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Lattice QCD thermodynamics up to the perturbative regime

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Abstract

We study the thermodynamics of the quark gluon plasma with lattice simulations in the continuum limit up to 1 GeV temperature where we show that a perturbative description already applies. We calculate the effect of the presence of charm quark in the equation of state and also describe the topological features of quantum chromodynamics. This conference contribution is based on Nature 539 (2016) no.7627, 69-71 and Phys.Rev. D92 (2015) no.11, 114505.

Keywords: lattice QCD, equation of state, fluctuations of conserved charges, topological susceptibility

1. Introduction

The topic of this conference contribution is motivated in two ways, one theoretical and one phenomenological. On the theoretical side, we can ask: When is a weak coupling description of the QGP accurate? In this contribution we use recent high temperature lattice results to check. In particular we calculate the equation of state, fluctuations of conserved charges and the topological susceptibility and compare with weak coupling estimates. The phenomenological motivation of this work comes from the fact that all of these quantities are important either for heavy ion physics or cosmology. For details of the lattice calculations, the reader is advised to consult the papers [4] and [5].

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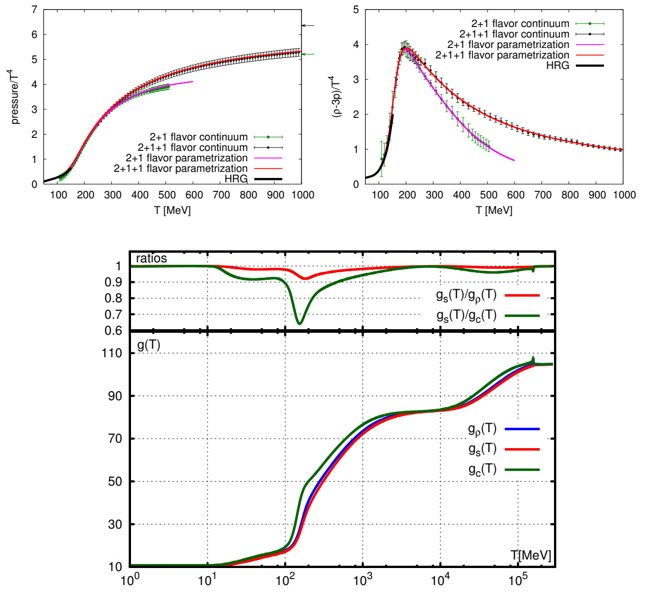

Fig. 1: Top: The QCD equation of state for 2 + 1 and 2 + 1 + 1 flavours of quarks. Left: pressure, Right: trace anomaly Bottom: The Standard Model equation of state. I.e. the effective degrees of freedom g_ρ for the energy density ($\rho = g_\rho \frac{\pi^2}{30} T^4$), g_s for the entropy density $s = g_s \frac{2\pi^2}{45} T^3$, and g_c for the heat capacity ($c = g_c \frac{2\pi^2}{15} T^3$) for the Standard Model. Neglecting the cosmological constant, the time dependence of the temperature in the early universe is given by these factors as: $\frac{dT}{dt} = -\frac{2\pi^{3/2}}{3\sqrt{5}} \frac{T^3}{M_{Pl}} \frac{\sqrt{g_\rho} g_s}{g_c}$, where M_{Pl} is the Planck mass. At temperatures $T < 1$ MeV the equilibrium equation of state becomes irrelevant for cosmology, because of neutrino decoupling. The EoS comes from our calculation up to $T = 100$ GeV. At higher temperatures the electroweak transition becomes relevant and we use the results of Ref. [10]. Note that for temperatures around the QCD transition non-perturbative QCD effects modify the EoS significantly, compared to the ideal gas limit, an approximation which is often used in cosmology, e.g. g_s/g_c is reduced from the SB limit by about 35%. Also note that g_s/g_c has four local minima: near the muon threshold, the QCD transition, the W, Z-boson thresholds and the electroweak transition. For parameterizations for the QCD regime or for the whole temperature range see [5].

2. The equation of state

The equation of state is of high importance for both heavy ion phenomenology, and cosmology, since it gives the expansion rate of the fireball created in the collision, and the universe respectively through the hydrodynamic and Friedmann equations respectively. The equation of state for 2+1 flavours of dynamical quarks is known for some time [6, 7]. Here we present the equation of state with the charm quark included (Fig. 1). The inclusion of the charm quark in the equation of state might effect hydrodynamic modeling of heavy ion collisions. For the case of cosmology, the inclusion of heavy quarks is absolutely necessary. Our lattice calculations also allow for a comparison with perturbative calculations of the heavy quark threshold effects. The pressure of the 2 + 1 + 1 and 2 + 1 flavour pressures are very well reproduced by a tree-level correction, given by $\frac{p^{(3+1)}(T)}{p^{(4)}(T)} = \frac{\mathrm{SB(3)+F_Q(m_c,T)}}{\mathrm{SB(4)}}$, where F_Q is the dimensionless free energy density of a free massive quark, and SB stands for the Stefan-Boltzmann limit. The small $O(g^2)$ correction calculated in [8] also matches the lattice results well. Note that while the ratios are well reproduced by these perturbative calculations, the pressures themselves are not. Knowing the threshold correction allows an extrapolation of the equation of state up to much higher temperatures with the help of perturbative results in the literature. [9] gives the result for massless quarks in the form $p = \# + \#g^2 + \#g^3 + \#g^4 + \#g^4 \log(g) + \#g^5 + \#g^6 \log(g) + \#g^6$. The g^6 term has a non-perturbative coefficient. After we introduce the tree-level correction coming from the charm quark mass, we fit it to the lattice results, and obtain $-3200 < q_c < -2700$. Next, keeping q_c fixed, we can introduce the bottom quark threshold at tree level to obtain $p^{(2+1+1+1)(T)} = p^{(2+1+1)}(T)\frac{\mathrm{SB(4)+F_Q(m_b/T)}}{\mathrm{SB(4)}}$.

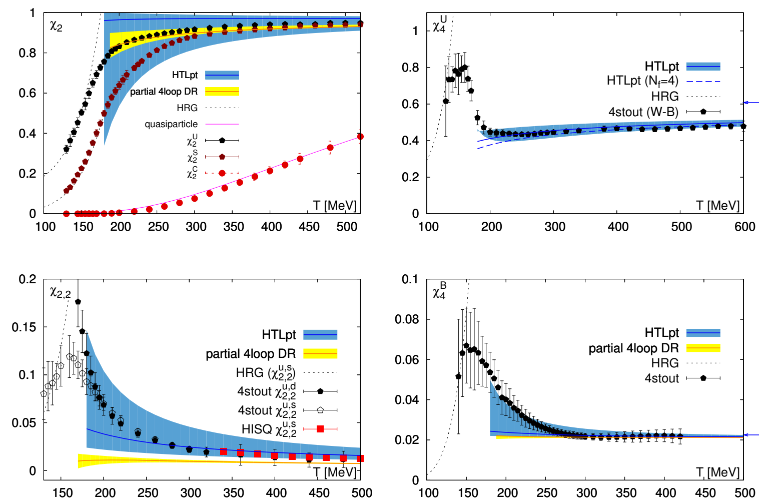

Fig. 2: Various second and fourth order susceptibilities calculated on the lattice and compared to HRG and resumed perturbation theory.

This allows for the estimation of the QCD equation of state in the 2 + 1 + 1 + 1 flavour case. Note, that the perturbative threshold correction is expected to work even better for the bottom quark compared to the charm quark. By also using the results for the electroweak transition from [10] this allows one to estimate the full cosmological equation of state in the standard model. This is seen in Fig. 2.

3. Fluctuations of conserved charges

The higher order susceptibilities or fluctuations with respect to quark chemical potentials are defined as $\chi^{u,d,s,c}_{i,j,k,l} = \frac{\partial^{i+j+k+l}(p/T^4)}{(\partial\hat{\mu}_u)^i(\partial\hat{\mu}_d)^j(\partial\hat{\mu}_s)^k(\partial\hat{\mu}_c)^l}$ and with respect to the baryon number, strangeness and electric charge as $\chi^{B,S,Q}_{i,j,k} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_S)^j(\partial\hat{\mu}_Q)^k}$,, where $\hat{\mu} = \mu/T$. These observables are interesting for their sensitivity to criticality, for probing the composition and distribution of the conserved charges in the QCD medium, and for providing stringent tests on the hadron resonance gas model at low and resumed perturbation theory at high temperatures. They can also be used for extrapolation of observables to small finite μ. Our lattice results and comparisons to the hadron resonance gas and various versions of resumed perturbation theory can be seen in Fig. 2. All the observables we consider show an agreement with the HRG model up to $T \sim 150-155$ MeV. In addition to the HRG we show a naive quasiparticle estimate for the charm susceptibility. The charm quark mass ($m_c^{QP} = 1430$ MeV) is empirical, and may depend on the range of the matching to the data. The quasiparticle models results are overestimating the lattice data below approx. 350MeV. The resumed perturbative results are from Ref.s [1, 2, 3]. For a similar lattice calculation, with results in agreement with ours, see also [12]. For an earlier study, with similar results, but less observables considered, see [11].

4. The topological susceptibility

The topological susceptibility is defined as $\chi_t = \frac{\langle Q^2 \rangle}{V_4}$ with $Q = \frac{1}{32\pi^2}\int d^4x F_{\mu\nu}\tilde{F}^{\mu\nu}$. In axion cosmology, the importance of this quantity comes from the temperature dependent axion mass, given by $f_A^2 m_A^2 = \chi_t$, where f_A is the axion scale. From the theoretical side, the topological susceptibility basically gives the instanton density. This suggests that a necessary condition for a perturbative treatment to be feasible is a small topological susceptibility. The topological susceptibility is indeed small, as can be seen in Fig. 3.

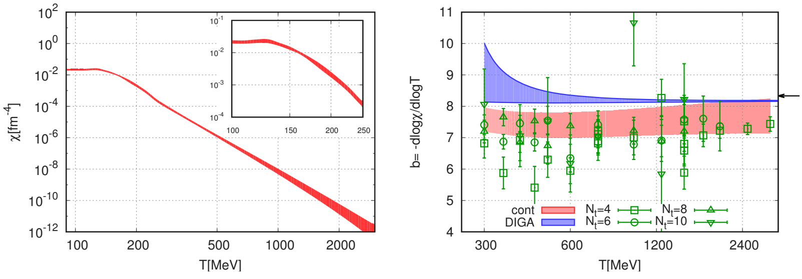

Fig. 3: Left: The topological susceptibility from our lattice calculations in [5]. Right: the exponent compared to the DIGA.

Here, we can again ask the question, is a weak coupling treatment, in this case the dilute instanton gas (DIGA), accurate? It turns out the exponent is in approximate agreement with DIGA for $T \sim 1$GeV, but the prefactor is off by an order of magnitude (the real prefactor is an order of magnitude larger compared to DIGA). The details of our lattice calculation can be found in [5]. Under some assumptions about cosmology, this leads to a prediction on the axion mass of roughly $m_A = 50\mu$eV. See [5]. For other lattice calculations of the topological susceptibility, see [13, 14]. Even though the method of calculation is different, our results agree with Ref. [13], but we cover a larger temperature range and have smaller errorbars. Our results do not agree with [14], most likely due to the incorrect continuum extrapolation in that paper. (For more details on the comparison, see [5])

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