for the $J = 3/2$ sector couple strongly with the only decay channel of $J/\psi N$. Besides, in Fig. 6, the results of the solid (green) lines are the ones with full Eq. (31) (labeled as 1+GT), and the results of the dashed (red) ones without the tree-level contributions (labeled as GT), where we can see the strong interference effects with the contributions of the direct decay diagram, Fig. 4(a), as found in Ref. [78].

V. CONCLUSIONS

Using the local hidden gauge Lagrangian, combined with the heavy quark spin symmetry and the chiral symmetry, we reinvestigated the interactions of $J/\psi N$ and its coupled channels, where we also took into account the contributions of the s-/u-channel diagrams ignored in the former works. We found that the s-/u-channel

contributions were not negligible; they were analogous with the case of $\bar{K}N$ interactions in the isospin $I = 1$ sector and differed from the isospin $I = 0$ sector of $\bar{K}N$ interactions and the VB interactions in the light quark sectors. As in the former works, the $\bar{D}\Sigma_c$, $\bar{D}^*\Sigma_c$, $\bar{D}\Sigma_c^*$, and $\bar{D}^*\Sigma_c^*$ bound states were found in the present work of which the masses and the widths were consistent with the former works within the uncertainties. Furthermore, we studied the $\Lambda_b^0 \rightarrow J/\psi K^- p$ and the $\Lambda_b^0 \rightarrow J/\psi \pi^- p$ decays with the scattering amplitudes of $J/\psi N$ obtained, and got a consistent line shape of the $J/\psi p$ mass distribution with the LHCb experiments [4,11,39] where we can associate the $P_c(4450)$ state as a $\bar{D}^*\Sigma_c$ bound state.

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