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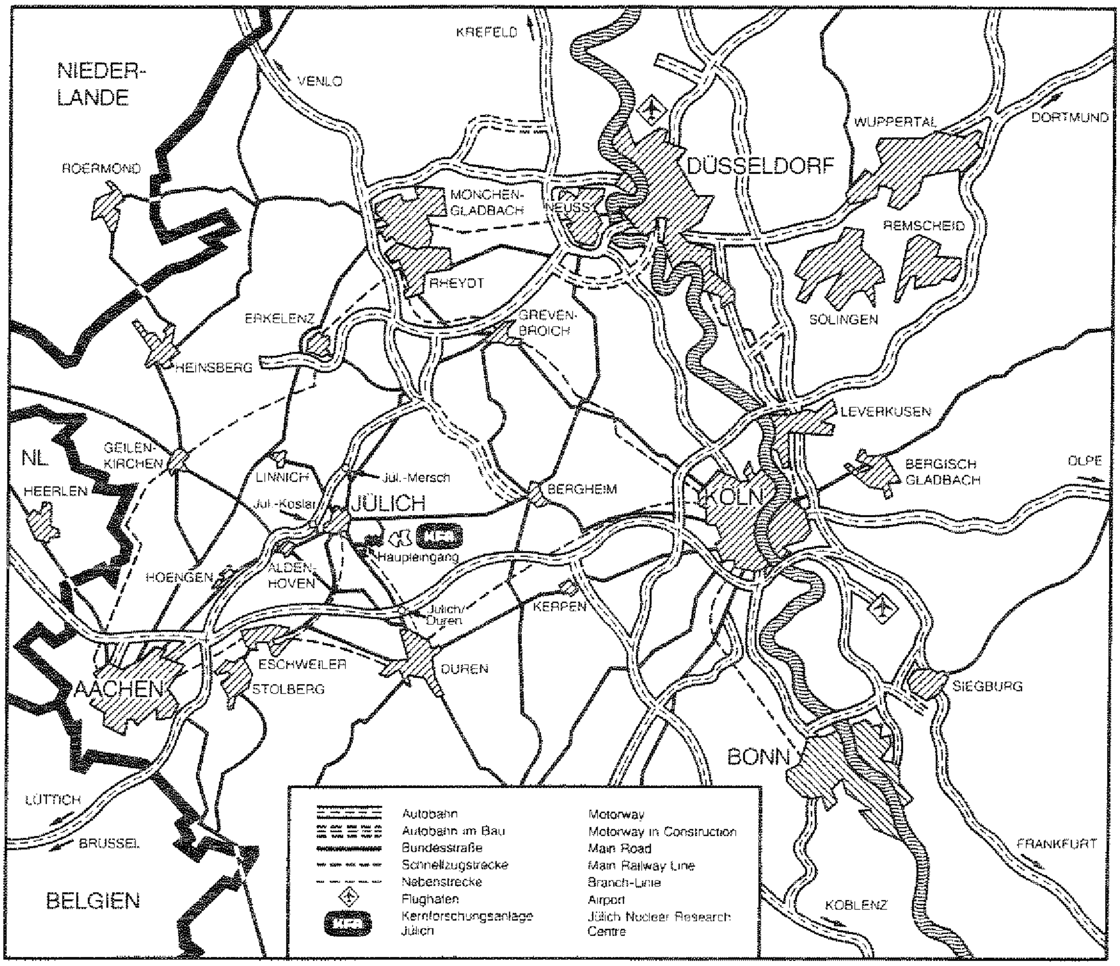
Jül-1972
February 1985
ISSN 0366-0885



Als Manuskript gedruckt

Berichte der Kernforschungsanlage Jülich – Nr. 1972
Institut für Plasmaphysik, Association EURATOM-KFA Jül-1972

Zu beziehen durch: ZENTRALBIBLIOTHEK der Kernforschungsanlage Jülich GmbH
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I show that under certain conditions the spectrum of density fluctuations associated with a toroidal version of the rippling instability presents a significant top-bottom asymmetry. The latter follows from the asymmetry of the radial component of the magnetic drift. The wave propagates in the direction of the electron diamagnetic velocity. Its frequency is about $1.71\ \eta_e \omega_e^*$ [ω_e^* = electron diamagnetic frequency; $\eta_e = (d\ln T_e/dr)/(d\ln N/dr)$].

Fluctuations of relatively large amplitudes have been detected in the edge plasma of many tokamaks[1-3]. The hypothesis[4-5] that they might arise from rippling instabilities[6] at first seemed to fail as these are efficiently suppressed by the diamagnetic drift in the cylincrical model[7] used by these authors. Recently new arguments in favour of the rippling instability arose indirectly from far infrared scattering diagnostics in TEXT, the Texas Tokamak Experiment; these showed a marked top-bottom asymmetry of the density fluctuation amplitude. Such an asymmetry, which reverses with the direction of the plasma current, is the signature of the radial component of the magnetic drift; it does not fit with a drift wave model, and implies that the poloidal mode number is smaller than the radial mode number. The second conclusion points at a toroidal version of the rippling branch.

I have accordingly solved the two-fluids equations in the parameter range where the rippling instability is expected to occur. I have found an unstable toroidal version of the otherwise stable cylindrical mode[7]. Remarkably the amplitude of the density fluctuation spectrum can present a marked top-bottom asymmetry, whereas the potential fluctuation spectrum is essentially symmetric.

The equations to be solved are:

$$\partial\tilde{n}/\partial t + \hat{n}\cdot\vec{\nabla}(u_{\parallel,J} + \tilde{n}\, U_{\parallel,J}) - 2\, \vec{u}_J\cdot\hat{b}\;\hat{b}\cdot\vec{\nabla}B/B = -i\,\omega_e^{*}\,\hat{\Phi} \qquad (1)$$

$$1 \qquad\qquad 1 \qquad\qquad \eta^{-1} \qquad\qquad \eta^{-1}$$

$$m_i\, N\left[\partial/\partial t - 2\,(cT_i\hat{b}\cdot\vec{\nabla}B/e\,B^2)\;\hat{p}\cdot\vec{\nabla}\right]u_{\parallel,i} = -\hat{n}\cdot\vec{\nabla}\,(p_i+p_e) \qquad (2a)$$

$$1 \qquad\qquad \eta^{-1} \qquad\qquad 1$$

$$\hat{n}\cdot\vec{\nabla}\,(\tilde{p}_e - \tilde{\Phi} + 0.71\,\tilde{t}_e) + (0.51/c_e^2\,\tau_e)\left[(u_{\parallel,e}-u_{\parallel,i}) + \tilde{n}U_{\parallel,e} - \frac{3}{2}\,\tilde{t}_e\,U_{\parallel,e}\right] = 0$$

$$1 \qquad\qquad \eta^{-3} \qquad (2b)$$

$$(\partial/\partial t + U_{\parallel,J}\,\hat{n}\cdot\vec{\nabla})\;(1.5\,\tilde{t}_J-\tilde{n}) + i\omega_e^{*}\,(1.5\,\eta_J - 1)\,\tilde{\Phi}$$

$$\eta^{-3} \qquad\qquad \eta^{-3} \qquad \eta^{-4}$$

$$-5\;(cT_J\hat{b}\cdot\vec{\nabla}B/e_JB^2)\;\hat{p}\cdot\nabla\tilde{t}_J = B\;\hat{n}\cdot\vec{\nabla}\left[(\chi_{\parallel,J}/NB)\;\hat{n}\cdot\vec{\nabla}\tilde{t}_J\right] \qquad (3)$$

$$\eta^{-4} \qquad\qquad 1\;;\;0$$

where $\tilde{n} = n/N$, $\tilde{t}_J = t_J/T_J$, $\tilde{\Phi} = e\,\Phi/T_e$, and

$$\vec{u}_J = (c/Be_JN)\;\hat{n}\times\vec{\nabla}(p_J+e_JN\phi). \qquad (4)$$

The definition of the units vectors are: $\hat{n} = \vec{B}/B$, $\hat{p}$ = normal to magnetic surface = direction of background temperature and density gradients, $\hat{b} = \hat{n}x\hat{p}$. The symbol $\|$ refers to the direction of the magnetic field. Equation (1) is obtained by projection of the curl of the momentum equation onto the unit vector $\hat{n}$. I have deliberately ignored ion ineratia and finite Larmor radius effects, but discuss their role in the last paragraph; parallel viscosity has been likewise neglected. I note that the divergence of the flow $\vec{\nabla}\cdot\vec{u}_J$ (species J) has been eliminated via the continuity equation. The terms $\vec{\nabla}\cdot\vec{u}, \hat{n}\cdot\vec{\nabla}u_{\|}$, and $2\vec{u}\cdot\hat{b}\ \hat{b}\cdot\vec{\nabla}B/B$ all arise from $[\hat{n}\cdot\vec{\nabla}x(\vec{u}x\vec{B})]$ under the assumption that $\hat{b}\cdot\vec{\nabla} << \hat{p}\cdot\vec{\nabla}$ ("radial" wavelength much shorter than "poloidal" wavelength). In Eq. (2a), I have assumed $U_{\|,i} \to 0$; the second term on the left-hand side arises from the gyroviscosity; another contribution from the gyrostress tensor has cancelled the convective derivative $\vec{U}_i\cdot\vec{\nabla}u_{\|,i}$. The last term of Eq. (2b) is due to the resistivity gradient and, as usual, drives the rippling instability. In the heat equation (3) the term $(\hat{b}\cdot\vec{\nabla}B/B)\ \hat{p}\cdot\vec{\nabla}\tilde{t}_J$ follows from the divergence of the contribution $5\ cNT_J\ \hat{n}x\vec{\nabla}t_J/2e_JB$ to the heat flux as defined by Braginskii[8]. Eq. (4) gives the perpendicular velocity appearing in (1); this expression has also been used in writing the convective derivatives $\vec{u}_J\cdot\vec{\nabla}T_J$ and $\vec{u}_J\cdot\vec{\nabla}N$ in the heat equation. $\chi_{\|,J}$ is the parallel heat conduction coefficient and τ_e the electron collision time[8]. Finally I have identified $\hat{b}\cdot\vec{\nabla}\tilde{n} = -i1B\tilde{n}/RB_\theta$ and $\omega_e^* = (1cT_e/ReB_\theta)\hat{p}\cdot\vec{\nabla}\ln N$; 1 is the toroidal mode number.

Solving Eqs. (1-4) usually ends up in a formidable eigenvalue problem; various limits have then to be considered to explore the stability of the different regions of parameter space. A later paper will be devoted to such a detailed analysis. Here my purpose, motivated by the TEXT results, is to show as simply as possible that strongly asymmetric density spectra can be excited under some specific conditions. Consistently with this goal I choose to scale the free parameters as to maximize the asymmetry and to eliminate those physical effects which, although always present to some degree, are not directly connected with it. By an iterative procedure, I came to the conclusion that the asymmetry will be important if

$$\langle k_\parallel \rangle U_{\parallel e}/\omega \sim 1/\eta^2 \tag{5}$$

(I assume $\eta_i \cong \eta_e$) where $k_\parallel$ is the parallel mode number of an oscillation with toroidal (ϕ) mode number 1; the latter is localized in the vicinity of a rational surface r: $q(r) = \oint d\theta (rB_\phi/RB_\theta)/2\pi = \oint d\theta\ \nu(\theta,r)/2\pi = -m/1$, where m is also an integer. Hence $k_\parallel = (1/R)\ x\ \partial\ln\nu/\partial r$, where x is the distance from the rational surface. ω is the wave frequency and <> implies that a characteristic value is considered. In order to minimize the role of parallel electron heat conduction (stabilizing) on the one hand, and the mathematical complexity which would result from a large growth rate ($\mathrm{Im}\,\omega \sim \mathrm{Re}\omega$) on the other hand, it is necessary to adopt the compromise $\omega \gg k_\parallel^2 c_e^2 \tau_e \gg \omega/\eta^2$; I thus assume

$$k_\parallel^2 c_e^2 \tau_e \sim \omega/\eta \qquad ; \qquad \eta \gg 1.$$

($c_J^2 = T_J/m_J$). Although the requirement $\eta \gg 1$ is somewhat artificial, the theory, developed on the basis of an expansion in η^{-1}, nevertheless shows the trend as well as a more complete but also more complicated solution involving many free parameters associated with as many different processes. Solving by an expansion procedure also requires knowledge of the characteristic radial length scale. The theory will show that

$$\langle \xi \rangle \equiv \langle \frac{x}{a_i} \rangle \sim \frac{c_i}{\omega R} \left(\frac{\omega}{k_\parallel U_{\parallel,e}}\right)^{1/2} \sim \eta \frac{c_i}{\omega R} \tag{7}$$

where $a_i = c_i/\Omega_i$ (Ω_i = gyrofrequency); the last equality follows from Eq. (5). It remains to scale the relevant and independent parameter $U_{\parallel,e}/c_i$ and $c_i/\omega R$. It is convenient to choose

$$U_{\parallel,e}/c_i \sim c_i/\omega R \sim 1$$

so that $<\xi> \sim \omega/\omega_e^* \sim \eta$ and $L_N/L_S \sim r/R \sim \eta^{-2}$; $L_S^{-1} = -(B_\theta/B_\phi)\, d\ln q/dr$ is the inverse of the shear length. Also $k_\| R \sim \eta^{-2}$ and $k_\| c_e \tau_e \sim k_\| c_i \tau_i \sim \eta (m_e/m_i)^{1/2}$. I assume $(m_e/m_i)^{1/2} \sim \eta^{-4}$; ion viscosity and parallel and perpendicular ion heat conduction can then be neglected. I now let $\tilde{n} \sim \tilde{t} \sim \tilde{\Phi} \sim u_{\|,J}/c_i \sim \lambda$ where $\lambda \to 0$ is the linearization parameter and scale the different terms of Eqs. (1-3) as shown.

I obtain successively from Eqs. (1), (2b) (the latter at order η^{-2}), and (3):

$$\oint d\theta\, \partial\tilde{n}^o/\partial t_o = \oint d\theta\, (1.71\, \tilde{t}_e^{(o)} - \tilde{\Phi}^{(o)}) = 0 \tag{9}$$

and

$$\oint d\theta\, \left[(\partial\tilde{t}_e^{(o)}/\partial t_o) + i\omega_e^* \eta_e\, \tilde{\Phi}^{(o)}\right] = 0 \tag{10a}$$

Hence the dispersion relation

$$i\partial/\partial t_o = \omega_o = 1.71\, \eta_e \omega_e^* . \tag{10b}$$

There follows also from Eqs. (3) and (2b) that

$$\partial\tilde{t}_e^{(o)}/\partial\theta = \partial(\tilde{n}^{(o)} - \tilde{\Phi}^{(o)})/\partial\theta \tag{11}$$

Introducing in (1) yields

$$\oint d\theta\, (1.71\, \eta_e \tilde{n}^{(1)} - \tilde{\Phi}^{(o)}) = 0 \tag{12}$$

whereas, still from (1), I obtain

$$\partial \left[(u_{\|,e}^{(o)} - u_{\|,i}^{(o)}) + \tilde{n}^{(o)} U_{\|,e} \right] / \partial\theta = 0. \tag{13}$$

Eqs. (2b) and (3) can be combined to yield

$$\left[(\partial/\partial t_1) - i\omega_e^* - 1.5\, \eta_e \omega_e^* \,(0.51\, U_{\|,e}/k_\| c_e^2 \tau_e) + (2\, k_\|^2 \chi_{\|,e}/3N) \right] \tilde{t}_e^{(o)}$$

$$= - \eta_e \omega_e^* \,(0.51/k_\| c_e^2 \tau_e)\, (u_{\|,e}^{(o)} - u_{\|,i}^{(o)} + \tilde{n}^{o}\, U_{\|,e}) \tag{14a}$$

It remains to eliminate $u_{\|,e}^{(o)} - u_{\|,i}^{(o)} + \tilde{n}^{(o)} U_{\|,e}$. I note first that there results from Eqs. (1) at order η^{-1}, (3) at order η^{-3}, and (11), that

$$\oint d\theta\, (\hat{b}\cdot\vec{\nabla}B/B) \{ \tilde{n}^{(o)}, \tilde{t}_i^{(o)}, \tilde{\Phi}^{(o)} \} = 0. \tag{15}$$

From (1) at order η^{-2} and (13) I also obtain

$$u_{\|,e}^{(o)} - u_{\|,i}^{(o)} + \tilde{n}^{(o)} U_{\|,e} = 2\, i\, \frac{cT_i}{eB} \frac{1}{k_\|}\, \hat{p}\cdot\vec{\nabla} \oint \frac{d\theta}{2\pi} \frac{\hat{b}\cdot\vec{\nabla}B}{B}$$

$$\left[(1 + \frac{T_e}{T_i})\, \tilde{n}^{(1)} + \tilde{t}_i^{(1)} \right] \tag{14b}$$

since [Eqs. (3) and (2b)]

$$\partial \tilde{t}_e^{(1)}/\partial\theta = \partial(\tilde{n}^{(1)} - \tilde{\Phi}^{(1)})/\partial\theta = 0.$$

The problem thus becomes to eliminate $\tilde{t}_i^{(1)}$ and $\tilde{n}^{(1)}$ in terms of the lowest order functions. The equations requested are Eq. (3) at order η^{-4} on the one hand and Eqs. (1) and (2a), both at order η^{-1}, on the other hand. Noting that $\partial^2\, (\hat{b}\cdot\vec{\nabla}B/B)/\partial\theta^2 = -\hat{b}\cdot\vec{\nabla}B/B$, one obtains

$$\oint d\Theta \frac{\hat{b}\cdot\vec{\nabla}B}{B}\left[\frac{3}{2}\tilde{t}_i^{(1)} - \left(1 + \frac{3}{2}\eta_i\frac{\omega_e^*}{\omega_o}\right)\tilde{n}^{(1)}\right] = 5\frac{cTi}{eB}\frac{i}{\omega_o}\hat{p}\cdot\vec{\nabla}\oint d\Theta\left(\frac{\hat{b}\cdot\vec{\nabla}B}{B}\right)^2 \hat{t}_i^{(o)} \tag{14c}$$

and

$$\oint d\Theta \frac{\hat{b}\cdot\vec{\nabla}B}{B}\left\{\tilde{n}^{(1)} - \left(\frac{c_i}{qR\omega_o}\right)^2\left[\left(1 + \frac{T_e}{T_i}\right)\tilde{n}^{(1)} + \tilde{t}_i^{(1)}\right]\right\}$$

$$= 2\frac{cTi}{eB}\frac{i}{\omega_o}\hat{p}\cdot\vec{\nabla}\oint d\Theta\left\{\left(\frac{\hat{b}\cdot\vec{\nabla}B}{B}\right)^2\left(\tilde{n}^{(o)} + \tilde{t}_i^{(o)} + \frac{T_e}{T_i}\tilde{\Phi}^{(o)}\right)\right.$$

$$\left. - \left(\frac{\hat{b}\cdot\vec{\nabla}B}{B}\frac{c_i}{qR\omega_o}\frac{\partial}{\partial\Theta}\right)^2\left[\left(1 + \frac{T_e}{T_i}\right)\tilde{n}^{(o)} + \tilde{t}_i^{(o)}\right]\right\} \tag{14d}$$

I note at this point that the system of Eqs. (9-15), to which one should add (from (3))

$$\tilde{t}_i^{(o)} = (2/3)\,\tilde{n}^{(o)} + (\eta_i\omega_e^*/\omega_o)\,\tilde{\Phi}^{(o)} \tag{16}$$

is not closed as yet. This non-closure reflects the freedom that I have of selecting the poloidal structure of the eigenfunction. Seeking for the most simple Θ dependence, and guided by (9) and (15), I assume

$$\tilde{n}^{(o)} = 0. \tag{17}$$

There immediately follows that $\tilde{t}_i^{(o)} = \tilde{t}_e^{(o)} = \tilde{\Phi}^{(o)}/1.71$ are independent from Θ (I assume $T_e = T_i = T$ and $\eta_e = \eta_i = \eta$ from now on). Solving for $\tilde{n}^{(1)}$, $\tilde{t}_i^{(1)}$, and introducing in (14b) and then in (14a) results in the following equation:

$$\left[A(\partial^2/\partial\xi^2) - B(\gamma/\omega_o)\xi^2 + C\xi\right]\tilde{\Phi}^{(o)} = 0. \tag{18}$$

Here, $A = 12.25\ (c_i/R\omega_o)^2/\left[1 - 3.25\ (c_i/qR\omega_o)^2\right]$, $B = k_\parallel'^2 a_i^2 c_e^2 \tau_e/0.51\ \omega_o$, $C = 3\ k_\parallel' U_{\parallel,e}\ a_i/\ 3{,}42\ \ \omega_o$, and

$$\gamma = (\partial/\partial t_1) - i\omega_e^* + 2\, k_\|^2 \chi_{\|,e}/3N \tag{19}$$

which shows the stabilizing role of parallel electron heat conduction. Here I treat γ as a constant. I find also

$$\frac{1}{2\pi}\oint d\theta\ (\hat{b}\cdot\vec{\nabla}B/B)\ \tilde{n}^{(1)} = i\ D\ (c_i/R^2\omega_o)\ \partial\tilde{\Phi}^{(o)}/\partial\xi \tag{20}$$

where $D = \left[1.58 + 0.97\ (c_i/qR\omega_o)^2\right] / \left[1 - 3.25\ (c_i/qR\omega_o)^2\right]$.

The solution of Eq. (18) is of the form

$$\tilde{\Phi}^{(o)} \propto \exp\left[-\Theta^{1/2}\ (\xi + \delta)^2/2\right] \tag{21}$$

with $(\gamma/\omega_o) = (-C/2B)\ (-C/2A)^{1/3}$, $\delta = (-C/2A)^{-1/3}$, and $\Theta^{1/2} = \delta^{-2}$.

It can be shown that there is no instability if A is negative. This result is not surprising for, in this case, the potentially defavourable effect of the magnetic drift is averaged over. If A is positive, the growth rate is given by

$$\gamma\tau_e = \frac{0.24}{A^{1/3}}\ \frac{m_e}{m_i}\ \left|\ \eta\ \frac{L_S}{L_N}\ \frac{U_\|^2}{c_i^2}\ \right|^{2/3} \tag{22}$$

whatever the sign of η. Furthermore,

$$\delta = -1.57\ A^{1/3}\ \left|\ \eta\ \frac{L_S}{L_N}\ \frac{c_i}{U_\|}\ \right|^{1/3} \text{sign}\ \eta. \tag{23}$$

The density fluctuation is now obtained from Eqs. (12) and (20). Assuming $\tilde{n}^{(1)}_{k_\theta}(\xi) = (\alpha+\beta \sin\theta)\, \tilde{\Phi}^{(o)}/1.71\, \eta_e$, I get $\alpha = 1$, $\beta = 2iD(c_i/R\omega_e^*)\, \partial\tilde{\Phi}^o/\tilde{\Phi}^o\partial\xi$ with the convention that $\theta = 0$ is the poloidal angle on the low field side in the equatorial plane. The density fluctuation spectrum is thus

$$|\tilde{n}_{k_\theta,k_r}|^2 \propto \left[1 + 2\,D\,(k_r L_N/k_\theta R)\,\sin\theta\right]^2 \exp(-\,\Theta^{-1/2}\,k_r^2 a_i^2) \qquad (24)$$

where $L_N^{-1} = d\ln N/dr$; $k_\theta = -lq/r$ is the poloidal mode number. Eq. (24) shows that a scan of the scattering volume along a vertical chord keeping k_r/k_θ fixed will display a top bottom asymmetry as observed in the Texas Tokamak Experiment[3].

In concluding, I note that ion inertia and gyroviscosity would enter Eq. (1) at order η^{-2}; since I made use of this equation up to that order (see Eq. (14b)) I stress that Eq. (18) is valid as long as $7.73\,(c_i/\omega_o R)^2 > 1$ as a more thorough calculation shows. In the opposite limit, one recovers the cylindrical result and the plasma is stable, the parameter A changing sign[7].

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