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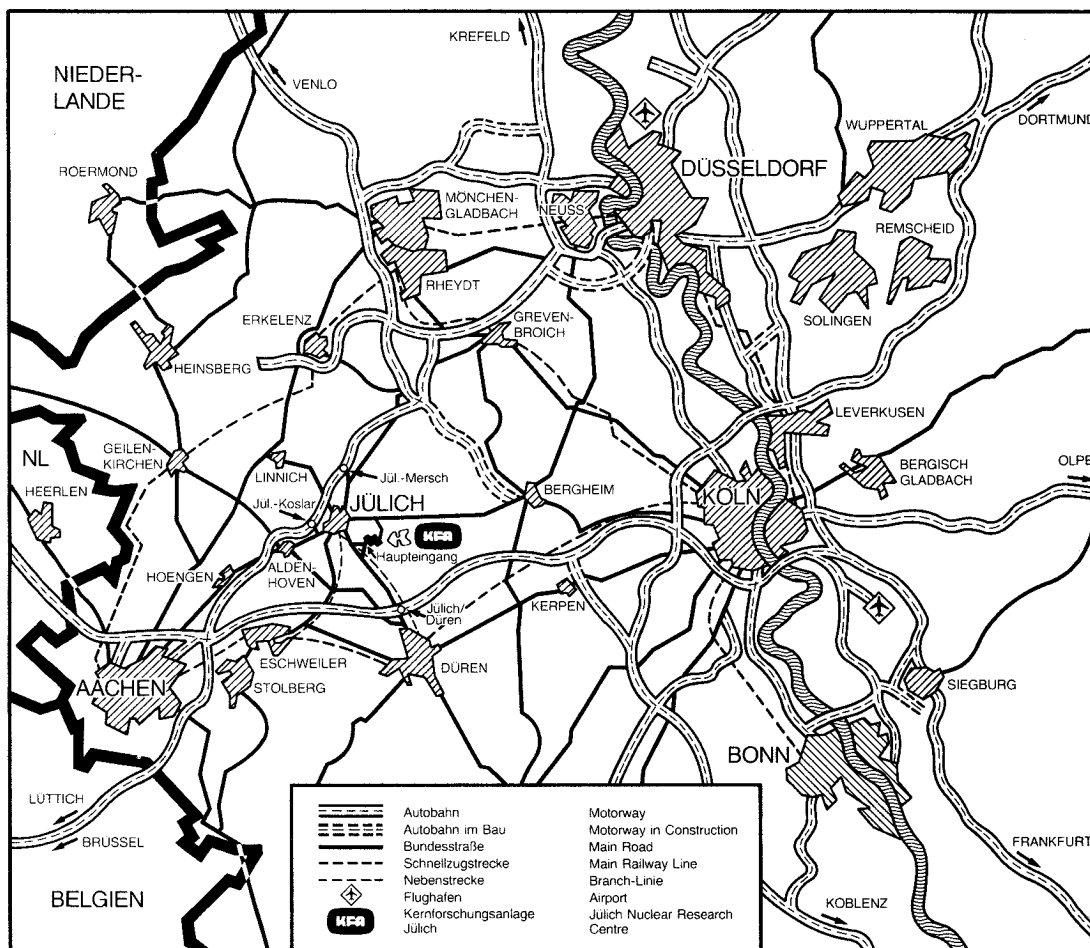
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**Study of Nuclear Medium Effects on the
Effective Interaction Based on the
One-Boson Exchange Model**

by

K. Nakayama

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Study of Nuclear Medium Effects on the Effective Interaction Based on the One-Boson Exchange Model

by

K. Nakayama

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Abstract

In this work, starting from a realistic nucleon-nucleon interaction based on the one-boson exchange model for the nuclear force, we attempted a microscopic derivation of the effective interaction which may be appropriate for nuclear structure as well as for nucleon-nucleus scattering problems.

Short-range correlations and medium polarization as well as relativistic effects on both particle-hole and Δ -hole interactions have been investigated. For the nucleon-nucleon case short-range correlations are basically restricted to S-states and affect mainly the central components of the effective interaction. In contrast, the Δ -nucleon interaction is essentially unaffected by short-range correlations due to the Pauli principle restrictions and the momentum mismatch between the central components of the correlation operator and the tensor component of the bare transition potential. Based on these analyses it is shown that short-range correlation effects can be summarized in a very simple correlation operator.

The medium polarization affects dominantly the scalar-isoscalar component of the effective interaction in the nucleon-nucleon sector and tends to cure the instability of the system caused by the strongly attractive interaction due to short-range correlations. In the Δ -nucleon sector the medium polarization enhances the Δ -nucleon coupling strength $(g'_0)_{\Delta N}$ much more than the corresponding nucleon-nucleon coupling g'_0 , the reason being again the Pauli principle.

Relativistic effects give a considerable repulsive contribution in the scalar-isoscalar component of the effective interaction; however, in the present calculation these effects alone are not yet able to cure the instability of nuclear matter. The Δ -nucleon coupling strength $(g'_0)_{\Delta N}$ is enhanced by $\sim 17\%$

with respect to the conventional G-matrix result (no inclusion of relativistic effects) and together core polarization effects enhance $(g'_0)_{\Delta N}$ by $\sim 35\%$. Relativistic effects also show a number of interesting features which may improve the agreement with experimental data.

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1. Introduction

One of the major difficulties in theoretical physics is that one has to deal with a many-body problem whose solution remains unsolved. Nevertheless, for a long time several physicists have developed various approximation schemes in order to reduce the complicated many-body problem to a much simpler and solvable few-body problem which still retains relevant properties of the many-body system in question.

A classical example of such a many-body problem is an electron gas¹⁾ interacting by means of the Coulomb force. It is well known that ordinary perturbation treatments of the Coulomb interaction between electrons fail because of the occurrence of unphysical divergences which arise from the long range of the Coulomb interaction and the resulting singular behaviour of the Coulomb potential. However, these divergences can be removed by performing a formal summation of a special class of Feynman diagrams to all orders in perturbation theory. In general, which class or classes of diagrams have to be summed, depends on the specific nature of the interaction and on the characteristics of the many-body system under consideration. In the case of the electron gas, the class of diagrams which has to be summed in order to obtain a well-behaved effective interaction is the so-called "ring diagram" displayed in Fig. 1. The bare Coulomb interaction behaves like $1/q^2$ but the higher order diagrams on the r.h.s. in Fig. 1, which account for the long-range effects, screens this out to $1/(q^2+a^2)$.

In a nuclear system the problem is much more complicated than merely a many-electron problem. The main reason is that whereas in the electron case the basic interaction is well known, such an interaction is unknown for nucleons

from first principles. Nucleons are not elementary particles but they are composed by quarks. Thus, the nucleon-nucleon (NN) interaction should, in principle, be calculated from Quantum Chromodynamics (QCD). However, although considerable progress has been made recently, the NN potentials derived from QCD are not yet of such a quality to be used in nuclear structure calculations. Even if one could solve the three quark system starting from a QCD Lagrangian, one has still to deal with the problem of a many-three quark system. Therefore, in practice, the nuclear many-body system was for a long time and is still treated as a system consisting of the point-like particles (nucleons) interacting by means of the so-called "realistic" interaction. Such an interaction has been constructed by several authors²⁻⁵⁾ to reproduce the NN scattering data up to the pion production threshold and the deuteron properties. It should be stressed, however, that this procedure is unable to determine such an interaction uniquely. Consequently, there exist several NN phase-shift-equivalent interactions.

One of the important characteristics of these realistic interactions (hereafter called bare NN potentials) is the strong repulsive behaviour at short distances (~ 0.4 fm) which is required to reproduce NN phase-shift data. Within the one-boson-exchange model⁴⁾ for the nuclear force this strong repulsion arises from the vector ω -meson exchange contribution. The attractive intermediate-range part of the NN potential can be understood as arising from the exchange of scalar mesons such as the σ -meson which simulates systems of two pions coupled to spin $S=0$ and isospin $T=0$. The long-range part of the NN potential results from the exchange of the π -meson. Due to this strongly repulsive behaviour of the NN potential the ordinary perturbation theory fails completely in a nuclear system. Therefore, a formal summation of a special class of diagrams to all orders in perturbation theory is required.

Nuclei are special many-body systems which present saturation properties. Electron scattering experiments reveal that the interior of nuclei has a more or less constant density of $\rho_0 \sim 0.17$ nucleons/fm³. This fact allows us to separate the nucleus into two distinct regions: the interior and the surface region (of course, for light nuclei such a distinction becomes difficult). To study general properties of nuclei it is useful to make use of this saturation property of nuclei and consider a model which is an infinite homogeneous system consisting of equal numbers of protons and neutrons and having a given density ρ which may be chosen to have the value of the empirical saturation density of nuclei. Moreover, since we are interested in the nuclear interaction, the Coulomb force between protons is switched off. Such a system is known as infinite nuclear matter or simply nuclear matter. An important quantity in nuclear matter is the binding energy per particle at a saturation density ρ_0 . The empirical value of the binding energy (~ -16 MeV) can be extracted from the volume term of the semi-empirical Weizsäcker mass formula.

In nuclear physics, we frequently consider nuclear matter because it is one of the simplest nuclear many-body systems. The most traditional theory of nuclear matter is that introduced by Brueckner⁷⁾. The Brueckner-Bethe theory⁸⁾ of nuclear matter is currently formulated using the hole-line expansion method^{9,10)}. The physical idea behind the hole-line expansion is as follows: since the NN potential is strong at short distances, one may consider a theory which makes the expansion according to the number of particles correlated at short distances. In other words, in such a theory the expansion parameter is the product of the range of the NN interaction and the density of the nuclear system. Of course, such an expansion may be badly convergent for a high density system. However, for nuclear matter in the vicinity of the empirical saturation density, this expansion is expected to be rapidly convergent^{9,10)}. In terms of

diagrams the above expansion is known as the hole-line expansion whose leading term, which accounts for the two particle correlations, is the Brueckner G-matrix displayed in Fig. 2. The next term in this expansion are the diagrams which account for the three-body correlations. In the hole-line expansion this gives the three-body cluster diagram displayed in Fig. 3.

The convergence properties of the hole-line expansion method have been studied by Day¹¹⁾ who performed a three-body Bethe-Faddeev calculation. In order to have some feeling, here we summarized his results for the binding energy in nuclear matter; they are displayed in Table 1. The G-matrix result has also been added. As we can see the G-matrix contribution which accounts for the two-body correlations is the dominant term giving $\sim 84\%$ of the total potential energy. The three-body clusters give an additional contribution of $\sim 13\%$ and therefore cannot be neglected. These results from the hole-line expansion method are consistent with those¹²⁾ obtained from a Jastrow-type variational approach¹³⁾.

In contrast to the hole-line expansion, the coupled-cluster equations¹⁴⁾, recently applied in ground state calculations by the Bochum group¹⁵⁾, imply that the diagrams should not be grouped according to the number of hole-lines, but a partial summation of particular diagrams involving three, four, five, ... hole-lines should be carried out. The dominant contribution for this partial summation is given by the so-called "generalized ring diagrams" because it includes all the forward-going ring diagrams, which are known to account for the important long-range correlations in the electron gas as discussed before. The first term of the generalized ring series are the three-body cluster diagrams of the hole-line expansion. The coupled-cluster method goes beyond the Brueckner-Bethe hole-line expansion approach; however, in either case the lowest or-

der approximation involves a two-body equation. The convergence of the coupled-cluster calculation in nuclear matter has been studied in Ref. 16.

The fundamental ingredient in understanding the properties of excited states of a nucleus is the effective interaction acting between nucleons inside the nucleus. One of the most successful theories in obtaining such an interaction is the Landau-Migdal theory¹⁷⁾ of interacting Fermi systems. In this theory the crucial assumption is the concept of quasi-particles. Landau assumed that the interacting Fermi system can be described in terms of elementary excitations which carry the quantum numbers of single-particle states of non-interacting Fermi systems. These elementary excitations are called quasi-particles and may be thought of as particles which are dressed through the interaction with all the other particles in the system. Since these quasi-particle states are not eigenstates of the system, the concept of quasi-particles starts to lose its meaning, when going away from the Fermi surface, due to the decrease of lifetime of these states.

In the special situation when quasi-particles are on the Fermi surface and in the limit of vanishing momentum transfer, the so-called "Landau limit", the interaction between these quasi-particles, depends only on the angle between their initial momenta. Therefore, in this situation, the quasi-particle interaction can be expanded in terms of Legendre polynomials whose expansion coefficients, which are known as the Landau parameters, can be connected with observable quantities. Originally, Landau¹⁸⁾ applied these ideas to liquid ^3He and the expansion coefficients of the quasi-particle interaction have been related to the compressibility, specific heat and magnetic susceptibility. Later, this Landau theory has been extended to nuclear systems by Migdal¹⁷⁾. The extension of the Landau theory to finite systems, such as nuclei, requires

additional assumptions. The main reason is that the momentum of a quasi-particle is no longer a conserved quantity in a finite system. In nuclei, some of the Landau parameters can be related to the compression modulus, effective mass and symmetry energy. The Landau-Migdal theory has been applied extensively by several authors¹⁹⁻²²⁾ with a considerable success in the quantitative description of excited states of nuclei; from these analyses Landau parameters have also been determined, which are summarized in Table 2.

At this point one has to stress that the Landau-Migdal theory is a semi-phenomenological one since it does not derive the quasi-particle interaction starting from the basic NN interaction. A deeper understanding of the properties of nuclei requires a derivation of the effective interaction from microscopic considerations. In fact, in the last decade a considerable effort has been devoted to deriving effective interactions from this point of view²³⁻³¹⁾. A natural way of deriving such interactions microscopically is to extend the approximation schemes developed in the description of ground state properties. However, a straightforward application of these methods such as the hole-line expansion is dangerous, in the sense that diagrams which are unimportant in describing ground state properties (e.g. binding energy) could be essential for a description of excited states.

Early attempts to a microscopic derivation of the effective interaction have been carried out at the Brueckner G-matrix level by several authors^{23,24)}. There, the Brueckner G-matrix interaction has been related to the semi-phenomenological Landau quasi-particle interaction. The effective particle-hole interaction in the Landau theory is given by the double functional differentiation of the ground state energy with respect to the quasi-particle occupation number. Using the Brueckner G-matrix contribution to the ground state energy,

one gets as the particle-hole interaction the G-matrix interaction itself plus additional terms, one of which is exactly the first order ring diagram, which is known as the rearrangement term. As a first attempt, the above procedure seems to be quite reasonable (although it is not a fully microscopic derivation of the effective interaction) since $\sim 80\%$ of the experimental binding energy may be ascribed to the Brueckner G-matrix as discussed before. In fact, in addition to its traditional role in the calculation of ground state properties of nuclei, a Brueckner G-matrix interaction (pion- and rho-exchange potential folded with a central short-range correlation function) has recently been successfully applied to the description of magnetic states in nuclei²²⁾. Also a simple analysis of the Landau parameters derived from the Brueckner G-matrix shows qualitatively how the different components of an effective interaction influence the properties of the low-lying excited states.

There is also a number of works³²⁾, where the effective interaction has been derived from a variational calculation. Probably, one of the most sophisticated many-body theories presently available is the coupled-cluster method developed by the Bochum group¹⁵⁾, which has been extended³³⁾ to the description of excited states in nuclei. However, the application of this method in nuclear structure calculations seems to be rather cumbersome. In fact, a recent application by Emrich and Zabolitzky³⁴⁾ to the description of natural parity states in ^{16}O shows considerable discrepancies to experimental energy levels because of approximations found to be necessary due to the numerical difficulties. Such approximations lead to the non-inclusion of some collective effects on the effective interaction which play an important role in the description of low-lying collective states in nuclei.

In the present work we have attempted a microscopic derivation of the effective interaction rather guided by our physical intuition than developing a sophisticated and consistent many-body theory. For this purpose the effects of some correlations originated by the nuclear medium have been investigated in nuclear matter. We started from a realistic NN interaction based on the meson exchange theory. Within our approach the Brueckner G-matrix may be considered as the basic ingredient for the full effective interaction. Here, one has to stress that any reasonable microscopic theory which starts from a realistic NN interaction, as in the present work, must take into account the effects of short-range correlations arising from the strongly repulsive behaviour of this interaction at short distances. The Brueckner G-matrix accounts exactly for these correlations. When evaluated in nuclear medium, it also includes the important effects arising from the Pauli blocking.

Therefore, chapter I has been devoted to a detailed analysis of the Brueckner G-matrix. Based on these analyses a momentum transfer and density dependent quasi-particle G-matrix interaction has been constructed. It is, however, well known that the G-matrix interaction alone violates the stability conditions^{17,35)} of the ground state against the deformation of the Fermi surface in nuclear matter due to its strongly attractive scalar-isoscalar component. In fact, the Landau parameter extracted from a nuclear matter G-matrix interaction^{27,30)} alone leads to a value of the compression modulus $K = 6 \left(\frac{k_F^2}{2m} \right) (1+2f_0) < 0$, while empirically K is ~ 210 MeV³⁶⁾. This indicates that in nuclei correlations of other nature than that accounted for by the Brueckner G-matrix play also an important role.

In chapter II we have also constructed an energy-, density- and momentum transfer-dependent effective G-matrix interaction which may be suitable for

use in scattering problems. With the development and operation of a new generation of accelerators in the last decade, large amounts of nucleon-nucleus scattering data³⁷⁻⁴¹⁾ at intermediate and high energies have become available. In these energy regions the reaction mechanisms are much clearer than at lower energies and are, in general, largely dominated by one-step processes. This situation offers an excellent opportunity to test effective interactions derived from basic considerations when nuclear form factors which contain nuclear structure information are known from other experiments. Alternatively, one can regard the effective interaction as known and focus attention on obtaining nuclear structure information. In fact, such studies have been made by several authors⁴²⁻⁴⁴⁾ within the last years. Our effective G-matrix interaction constructed in chapter II may also be useful in such analyses.

In the recent years considerable emphasis has been given in considering the nucleus as a relativistic system. In fact, there is a number of works^{45,46)} which show some of the evidences for the relativistic behaviour of nuclei. One of the most applied relativistic models of nuclear dynamics⁴⁵⁻⁴⁸⁾ is known in the literature under the name of Dirac phenomenology. This model is characterized by having several free parameters which are adjusted to fit nuclear data. Quite recently, Anastasio et al.⁴⁹⁾ have developed the so-called "Relativistic Brueckner-Hartree-Fock" theory which distinguishes by having no free parameters other than those introduced in fitting free-space NN scattering data. These authors have shown that "relativistic effects" cause basically a decoupling of the scalar field in the nuclear interior and are very important for several aspects of nuclear structure physics. The microscopic derivation of the effective interaction involves a calculation where there is a large cancellation between strong repulsive ω -meson and attractive σ -meson exchange potentials in the scalar-isoscalar channel. Due to the decoupling of the scalar

field the resulting effective interaction presents less attractive scalar-isoscalar components than that which does not account for the relativistic effects and therefore tends to cure the instability of nuclear matter. In chapter III we have also investigated the relativistic effects on the effective interaction within a simple model where the nuclear medium effects on the bare NN potential can be displayed in a very transparent way.

Nuclei are systems which present collective excitation phenomena. It is a well known fact that these collective excitations can couple to single-particle states and change observable quantities in an important way. This effect is called core-polarization effect and builds up correlations of long-range character as in the case of the electron gas. A proper inclusion of core-polarization effects on the effective interaction has been carried out by Babu and Brown²⁶⁾ in the limit of vanishing momentum transfer. Sjöberg²⁷⁾ has applied this in nuclear matter and showed a strong reduction of the scalar-isoscalar component of the effective interaction, stabilizing the system. A particle clothed with collective excitations is substantially larger than the bare particle and harder to compress; consequently, it increases the compression modulus. Chapter IV has been devoted to study the role of the core-polarization effects on the effective particle-hole interaction including the relativistic effects. There we have also sketched briefly a method to derive an effective interaction starting from the Bethe-Salpeter equation⁵⁰⁾. Within this method the equation obtained by Babu and Brown²⁶⁾ as well as the Bethe-Goldstone equation appear as a special limit of a set of equations which couple particle-particle and particle-hole interactions. However, since the core-polarization effects are non-linear phenomena and make the resulting equations complicated to be solved, we have restricted ourselves to the limit of vanishing

momentum transfer in order to study these effects on the effective interaction.

In connection with the effective nucleon-nucleon interaction we have also investigated medium effects on the $\Delta(1232)$ -nucleon interaction. The recent discovery of the giant Gamow-Teller resonance³⁷⁻³⁹⁾ has caused intensive discussions and controversies in understanding spin-isospin excitation modes in nuclei. Although this collective mode, which is the spin-isospin nuclear sound, has been predicted in 1963⁵¹⁾, it was detected experimentally only in 1975 by Doering et al.³⁷⁾ through the intermediate energy (p,n)-reaction. While the excitation energy can be well reproduced by the usual random-phase-approximation (RPA)⁵²⁾, the corresponding transition strength is overestimated by this type of calculations. Since then, two different mechanisms have been suggested to explain this so to speak "quenching" of the Gamow-Teller strength. One of them is the excitation of energetically high-lying two particle-two hole states⁵³⁾ mixing with the low-lying one particle-one hole states and shifting the Gamow-Teller strength to a much higher energy region than the giant Gamow-Teller resonance. The other quenching mechanism, which is the most exciting one, is that due to the excitation of subnuclear degrees of freedom⁵⁴⁾, specifically the Δ_{33} -resonance. Here, $\Delta(1232)$ -isobar-nucleon-hole configurations can couple to proton-particle-neutron-hole configurations and remove the Gamow-Teller strength to the high energy Δ -isobar region. However, the magnitude of the reduction of Gamow-Teller strength due to the Δ -isobar-nucleon hole admixture in the nuclear wave function depends strongly on the Δ -isobar-nucleon coupling strength. Therefore, the determination of this coupling strength is of crucial importance, not only for understanding the role of Δ -isobars in nuclei but also for our understanding of spin-isospin excitations in nuclei. Also the recent work by Osterfeld et al.⁵⁵⁾ shows that a considerable fraction

of Gamow-Teller strength is located in the physical background below and beyond the giant Gamow-Teller state and therefore may escape experimental identification as Gamow-Teller strength.

It should be mentioned that our major aim in the present work was not to perform a numerically involved calculation of the effective interaction (except perhaps for the G-matrix interaction) but rather to understand the underlying physics inside the nuclear medium in a simple way.

CHAPTER I

Effective G-Matrix Interaction (Nuclear Structure)

Since the G-matrix interaction is the main ingredient of the full effective interaction, the present chapter has been devoted to detailed analyses of it. In addition, for its further applications a quantitatively accurate effective G-matrix interaction has been constructed.

First, we have studied the effects of correlations (Pauli blocking and short-range correlations) on each channel of the G-matrix interaction. Such a study provides us with a better understanding of the Brueckner G-matrix and suggests further approximations to it.

Second, we want to construct a G-matrix interaction which may be used in a description of low-lying excited states of nuclei. A number of authors⁵⁶⁻⁵⁸⁾ have constructed effective G-matrix interactions by averaging over the relative momentum with a weight function $W(k)$ which gives the probability of finding the relative momentum k in a Fermi sphere. In this work, however, we use a different approach, since the low-lying excited states are mainly built up by the excitation of particles near the Fermi surface and therefore require quite different weight function for averaging over the initial and final relative momenta. The density dependence of the effective G-matrix interaction is frequently mapped out by assuming the local density approximation (LDA). Since the averaging procedure of the relative momentum relates k to the Fermi-momentum k_F , the density dependence within the LDA may be completely masked by the k_F dependence of the averaged relative momentum if the G-matrix interaction depends strongly on the relative momentum. In order to avoid this problem, we have used a slightly different procedure for extracting the density dependence of the G-matrix interaction.

Presently several phase-shift-equivalent NN potentials are available in the literature^{3,4,59}). One aim of this chapter is to study the sensitivity of the G-matrix interaction to these potentials, since the study of low-lying excited states may provide a good way of distinguishing different phase-shift-equivalent NN potentials.

Based on these analyses we also made an extremely simple model to obtain the effective G-matrix interaction. The quantitative success of this model suggests that the G-matrix interaction may be obtained in a very simple way.

Since the $\Delta(1232)$ -isobar has been suggested to play an important role in the quenching of the magnetic states in nuclei, we have also devoted the present chapter to study the correlation effects on the Δ -nucleon G-matrix interaction.

The effective G-matrix interaction constructed in this chapter has been also tested in a realistic calculation (random-phase approximation) of low-lying natural parity states in nuclei such as ^{16}O and ^{208}Pb .

In order to do realistic nuclear structure calculations with a G-matrix it is convenient to determine its operator structure as is done⁴²) for the T-matrix in the free NN scattering problem. Recently, Dickhoff⁶⁰) has suggested a similar method for the G-matrix at low momentum transfer. In each case the method consists of a parametrization of the G-matrix elements with a convenient set of operators in spin-isospin space which are local in coordinate space. Also in Ref. 61 a method for extracting the matrix elements of each component of an interaction is given. In this work we derive the operator structure of the G-matrix, each term of which is expressed as a sum of its partial-wave matrix elements multiplied by the corresponding geometrical (angular momentum recou-

pling) factors. The spin (isospin) dependence is expressed in terms of combinations of the usual Pauli-spin(isospin) matrices σ_1 and σ_2 (τ_1 and τ_2). Such combinations determine the tensorial rank of the G-operator. This method of extracting the operator structure is simply the partial-wave decomposition of a two-body operator. It is completely general and can be applied, in principle, to any two-body interaction. The G-operator thus obtained is, in principle, a non-local, total momentum and density dependent two-body interaction. The latter two dependences enter through the Pauli-projection operator. The present method for extracting the operator structure of the Brueckner G-matrix is substantially simpler than the methods used so far.

This chapter is organized as follows: In section 1.1 a brief review of the relevant parts of Brueckner's theory is presented. Section 1.2 is devoted to the derivation of the operator structure of a two-body interaction. In section 1.3 we have extracted the operator structure of the Brueckner G-matrix, based on the one-boson-exchange potential (OBEP), when all of the particles in the initial and final states are on the Fermi surface. A prescription to construct a symmetric part of the G-matrix interaction is also suggested. In sub-section 1.4a a detailed study of the effects of the Pauli-blocking and short-range correlations is presented. Based on this analysis, a momentum transfer and density dependent quasi-particle interaction is constructed in sub-section 1.4b and, for further convenience, it has been parametrized in terms of simple functions. In sub-section 1.4c a study of the sensitivity of the quasi-particle interaction to the different phase-shift-equivalent NN potentials is presented. In section 1.5 the effective G-matrix interaction has been constructed within a very simple model. Section 1.6 has been devoted to study the correlation effects on the Δ -nucleon G-matrix interaction. Finally, in section 1.7 an application of the effective G-matrix interaction is performed. In ap-

pendices A and B respectively some details of the derivation of the operator structure and of the Landau parameters are presented.

1.1 The Bethe-Goldstone Equation

In this section a brief review of the Bethe-Goldstone (BG) equation is presented.

The BG-equation for the G-matrix reads

$$G(\omega) = V + V \frac{Q}{\omega - H_0 + i\epsilon} G(\omega) . \quad (1.1.1)$$

Here, V denotes the bare NN potential. The Pauli projection operator Q restricts the two nucleon intermediate states to be outside the Fermi distribution F :

$$Q|\alpha\beta\rangle = \begin{cases} |\alpha\beta\rangle & \text{for } \alpha \text{ and } \beta \text{ outside } F \\ 0 & \text{for } \alpha \text{ or } \beta \text{ inside } F \end{cases} . \quad (1.1.2)$$

The Hamiltonian H_0 includes the kinetic energy $\frac{\hbar^2 p^2}{2m}$ plus a selfconsistent single-particle potential U . Acting on product states it gives

$$H_0|\alpha\beta\rangle = (E_\alpha + E_\beta)|\alpha\beta\rangle, \quad (1.1.3)$$

$$E_\alpha = \langle \alpha | \frac{\hbar^2 p^2}{2m} + U | \alpha \rangle .$$

ω is the starting energy which will be discussed later in connection with the selfconsistent problem and the choice of ϵ is determined by the boundary conditions.

Introducing the wave operator Ω defined as

$$G(\omega) \equiv V \Omega(\omega) , \quad (1.1.4)$$

a comparison with eq. (1.1.1) leads to the relation

$$\Omega(\omega) = 1 + \frac{Q}{\omega - H_0 + i\epsilon} G(\omega) . \quad (1.1.5)$$

The relative and center-of-mass (C.M.) momenta are:

$$\begin{aligned} 2\vec{k}_{\alpha\beta} &= \vec{k}_{\alpha} - \vec{k}_{\beta} , \\ 2\vec{K}_{\alpha\beta} &= \vec{k}_{\alpha} + \vec{k}_{\beta} . \end{aligned} \quad (1.1.6)$$

(The state subscripts will often be omitted). The G-matrix equation in momentum space is then written as

$$G(\vec{K}, \vec{k}, \vec{k}_0) = V(\vec{k}, \vec{k}_0) + \int d\vec{k}' \frac{V(\vec{K}, \vec{k}') Q(\vec{K}, \vec{k}') G(\vec{K}, \vec{k}', \vec{k}_0)}{E(\vec{K}, \vec{k}_0) - E(\vec{K}, \vec{k}') + i\epsilon} , \quad (1.1.7)$$

where $\vec{k}_0$ and $\vec{k}$ are the initial and final relative momenta. In Fig. 2 a diagrammatic representation of the G-matrix is given. See also Fig. 4 for a graphical representation of the G-matrix in the particle-particle channel. In infinite nuclear matter where the C.M. momentum is conserved the energy denominator reads

$$E(\vec{k}, \vec{k}_0) - E(\vec{k}, \vec{k}') = \frac{\hbar^2}{m} (k_0^2 - k'^2) + \{U(|\vec{k} + \vec{k}_0|) + U(|\vec{k} - \vec{k}_0|) - U(|\vec{k} + \vec{k}'|) - U(|\vec{k} - \vec{k}'|)\} . \quad (1.1.8)$$

The energy denominator and the Pauli function Q generally depend on the angle between $\vec{k}$ and $\vec{k}'$, and $\vec{k}$ and $\vec{k}_0$. This angular dependence causes coupling between states of different relative angular momentum. A partial wave decomposition of (1.1.7) is therefore rather complicated. To avoid this problem, an angle-average Pauli operator

$$\bar{Q}(K, k) = \frac{1}{4\pi} \int d\hat{k} Q(\vec{k}, \vec{k}) \quad (1.1.9)$$

is usually used, especially in binding energy calculations of normal nuclear matter. The accuracy of the above approximation to the Pauli operator has been studied explicitly by Legindgaard⁶²⁾. The remaining dependence on angle is handled by either Brueckner's prescription⁶³⁾

$$|\vec{k} \pm \vec{k}|^2 = K^2 + k^2 \pm \frac{2}{\sqrt{3}} \bar{Q}^{3/2}(K, k) kK , \quad (1.1.10)$$

or by the effective mass approximation

$$E(K, k) = \frac{\hbar^2}{m^*} (K^2 + k^2) - 2U_0 . \quad (1.1.11)$$

The former approximation is adopted here; the latter one may be used to study the sensitivity of each choice. The effective mass approximation is particularly useful for understanding the weak dependence of the G -matrix on the C.M. momentum.

The single-particle potential for hole states is determined by the interaction of each nucleon with the others within the Fermi distribution; namely,

$$\langle \mu | U | \mu \rangle = U(k_\mu) = \sum_{\nu \in F} \langle \mu \nu | G(\omega) | \mu \nu - \nu \mu \rangle. \quad (1.1.12)$$

The starting energy ω is chosen to be $\omega = E_\mu + E_\nu = E(K, k_0)$, which presents a selfconsistency problem. The above definition⁶⁴⁾ of U is based on the requirement that U cancels the bubble, or self-energy insertions, that occur on hole lines in higher-order terms of the Brueckner-Goldstone expansion. Although the potential for single-particle states outside the Fermi distribution may be considered by means of either "continuous choice"⁶⁵⁾ or more fundamental model-space calculations⁶⁶⁾, we will neglect it throughout this chapter.

The total binding energy of nuclear matter is then given as

$$B = \sum_{\mu \in F} \langle \mu | \frac{\hbar^2 p^2}{2m} | \mu \rangle + \frac{1}{2} \sum_{\mu, \nu \in F} \langle \mu \nu | G(\omega) | \mu \nu - \nu \mu \rangle. \quad (1.1.13)$$

With the Pauli operator Q and Brueckner's prescription for angle averaging between relative and C.M. momenta (eq. (1.1.10)), the standard partial-wave decomposition

$$\begin{aligned} \langle S M_S' | G(K, \vec{k}, \vec{k}_0) | S M_S \rangle &= \frac{2}{\pi} \sum_{L' L} \sum_J \sum_{M_J} i^{L-L'} Y_{L'}^{M_J-M_S'}(\hat{k}) Y_L^{M_J-M_S}(\hat{k}_0)^* \\ &\times (S M_S' L' M_J - M_S' | J M_J) (S M_S L M_J - M_S | J M_J) G_{L' L}^{JST}(K, k, k_0) P_T \end{aligned} \quad (1.1.14)$$

can be used in order to reduce eq. (1.1.7) to a set of one-dimensional coupled-channels integral equations. In the above relation $J, L(L'), S$ and T represent the total angular momentum, the orbital angular momentum and the total

spin and isospin respectively. M_J and $M_S(M_S')$ are the projections of total and spin angular momentum. P_T denotes the isospin projection operator. The resulting integral equation is

$$G_{L'L}^{JST}(K, k, k_0) = V_{L'L}^{JST}(k, k_0) + \frac{2}{\pi} \sum_L \int_0^\infty k'^2 dk' \frac{V_{L'L}^{JST}(k, k') \bar{Q}(K, k') G_{L'L}^{JST}(K, k', k_0)}{E(K, k_0) - E(K, k') + i\epsilon} . \quad (1.1.15)$$

The selfconsistent single particle potential for hole states in symmetric nuclear matter is obtained by substituting (1.1.14) into (1.1.12) which gives

$$U(k_\mu) = \frac{1}{4\pi^2} \int_F d\vec{k}_v \sum_L (2J+1)(2T+1) G_{LL}^{JST}(K, k_0, k_0) , \quad (1.1.16)$$

where $k_0 = |\vec{k}_{\mu\nu}|$, and $K = |\vec{k}_{\mu\nu}|$ as defined in (1.1.6)

Similarly, the partial-wave expansion of the wave operator defined by eq. (1.1.4) is given by the expression (1.1.14) by replacing $G \rightarrow \Omega$, where

$$\Omega_{L'L}^{JST}(K, k, k_0) = \frac{\pi}{2} \frac{\delta(k-k_0)}{kk_0} \delta_{L'L} + \frac{\bar{Q}(K, k) G_{L'L}^{JST}(K, k, k_0)}{E(K, k_0) - E(K, k) + i\epsilon} . \quad (1.1.17)$$

For a given two-body state the second term of the r.h.s. of the above equation represents the effects of both Pauli principle and short-range correlations. The latter arise from the strongly repulsive behaviour of the bare NN potential V . This term is related to the partial wave contribution of the wound integral^{9,67)} $\mathcal{K}_{LL}^{JST}$, which is the expansion parameter of the theory, by

$$\mathcal{K}_{LL}^{JST} = \frac{2}{3\pi^2} k_F^3 (2J+1)(2T+1) \int_0^\infty dk \left| F_{LL}^{JST}(K, k, k_0) \right|^2 , \quad (1.1.18)$$

where

$$F_{LL}^{JST}(K, k, k_0) \equiv k \frac{Q(K, k) G_{L'L}^{JST}(K, k, k_0)}{E(K, k_0) - E(K, k) + i\epsilon} . \quad (1.1.19)$$

F_{LL}^{JST} is called the defect function. In eq. (1.1.18) a spherical Fermi distribution of radius k_F is assumed. Either eq. (1.1.17) or (1.1.18) may be used here as a measure of short-range correlation and Pauli blocking effects.

Finally, a number of methods which can be used to solve eq. (1.1.15) have been suggested by several authors^{62,67,68}). In this work we used the matrix inversion method proposed by Haftel and Tabakin⁶⁷).

1.2 Operator Structure of a Two-Body Interaction

For subsequent calculations it is especially convenient to determine the operator structure of the G-matrix and this section is devoted to deriving this structure for a two-body interaction between particles of spin $1/2$. Similar procedures may be used for particles with different spins (e.g., Δ -isobar-nucleon).

We start by considering the usual partial-wave decomposition of a two-body interaction in momentum space (see eq. (1.1.14))

$$\begin{aligned} \langle SM'_S | V(\vec{k}', \vec{k}) | SM_S \rangle &= \frac{2}{\pi} \sum_{L'L} \sum_J \sum_{M_J} i^{L-L'} Y_{L'L}^{M_J-M'_S}(\hat{k}') Y_{L'L}^{M_J-M_S}(\hat{k})^* \\ &\times (SM'_S L' M_J - M'_S | JM_J) (SM_S L M_J - M_S | JM_J) V_{L'L}^{JST}(k', k) P_T . \end{aligned} \quad (1.2.1)$$

From the above expansion, $V(\vec{k}', \vec{k})$ can be extracted by writing it as

$$V(\vec{k}', \vec{k}) = \sum_{M_S M'_S} |SM'_S\rangle \langle SM'_S| V(\vec{k}', \vec{k}) |SM_S\rangle \langle SM_S|. \quad (1.2.2)$$

We note that under rotations, $|SM_S\rangle$ and $(-)^{S+M_S} \langle SM_S|$, behave like components of a spherical tensor.

Substituting expression (1.2.1) into (1.2.2) and recoupling the spherical tensors one obtains

$$\begin{aligned} V(\vec{k}', \vec{k}) &= \frac{2}{\pi} \sum_{L' L J \ell} i^{L-L'} (-)^{J+S} [J]^2 \left\{ \begin{matrix} S & L' & J \\ L & S & \ell \end{matrix} \right\} V_{L'L}^{JST}(k', k) \\ &\times [A_S \otimes B_S]^\ell \cdot [Y_{L'}(\hat{k}') \otimes Y_L(\hat{k})]^\ell P_T, \end{aligned} \quad (1.2.3)$$

where we have introduced the notation

$$\begin{aligned} A_S^M &\equiv |SM_S\rangle, \\ B_S^{-M} &\equiv (-)^{S+M} \langle SM_S|, \end{aligned} \quad (1.2.4)$$

$$[J] \equiv \sqrt{2J+1},$$

and ℓ denotes the rank of the operator both in spin and coordinate spaces.

We now want to express the quantity $[A_S \otimes B_S]^\ell$ in terms of the usual Pauli-spin matrix σ . For this purpose, we shall evaluate each of its components. We first remember that

$$[A_S \otimes B_S]_m^\ell = \sum_{m_S m'_S} (S m_S m'_S | \ell m) (-)^{S-m'_S} |S m_S\rangle \langle S -m'_S|, \quad (1.2.5)$$

and consequently

$$\langle SM'_S | [A_S \otimes B_S]^\ell_m | SM_S \rangle = (-)^{S+M_S} \langle SM'_S S-M_S | \ell m \rangle . \quad (1.2.6)$$

There are four cases to be considered:

i) S=0, $\ell=0$:

Using directly eq. (1.2.5) one has

$$[A_0 \otimes B_0]^0 = |00\rangle\langle 00| \equiv P_{S=0} . \quad (1.2.7a)$$

ii) S=1, $\ell=0$:

As before,

$$\begin{aligned} [A_1 \otimes B_1]^0 &= \sum_{m_S} (1m_S 1-m_S | 00) (-)^{1+m_S} |1m_S\rangle\langle 1m_S| \\ &= \frac{1}{\sqrt{3}} \sum_{m_S} |1m_S\rangle\langle 1m_S| \equiv \frac{1}{\sqrt{3}} P_{S=1} . \end{aligned} \quad (1.2.7b)$$

P_S is the spin singlet (triplet) projection operator as S=0(1).

iii) S=1, $\ell=1$:

In this case, using eq. (1.2.6) we obtain

$$\langle 1M'_S | [A_1 \otimes B_1]^1_m | 1M_S \rangle = (-)^{1+M_S} \langle 1M'_S 1-M_S | 1m \rangle .$$

We now note that the matrix element of $\sigma_1 + \sigma_2$ (the only spin-space operator of rank 1 which is symmetric in the particle labels) can be expressed as

$$\langle 1M'_S | (\sigma_1 + \sigma_2)_m | 1M_S \rangle = 2\sqrt{2} (-)^{1+M_S} (1M'_S 1-M_S | 1m) .$$

Comparing these results gives

$$[A_1 \otimes B_1]_m^1 = \frac{1}{2\sqrt{2}} (\sigma_1 + \sigma_2)_m . \quad (1.2.7c)$$

iv) S=1, l=2:

From eq. (1.2.6),

$$\langle 1M'_S | [A_1 \otimes B_1]_m^2 | 1M_S \rangle = (-)^{1+M_S} (1M'_S 1-M_S | 2m) .$$

We also note that

$$\langle 1M'_S | (\sigma_1 \otimes \sigma_2)_m^2 | 1M_S \rangle = 2(-)^{1+M_S} (1M'_S 1-M_S | 2m) .$$

Therefore,

$$[A_1 \otimes B_1]_m^2 = \frac{1}{2} (\sigma_1 \otimes \sigma_2)_m^2 . \quad (1.2.7d)$$

Collecting the results i)-iv) and substituting them into eq. (1.2.3) gives

$$\begin{aligned} \mathcal{V}(\vec{k}', \vec{k}) &= \frac{2}{\pi} \sum_{L' L J} i^{L-L'} \{ (-)^J [J]^2 \begin{Bmatrix} 0 & L' & J \\ L & 0 & 0 \end{Bmatrix} \mathcal{W}_{L' L}^{J0T}(k', k) [Y_{L'}(\hat{k}') \otimes Y_L(\hat{k})]^0 \}_{P_{S=0}} \\ &+ (-)^{J+1} [J]^2 \begin{Bmatrix} 1 & L' & J \\ L & 1 & 0 \end{Bmatrix} \mathcal{W}_{L' L}^{J1T}(k', k) [Y_{L'}(\hat{k}') \otimes Y_L(\hat{k})]^0 \frac{1}{\sqrt{3}} \}_{P_{S=1}} \\ &+ (-)^{J+1} [J]^2 \begin{Bmatrix} 1 & L' & J \\ L & 1 & 1 \end{Bmatrix} \mathcal{W}_{L' L}^{J1T}(k', k) [Y_{L'}(\hat{k}') \otimes Y_L(\hat{k})]^1 \cdot (\sigma_1 + \sigma_2) \frac{1}{2\sqrt{2}} \\ &+ (-)^{J+1} [J]^2 \begin{Bmatrix} 1 & L' & J \\ L & 1 & 2 \end{Bmatrix} \mathcal{W}_{L' L}^{J1T}(k', k) [Y_{L'}(\hat{k}') \otimes Y_L(\hat{k})]^2 \cdot (\sigma_1 \otimes \sigma_2)^2 \frac{1}{2} \}_{P_T} . \end{aligned} \quad (1.2.8)$$

The above expression relates the operator structure of a two-body interaction to its partial-wave components V_L^{jST} for particles with spin $1/2$. The first two terms are the singlet- and triplet-central components, respectively. The third term corresponds to the spin-orbit force, while the last term is the tensor interaction. The quantity $[Y_L(\hat{k}') \otimes Y_L(\hat{k})]^L$ can also be expressed in terms of associated Legendre-polynomials by choosing a special coordinate system; the 6-j symbols can also be reduced to relatively simple functions of L , L' and J .

In anticipation of specific applications, it is useful to express the tensor component in eq. (1.2.8) in terms of the usual tensor operator $S_{12}(\hat{p}) \equiv 3\sigma_1 \cdot \hat{p} \sigma_2 \cdot \hat{p} - \sigma_1 \cdot \sigma_2$, where $\hat{p}$ is an arbitrary unit vector. This can be done by noticing that the most general two-body interaction, for spin $1/2$ identical particles, consistent with invariance principles, may be written in momentum space as⁶⁹⁾ (suppressing isospin for simplicity)

$$V = \alpha + \beta \sigma_1 \cdot \hat{n} \sigma_2 \cdot \hat{n} + i\gamma[\sigma_1 + \sigma_2] \cdot \hat{n} + \delta \sigma_1 \cdot \hat{q} \sigma_2 \cdot \hat{q} + \epsilon \sigma_1 \cdot \hat{Q} \sigma_2 \cdot \hat{Q} + (\hat{q} \cdot \hat{Q})\omega[\sigma_1 \cdot \hat{q} \sigma_2 \cdot \hat{Q} + \sigma_1 \cdot \hat{Q} \sigma_2 \cdot \hat{q}] , \quad (1.2.9)$$

where, in terms of the relative momenta $\vec{k}$ and $\vec{k}'$ the three vectors $\vec{q}$, $\vec{Q}$ and $\vec{n}$ can be defined as

$$\begin{aligned} \vec{q} &\equiv \vec{k} - \vec{k}' , \\ \vec{Q} &\equiv \vec{k} + \vec{k}' , \\ \vec{n} &\equiv \vec{k} \wedge \vec{k}' = \frac{\vec{q} \wedge \vec{Q}}{2} . \end{aligned} \quad (1.2.10)$$

Moreover, the coefficients $\alpha, \beta, \gamma, \delta, \epsilon$ and ω can be expressed by means of the relations

$$\alpha = \frac{1}{4} \text{Tr}[\mathcal{V}] ,$$

$$\beta = \frac{1}{4} \text{Tr}[\mathcal{V} \sigma_1 \cdot \hat{n} \sigma_2 \cdot \hat{n}] ,$$

$$\gamma = \frac{1}{8\pi} \text{Tr}[\mathcal{V} (\sigma_1 + \sigma_2) \cdot \hat{n}] ,$$

$$\begin{aligned} \delta = \frac{1}{4\sin^4\varphi} \text{Tr}[\mathcal{V} \sigma_1 \cdot \hat{q} \sigma_2 \cdot \hat{q} + \cos^2\varphi \sigma_1 \cdot \hat{Q} \sigma_2 \cdot \hat{Q} \\ - \cos\varphi [\sigma_1 \cdot \hat{q} \sigma_2 \cdot \hat{Q} + \sigma_1 \cdot \hat{Q} \sigma_2 \cdot \hat{q}]] , \end{aligned} \quad (1.2.11)$$

$$\begin{aligned} \varepsilon = \frac{1}{4\sin^4\varphi} \text{Tr}[\mathcal{V} (\cos^2\varphi \sigma_1 \cdot \hat{q} \sigma_2 \cdot \hat{q} + \sigma_1 \cdot \hat{Q} \sigma_2 \cdot \hat{Q} \\ - \cos\varphi [\sigma_1 \cdot \hat{q} \sigma_2 \cdot \hat{Q} + \sigma_1 \cdot \hat{Q} \sigma_2 \cdot \hat{q}])] , \end{aligned}$$

$$\begin{aligned} \omega = \frac{1}{4\sin^4\varphi} \text{Tr}[\mathcal{V} (-\sigma_1 \cdot \hat{q} \sigma_2 \cdot \hat{q} - \sigma_1 \cdot \hat{Q} \sigma_2 \cdot \hat{Q} \\ + \frac{1+\cos^2\varphi}{2\cos\varphi} [\sigma_1 \cdot \hat{q} \sigma_2 \cdot \hat{Q} + \sigma_1 \cdot \hat{Q} \sigma_2 \cdot \hat{q}])] . \end{aligned}$$

Here the trace over the Pauli spin matrices σ_1 and σ_2 is implied. φ denotes the angle between $\vec{q}$ and $\vec{Q}$. The last term in eq. (1.2.9) vanishes when $\hat{q} \cdot \hat{Q} \equiv \cos\varphi = 0$ as a consequence of time reversal invariance. (This condition holds when $|\vec{k}'| = |\vec{k}|$).

Eq. (1.2.9) can be rewritten as

$$\begin{aligned} \mathcal{V} = \alpha' P_{s=0} + \eta P_{s=1} + i\gamma (\sigma_1 + \sigma_2) \cdot \hat{n} \\ + \delta' S_{12}(\hat{q}) + \varepsilon' S_{12}(\hat{Q}) + (\hat{q} \cdot \hat{Q}) \omega' [\sigma_1 \cdot \hat{q} \sigma_2 \cdot \hat{Q} + \sigma_1 \cdot \hat{Q} \sigma_2 \cdot \hat{q}] . \end{aligned} \quad (1.2.12)$$

The new coefficients $\alpha', \eta, \delta', \epsilon'$ and ω' are related to the old ones (eq. (1.2.11)) by the relations

$$\begin{aligned}\alpha' &= \alpha - \left(\frac{\beta}{\sin^2} + \delta + \epsilon \right) , \\ \eta &= \alpha + \frac{1}{3} \left(\frac{\beta}{\sin^2} + \delta + \epsilon \right) , \\ \delta' &= \frac{1}{3} \left(\delta - \frac{\beta}{\sin^2} \right) , \\ \epsilon' &= \frac{1}{3} \left(\epsilon - \frac{\beta}{\sin^2} \right) , \\ \omega' &= \omega + \frac{1}{\sin^2} \beta .\end{aligned}\tag{1.2.13}$$

Eqs. (1.2.11), (1.2.12) and (1.2.13) allow us to express the two-body interaction (eq. (1.2.8)) in terms of standard tensors. The coefficients α' , η and γ are immediately found by inspection (see also appendix A). An explicit derivation of the coefficients δ' and ϵ' in the limit $|\vec{k}| = |\vec{k}'|$ is given in appendix A.

1.3 The G-Operator as an Effective Interaction Near the Fermi Surface

In the previous section a method for extracting the operator structure of a general two-body interaction was presented. In this section, the method is applied to the G-matrix of section 1.1 in the limit when all particles, in the initial and final states, are on the Fermi sphere. Such conditions are useful for relating this work to the Landau quasi-particle assumption^{17,18,35}). The

G-operator thus obtained may be used as a starting point to construct an effective quasi-particle interaction.

Through this chapter we will assume that each particle, in the initial and final states, has momentum k_F . We will call the G-operator obtained under this assumption the quasi-particle interaction. Eq. (1.1.6) then tells us that both initial and final relative momenta, $\vec{k}_0$ and $\vec{k}$, as well as the C.M. momentum $\vec{K}$ range from 0 to k_F . Moreover, the conservation of the C.M. momentum gives $|\vec{k}| = |\vec{k}_0|$, which implies that the (direct) momentum transfer $\vec{q}$ and the exchange momentum transfer $\vec{Q}$,

$$\begin{aligned}\vec{q} &= \vec{k}_0 - \vec{k} , \\ \vec{Q} &= \vec{k}_0 + \vec{k} ,\end{aligned}\tag{1.3.1}$$

are orthogonal. Consequently, the last term in eq. (1.2.13) vanishes. Thus, eq. (1.2.12) and eq. (1.2.8) give, denoting the G-operator within our assumption by $\mathcal{F}$ and using the notation G^{ST} ,

$$\begin{aligned}\mathcal{F}(\vec{k}, \vec{k}_0; k_F) &= \{G^{0T}P_{s=0} + G^{1T}P_{s=1} + iG_{LS}^{1T}(\sigma_1 + \sigma_2) \cdot \hat{n} \\ &\quad G_t^{1T}S_{12}(\hat{q}) + G_t^{1T}S_{12}(\hat{Q})\}P_T ,\end{aligned}\tag{1.3.2}$$

where the coefficients G^{0T} , G^{1T} , G_{LS}^{1T} , G_t^{1T} and G_t^{1T} are now functions of $\vec{k}_0$, $\vec{k}$, and the parameter k_F . In an LDA k_F gives the density dependence of the G-operator. From eq. (1.1.6) the C.M. momentum K is, in this case, ($k_\alpha = k_\beta = k_F$), related to the relative momentum $k_0 = k = |\vec{k}|$ by means of

$$K = k_F \sqrt{1 - (k_0/k_F)^2} .\tag{1.3.3}$$

From eq. (1.2.8) and appendix A, the above coefficients are given by

$$\begin{aligned}
 G^{0T} &= \frac{1}{2\pi^2} \sum_L [L]^2 G_{LL}^{L0T}(K, k, k_0) P_L(\hat{k} \cdot \hat{k}_0), \\
 G^{1T} &= \frac{1}{2\pi^2} \frac{1}{3} \sum_{LJ} [J]^2 G_{LL}^{J1T}(K, k, k_0) P_L(\hat{k} \cdot \hat{k}_0), \\
 G_{LS}^{1T} &= \frac{1}{4\pi^2} \sqrt{\frac{3}{2}} \sum_{LJ} (-)^{L+J+1} [J]^2 \frac{[L]}{\sqrt{L(L+1)}} \left\{ \begin{matrix} 1 & L & J \\ L & 1 & 1 \end{matrix} \right\} G_{LL}^{J1T}(K, k, k_0) P_L^1(\hat{k} \cdot \hat{k}_0), \\
 G_t^{1T} &= \frac{1}{36\pi^2} \sqrt{\frac{24\pi}{5}} \sum_{L'LJ} i^{L-L'} (-)^{J+1} [J]^2 [LL'] \left\{ \begin{matrix} 1 & L' & J \\ L & 1 & 2 \end{matrix} \right\} G_{L'L}^{J1T}(K, k, k_0) \quad (1.3.4) \\
 &\quad \times \sum_{M \geq 0} (L'ML0|2M) \sqrt{\frac{(L'-M)!}{(L'+M)!}} P_{L'}^M(\hat{k} \cdot \hat{k}_0) (2-\delta_{M0}) \\
 &\quad \times \{ 2Y_{2M}(\frac{\pi-\theta}{2}, 0) + (-)^M Y_{2M}(\frac{\theta}{2}, 0) \},
 \end{aligned}$$

$$\begin{aligned}
 G_{t'}^{1T} &= \frac{1}{36\pi^2} \sqrt{\frac{24\pi}{5}} \sum_{L'LJ} i^{L-L'} (-)^{J+1} [J]^2 [LL'] \left\{ \begin{matrix} 1 & L' & J \\ L & 1 & 2 \end{matrix} \right\} G_{L'L}^{J1T}(K, k, k_0) \\
 &\quad \times \sum_{M \geq 0} (L'ML0|2M) \sqrt{\frac{(L'-M)!}{(L'+M)!}} P_{L'}^M(\hat{k} \cdot \hat{k}_0) (2-\delta_{M0}) \{ Y_{2M}(\frac{\pi-\theta}{2}, 0) + 2(-)^M Y_{2M}(\frac{\theta}{2}, 0) \}.
 \end{aligned}$$

It is useful to have $\mathcal{F}$ in the spin and isospin transfer representation. Eq. (1.3.2) may be decomposed into isovector and isoscalar channels,

$$\begin{aligned}
 \mathcal{F}(\vec{k}, \vec{k}_0; k_F) &= \{ G^{01} P_{s=0} + G^{11} P_{s=1} + i G_{LS}^{11} (\sigma_1 + \sigma_2) \cdot \hat{n} + G_t^{11} S_{12}(\hat{q}) + G_{t'}^{11} S_{12}(\hat{Q}) \} P_{T=1} \\
 &\quad + \{ G^{00} P_{s=0} + G^{10} P_{s=1} + i G_{LS}^{10} (\sigma_1 + \sigma_2) \cdot \hat{n} + G_t^{10} S_{12}(\hat{q}) + G_{t'}^{10} S_{12}(\hat{Q}) \} P_{T=0}. \quad (1.3.5)
 \end{aligned}$$

Using the explicit form of the spin and isospin projection operator

$$P_{S=0} = \frac{1}{4} (1 - \sigma_1 \cdot \sigma_2) , \quad P_{S=1} = \frac{1}{4} (3 + \sigma_1 \cdot \sigma_2) , \quad (1.3.6)$$

$$P_{T=0} = \frac{1}{4} (1 - \tau_1 \cdot \tau_2) , \quad P_{T=1} = \frac{1}{4} (3 + \tau_1 \cdot \tau_2) ,$$

eq. (1.3.5) can easily be reduced to the familiar form

$$\begin{aligned} \mathcal{F}(\vec{k}, \vec{k}_0; k_F) = & F + F' \tau_1 \cdot \tau_2 + G \sigma_1 \cdot \sigma_2 + G' \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 \\ & + iS(\sigma_1 + \sigma_2) \cdot \hat{n} + iS'(\sigma_1 + \sigma_2) \cdot \hat{n} \tau_1 \cdot \tau_2 \\ & + T S_{12}(\hat{q}) + T' S_{12}(\hat{q}) \tau_1 \cdot \tau_2 \\ & + H S_{12}(\hat{Q}) + H' S_{12}(\hat{Q}) \tau_1 \cdot \tau_2 , \end{aligned} \quad (1.3.7)$$

where the new coefficients are given by

$$\begin{aligned} F &= \frac{1}{16} \{ 3G^{01} + G^{00} + 3G^{10} + 9G^{11} \} , \\ G &= \frac{1}{16} \{ -3G^{01} - G^{00} + G^{10} + 3G^{11} \} , \\ F' &= \frac{1}{16} \{ G^{01} - G^{00} - 3G^{10} + 3G^{11} \} , \\ G' &= \frac{1}{16} \{ -G^{01} + G^{00} - G^{10} + G^{11} \} , \\ S &= \frac{1}{4} \{ G_{LS}^{10} + 3G_{LS}^{11} \} , \quad S' = \frac{1}{4} \{ -G_{LS}^{10} + G_{LS}^{11} \} , \\ T &= \frac{1}{4} \{ G_t^{10} + 3G_t^{11} \} , \quad T' = \frac{1}{4} \{ -G_t^{10} + G_t^{11} \} , \end{aligned} \quad (1.3.8)$$

$$H = \frac{1}{4} \{G_t^{10} + 3G_t^{11}\}, \quad H' = \frac{1}{4} \{-G_t^{10} + G_t^{11}\}.$$

It is also convenient to express $\mathcal{F}$ as a function of the momentum transfer $\vec{q}$ and exchange momentum transfer $\vec{Q}$. This can easily be done by means of the transformation (1.3.1). Moreover, their magnitudes are related to k_0 by

$$\begin{aligned} q &= 2k_0 \sin \frac{\theta}{2}, \\ Q &= 2k_0 \cos \frac{\theta}{2}, \end{aligned} \tag{1.3.9}$$

which shows that q and Q range from 0 to $2k_0$. θ denotes the angle between the initial and final relative momenta.

It should be stressed that the G -operator given by eq. (1.3.4) is properly antisymmetrized when the BG equation is solved only for Pauli-allowed states, i.e., $L+S+T$ is odd. Note that when $L+S+T$ is restricted to be odd, the coefficients G^{ST} in eq. (1.3.4) may also be classified according to their spin (S) and parity. For example, G^{00} corresponds to the singlet-odd ($S0$), G^{01} to the singlet-even (SE), G^{10} to the triplet-even (TE) and G^{11} to the triplet-odd (TO) central component of the G -matrix interaction. As a result the G -operator, for fixed values of S and T , is only defined in states of even or odd parity. For the purpose of comparing the G -operator with the associated bare potential and for studying the intrinsic non-locality induced by correlation effects it is convenient to define the G -operator in Pauli-forbidden states ($L+S+T = \text{even}$) by solving the BG equation (including the Pauli-projection operator $\mathcal{Q}$) in these states using the potential defined for the Pauli-allowed states having the same values of S and T . We define a direct G -operator $\mathcal{F}_D$ in the S, T channel by summing over both even and odd L and L' in eq. (1.3.4).

Since any arbitrary contribution may be added to the G-operator which vanishes after antisymmetrization, there is an unavoidable ambiguity in the definition of $\mathcal{F}_D$. For example, any constant term (in momentum space) may be added to the direct term in the central-odd states, since it will be cancelled by a constant term arising from the exchange contribution (a zero-range interaction cannot act in odd states). In principle, even the bare NN-potential is subject to this same ambiguity. This is obvious for phenomenological potential, such as the Reid-soft-core (RSC) potential³⁾, which consists of a more or less arbitrary ansatz for the potential expression with parameters determined by fitting the empirical NN scattering data. The structure of meson-theoretic potentials⁴⁾, on the other hand, appears to be uniquely determined to within form factors from the meson-nucleon interaction Lagrangians. The coupling constants and meson-masses, however, are not derived from an underlying quark theory or meson-nucleon scattering experiments, but are obtained from a fit to NN scattering data. This implies a possible ambiguity. Since only antisymmetrized matrix elements contribute to the determination of observables, this ambiguity should be irrelevant. Likewise, in the case of the G-operator, this type of ambiguity should not matter for calculating observables.

The method applied in this section to extract the operator structure of the Brueckner G-matrix may also be applied to the wave-operator Ω defined in section 1.1. The structure of the relations deduced above for G is also valid for Ω . In terms of the wave operator, $\mathcal{F}$ is given by

$$\mathcal{F}(\vec{k}, \vec{k}_0; k_F) = \int d\vec{k}' V(\vec{k}, \vec{k}') \Omega(\vec{k}', \vec{k}_0; k_F) . \quad (1.3.10)$$

Therefore one needs not only the on-energy shell ($|\vec{k}'| = |\vec{k}_0|$) but also the off-energy shell ($|\vec{k}'| \neq |\vec{k}_0|$) matrix elements of the wave operator.

1.4 Construction and Characteristics of the Quasi-Particle Interaction

In this section, the quasi-particle interaction ($\mathcal{F}$) is extracted according to the prescription described in the previous sections. Several NN phase-shift-equivalent OBEP, given by the Bonn group, were used as the bare NN potentials. Care must be taken when one uses a phenomenological potential because it is usually defined only for lower partial waves (e.g. $J \leq 2$ for the RSC potential). The exact operator, however, requires, in principle, all higher partial-waves (see eq. (1.3.4)). In particular, for the OPEP, convergence is only reached when one includes $L \geq 10$. Even the heavier ω -, ρ - and σ -mesons require $L \geq 6$ to ensure reliable results. In the present chapter all partial-waves satisfying $J \leq 13$ were considered ($\vec{J} = \vec{S} + \vec{L}$).

The results for the selfconsistent single-particle potential (see eq. (1.1.16)) at $k_\mu = k_F$ are shown in Fig. 5 as a function of the Fermi-momentum k_F . The OBEP given by Holinde⁵⁹), hereafter referred to as HM3A ($\Lambda_\pi = 1530$ MeV version) was used as the bare NN potential.

1.4a Effects of Correlations

In this sub-section some of the effects on the quasi-particle interaction arising from correlations are considered.

The quasi-particle interaction given by eq. (1.3.7) depends on the momentum transfer $\vec{q}$, the exchange momentum transfer $\vec{Q}$ (through the transformation (1.3.1)) and the Fermi-momentum k_F . The latter enters as a parameter through the Pauli projection operator Q . An interaction whose direct matrix elements

depend on $\vec{Q}$ is non-local in a coordinate representation and makes further applications rather cumbersome. However, this non-locality, which in our case is also present in the bare NN potential[†], is usually neglected. A discussion of this matter is given by Negele⁵⁶⁾ for the case in which the bare NN potential is local. Here, we can quantitatively check the non-locality by examining the $\vec{Q}$ -dependence of the interaction $\mathcal{F}_D$, whose construction has been described in section 1.3. HM3A was used as the bare NN potential. In Fig. 6 the Q -dependence of $\mathcal{F}_D$ at $q=0$ and at normal density ($k_F = 1.36 \text{ fm}^{-1}$) is shown. The solid lines refer to the interaction $\mathcal{F}_D$, while the dashed lines refer to the bare NN potential normalized at $Q=0$ for comparison. One can see that the strongest Q -dependence (non-locality) is observed in the central channel F_D (Fig. 6a), which has a variation of $\sim 200 \text{ MeV fm}^3$ (20 %) in the range $0 < Q < 2k_F$. This variation corresponds to ~ 50 % of the variation observed within the same range of the momentum transfer q when Q is fixed. The strong Q -dependence in the central channel of the bare NN potential is mainly due to the ω - and σ -meson exchange potentials. Note that the additional Q -dependence introduced by correlations is small compared with the non-locality of the bare NN potential. The deviation observed at $Q \sim 2k_F$ is due to the Pauli projection operator Q which changes rapidly when $Q \sim 2k_F$. The isospin F_D^I and the spin G_D channels depend weakly on the momentum Q compared with the F_D channel as shown in Figs. 6b and 6c respectively. Their variations are $\sim 20 \text{ MeV fm}^3$ (10 %) in the range $0 < Q < 2k_F$. The effect of correlations is also small. The spin-isospin G_D^I channel has practically no Q -dependence (Fig. 6d) and the effect of correlations is negligible. The remaining channels of the interaction $\mathcal{F}_D$ do not present any significant Q -dependence and therefore are not displayed. Note that in the

[†] The OBEP is, in principle, a non-local potential. The RSC potential is made local only by having a different form for each two-body state.

spin-orbit channels a part of the non-locality is described by the operator $\vec{L} \equiv -i\vec{r}\wedge\vec{\nabla}$, which in momentum space becomes proportional to $\frac{1}{q} [\frac{\vec{q}\wedge\vec{Q}}{2}] = \frac{1}{q} \vec{n}$. Since the operator $\vec{L}$ is usually diagonalized in a $|JLS\rangle$ basis, the relevant Q -dependence (non-locality) in the spin-orbit channels may be studied by considering terms apart from the factor $\frac{1}{q} \vec{n}$ instead of the unit vector $\vec{n}$. For higher values of q the Q -dependence becomes much weaker in all the channels.

From these observations one may conclude that the exact G -matrix interaction may be reasonably well approximated by a local operator which depends on the momentum transfer $\vec{q}$, but not on the exchange momentum $\vec{Q}$, except in the central channel F_D .

In Fig. 7, the central components of the antisymmetrized quasi-particle interaction $\mathcal{F}$, at normal density and at the initial relative momentum $k_0 = 0.95 \text{ fm}^{-1}$, are displayed as a function of the momentum transfer q (solid lines). Note that both q and Q range from 0 to $2k_0$ according to eq. (1.3.9). In Fig. 7, the corresponding terms of the bare NN potential (HM3A) are also plotted. The strongest effect of correlations is observed in the central channel F due to the direct contributions of heavier ω -, σ - and φ -mesons in this channel. One can also analyze the correlations in each partial-wave. For this purpose, the partial-wave contributions of the wound integral as defined by eq. (1.1.18) are shown in Table 3. The most important contributions arise from $^1S_0(T=1)$ and $^3S_1 \rightarrow ^3D_1(T=0)$ states, which contribute to G^{01} and G^{10} in eq. (1.3.8), respectively. The corresponding integrand, the defect function F_{LL}^{JST} given by eq. (1.1.19) is shown in Figs. 8a, 8b and 8c (solid lines). Both, F_{00}^{001} and F_{00}^{110} , are negative and peak around 3 fm^{-1} , while F_{02}^{110} is positive and peaks around 2 fm^{-1} . In addition, the partial-wave contributions of the bare NN potential v_{00}^{001} corresponding to $^1S_0$ and v_{00}^{110} corresponding to $^3S_1 \rightarrow ^3D_1$ ($L=L'=0$) states

are both repulsive, while the contribution V_{02}^{110} corresponding to $^3S_1$ - $^3D_1$ ($L=0, L'=2$) is negative as shown also in Figs. 8a, 8b and 8c (dashed lines), respectively. The partial-wave contributions G_L^{JST} are obtained by integrating the products of the partial-wave contributions of the bare NN potential and the defect functions (see eq. (1.1.15) and (1.1.17)). Note that the product of V_{00}^{001} and F_{00}^{001} in Fig. 8a contributes to G^{01} , while the products of V_{00}^{110} and F_{00}^{110} and of V_{02}^{110} and F_{02}^{110} in Figs. 8b and 8c, respectively, contribute to G^{10} . Therefore, the effect of correlations is to give a negative contribution to the partial-wave contributions G_{00}^{001} and G_{00}^{110} in the $^1S_0$ and $^3S_1$ - $^3D_1$ ($L=L'=0$) states, respectively. Moreover, since the contributions in G_{00}^{110} which arise from F_{00}^{110} and F_{02}^{110} add coherently, the effects of correlations are much larger in the $^3S_1$ - $^3D_1$ ($L=L'=0$) state than in the $^1S_0$ state (compare Figs. 8a and 8b and 8c). Now, by inspection of eq. (1.3.8), one can immediately see that the strongest correlations should be in the central channel F and, since they are negative, they must give an attraction. The isospin channel F' is less affected by correlations than the central channel. This fact can again be understood from the partial-wave considerations. Since the contributions to G^{01} and G^{10} arising from the correlations in $^1S_0$ and $^3S_1$ - $^3D_1$ states are both negative, inspection of eq. (1.3.8) shows that there is a cancellation between these contributions in the isospin channel. Moreover, since the correlations are larger (in magnitude) in $^3S_1$ - $^3D_1$ states than in the $^1S_0$ state, the effect of them is to give a repulsion in the isospin channel. A similar analysis can be made for the spin G and the spin-isospin G' channels. In order to make a more quantitative analysis, we shall denote by ΔG^{ST} the contributions arising only from the correlations,

$$\Delta G^{ST} = G^{ST} - V^{ST} . \quad (1.4a.1)$$

To a good approximation, one has

$$\Delta G^{10} \approx 2\Delta G^{01} \quad (1.4a.2)$$

as is shown by the values of the corresponding wound integrals (Table 3).

A quantitative estimate for ΔG^{01} may be obtained as follows: the quantities ΔG^{00} and ΔG^{11} due to correlations in the odd partial waves are small compared with the correlations in the even partial waves, and consequently may be neglected in eq. (1.3.8) for our purpose. Therefore a reasonably accurate estimate of the difference between the G-matrix and the bare potential in the spin-isospin channel is (eq. (1.3.8)):

$$\Delta G' = \frac{1}{16} (-\Delta G^{01} - \Delta G^{10}) . \quad (1.4a.3)$$

The value $\Delta G'$ may be extracted from Fig. 7. Using eq. (1.4a.1) and (1.4a.2) one gets

$$\Delta G^{01} = -853 \text{ MeV fm}^3$$

at $q=0$. Using this value we obtain $F \sim -250$, $F' \sim 73$ and $G \sim 62 \text{ MeV fm}^3$, which should be compared with the exact values $F = -304$, $F' = 67$ and $G = 38 \text{ MeV fm}^3$, respectively. The small discrepancies, especially in the central channel F , can be ascribed to our omission of the contribution ΔG^{00} and ΔG^{11} from the odd states.

In Fig. 9, both isoscalar S and isovector S' spin-orbit components are displayed. The dashed lines correspond to the bare NN potential, while the solid

lines refer to the quasi-particle interaction. The isoscalar channel is generated mainly by the ω -meson and the isovector channel by the ρ -meson. Both channels are weakly influenced by correlations which are most important in $L=0$ states (see Table 3), and these do not contribute to the spin-orbit forces. In nuclear structure calculations the spin-orbit terms have usually been neglected because of the argument that they are small compared to the other channels of the interaction. One has to mention, however, that they are at least as strong as the tensor interaction, especially for momentum transfers above 1.0 fm^{-1} . (Note that, as mentioned before, the operator $\vec{L}$ in momentum space is proportional to $\frac{1}{q} \left[\frac{\vec{q} \Delta \vec{Q}}{2} \right] = \frac{1}{q} \vec{h}$.)

Isoscalar (T and H) and isovector (T' and H') tensor forces are shown in Fig. 10. Again, dashed lines refer to the bare NN potential and the solid lines correspond to the quasi-particle G-matrix interaction. In spite of the strong correlations in both $^3S_1$ - $^3D_1$ ($L=L'=0$) and $^3S_1$ - $^3D_1$ ($L=0, L'=2$) states (see Table 3), the net effect is very small for the following reason: (i) In spite of the enormous correlations in both ($L=0, L'=2$) and ($L=L'=0$) states their contributions are largely suppressed by the momentum mismatch between $kV_{20}^{110}(k_0, k)$ and $F_{00}^{110}(k, k_0)$ and $kV_{22}^{110}(k_0, k)$ and $F_{02}^{110}(k, k_0)$, respectively (see Figs. 8d and 8e). In Fig. 8d, kV_{20}^{110} peaks at $k \sim 0.5 \text{ fm}^{-1}$ while F_{00}^{110} peaks around $k \sim 3 \text{ fm}^{-1}$. In Fig. 8e, kV_{22}^{110} oscillates in the region of the momentum k where F_{02}^{110} peaks. (ii) Furthermore, since the partial wave components V_{22}^{110} and V_{20}^{110} of the bare potential both are repulsive in the region of the momentum k where they have small overlaps with the corresponding partial wave contributions of the defect function F_{02}^{110} and F_{00}^{110} , whereas F_{02}^{110} is positive and F_{00}^{110} is negative, the small contributions to the G-matrix have opposite signs, resulting in cancellation. One has to mention that this situation persists for different values of the Fermi-momentum k_F , although less cancellation is observed for values of

k_F smaller than 1.36 fm^{-1} . Different versions of OBEP give also similar results. The symmetric bare NN potential (direct term only) has practically no contribution to the tensor components, except in the isovector T' channel, which is basically given by the π - and ρ -meson exchange. This means that one can describe the tensor part of the G-matrix to a good approximation by the tensor component of the bare π - plus ρ -meson exchange potential, at least in the range of the Fermi-momentum $1.0 \lesssim k_F \text{ (fm}^{-1}\text{)} \lesssim 1.36$ and for the momentum transfer q up to $\sim 2 \text{ fm}^{-1}$.

Finally, to illustrate how much the correlations are restricted to the $^1S_0$ and $^3S_1$ - $^3D_1$ states, comparison between the interaction which includes correlations from only the $^1S_0$ and $^3S_1$ - $^3D_1$ states (dashed lines) and the exact interaction (solid lines) is made in Fig. 11. First of all the small discrepancies observed are consistent with our previous analysis, showing that the correlations, in even states, are confined primarily to $^1S_0$ and $^3S_1$ - $^3D_1$ states. The effect of correlations from the odd states is 50 MeV fm^3 in the central channel F and 20 MeV fm^3 in the spin G and isospin F' channels, which correspond to 15 % and 25 % of the exact results, respectively. The influence of odd-state correlations on the spin-isospin G' channel is extremely small. This is because the effect of correlations from the singlet-odd states (ΔG^{00}) is repulsive, while that for the triplet odd states (ΔG^{11}) is attractive. Moreover, they are of nearly the same magnitude and therefore cancel each other to a large extent in the spin-isospin G' channel (see eq. (1.3.8)).

1.4b Momentum Transfer and Density Dependent Quasi-Particle Interaction

In the previous sub-section the non-locality of the quasi-particle interaction ρ has been studied. Although correlations due to the medium introduce relatively little non-locality, the resulting interaction exhibits a considerable ρ dependence in the central channel. This occurs because the bare NN potential used (HM3A) has such a behaviour. Therefore, in the central channel, a local approximation to the quasi-particle interaction may not be a good approximation and should be used with caution. In spite of this fact, we would like to construct an interaction which can be used in nuclear structure calculations. Note that in the previous section we considered the Q -dependence at $q=0$ by varying the initial relative momentum k_0 (see eq. (1.3.1)). We now use an average value of k_0 (instead of averaging over the momentum Q) to construct an effective G -matrix interaction. This procedure is a natural extension of the scattering problem, in which k_0 is related to the incident momentum of the projectile. In the averaging procedure we have set the weight function equal to unity, since in the present case ($k_\alpha = k_\beta = k_F$) k_0 can, for fixed k_F , only be varied by varying the angle between the momenta of the particles in the initial state (see eq. (1.1.6)). The averaged value k_0 is given as a function of the Fermi momentum k_F ,

$$\langle k_0 \rangle \sim 0.7 k_F . \quad (1.4b.1)$$

The price we pay in using such an approximation will be discussed later.

The quasi-particle interaction obtained with the above approximation still depends on $\vec{q}$ and $\vec{Q}$; however, these variables are no longer independent (see eq. (1.3.9)).

For convenience in further applications of our antisymmetrized quasi-particle interaction $\mathcal{F}$, we now parametrize it in terms of Yukawa-type interactions following the suggestions of Refs. 42 and 60. The parameters are the masses m_i and the coupling strengths A_i of the effective mesons. The latter depend only on the density. Instead of parametrizing directly in the spin and isospin transfer representation, here we have parametrized the coefficients G^{ST} (see eq. (1.3.5)) for convenience. Explicitly one has

$$\begin{aligned}
 G^{00} &= A^{S0} \left\{ \frac{m_{S0}^2}{q^2 + m_{S0}^2} - \frac{m_{S0}^2}{Q^2 + m_{S0}^2} \right\}, \\
 G^{01} &= A^{SE} \left\{ \frac{m_{SE}^2}{q^2 + m_{SE}^2} + \frac{m_{SE}^2}{Q^2 + m_{SE}^2} \right\}, \\
 G^{10} &= A^{TE} \left\{ \frac{m_{TE}^2}{q^2 + m_{TE}^2} + \frac{m_{TE}^2}{Q^2 + m_{TE}^2} \right\}, \\
 G^{11} &= A^{T0} \left\{ \frac{m_{T0}^2}{q^2 + m_{T0}^2} - \frac{m_{T0}^2}{Q^2 + m_{T0}^2} \right\}, \\
 G_{LS}^{10} &= \frac{A^{LSE}}{4} \left\{ \frac{qQ}{q^2 + m_{LSE}^2} - \frac{qQ}{Q^2 + m_{LSE}^2} \right\}, \\
 G_{LS}^{11} &= \frac{A^{LS0}}{4} \left\{ \frac{qQ}{q^2 + m_{LS0}^2} + \frac{qQ}{Q^2 + m_{LS0}^2} \right\}, \\
 G_t^{10} &= A^{TNE} \frac{q^2}{q^2 + m_{TNE}^2} \left(\frac{\Lambda_{TNE}^2 - m_{TNE}^2}{\Lambda_{TNE}^2 + q^2} \right); \quad G_{t'}^{10} = A^{TNE} \frac{Q^2}{Q^2 + m_{TNE}^2} \left(\frac{\Lambda_{TNE}^2 - m_{TNE}^2}{\Lambda_{TNE}^2 + Q^2} \right), \\
 G_t^{11} &= A^{TNO} \frac{q^2}{q^2 + m_{TNO}^2} \left(\frac{\Lambda_{TNO}^2 - m_{TNO}^2}{\Lambda_{TNO}^2 + q^2} \right); \quad G_{t'}^{11} = -A^{TNO} \frac{Q^2}{Q^2 + m_{TNO}^2} \left(\frac{\Lambda_{TNO}^2 - m_{TNO}^2}{\Lambda_{TNO}^2 + Q^2} \right),
 \end{aligned} \tag{1.4b.2}$$

where the form factor $\left(\frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 + q^2} \right)$ has been included for the tensor components according to the suggestion of Ref. 70. The OPEP with the contact term removed has been included in the central channels, while in the tensor terms the regu-

larized-OPEP⁷⁰⁾ (which includes the form factor) has been added. Apart from the OPEP a superposition of only two Yukawas has been allowed in each channel of the interaction for simplicity. The masses of the two mesons exchanged in each channel, which were chosen to be $m_1 = 4.0$ and $m_2 = 2.5 \text{ fm}^{-1}$, are kept the same for all channels. Also for the cut-off parameters Λ_{TNE} and Λ_{TNO} a common value, $\Lambda = 960 \text{ MeV}$, was chosen. The coupling strengths A_i (other than the OPEP) of the Yukawa-type form were varied until a good fit to the exact G-matrix interaction was obtained. The results, at normal density, are tabulated in Table 4. The quality of the fit is excellent ($\lesssim 0.2 \%$) in the momentum transfer range $0 < q \leq 2k_0$ ($k_0 = 0.7 k_F$). The usual spin and isospin transfer representation of the interaction can be easily obtained by means of eq. (1.3.8).

The next step is to extract the density dependence of the quasi-particle interaction $\mathcal{F}$. In the present work, the density dependence of the G-matrix is mapped out by solving the BG-equation at several densities and then using the LDA. The density enters parametrically as a function of k_F . Note that the average value of initial relative momentum k_0 which enters in the G-matrix as a parameter depends on k_F (see eq. (1.4b.1)). One has also to remember that the C.M.-momentum K is also a function of k_F (see eq. (1.3.3)) so that a variation of k_F implies a variation in both k_0 and K . Therefore, the extraction of the density dependence through k_F introduces a k_0 -dependence, on which our G-matrix interaction depends strongly as has been discussed in sub-section 1.4a. In finite nuclei, however, the density dependence is contained in the single-particle wave functions $|p\rangle$ which enter through expressions of the form

$$\sum_{pp'} \frac{\langle p_1 p_2 | V | pp' \rangle \langle pp' | G | p_1 p_2 \rangle}{\epsilon_{p_1} + \epsilon_{p_2} - \epsilon_p - \epsilon_{p'}}. \quad (1.4b.3)$$

The prime in the summation denotes the restricted sum over the unoccupied intermediate single-particle states p and p' due to the Pauli blocking. It is quite clear that the single-particle energies ϵ in the denominator (in particular, the starting energy $\epsilon_{p_1} + \epsilon_{p_2}$) depend on the system but not on the density of the system. This means that both k_0 and K should, in principle, be independent of the density. Therefore, in extracting the density dependence of the quasi-particle interaction, we decided to fix the kinematical variables at their values obtained at normal density ($k_0 \approx 0.7$ $k_F \approx K$). This assumption precludes the extraction of the density dependence of the quasi-particle interaction for lower densities, since the value of k_F is limited by the condition $k_F > k_0$. In spite of this limitation, the density dependence of the G-matrix interaction can still be extracted over the major part of the region of interest ($\rho \gtrsim 0.35 \rho_0$; ρ_0 = normal density).

In Fig. 12, the quasi-particle interaction $\mathcal{F}$ is shown for $k_F = 1.36, 1.25, 1.10$ and 0.95 fm^{-1} as a function of the momentum transfer q . The central channel F is largely affected by a variation of the density (Fig. 12a), since this channel is strongly influenced by correlations. The strongest k_F dependence occurs in the low momentum transfer region and, in particular, at $q=0$ it has $\sim 40\%$ of variation (with respect to the value at normal density) in the range $0.95 < k_F (\text{fm}^{-1}) < 1.36$; it becomes weaker with increasing q . The figure also shows that the central channel is more attractive for low density. This occurs because the Pauli-operator restricts the intermediate states less at low densities and makes the effect of correlations much stronger. Now, since we know that correlations provide the attraction in this channel (see the discussions in sub-section 1.4a), one can easily understand the behaviour observed in the figure. The isospin channel F' (Fig. 12b) is drastically affected by a variation of the density. In contrast to the central channel, the effect on F' is

to shift it by a constant in the region $0 < q < 2k_0$. Although its variation in absolute magnitude corresponds to $\sim 50\%$ of that of the central channel F , it changes by a factor ~ 2 with respect to its value at normal density within the range $0.95 < k_F (\text{fm}^{-1}) < 1.36$. The reason is that, as we will show later in Table 4, the triplet-even (G^{10}) channel has much stronger density dependence than the singlet-even (G^{01}) channel changing their ratio rapidly. Therefore the cancellation in eq. (1.3.8) becomes less with decreasing density (note also the factor 3 in front of G^{10}). The same considerations which have been used for the central channel explain the increasing repulsion in the F' channel with decreasing density. In the spin G (Fig. 12c) channel a negligible density dependence is observed. The spin-isospin G' (Fig. 12d) channel is also very weakly density dependent. The remaining components of the interaction have essentially no density dependence due to the very weak effect of correlations in those channels (see the discussion of sub-section 1.4a), and therefore are not shown.

For further applications of our quasi-particle interaction $\mathcal{F}$, it is also convenient to obtain a simple parametrization of its density dependence. Again we considered the coefficients G^{ST} . In order to choose a reasonable functional form for the density dependence, we analyzed the analytical G -matrix given in Ref. 67. For a repulsive separable potential of the form

$$V_{00}(k, k_0) = g_0(k)g_0(k_0) , \quad (1.4b.4)$$

one has

$$G_{00}(K, k, k_0; k_F) = g_0(k)g_0(k_0) \frac{1}{1 - I(K, k_0; k_F)} , \quad (1.4b.5)$$

where,

$$I(K, k_0; k_F) = \frac{2}{\pi} \int_0^\infty dk' \frac{k'^2 \bar{Q}(K, k') g_0^2(k')}{E(K, k_0) - E(K, k')} . \quad (1.4b.6)$$

For

$$g_0(k) = \alpha/(k^2+a^2) \quad (1.4b.7)$$

the above integral is given by a complicated expression. However, for typical values of the parameters⁶⁷⁾, it has nearly a quadratic dependence on k_F as is shown in Fig. 13. These considerations suggest an expression of the form

$$F_i(k_F) = \frac{1}{1+\delta_i+\gamma_i k_F+\beta_i k_F^2} \quad (1.4b.8)$$

for the density dependence of G^{ST} . The parameters are δ_i , γ_i and β_i which are adjusted to reproduce the density dependence of our quasi-particle interaction. In order to extract a local effective interaction, one has to replace all the coupling strengths A_i (see eq. (1.4b.2)) by the density dependent coupling strengths

$$\frac{A_i(k_F=k_F^0)}{1+\gamma_i(k_F-k_F^0)+\beta_i(k_F-k_F^0)^2}, \quad (1.4b.9)$$

where k_F^0 denotes the value of k_F at normal density. In the fitting procedure, we have considered all of the values of k_F in steps of $\sim 0.05 \text{ fm}^{-1}$ ranging from 0.95 to 1.36 fm^{-1} . Moreover, because of our assumption we have no way of extracting the density dependence for $k_F < k_0$; consequently, we have only demanded that the resulting parameters γ_i and β_i give a smooth function of density in the region of interest ($k_F > 0$). Therefore, care must be taken in extrapolating our results to $k_F < k_0$. The results for γ_i and β_i are also displayed in Table 4. The singlet-even G^{01} and singlet-odd G^{00} channels are reproduced within 0.4 %, whereas triplet-even G^{10} and triplet-odd G^{11} channels are reproduced within 1 %. The reason why the fit of the triplet-even channel is poorer than the fit in the singlet-even channel is as follows: The density dependence

in the triplet-even state is essentially introduced by the tensor force in the ${}^3S_1$ - ${}^3D_1$ ($L=L'=0$) partial-wave contribution which is a sum of various terms each of them involving different partial wave in the intermediate state (tensor forces couple different partial waves). For instance, if we assume, for simplicity,

$$V_{LL'}(k, k_0) = \begin{cases} g_L(k)g_{L'}(k_0) & L \neq L' \\ 0 & L = L' \end{cases}, \quad (1.4b.10)$$

the partial-wave contributions of the G-matrix in the ${}^3S_1$ - ${}^3D_1$ ($L=L'=0$) state read explicitly:

$$G_{00}(K, k, k_0; k_F) = g_0(k)g_0(k_0) \frac{I(K, k_0; k_F)}{1 - I^2(K, k_0; k_F)}. \quad (1.4b.11)$$

Eq. (1.4b.11) shows that the tensor force can introduce a density dependence different from that of eq. (1.4b.5) and therefore the parametrization (1.4b.9) is not necessarily optimal. However, in order to avoid complications, we used the same functional form for the density dependence in each of the channels. The density dependence in the triplet-odd channel is almost negligible compared to the other channels. Several authors^{56,58)} have suggested either linear or quadratic dependence of G-matrix interaction on k_F . From the present considerations one can see that this corresponds only to the first term of the expansion of eq. (1.4b.9). Also from the results for γ_i and β_i (see Table 4) one can understand why such a simple expression can reasonably reproduce the exact density dependence of a G-matrix.

We also parametrized the density dependence of our quasi-particle G-matrix interaction with a quadratic form

$$A_i(k_F=k_F^0)[1+\gamma_i'(k_F-k_F^0)+\beta_i'(k_F-k_F^0)^2] , \quad (1.4b.11)$$

where k_F^0 again denotes the value of k_F at normal density. The results are displayed in Table 5. Although the above quadratic form gives acceptable results ($\sim 4\%$ in both G^{10} and G^{01} channels) for $0.95 < k_F(\text{fm}^{-1}) < 1.36$ its quality of the fit is clearly poorer than that obtained using the form given by eq. (1.4b.9). Moreover, it becomes even poorer with decreasing density.

1.4c Landau Parameters from the G-Matrix Interaction

The aim of this sub-section is twofold. First, we want to illustrate some of the aspects of the local effective interaction constructed in the previous section. Second, we also want to study the sensitivity of the G-matrix interaction to the choice of bare NN potential. Several NN potentials based on the meson-exchange are now available in the literature. These potentials are equivalent in the sense that all of them fit the NN phase-shift data. However, they may differ from each other when applied to the description of excited states of nuclei (taking properly into account the correlations induced by the medium). To study the sensitivity to the choice of NN potential we have extracted the usual Landau parameters^{17,35)} since they represent some of the most important features of the interaction.

The Landau parameters, at normal density, were extracted from our local effective interaction following the prescription given in Appendix B. Only the lowest three partial-wave contributions were calculated.

The results from our local effective interaction using the HM3A version of the OBEP (see sub-section 1.4b) are displayed in parentheses in the first column of Table 6. They reproduce closely the exact values which are also shown in the same column. The discrepancies are attributed to our approximation in averaging over the initial relative momentum k_0 in getting the local effective interaction. The Landau parameters in the central channels (f_ℓ, f'_ℓ, g_ℓ and g'_ℓ) are most affected by this approximation, since a significant k_0 dependence is present in the central channel F (see Fig. 6). Note that all of the parameters, f_ℓ, f'_ℓ, g_ℓ and g'_ℓ , are affected because the exchange term of the central channel interaction F contributes to each of them. As a matter of fact, for the tensor terms, much better agreement is obtained since there is no significant k_0 dependence in these channels.

The exact results from the OBEP⁵⁹⁾ ($\Lambda_\pi = 1265$ MeV and $\Lambda_\pi = 1000$ MeV versions, hereafter denoted by HM3B and HM3C, respectively) are also displayed in columns two and three of Table 6. These parameters reveal interesting features. Let us concentrate on the results for $\ell=0$, i.e., $f_0, f'_0, g_0, g'_0, h_0$ and h'_0 . The first important characteristic which emerges from the table is that each G-matrix interaction violates the well known stability condition^{17,35)}

$$2f_\ell > -(2\ell+1) \quad (1.4c.1)$$

for $\ell=0$. This means that such an interaction must be used with care in nuclear structure calculations such as the random phase approximation. However, it is well known that the G-matrix is only a part of the effective quasi-particle interaction. Higher order terms, such as the induced interaction, play an important role and tend to stabilize the full effective interaction^{23,27,30,31)}. Also several authors⁴⁹⁾ have shown that inclusion of relativistic effects

tends to stabilize the G-matrix results since it introduces a density dependence in the meson coupling strengths. Second, phase-shift-equivalent NN potentials lead to quite different results for the Landau-parameters. For example, the Landau parameter f_0 is systematically decreasing from HM3A to HM3C, while the parameter f'_0 increases by $\sim 48\%$ from the first to the third column. The parameter g_0 is decreasing and g'_0 remains almost unchanged in going from column one to column three. In the tensor channels, both h_0 and h'_0 decrease systematically from column 1 to column 3. How can one understand this behaviour? The only difference in the results comes from the values of the pion cut-off parameter Λ_π , the sigma-coupling strength g_σ^2 and the delta-coupling strength g_δ^2 (59), which are summarized in Fig. 14. First of all, the cut-off parameter Λ_π is varied from $\Lambda_\pi = 1530$ MeV to 1000 MeV through HM3A to HM3C. In order to compensate this variation, the sigma-meson coupling strength is increased, while the delta-meson coupling strength is decreased. The pion cut-off parameter introduces a small enhancement of the tensor piece of the pion potential in going from HM3A to HM3C (for momentum transfer above 1.5 fm^{-1}), while the spin-isospin channel remains almost unchanged. It is also worthwhile to remember that the σ -meson is a scalar-isoscalar meson, while the δ -meson is a scalar-isovector one. Now, let us make a very simple model for our G-matrix interaction. We shall assume that the changed piece of the interaction is given by

$$\mathcal{F}_{\text{CHANGED}} \sim \Omega \{V_\delta + V_\sigma\} + V_\pi^{T(2)}, \quad (1.4c.2)$$

where Ω is a factor which simulates the effect of short-range correlations. V_σ and V_δ are the σ - and δ -exchange potentials, while $V_\pi^{T(2)}$ denotes the twice iterated tensor term of the pion-exchange potential

$$V_{\sigma}(q) = -g_{\sigma}^2 \frac{1}{q^2 + \mu_{\sigma}^2} \Gamma_{\sigma}(q) ,$$

$$V_{\delta}(q) = -g_{\delta}^2 \frac{1}{q^2 + \mu_{\delta}^2} \Gamma_{\delta}(q) \tau_1 \cdot \tau_2 , \quad (1.4c.3)$$

$$V_{\pi}^T(\vec{q}) = -\frac{g_{\pi}^2}{\mu_{\pi}^2} \frac{q^2}{q^2 + \mu_{\pi}^2} \Gamma_{\pi}(q) S_{12}(q) \tau_1 \cdot \tau_2 ,$$

with the form factor

$$\Gamma_i(q) \equiv \left[\frac{\Lambda_i^2 - \mu_i^2}{\Lambda_i^2 + q^2} \right]^2 , \quad i=\sigma, \delta \text{ and } \pi .$$

Including the exchange term, one has in the Landau limit ($q \rightarrow 0$)

$$\begin{aligned} \delta \mathcal{F}(q=0) \sim & \left[\Omega \left\{ \frac{3}{4} \left(\frac{\tilde{g}_{\delta}}{\mu_{\delta}} \right)^2 - \left(\frac{g_{\sigma}}{\mu_{\sigma}} \right)^2 + \frac{1}{4} \left(\frac{\tilde{g}_{\sigma}}{\mu_{\sigma}} \right)^2 \right\} - 18 V_{\pi}^{T(2)} \right] \\ & + \left[\Omega \left\{ - \left(\frac{g_{\delta}}{\mu_{\delta}} \right)^2 - \frac{1}{4} \left(\frac{\tilde{g}_{\delta}}{\mu_{\delta}} \right)^2 + \frac{1}{4} \left(\frac{\tilde{g}_{\sigma}}{\mu_{\sigma}} \right)^2 \right\} + 12 V_{\pi}^{T(2)} \right] \tau_1 \cdot \tau_2 \\ & + \left[\Omega \left\{ \frac{3}{4} \left(\frac{\tilde{g}_{\delta}}{\mu_{\delta}} \right)^2 + \frac{1}{4} \left(\frac{\tilde{g}_{\sigma}}{\mu_{\sigma}} \right)^2 \right\} - 6 V_{\pi}^{T(2)} \right] \sigma_1 \cdot \sigma_2 \\ & + \left[\Omega \left\{ - \frac{1}{4} \left(\frac{\tilde{g}_{\delta}}{\mu_{\delta}} \right)^2 + \frac{1}{4} \left(\frac{\tilde{g}_{\sigma}}{\mu_{\sigma}} \right)^2 \right\} + 4 V_{\pi}^{T(2)} \right] \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 , \end{aligned} \quad (1.4c.4)$$

excluding the tensor channels. Here, $(\frac{\tilde{g}}{\mu})^2$ denotes the contribution to the Landau parameter from the exchange term. The form factors Γ_i have been omitted for simplicity. Factors which come from the spin-isospin algebra are explicitly displayed. From eq. (1.4c.4), one can immediately understand the general trend of the Landau parameters in Table 6. For instance, in the isospin channel the first term $-(\frac{g_{\delta}}{\mu_{\delta}})^2$ increases from HM3A to HM3C, since the coupling strength g_{δ}^2 decreases. The second term $-\frac{1}{4}(\frac{\tilde{g}_{\delta}}{\mu_{\delta}})^2$ also increases since this is

the exchange contribution from the former one. The next term $\frac{1}{4}(\frac{\tilde{g}_\sigma}{\mu_\sigma})^2$ increases from HM3A to HM3C because the coupling strength g_σ^2 increases. Therefore the variation in the σ - and δ -meson coupling strengths add coherently to increase the Landau parameter f'_0 from HM3A to HM3C. The last term $12V_\pi^{T(2)}$ decreases from HM3A to HM3C because $V_\pi^{T(2)}$ is a negative quantity (due to the energy denominator) and its magnitude increases with decreasing Λ_π . However, in the present study, its variation is not large enough to cancel the contribution from the variation of σ - and δ -exchange and the final result is still to increase the parameter f'_0 from HM3A to HM3C. Similar analyses can also explain the general tendency of the remaining channels, although there is some cancellation between different contributions and the general tendency is no longer as clear as in the isospin channel. Therefore, a quantitative analysis is called for. To do this, we shall take all the values of the parameters from Ref. 58. Moreover, the effect of the short-range correlation on the rho-meson exchange contribution ($\mu_\rho \sim 760$ MeV) is known to reduce the spin-isospin channel by a factor of 0.4²²⁾. Therefore, we assume the same reduction factor $\Omega = 0.4$ for both δ - and σ -exchange potential which have masses $\mu_\delta = 960$ MeV and $\mu_\sigma = 550$ MeV respectively. The variation of the second-order tensor contribution $V_\pi^{T(2)}$ has been estimated from the variation in the isovector-tensor channel h'_0 since in this channel only that term can contribute. The difference between the third and first columns of Table 6 shows a value $\Delta V_\pi^{T(2)} \sim -0.02$. The same result is also valid for the tensor channel h_0 . Using the above result and following again the prescription of Appendix B, we obtained the results displayed in Table 7. Our estimate shows that the variation in the central channel Δf_0 is underestimated by ~ 50 %. However, we already know that this channel is strongly influenced by short-range correlations and therefore our model ($\Omega \sim 0.4$) is too simple to quantitatively account for this effect.

For the isospin channel $\Delta f'_0$, our simple model almost reproduces the exact variation. This occurs because short-range correlations are less important than in the central channel, and in addition the δ -meson contributes directly in this channel. In the spin channel, the variation Δg_0 is a positive number, while the exact result has the opposite sign. It is an extremely small number due to the delicate cancellation between δ -, σ - and π -exchange contributions. For the spin-isospin channel, our schematic results for $\Delta g'_0$ are larger than the exact one. However, both results are very small.

The qualitative success of our simple model shows that the G-matrix has a relatively simple structure, even though formally it appears to be a highly renormalized object.

1.5 A Simple Model for the Effective G-Matrix Interaction

In the previous sections we have obtained an effective G-matrix interaction by solving the BG-equation and its characteristics have been studied in detail. In particular, we have shown that the effects of correlations are largely restricted to S-states and present a relatively simple structure. These facts motivate us to obtain the quasi-particle G-matrix interaction within a very simple model.

The starting point of our consideration is eq. (1.3.10) where the quasi-particle G-matrix interaction $\mathcal{F}$ is expressed in terms of convolution of the bare NN potential V and the wave operator Ω . The idea here is to make a very simple model for the wave operator $\Omega(\vec{k}', \vec{k}_0; k_F)$ based on the analyses of the previous sections.

The different components Ω^{ST} of the wave operator can be expressed in terms of its partial-wave matrix elements as developed in section 1.2 (see also appendix A). The latter is given by means of eq. (1.1.17). We will now consider only the contributions which arise from the second term of the r.h.s. of eq. (1.1.17). The first term is a trivial one, which gives rise to the delta function $\delta(\vec{k}' - \vec{k}_0)$ for the wave operator $\Omega(\vec{k}', \vec{k}_0; k_F)$. Since the dominant correlation effects are present in the $1S_0$ and $3S_1-3D_1$ ($L=L'=0$ and $L=0, L'=2$) states, the wave operator is well approximated by

$$\Omega \approx \Omega^{01} P_{S=0} P_{T=1} + \Omega^{10} P_{S=1} P_{T=0} + \Omega_t^{10} S_{12}(\hat{k}') P_{T=0}, \quad (1.5.1)$$

with

$$\begin{aligned} \Omega^{01} &\approx \frac{1}{2\pi^2} \Omega_{00}^{001}(K, k', k_0), \\ \Omega^{10} &\approx \frac{1}{2\pi^2} \Omega_{00}^{110}(K, k', k_0), \end{aligned} \quad (1.5.2)$$

$$\Omega_t^{10} \approx -\frac{1}{2\sqrt{2}} \frac{1}{2\pi^2} \Omega_{20}^{110}(K, k', k_0).$$

Eq. (1.5.1) can also be rewritten as

$$\begin{aligned} \Omega &= \Omega_0 + \Omega_\tau \tau_1 \cdot \tau_2 + \Omega_\sigma \sigma_1 \cdot \sigma_2 + \Omega_{\sigma\tau} \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 \\ &+ \Omega_t S_{12}(\hat{k}') + \Omega_{t'} S_{12}(\hat{k}') \tau_1 \cdot \tau_2, \end{aligned} \quad (1.5.3)$$

where

$$\begin{aligned} \Omega_0 &= \frac{3}{16} (\Omega^{01} + \Omega^{10}), & \Omega_\tau &= \frac{1}{16} (\Omega^{01} - 3\Omega^{10}), \\ \Omega_\sigma &= \frac{1}{16} (-3\Omega^{01} + \Omega^{10}), & \Omega_{\sigma\tau} &= \frac{1}{16} (-\Omega^{01} - \Omega^{10}), \\ \Omega_t &= \frac{1}{4} \Omega_t^{10}, & \Omega_{t'} &= -\frac{1}{4} \Omega_t^{10}. \end{aligned} \quad (1.5.4)$$

Although the above approximation for the wave operator is an excellent one in all the ranges of the density $\rho \gtrsim 0.3 \rho_0$, we will restrict ourselves in this section only to the case of the normal density ρ_0 ($k_F = 1.36 \text{ fm}^{-1}$).

The partial-wave matrix elements $\Omega_{L'L}^{JST}$ are related to the defect functions F_{LL}^{JST} as defined by eq. (1.1.19), the dominant contributions of which are shown in Fig. 5 as a function of the intermediate relative momentum k' and at $k_F = 1.36 \text{ fm}^{-1}$. As one can see, F_{00}^{001} and F_{00}^{110} are very similar in the shape. This suggests to make the approximation

$$r F_{00}^{110} \sim F_{00}^{001} . \quad (1.5.5)$$

Moreover, although the defect function F_{02}^{110} is larger than F_{00}^{001} and F_{00}^{110} (see Fig. 8), the corresponding partial-wave contribution of the bare potential V_{02}^{110} , which gives rise to a contribution to the central components of the G-matrix interaction, is relatively small (by a factor $\sim 1/4$) compared to the corresponding contributions V_{00}^{001} and V_{00}^{110} . A similar observation holds for the contribution to the tensor component of the G-matrix interaction. In this case there is also a large momentum mismatch between the defect function and the corresponding bare potential. Therefore, in the present model we will entirely neglect the partial-wave contribution F_{02}^{110} . With these two approximations, the wave operator given by eq. (1.5.3) has only the central components with Ω^{01} equal to $r \Omega^{10}$ in eq. (1.5.4).

We now approximate the partial-wave contribution F_{00}^{001} by a simple expression of the form

$$F_{00}^{001}(K, k', k_0) = k' \Omega_{00}^{001}(K, k', k_0) \sim k' c_0 \delta(k' - k_c) , \quad (1.5.6)$$

where c_0 and k_c are parameters which, in principle, should be adjusted in order to well simulate the exact defect function F_{00}^{001} . (Note that these parameters are, in principle, functions of density through the Fermi momentum k_F .) In sub-section 1.4a we have shown that correlation effects essentially do not depend on the angle θ between the initial and final relative momenta $\vec{k}_0$ and $\vec{k}$ (see Fig. 7). Eq. (1.5.6) gives rise to the wave operator which is independent of $\vec{k}_0$.

The expression (1.3.10) for the quasi-particle interaction $\mathcal{F}$ may be transformed into the form

$$(\vec{q}, \vec{Q}) = \int d\vec{q}' V(\vec{q}-\vec{q}', \vec{Q}-\vec{q}') \Omega(\vec{q}', \vec{Q}+\vec{q}-\vec{q}') , \quad (1.5.7)$$

where the wave operator now includes also the trivial term proportional to the three dimensional δ -function. $\vec{q}'$ and $\vec{Q}'$ in eq. (1.5.7) are defined by

$$\begin{aligned} \vec{q}' &\equiv \vec{k}_0 - \vec{k} , \\ \vec{Q}' &\equiv \vec{k}_0 + \vec{k} . \end{aligned} \quad (1.5.8)$$

In sub-section 1.4a we have explicitly shown that the exchange momentum transfer Q dependence of the G -matrix interaction is mainly due to the bare NN potential which has already such a dependence. The higher order terms introduce a negligible Q dependence. Here we assume that the wave operator is a function only of the momentum transfer $\vec{q}'$ and neglect its exchange momentum transfer $\vec{Q}' = \vec{Q} + \vec{q} - \vec{q}'$ dependence, so that

$$\Omega(\vec{q}', Q + \vec{q} - \vec{q}') \sim \Omega(\vec{q}') . \quad (1.5.9)$$

In eq. (1.5.6) the partial-wave contribution Ω_{00}^{001} of the wave operator (second term of the r.h.s. of eq. (1.1.17)) is expressed in terms of relative momenta k' and k_0 . Therefore it requires a transformation to express in terms of momentum transfers q' and Q' , as in eq. (1.5.7). This, however, complicates the situation. Therefore, we simply assume that

$$F_{00}^{001}(K, q', Q') = k' \Omega_{00}^{001}(K, q', Q') \sim k' c \delta(q' - q_c) . \quad (1.5.10)$$

The new parameters c and q_c should be adjusted by fitting directly the exact results for the quasi-particle G-matrix interaction $\mathcal{F}$ of sub-section 1.4a. In the limit of $k_0=0$, the above expression is identical to eq. (1.5.6) since in this case $q'=k'$.

The above considerations allow us to express the quasi-particle G-matrix interaction $\mathcal{F}$ as

$$\mathcal{F}(\vec{q}, \vec{Q}) = \int d\vec{q}' V(\vec{q} - \vec{q}', \vec{Q} - \vec{q}') \Omega(\vec{q}') , \quad (1.5.11)$$

where

$$\Omega(\vec{q}') = \Omega_0(\vec{q}') + \Omega_\tau(q') \tau_1 \cdot \tau_2 + \Omega_\sigma(q') \sigma_1 \cdot \sigma_2 + \Omega_{\sigma\tau}(q') \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 , \quad (1.5.12)$$

with

$$\begin{aligned} \Omega_0(\vec{q}') &= \delta(\vec{q}') + \frac{3}{32\pi^2} (1+r) c \delta(q' - q_c) , \\ \Omega_\tau(q') &= \frac{1}{32\pi^2} (1-3r) c \delta(q' - q_c) , \\ \Omega_\sigma(q') &= \frac{1}{32\pi^2} (-3+r) c \delta(q' - q_c) , \\ \Omega_{\sigma\tau}(q') &= \frac{-1}{32\pi^2} (1+r) c \delta(q' - q_c) . \end{aligned} \quad (1.5.13)$$

The integral in eq. (1.5.11) can be straightforwardly carried out with the wave operator given by eqs. (1.5.12) and (1.5.13).

A further approximation which may be done in the above scheme is to keep only the scalar-isoscalar component $\Omega_0(\vec{q}')$ in eq. (1.5.12), since this term is the largest component of the wave operator. In doing so, one gets

$$\Omega(\vec{q}') \sim \delta(\vec{q}') + \frac{3}{32\pi^2} c \delta(q' - q_c) . \quad (1.5.14)$$

In the above expression the parameter c has been redefined to incorporate the factor $(1+r)$. With an appropriate value for the parameter c ($c = -\frac{8\pi}{3q_c^2}$), the above equation reduces to

$$\Omega(\vec{q}') = \delta(\vec{q}') - \frac{1}{4\pi q_c^2} \delta(q' - q_c) , \quad (1.5.14')$$

the Fourier transformation of which is

$$\Omega(\vec{r}) = 1 - j_0(q_c r) , \quad (1.5.15)$$

which satisfies the condition that it vanishes in the limit of $r \rightarrow 0$. This form for the wave operator has been introduced by Brown et al.⁷²⁾ and folding with a one-pion and one-rho exchange potential it has been applied in connection with the pion condensation physics^{72,73)}. There, the parameter $q_c \approx 3.93 \text{ fm}^{-1}$ has been adjusted to reproduce the dominant Fourier components of a realistic two-body correlation function calculated with the RSC³⁾. In the above equation j_0 is the usual spherical Bessel function for $l=0$. Since then it has been extensively used by several authors^{21,74)} especially in the study of the spin-isospin excitation modes in nuclei.

In the following we will consider only the σ -, ω -, ρ - and π -meson exchange contributions for the NN potential, keeping only the dominant terms. Explicitly, one has

$$\begin{aligned}
 V(\vec{q}, \vec{Q}) = & -4\pi g_\sigma^2 \frac{1}{q^2 + m_\sigma^2} \left\{ 1 + \frac{q^2 - Q^2}{8m^2} + \frac{i}{4m^2} \left(\frac{\sigma_1 + \sigma_2}{2} \right) (\vec{q} \wedge \vec{Q}) \right\} \\
 & + 4\pi g_\omega^2 \frac{1}{q^2 + m_\omega^2} \left\{ 1 - \frac{q^2 - 3Q^2}{8m^2} - \frac{3i}{4m^2} \left(\frac{\sigma_1 + \sigma_2}{2} \right) (\vec{q} \wedge \vec{Q}) \right\} \\
 & - \frac{4\pi}{3} g_\pi^2 \frac{1}{4m^2} \frac{q^2}{q^2 + m_\pi^2} \{ \sigma_1 \cdot \sigma_2 + S_{12}(q) \} \tau_1 \cdot \tau_2 \\
 & - \frac{4\pi}{3} g_\rho^2 \left(1 + \frac{f_\rho}{g_\rho} \right)^2 \frac{1}{4m^2} \frac{q^2}{q^2 + m_\rho^2} \{ 2\sigma_1 \cdot \sigma_2 - S_{12}(q) \} \tau_1 \cdot \tau_2 .
 \end{aligned} \tag{1.5.16}$$

In the above expression the form factors have not been written down explicitly.

The central components of the quasi-particle G-matrix interaction $\mathcal{F}$ within the present model, using the very crude approximation for the wave operator as given by eq. (1.5.14), are displayed in Fig. 15 (dashed lines). The parameters q_c and c have been adjusted to fit (not the best fit) the exact results of sub-section 1.4a (see Fig. 7). This yields $q_c \approx 2.0 \text{ fm}^{-1}$ and $c \approx -2.10 \sim -\frac{8\pi}{3q_c^2}$. The spin-isospin channel (G') is rather insensitive to the value of q_c . In fact, variation of q_c from 2.0 to 4.0 fm^{-1} , keeping $c = -\frac{8\pi}{3q_c^2}$, leads to the variation of G' by $\sim 23\%$. Of course, $\mathcal{F}$ has been properly antisymmetrized. It is a pleasure to see that our simple model, even with the cruder approximation for the wave operator, is already able to account for the major feature of the exact G-matrix interaction (solid lines). The deviation observed in the isospin F' -channel is partially due to the fact that we have entirely neglected the contributions of other mesons which are present in the

HM3A version of the OBEP. In particular, the δ -meson contributes directly to this channel. In addition, the other components than the scalar-isoscalar one of the wave operator also play a role.

The spin-orbit as well as the tensor components of the effective interaction are basically unaffected by correlation effects within the present model in accordance with the exact results of sub-section 1.4a. Therefore they are not shown.

The considerable success of our simplest model for the wave operator Ω indicates how easily the Pauli-blocking and short-range correlation effects in the nuclear medium can be taken into account. However, a more quantitative description requires a somewhat more complete wave operator which involves other channels than the scalar-isoscalar one, such as given by eqs. (1.5.12) and (1.5.13). Furthermore, the bare NN potential must also account for the contributions of other mesons than that considered in eq. (1.5.16). These improvements constitute no difficulties at all.

Finally, in Fig. 16 a comparison between the effective force used by the Jülich-Stony Brook group⁷⁵⁾ (dashed lines) and the exact G-matrix results (using the HEA version⁷⁶⁾ of the OBEP) (solid lines) in the spin-isospin and isovector-tensor channels are made. As one can see, the Jülich-Stony Brook ansatz for the effective force with $\delta g_0' = 0$ gives quite close results to the exact G-matrix interaction. This has to be expected, since the former interaction is given by

$$F_{\sigma\tau}(\vec{q}) = \int d\vec{q}' V_{\sigma\tau}(\vec{q}-\vec{q}')\Omega(\vec{q}') + \delta g_0' \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 , \quad (1.5.17)$$

where $V_{\sigma\tau}(\vec{q})$ denotes the sum of the one-pion and one-rho exchange potential and $\Omega(\vec{q}')$ is given by eq. (1.5.14') with $q_c = 3.93 \text{ fm}^{-1}$. $\delta g'_0$ is a constant term which summarizes higher order effects.

1.6 $\Delta(1232)$ -Nucleon G-Matrix Interaction

In the previous sections a quasi-particle G-matrix interaction has been derived. In particular, effects of correlations have been analyzed in quite detail. In this section we will make use of those results in order to construct a Δ -nucleon G-matrix interaction in a simple way.

Inclusion of the Δ -isobar degree of freedom introduces, in principle, many complications due to the selfconsistent coupling between isobars and nucleons, which leads to a set of coupled BG-equations. However, in Ref. 77 we have explicitly shown that the graphs which have Δ -isobars in the intermediate states give negligible contributions basically due to the large energy denominator. (Note that the mass difference between Δ -isobar and nucleon is about 300 MeV.) The results from those calculations for the Landau parameter in the spin-isospin channel of the NN as well as of the Δ -nucleon interaction are summarized in Table 8, which show very small contributions of the intermediate Δ -isobar graphs compared to the corresponding intermediate nucleon graphs. Once the diagrams which contain Δ -isobars in the intermediate states are neglected, the coupled BG-equations become uncoupled and the G-matrix in the Δ -nucleon sector ($G_{\Delta N}$) can be written as

$$G_{\Delta N} \approx V_{\Delta N} \left(1 + \frac{Q}{e} G_{NN} \right) = V_{\Delta N} \Omega_{NN} . \quad (1.6.1)$$

In the above equation, $V_{\Delta N}$ denotes the Δ -nucleon transition potential which we will assume to be given by the π - plus ρ -meson exchange potential in the spin-isospin $(\sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2)$ and isovector tensor $(S_{12}(\hat{q}) \tau_1 \cdot \tau_2)$ channels, replacing $\sigma_1(\tau_1)$ by the transition operator $S_1(T_1)$ and the coupling strength $g_{NN\pi}(g_{NN\rho})$ by the corresponding coupling $g_{\Delta N\pi}(g_{\Delta N\rho})$. Moreover, we will assume the experimental value $(g_{\Delta N\pi}/g_{NN\pi}) = 2$; the same value for the ratio $(g_{\Delta N\rho}/g_{NN\rho})$ is also assumed. In eq. (1.6.1), Ω_{NN} denotes the wave operator in the NN sector. A diagrammatic representation of eq. (1.6.1) is shown in Fig. 17.

In the following we will give a simple argument based on our results of section 1.4 that the Δ -nucleon G-matrix interaction in eq. (1.6.1) is to a good approximation given by the bare transition potential $V_{\Delta N}$. In other words, the Δ -isobar does not feel the effects of correlations given by Ω_{NN} . The basic idea of this argument was recently given by Arima et al.⁷⁸⁾ and stands on the considerations of the Pauli-principle. Due to the fact that nucleons carry spin and isospin $1/2$ while the isobar has both spin and isospin $3/2$, the two interacting nucleons must have total isospin $T=1$ and, in the spin-isospin channel, total spin $S=1$. This implies that all even relative angular momenta of the interacting nucleons are suppressed in the spin-isospin channel, while in the isovector-tensor channel all even or odd relative momenta are suppressed according to spin S being odd or even, respectively. The above consideration may easily be visualized in Fig. 17 where the total isospin $T=1$ must be conserved through the diagram. Now, in sub-section 1.4a, we have shown that the effects of correlations are essentially confined to the $1S_0$ and $3S_1$ - $3D_1$ states (see Fig. 8). The latter cannot contribute to the Δ -nucleon interaction because it violates the condition $T=1$. The singlet-even $1S_0$ states has $T=1$ and therefore it contributes to the Δ -nucleon interaction. The only partial-wave contribution to the transition potential which can match the $1S_0$ state is the

tensor $^1S_0$ - $^5D_0$ state and consequently it can only contribute to the tensor component of the Δ -nucleon interaction. Now, the defect function $F_{00}^{001}(k, k_0)$ in the $^1S_0$ state peaks at the relative momentum $k \sim 3 \text{ fm}^{-1}$ while the tensor transition potential in the $^1S_0$ - $^5D_0$ state peaks at the momentum $k \sim 0.6 \text{ fm}^{-1}$ as shown in Fig. 18. Therefore, due to this momentum mismatch, the effects of correlations from the $^1S_0$ state are also largely suppressed.

The above considerations are the basic idea why the Δ -nucleon G-matrix interaction can be given to an excellent approximation by the bare transition potential $V_{\Delta N}$.

1.7 Test of the Quasi-Particle Nuclear Matter G-Matrix Interaction

In section 1.4 the momentum transfer- and density-dependent quasi-particle G-matrix interaction has been constructed from a nuclear matter calculation. We believe that this effective interaction preserves the most essential features of the exact G-matrix interaction in a nuclear structure calculation. In order to simulate the finite system its density dependence has been mapped out through LDA.

One of the main differences between an infinite and a finite system is that the total momentum of a particle in an infinite system is a conserved quantity, while in a finite system this is no longer true. Each two-body matrix element in a finite system may involve different values of the total momentum. The quasi-particle G-matrix interaction extracted from nuclear matter calculations in the previous sections involves two-body matrix elements all of them having the same value of the total momentum.

In sub-section 1.4c we have shown that the G-matrix interaction leads to an instability in nuclear matter due to the too strong attraction in the scalar-isoscalar channel. The Landau parameter f_0 extracted from the G-matrix interaction violates drastically the stability condition given by eq. (1.4c.1). However, the instability in nuclear matter does not necessarily imply an instability in finite nuclei. In fact, in Ref. 79 the G-matrix was found to work reasonably well in ^{160}O . In Ref. 80, an effective interaction obtained from a variational calculation has also been applied in ^{160}O .

The above considerations clearly indicate that the effective interaction such as derived from a nuclear matter calculation should be tested about its reliability in a realistic nuclear structure calculation. In this section we are especially interested in verifying to which extent our effective G-matrix interaction leads to an instability in finite nuclei, since this is the most critical aspect of the G-matrix as an effective interaction.

In the present RPA calculation we have used the parametrized form of the quasi-particle G-matrix interaction as obtained in sub-section 1.4b. The HEA version⁷⁶⁾ of the OBEP has been used. In Table 8 we have displayed the results of the parametrization at $k_F = 1.36$ and 0.95 fm^{-1} . The spin-orbit terms S and S' have been entirely neglected in the present calculation.

The single-particle states were generated using a Woods-Saxon potential, the parameters^{19,81)} of which were adjusted to reproduce the experimental single-particle energies in the vicinity of the Fermi level and the mean square charge radius. We checked to which extent the results depend on the size of the configuration space. The largest spaces used in this work include all single particle orbits below the Fermi level and up to four major shells above the Fermi level.

The starting energy is the sum of the single particle energies of the hole states involved in a particular matrix element. In order to simplify numerical calculations, one may choose a constant average starting energy. To the extent that only single particle states within one major shell below the Fermi level contribute, a first guess for the starting energy is $\omega = -2 \times 41 \times A^{-1/3}$, which means $\omega = -32.5$ MeV in ^{160} and $\omega = -13.8$ MeV in ^{208}Pb . Since the levels in the second major shell below the Fermi level cannot be neglected, however, it might be more reasonable to use lower values for the starting energy. Therefore, we have chosen $\omega = -40$ MeV in ^{160} . The dependence on the starting energy of the G-matrix interaction has been studied by looking at the RPA results for different values of the starting energy. The RPA results change by $\sim 10\%$ when we vary the starting energy ω from -80 to -20 MeV. For the 0^+ state in ^{160} this change is 29% . The value $\omega = -40$ MeV has been kept in ^{208}Pb . By choosing this rather low value of the starting energy, the effects of the short-range correlations are reduced with respect to more realistic values such as $\omega = -13.8$ MeV. In particular, the large attraction in the scalar-isoscalar channel due to the iterated tensor force⁷⁵⁾ is reduced. Note that the conclusions concerning the instability in ^{208}Pb are valid even for the present extreme choice of the starting energy.

In Table 10 the results for the lowest excited natural parity $T=0$ states (0^+ , 2^+ and 3^-) in ^{160} are displayed. The first column refers to the experimental energies and the corresponding electric transition probabilities. In order to show the influence of the single-particle basis, in the second column we show the results obtained using a harmonic oscillator basis with the harmonic oscillator constant $b = 1.726$ fm. The density dependence of the G-matrix interaction has been neglected in this case, keeping the value $k_F = 1.36$ fm $^{-1}$.

In contrast to the nuclear matter case, the G-matrix interaction does not lead to an instability in ^{16}O . The difference in our results compared to the recent results of Ref. 79 is essentially due to the different density, starting energy and configuration space used. In the third column we display the results which differ from those of the second column only by the different single-particle basis, i.e., in this case the Woods-Saxon basis is used. In the case of the breathing mode, the harmonic oscillator basis is particularly bad since the $2p_{1/2}$ and $2p_{3/2}$ orbits are far above the particle emission threshold. As a consequence, the excitation energy is considerably below the energy $E_{0+} = 80 A^{-1/3}$ which corresponds to the centroid energy of the monopole resonance in heavy nuclei. Using the more realistic Woods-Saxon basis, however, the monopole excitation energy is found at 22.30 MeV.

The 2^+ and 3^- resonances are too high in energy as compared to the experimental energies. The reason for this is due to the density dependence of the G-matrix interaction which becomes more attractive for lower densities. The calculation in column 3 was performed with a G-matrix evaluated for nuclear matter density corresponding to $k_F = 1.36 \text{ fm}^{-1}$. This may be adequate for a volume mode, such as the breathing mode, but certainly not for the surface modes 2^+ and 3^- . In the vicinity of the peaks of the transition densities of the 2^+ and 3^- modes the density has fallen to $\sim 35\%$ of the normal density. This corresponds to $k_F \sim 0.95 \text{ fm}^{-1}$. Therefore, for the 2^+ and 3^- states $k_F = 0.95 \text{ fm}^{-1}$ may be a more appropriate value. The corresponding results are shown in the fourth column and show a much better agreement with the experimental data than in the former case.

In Table 11 the results for the lowest 0^+ , 2^+ , 3^- and 5^- states in ^{208}Pb are displayed. The first column shows the experimental results. In the second col-

umn the results using the G-matrix interaction with $k_F = 1.36 \text{ fm}^{-1}$ are shown. All the states considered, except the 5^- , are unstable (i.e., imaginary eigenvalues). Note that even the 5^- state gets unstable if one considers the density dependence of the G-matrix interaction as is shown in the third column. In particular, more realistic values of the starting energy would further enhance the instability. These results clearly show that the G-matrix interaction alone is not enough for the description of the excited state of ^{208}Pb .

Strictly speaking, the above calculation presents an inconsistency, since we have used "experimental" single-particle energies instead of selfconsistent ones. Certainly, the former ones include 2p-2h effects which should also be included consistently in the residual interaction. The 2p-2h effects may be interpreted as dressing of the particle ($P^P(E)$) and ($P^h(E)$) energies due to phonon coupling and a screening of the interaction ($I^{Ph}(E)$) which is usually referred to as "induced interaction"^{26,27}). The latter process is investigated within a microscopic framework in chapter IV. Here, we will discuss these effects, in particular the particle (hole)-phonon coupling effects, within a simple phenomenological approach.

The 2p-2h effects in the lowest order are given by

$$\begin{aligned}
 P^P(E) &= \delta_{ij} \sum_{\ell p' h'} \frac{\langle m h' | G | \ell p' \rangle \langle \ell p' | G | n h' \rangle}{E - (\epsilon_\ell - \epsilon_i + \epsilon_{p'} - \epsilon_{h'})}, \\
 P^h(E) &= \delta_{mn} \sum_{k p' h'} \frac{\langle j p | G | k h' \rangle \langle k h' | G | i p' \rangle}{E - (\epsilon_n - \epsilon_k + \epsilon_{p'} - \epsilon_{h'})}, \\
 I^{Ph}(E) &= -2 \sum_{p' h'} \frac{\langle m i^{-1} | G | p' h' \rangle \langle p' h' | G | n j^{-1} \rangle}{E - (\epsilon_m - \epsilon_j + \epsilon_{p'} - \epsilon_{h'})}.
 \end{aligned} \tag{1.7.1}$$

Antisymmetrization has not been written down explicitly. A diagrammatic representation of eq. (1.7.1) is shown in Fig. 19.

In an electron gas, these terms are known to cancel each other like $(q/k_F)^2$. In the field of giant resonances, 2p-2h effects so far have mainly been studied with phenomenological interactions. Neglecting the coupling among different 2p-2h configurations, the energy E of an excited state, assuming degenerate particle and hole energies, is given by

$$E = \epsilon^{HF} + p^p(E) + p^h(E) + I^{ph}(E) + G^{ph} . \quad (1.7.2)$$

Here ϵ^{HF} denotes the Hartree-Fock single particle-hole energy gap, while G^{ph} refers to the G-matrix interaction. In order to map eq. (1.7.2) onto the random-phase approximation, the explicit energy dependence of the 2p-2h contributions has to be linearized, for instance by expanding around a suitable energy, as suggested by Sommermann and Mahaux⁸⁴). This way one obtains

$$E = \epsilon^{exp} + F^{ph} , \quad (1.7.3)$$

where ϵ^{exp} refers to the experimental particle-hole energies, while the effective particle-hole interaction is given by

$$F^{ph} = Z^2 \{ G^{ph} + I^{ph} \} \quad (1.7.4)$$

with the z-factor

$$Z = [1 + \frac{\partial}{\partial E} (p^p + p^h + I^{ph})]^{-1/2} . \quad (1.7.5)$$

From the calculations of Ref. 85, one may extract values for Z^2 which range from 0.6 to 0.8. Clearly, the effect of the z -factor is to reduce the strength of the G -matrix and thus help to stabilize the excitation energies.

The results using $Z^2 = 0.6$ and still setting $I^{ph} = 0$ in eq. (1.7.4) are shown in the fourth column of Table 11 for ^{208}Pb . One can see that even with this strong reduction factor, the G -matrix interaction alone is too attractive in the case of the two collective states, i.e., the 3^- state at 2.62 MeV and the 0^+ breathing mode. The induced interaction provides an additional repulsion (see chapter IV). Both the 2^+ and the 5^- state are dominated by one or two one-particle one-hole configurations and consequently may be expected to be less affected by $2p$ - $2h$ effects than collective modes. Since in ^{160} , there are very few collective modes in comparison with ^{208}Pb , the induced interaction as well as the selfenergy insertion graphs may be expected to play a minor role in this nucleus, which qualitatively explains the stability found in ^{160} considering only the G -matrix interaction.

In summary, we have tested the nuclear matter G -matrix interaction in finite system, showing explicitly that it leads to an instability in heavier nuclei in the case of states of natural parity, i.e. heavier nuclei show a behaviour similar to nuclear matter. Consistent inclusion of $2p$ - $2h$ effects cures this problem.

CHAPTER II

Effective G-Matrix Interaction (Nucleon-Nucleus Scattering)

Recently, Love and Franey⁴²⁾ have obtained a t-matrix interaction, extracted directly by a fit to NN amplitudes which describe available NN data and have applied this t-matrix interaction with considerable success in the description of nucleon-nucleus scattering at intermediate energies where the reaction mechanism is much simpler than at lower energies. The main advantage of this method is that it does not require the basic NN interaction at all and therefore can be extended to energies above the pion production threshold. In principle, the only limitation is the availability of NN scattering data. The effective interaction obtained from this procedure is a function of the energy of (bombarding energy) and the momentum transfer to the incident particle. It does not contain the dependence on the density of the system (target nucleus) since it is extracted by fitting the free NN scattering data. It is also a phenomenological interaction in the sense that it is not derived from a basic NN interaction.

In this chapter we describe the construction of a G-matrix interaction based on OBEP which may also be suitable for use in nucleon-nucleus scattering. This interaction contains, in addition to the energy and the momentum transfer dependence, also a dependence on the density through the LDA as in chapter I. Such a similar G-matrix interaction has also been constructed by other authors^{65,86)}. The density-dependent corrections to the free t-matrix have been discussed in some detail in Ref. 43.

Section 2.1 is devoted to a brief discussion on the t-matrix interaction. In section 2.2 we describe the construction and analysis of our G-matrix interac-

tion which converges smoothly to the t-matrix at vanishing density. Section 2.3 is devoted to the study of the energy and density dependences of the isovector tensor interaction. There, an apparent contradiction between experimental evidences on the effective isovector tensor force observed in nuclear structure and in nuclear scattering has been explained. Finally, in section 2.4 a few illustrative applications are made.

2.1 The Lippmann-Schwinger Equation

In this section we consider briefly the Lippmann-Schwinger (LS) equation for the t-matrix. This equation differs from the BG-equation of the previous chapter in two important ways.

The LS integral equation for the scattering of two free nucleons of the c.m. energy ω and NN potential V is

$$T(\omega) = V + V \frac{1}{\omega - H_0 + i\epsilon} T(\omega) . \quad (2.1.1)$$

Here, the first important difference from the BG-equation is obvious. In the BG-equation the nuclear medium produces the Pauli exclusion effect contained in Q , and this effect is absent in the LS-equation. Another difference arises from the energy denominator, since in eq. (2.1.1) the Hamiltonian H_0 includes only the kinetic energy $\frac{\hbar^2 p^2}{2m}$.

Since we have considered the BG-equation in nuclear matter (of constant density) and have also used the angle-averaged propagator, the essential kinematical symmetries of the t- and the G-matrices are the same (apart from the iso-

tropic K dependence which violates Galilean invariance). Consequently, the partial wave decomposition and technique for solving the G -matrix may be carried over directly to the t -matrix. Indeed, our t -matrix is just the $k_F = 0$ solution of the BG-equation.

2.2 Construction of the G -Matrix Interaction for the Scattering Problem

A physical picture of the scattering problem is that one of the two nucleons interacting in the initial state belongs to the incident beam (above the Fermi level) and strikes the other nucleon in the target nucleus (below the Fermi level) with a given incident energy E_p . The incident energy (or bombarding energy) E_p is related to the local momentum k_μ of the incident particle by

$$E_p = \frac{\hbar^2 k_\mu^2}{2m} + U(k_\mu) , \quad (2.2.1)$$

where $U(k_\mu)$ is the potential energy, felt by the incident particle with a local momentum k_μ , due to the surrounding nucleons inside the target nucleus.

In the present work, we will assume that the potential energy U in eq. (1.3.1) is given by the real part of the self-consistent potential defined in eq. (1.1.12) of chapter I. Note that for positive starting energy, the G -matrix interaction becomes a complex quantity due to the existence of a pole in the energy denominator. Whether such an approximation for the potential energy is reasonable or not has to be verified later. In chapter I, the self-consistent potential has been calculated within the standard assumption⁶⁴⁾ that for particles above the Fermi distribution $U(k_\mu) = 0$. In the present chapter, however, we have used the so-called "continuous choice" introduced in Ref. 65.

With these assumptions for the potential energy U , the local momentum k_μ of the incident particle can be determined by means of eq. (1.3.1) for a given incident energy E_p .

The momentum k_ν (in the initial state) of the nucleon below the Fermi distribution which interacts with the incident particle is, in the present work, assumed to be given by the averaged value

$$\langle k_\nu \rangle \sim \frac{3}{4} k_F . \quad (2.2.2)$$

Once the momenta of the interacting particles in the initial state are fixed, the initial relative momentum k_0 and the c.m. momentum K defined in eq. (1.1.6) may be determined. We have again averaged over the angle between momenta of the interacting particles which yields

$$\langle k_0 \rangle = \langle K \rangle = \frac{1}{12k_\mu k_\nu} \{ (k_\mu^2 + k_\nu^2 + 2k_\mu k_\nu)^{3/2} - (k_\mu^2 + k_\nu^2 - 2k_\mu k_\nu)^{3/2} \} . \quad (2.2.3)$$

The final relative momentum $\vec{k}$ is assumed to have $|\vec{k}| = |\vec{k}_0|$, as in the previous chapter.

With the above assumptions, the G -operator for the positive starting energy ω can be extracted in exactly the same way as in section 1.3 of chapter I. The only difference is that here the G -matrix interaction is a complex quantity; however, all the formulae in section 1.3 hold for both real and imaginary parts of the interaction.

In order to illustrate some of the general characteristics of our interaction, in Figs. 20-22 the moduli of the components of the G -matrix interaction (direct + exchange) are displayed as a function of the momentum transfer q . The

HEA version⁷⁶⁾ of the OBEP has been used. Since we choose $|\vec{k}| = |\vec{k}_0|$, the exchange momentum transfer Q also varies according to eq. (1.3.9). All partial waves satisfying $L \leq 14$ were considered.

In Fig. 20 the central components are shown at a bombarding energy of $E_p = 50$ MeV for densities $\rho=0$ (dashed curves), $0.34 \rho_0$ (dash-dotted curves), $0.5 \rho_0$ (long-dashed curves) and ρ_0 (double-dashed curves); $\rho_0 = 0.17$ particles per fm^3 . The first characteristic which emerges from this figure is that the scalar-isoscalar (F) channel dominates in the low momentum transfer region (compare different channels at a fixed density). The isospin F'-channel becomes comparable to the spin-isospin G'-channel only at low densities. This fact is very important in connection with the excitation of the isobaric analog states (IAS) of the ground state of nuclei. It is known that the IAS can be best observed at low incident energies. As the bombarding energy increases, the isovector nuclear excitations at forward angles become dominated by the spin-isospin modes, such as the Gamow-Teller resonance. This behaviour is due to the strong energy dependence of the isospin component of the effective interaction as has been pointed out in Refs. 42,75 (see also Fig. 22). The results of Fig. 20 show that in addition to the energy dependence of the isospin channel, the excitation of the IAS is favoured because this excitation is a surface mode. It should be stressed here that the comparison between different curves for a given component of the force in Figs. 20-22 may be masked by the fact that those curves, for a given component and for a given value of the momentum transfer q , may have different values of the exchange momentum Q , since the initial relative momentum k_0 varies with density and bombarding energy. Nevertheless, Fig. 20 suggests a strong density dependence of the scalar-isoscalar component.

The central components of the effective G-matrix interaction at $E_p = 210$ MeV and for densities $\rho=0$ (dashed lines) and ρ_0 (double-dashed lines) are shown in Fig. 21. Again the scalar-isoscalar component dominates at low momentum transfers. In contrast to the case of low bombarding energies, the isovector component is dominated by the spin-isospin channel in the small momentum transfer region even at low densities. At this bombarding energy the density dependence is very weak in each channel of the effective interaction. The reason for this is that as the incident energy increases the Pauli blocking effects become weaker since more phase-space for scattering becomes available. Furthermore the energy denominator at higher energies is dominated by the kinetic energy. Therefore, the G-matrix approaches the t-matrix with increasing incident energy.

Fig. 22 shows the central components of the effective G-matrix interaction at vanishing density (t-matrix) and for incident energies $E_p = 50$ (dashed curves), 100 (dash-dotted curves), 140 (long-dashed curves) and 210 MeV (double-dashed curves). The scalar-isoscalar channel again dominates at low momentum transfers and has the strongest energy dependence. The isospin channel shows also a considerable energy dependence and this explains why the isospin modes, such as the IAS, are much more easily excited at low bombarding energies. At high densities the energy dependence of the effective interaction becomes very weak and is not shown here.

The density and energy dependences of the effective G-matrix interaction can be understood in terms of considerations similar to those in section 1.4 of chapter I. Here, however, the situation is slightly more involved because at higher bombarding energies higher partial wave states become important. Consequently, the next section is devoted to a detailed study of the energy and density dependences of the effective tensor force.

2.3 Energy and Density Dependences of the Effective Isovector Tensor Interaction

A unified understanding of the magnitude and shape of the isovector tensor force is a long standing problem of extreme importance in both nuclear structure and nuclear reaction physics. The experimental evidences which have been obtained on the isovector tensor force in the past few years seem to contradict each other. Systematic analyses of spin-isospin excitations in nuclei have successfully relied on the assumption that the interaction in this channel is dominated by the one-pion and one-rho exchange potential whereas the effect of all the other mesons is summarized in a state-independent central Jastrow correlation function²¹⁾. Under this assumption the isovector tensor force in the nuclear medium is essentially unaffected by correlations and consequently, it does not depend on either the starting energy or density. In chapter I we have shown explicitly that the tensor component of the effective nuclear matter G-matrix interaction between two nucleons below the Fermi level in the initial state is practically unaffected by correlations due to the momentum mismatch between the defect function and the corresponding bare interaction. On the other hand, the real and dominant part of the isovector tensor component of the t-matrix interaction, extracted from the recent nucleon-nucleon amplitudes of Arndt and Roper⁸⁷⁾ using the methods of Refs. 42,88, shows a strong energy dependence. This means that the isovector tensor component of the t-matrix interaction is strongly affected by short-range correlations, which is quite surprising in view of the results of chapter I.

This section is devoted to explaining the behaviour of the t- and the G-matrix tensor interactions. We especially discuss why the t-matrix isovector tensor interaction is strongly energy-dependent and why it converges to the bare one-

pion and one-rho exchange at high bombarding energies. Furthermore, we want to discuss the density dependence of the G-matrix isovector tensor interaction, where the Pauli principle plays an important role. All of our numerical results in this section are again based on the one-boson exchange potential HEA⁷⁶). It should be stressed, however, that our conclusions do not depend on the details of the particular bare interaction used.

In section 1.4 we have shown that the effects of short-range correlations and Pauli blocking are largely restricted to the relative $^1S_0$ and relative $^3S_1$ - $^3D_1$ states. For the tensor interaction, this situation persists for the case where one of the two nucleons in the initial state is above the Fermi level, even at incident energies up to 250 MeV. The relative $^3S_1$ - $^3D_1$ states can contribute in three different ways to the tensor force. One way is that which causes a transition from an initial relative S- to a final relative D-state; another is from a relative D- to a relative S-state and the last one is that from a relative D- to a relative D-state. The largest defect functions are those which cause a transition from an S- to a D-state; however, the net effect for this transition is very small due mainly to the momentum mismatch between those defect functions and the corresponding bare interactions. The magnitude of these interactions is also very small (see section 1.4). On the other hand, although the defect functions which cause a transition from an initial D- to a final S-state are relatively small compared to those contributing to the transition from an S- to a D-state, the corresponding bare interactions are very large and cause the transition from a D- to an S-state to be the dominant contribution for the correlated part (second term of the r.h.s. of eq. (1.1.15) of both the t- and the G-matrix tensor force. The transition from a D- to an S-state may proceed either via an S- or via a D-intermediate state (see eq. (1.1.15)). Both give comparable contributions.

In the following we will give a simple explanation for the energy and density dependences of the effective tensor force. For this purpose we will restrict the discussion of only the partial-wave contribution to the tensor force (correlated part) which causes a transition from a D- to an S-state via an intermediate D-state. Similar considerations hold for the transition via an intermediate S-state.

In the upper part of Fig. 23, the product $k'V_{02}(k_0, k')F_{22}(k_0; k', k_0)$ which describes the correlated contribution (see eq. (1.1.15)) of the transition from a relative D- to a relative S-state via a relative D-state in the vacuum is shown as a function of the intermediate momentum k' . We consider two different incident energies of $E_p = 50$ (curve a) and 210 MeV (curve b) to illustrate the points. The position of the pole is determined by the energy denominator of the defect function (eq. (1.1.19)). At $E_p = 50$ MeV, the correlated contribution to the D-to-S t-matrix element is positive, since the pole occurs at a relatively small value of k' . As the bombarding energy increases, three qualitative changes occur: (i) the energy denominator increases in the region of k' where the numerator of the defect function is significant, which implies a reduction of the absolute magnitude of the defect function. (ii) The magnitude of $V_{02}(k_0, k')$ decreases, as k_0 increases, in the region of k' where the defect function $F_{22}(k_0; k', k_0)$ is significant. Therefore the absolute magnitude of the product $k'V_{02}(k_0, k')F_{22}(k_0; k', k_0)$ is further reduced. (iii) The pole of the principal value integral moves to a larger value of k' and thus produces a larger cancellation in the principal value integral. The contribution via an intermediate S-state, $k'V_{00}(k_0, k')F_{02}(k_0; k', k_0)$, even becomes attractive due to this pole structure and tends to cancel the repulsive contribution via an intermediate D-state. These effects combine coherently and reduce the influence of correlations with increasing energy. As a result, the influence of

correlations on the t-matrix tensor interaction decreases with increasing bombarding energy.

In the nuclear medium, Pauli blocking and the selfconsistent potential start to play extra roles relative to the t-matrix case, introducing a density dependence in the effective interaction. The middle part of Fig. 23 shows the product $k'V_{02}(k_0, k')F_{22}(k_0; k', k_0)$ as a function of k' at $E_p = 50$ MeV for several values of the density ρ . As one can see, this product decreases strongly with increasing density. The major reason for this behaviour is due to the Pauli operator and the energy denominator. For small incident energies, the defect function for momenta less than $k' = (k_F^2 - k_0^2)^{1/2}$ is entirely suppressed due to the Pauli blocking and is reduced for momenta $k' < k_0 + k_F$. Furthermore, the energy denominator increases with increasing density, since in the nuclear medium the energy of a nucleon consists of the sum of the kinetic energy and of the potential energy due to the selfconsistent Brueckner-Hartree-Fock potential which increases in absolute magnitude with increasing density. In addition, the defect function $F_{22}(k_0; k', k_0)$ starts to oscillate around zero as the density increases and produces a large cancellation of the product $k'V_{02}(k_0, k')F_{22}(k_0; k', k_0)$ as curve (3), corresponding to the normal nuclear matter density ρ_0 , illustrates. The position of the pole, on the other hand, changes smoothly with density. (Note that the effect of the energy denominator on the defect function (eq. (1.1.19) is not linear.) Therefore at low incident energies, a strong density dependence of the effective tensor force should be expected. Moreover, at high densities the influence of correlations on the tensor force is strongly reduced.

In the lower part of Fig. 23 the same quantity as in the middle part is shown, now at the much lower incident energy of $E_p = 20$ MeV. At this energy the prod-

uct $k'V_{02}(k_0, k')F_{22}(k_0; k', k_0)$ decreases rapidly with density in the very low density region ($\rho \leq 0.3 \rho_0$). For larger values of the density, it is practically density independent, showing characteristics similar to the case of $E_p = 50$ MeV at high densities. The change in its behaviour at small ρ compared to the case of $E_p = 50$ MeV is mainly caused by the energy denominator, which becomes larger as the incident energy decreases for a fixed density. For instance, at $\rho = 0.5 \rho_0$, the energy denominator increases by a factor ~ 2 in the region of $1.0 < k' \text{ (fm}^{-1}\text{)} < 2.0$ going from $E_p = 50$ to 20 MeV.

At higher incident energies, the density dependence of the effective tensor force is weak and the G-matrix approaches the t-matrix⁺. This occurs for two reasons: (i) As the energy of the incident nucleon increases beyond the Fermi momentum the influence of Pauli blocking is reduced, since intermediate states become available at momenta up to $k' = k_0 - k_F$ and (ii) the energy denominator becomes similar to that of the t-matrix as the bombarding energy increases.

In Fig. 24, the real part of the isovector tensor t-matrix interaction is shown as a function of momentum transfer q for different bombarding energies. The results are compared with the isovector tensor interaction (dotted curves) extracted directly from SP84 amplitudes⁸⁷⁾, which shows a similar energy dependence. The magnitude of the interaction is seen to increase with increasing bombarding energy, especially for values of the momentum transfer beyond $q = 1 \text{ fm}^{-1}$. This behaviour follows from the energy dependence of the correlated part of the effective interaction (see upper part of Fig. 23), as discussed before. With increasing energy, the repulsive correlated contributions are reduced and as a consequence, the isovector tensor t-matrix interaction

⁺ Note, that not only the tensor force, but also the other components of the effective G-matrix interaction must become weakly density-dependent at high bombarding energies.

becomes close to the attractive one-pion plus one-rho exchange potential (solid line) for high energies of the incident nucleon.

The density dependence of the isovector tensor interaction is shown in Fig. 25. At low energies ($E_p = 20, 50$ MeV), a strong density dependence of the isovector tensor interaction is found, as one expects from the behaviour of the correlated part (see lower part of Fig. 23) of the effective G-matrix tensor interaction. At very low energies ($E_p = 20$ MeV) the isovector tensor force shows a rapid variation with density in the very low density region up to $\rho \sim 0.3 \rho_0$. At higher densities it remains practically unchanged, showing no correlation effects. In the lower part of Fig. 25, we show the density dependence of the isovector tensor force for higher energies ($E_p = 210$ MeV). The density dependence is weak because in this case the G-matrix approaches the t-matrix, which describes the interaction between two nucleons in free space.

In summary, for the case in which one of the two nucleons in the initial state is above the Fermi level, the real (and dominant) part of the effective isovector tensor interaction has an energy dependence which becomes weaker as the density increases. Its absolute magnitude increases with increasing bombarding energy. It also exhibits a strong density dependence at low incident energies and its magnitude increases with increasing density. Either at high densities or at high bombarding energies, the effective isovector tensor interaction approaches closely that given by the bare one-pion plus one-rho exchange interaction.

It should be noted that as soon as the effective isovector tensor interaction starts to deviate from the bare one-pion plus one-rho exchange interaction the isoscalar tensor component starts to grow, since this component is essentially generated by higher order terms. In particular, at low incident energies and

at low densities the isoscalar tensor interaction becomes even larger than the isovector tensor force. Fig. 26 illustrates this point. There we show the modulus of each component of the effective tensor force at $E_p = 50$ MeV for several densities. The imaginary part of the tensor force also becomes comparable in magnitude to its real part at very low incident energies and densities.

The isovector tensor G-matrix interaction between two nucleons below the Fermi level shows a very weak density dependence, at least up to $q \sim 2 \text{ fm}^{-1}$, for values of the density ranging between $0.35 \rho_0 \leq \rho \leq \rho_0$, as was pointed out in chapter I. However, it is extremely dangerous to extrapolate this finding to lower densities. In fact, our present result for the case in which one nucleon is above the Fermi surface shows a very similar behaviour for $0.35 \rho_0 \leq \rho \leq \rho_0$ at very low incident energies and therefore strongly suggests a rapid reduction of the isovector tensor force at lower densities when both nucleons are below the Fermi level. At smaller densities, a problem arises in nuclear matter calculations, since the initial momenta of two interacting nucleons which are below the Fermi level has to vanish with vanishing density. This prevents a comparison between two G-matrices with common starting energies, but evaluated at different densities. Therefore from the nuclear matter calculation, as carried out in the present work, no unique conclusions concerning the density dependence of the effective isovector tensor interaction between two bound nucleons in the surface region of a finite nucleus can be drawn.

2.4 Application of the Effective G-Matrix Interaction to Nucleon-Nucleus Scattering

In this section we show a few applications of the G-matrix interaction constructed in the previous section within the distorted wave impulse approximation (DWIA). It is not our purpose here to provide an interpretation or analysis of the experimental data but rather to get a feeling for how the present G-matrix interaction works within the DWIA.

In the present application the optical potential has been calculated in a folding model⁸⁹⁾ using the G-matrix interaction. Therefore, the calculation is consistent in the sense that the same NN interaction (albeit possibly different components) is used to generate both the optical and transition potentials⁹⁰⁾.

In the present section we also use the parametrized form of the NN G-matrix interaction of the previous section. The same form for the parametrization as described in Ref. 42 has been used for each density and incident energy. The NN G-matrix interaction has been transformed to the nucleon-nucleus system which is appropriate for the nucleon-nucleus scattering problem. The knock-on exchange contribution of the nucleon-nucleus G-matrix interaction is treated within a well defined⁴²⁾ short-range approximation. The details may be found in Ref. 42.

Our first example is the natural parity excitation 2^+ ($E_x = 4.44$ MeV, $T=0$) in ^{12}C at $E_p = 200$ MeV. This transition has been chosen because the transition density as well as the ground state density in ^{12}C are well established over a wide range of momentum transfer. In addition, (p,p') data⁴¹⁾ for this transition are also available at several energies. Here we use the transition densi-

ty as described in Ref. 90. The G-matrix interaction calculated at $E_p = 210$ MeV has been used. In Fig. 27 a comparison is made between the experimental (crosses) and our (solid line) results for the elastic scattering differential cross section, which provides an indication of the ability of the G-matrix interaction to generate reliable distorted waves.

In Fig. 28 the differential cross section for the 2^+ transition is shown. At small angles ($\theta_{cm} \lesssim 30^\circ$) the calculated cross section clearly overshoots the experimental one, while at large angles it is too small. Beyond $\theta_{cm} \sim 40^\circ$, the calculated result should be interpreted with caution, since the transition density used in the present calculation is not adjusted to the (e,e') experimental form factor for momentum transfers corresponding to $\theta_{cm} \sim 40^\circ$ at $E_p = 210$ MeV.

Quite generally, isoscalar natural parity states in nuclei with 0^+ ground states are essentially excited by the scalar-isoscalar (F) and isoscalar spin-orbit (S) components of the effective interaction. At small angles the scalar-isoscalar component dominates; the spin-orbit term becomes also important beyond $\theta_{cm} \sim 30^\circ$. The present result suggests that the scalar-isoscalar component of our G-matrix interaction is too strong at small momentum transfers (remember that the scattering angle is related to the momentum transfer). At higher momentum transfers one needs a stronger effective interaction in order to reproduce the experimental results. As we will show in chapter III, the inclusion of dynamical relativistic effects on the effective interaction may improve the agreement with the experimental data. In Fig. 29 we show a comparison of the differential cross sections obtained using different effective interactions. The solid lines are again our results; the dash-dotted lines correspond to the results from the G-matrix interaction of von Geramb⁹¹⁾ based on

the Paris potential, while the dashed lines correspond to the results obtained from the t-matrix interaction of Ref. 92. Here we used a phenomenological optical potential, the parameters of which are taken from Ref. 41. These interactions give similar results for both cross section and analyzing power, although they differ in details. As we mentioned in section 2.2, the G-matrix interaction at $E_p \sim 200$ MeV is already very close to the t-matrix interaction.

The nucleon charge-exchange reactions (p,n) and (n,p) are of special interest because of their selectivity which allows only isovector nuclear excitations. Concentration on this type of reaction makes the isoscalar part of the effective interaction, which dominates the (p,p') spectrum at small momentum transfers (such as in the previous example) inoperative in the charge-exchange process; it still can enter in calculating the distorted waves. Moreover, most of the existing (p,n) data at intermediate energies³⁷⁻³⁹⁾ have been taken at forward angles where the momentum transfer is small and therefore the non-central components of the effective interaction are relatively small. This offers an opportunity to test primarily the central isovector components of the effective interactions. Here we consider the 0^+ isobaric analog state (IAS) of the ground state of ^{90}Zr at $E_p = 120$ MeV which is excited by the isospin F' component of the effective interaction. The G-matrix interaction at $E_p = 140$ MeV has been used. The same nuclear transition density ($g_{9/2}^{10}$) as described in Ref. 42 has been used. The elastic scattering differential cross section (in a consistent folding model) is reproduced with a quality similar to the case of ^{12}C and therefore is not shown here. The result of the consistent calculation of the 0^+ isobaric analog transition is displayed in Fig. 30. Here our result has been normalized to the experimental point³⁸⁾ at the smallest available angle. The normalization factor is $N = 1.54$ which suggests, in principle, that our interaction is too weak in the isospin (F') channel. However, this channel be-

comes stronger with decreasing incident energy. In particular, the magnitude of the isospin channel of the t -matrix interaction at vanishing momentum transfer varies by a factor $\sim 4/3$ from $E_p = 140$ to 100 MeV (see Fig. 22). Since the cross section in this case is basically related to the square of the magnitude of the isospin F' component at $q=0$, use of the $E_p = 100$ MeV G -matrix interaction, instead of that at $E_p = 140$ MeV, might significantly reduce the normalization factor. This consideration shows that it is dangerous to draw conclusions concerning the strength of the isospin channel of the effective G -matrix interaction from the result of Fig. 30. In Fig. 31 we show the result for the differential cross section using now the phenomenological optical potential⁹³⁾ (solid line). In this case the shape of the cross section is better reproduced than in the consistent calculation. In addition, the normalization factor is reduced from $N = 1.54$ to 0.90, which means that our result now slightly overestimates the experimental cross section. These results show that the differential cross section in the present example is very sensitive to the distorted waves. In Fig. 31 we have also plotted the results from the calculations using the G -matrix interaction of von Geramb⁹⁰⁾ (dash-dotted line) and the t -matrix interaction of Love and Franey⁹¹⁾ (dashed line), both with the phenomenological optical potential. The corresponding normalization factors are $N = 1.18$ and 0.75, respectively. As one can see from this example, our interaction reproduces the shape of the experimental differential cross section better than the other interactions.

The results of the present section show that the effective G -matrix interaction works reasonably in the description of nucleon-nucleus scattering at intermediate bombarding energies. Inclusion of relativistic effects may improve the agreement with experimental data. At lower incident energies, however, inclusion of medium polarization effects as in the case of nuclear structure calculations may be required.

CHAPTER III

Effective G-Matrix Interaction Including Relativistic Effects

In chapter I the effective interaction has been constructed from the Brueckner G-matrix. The G-matrix interaction was obtained by solving the BG-equation, where we started from a realistic NN potential based on the OBE model developed by the Bonn group⁴⁾. These potentials are given in a covariant form and corrections in the BG-equation due to the relativistic kinematics are included by the so-called "minimal relativity"⁹⁴⁾ which ensures the fact that the scattering amplitude should satisfy the relativistic elastic unitarity relation.

Recently, Anastasio et al.⁴⁹⁾ have shown that "relativistic effects" are important for several aspects of nuclear structure physics. They especially emphasize that nuclear medium effects have to be included consistently not only in the single-particle energies but also in the nucleon spinors. These authors found a strong density dependence in the scalar-isoscalar channel of the G-matrix interaction, which reduces the attraction in this channel considerably and tends to satisfy the stability condition given by eq. (1.4c.1). The reason for this strong density dependence has been attributed to a decoupling of the σ -meson field in the nuclear interior.

In chapter I the G-matrix calculations have been performed taking into account the nuclear medium effects (through the nucleon self-energy) only in the single-particle energies. In the present chapter we have accounted for the nuclear medium effects not only on the single-particle energies but also on the nucleon spinors.

3.1 Medium Effects on the Bare NN Potential

The question whether the relativistic effects in the nuclear many-body theory are relevant or not may be tackled by looking at the proper description of the nucleon wave function in nuclear matter. Within the Schrödinger approach, the nucleon wave function in nuclear matter is always a product of a plane wave with a Pauli spinor and an isospinor. In the Dirac theory, however, the nucleon spinor is a Dirac spinor which obeys the Dirac equation of the form

$$[\alpha \cdot \vec{p} + \beta m + \Sigma(\vec{p})]f(\vec{p}, s) = E f(\vec{p}, s) , \quad (3.1.1)$$

where $\Sigma(\vec{p})$ stands for the nucleon self-energy. Throughout this chapter Bjorken and Drell's⁹⁵⁾ notation is adopted. It is clear that, in contrast to the case of the Schrödinger approach, the nucleon spinor which obeys the above equation may be affected by the nucleon self-energy $\Sigma(\vec{p})$.

The OBEP developed by the Bonn group⁴⁾ may be written as (in the c.m. system)

$$V(\vec{p}', \vec{p}) = \sum_{\alpha=\sigma, \omega, \pi, \rho, \dots} V_{\alpha}(\vec{p}', \vec{p}) \quad (3.1.2a)$$

with

$$V_{\alpha}(\vec{p}', \vec{p}) = [f(-\vec{p}', s_1') \Gamma_{\alpha}^{(1)} f(-\vec{p}, s_1)] P_{\alpha} [f(\vec{p}', s_2') \Gamma_{\alpha}^{(2)} f(\vec{p}, s_2)] . \quad (3.1.2b)$$

where $\Gamma_{\alpha}^{(i)}$ denotes the meson-nucleon vertices and P_{α} is the meson propagator. The minimal relativity factor as well as the form factor have not been written down explicitly in the above equation. The meson-nucleon coupling strengths as well as the corresponding meson masses are parameters which are fixed by fitting experimental NN phase-shift data. The latter are related to the transi-

tion matrix elements which are obtained by solving the Lippmann-Schwinger type equation. $f(\vec{p}, s)$ in this case denotes the nucleon spinor in the positive energy state and, of course, obeys the free Dirac equation (setting $\Sigma(\vec{p}) = 0$ in eq. (3.1.1)).

In the conventional nuclear many-body calculation, such as in the previous chapter, the bare NN potential given by eq. (3.1.2) has been used keeping always the nucleon spinor $f(\vec{p}, s)$ as a solution of the free Dirac equation. Therefore, within this procedure nuclear medium effects on the nucleon spinors are entirely neglected, although they are usually taken into account in the single-particle energies.

In the work of Ref. 49 nuclear medium effects on the nucleon spinors have been taken into account by mixing the annihilation of an antiparticle with the creation of a particle. In other words, the positive energy nucleon spinor $f(\vec{p}, s)$ is expanded in terms of the positive ($u(\vec{p}, s)$) and negative ($\omega(\vec{p}, s)$) energy solutions of the free Dirac equation

$$f(\vec{p}, s) = \sum_n \{a_n u_n(\vec{p}, s) + b_n \omega_n(\vec{p}, s)\} . \quad (3.1.3)$$

Since for $\Sigma(\vec{p}) \neq 0$ the coefficient b_n does not vanish, it is clear that using $f(\vec{p}, s)$ given by eq. (3.1.3) we are going to include effects of excitations of anti-particles in the bare NN potential. Equally well, one may work with positive energy spinors only, but allowing that the ratio between the large and the small component of a Dirac spinor in a medium may differ from the corresponding ratio of a free Dirac spinor. This latter method, which is used in the present work, offers the advantage that medium effects on the bare NN potential can be displayed in a very transparent way.

The nucleon self-energy must be determined from a selfconsistent calculation. This may be done within the Brueckner-Hartree-Fock approximation and has been performed up to one iteration in Ref. 49. However, as far as the effective interaction is concerned, the major effect of $\Sigma(\vec{p})$ may be taken into account within the approximation

$$\Sigma(\vec{p}) = \Sigma = \beta U_S + U_0 . \quad (3.1.4)$$

The quantities U_S and U_0 may be determined within the mean field approximation as has been done by Walecka⁴⁷⁾. They are given by

$$U_S = - \frac{4}{(2\pi)^3} \left(\frac{g_S}{m_S} \right)^2 \int_{|\vec{k}| \leq k_F} \frac{\tilde{m}}{\sqrt{\tilde{m}^2 + k^2}} d\vec{k} , \quad (3.1.5)$$

$$U_0 = \frac{4}{(2\pi)^3} \left(\frac{g_V}{m_V} \right)^2 \int_{|\vec{k}| \leq k_F} d\vec{k} ,$$

where $\tilde{m}$ is defined in eq. (3.1.8). The parameters (g_S/m_S) and (g_V/m_V) have been chosen⁴⁷⁾ to reproduce the correct binding energy and density of nuclear matter. Using those values one gets

$$U_S = -371.5 \text{ MeV}, U_0 = 289.8 \text{ MeV} . \quad (3.1.6)$$

When the above approximation for the self-energy is used, eq. (3.1.1) can be immediately solved to yield

$$f(\vec{p}, s) = \sqrt{\frac{\tilde{E} + \tilde{m}}{2\tilde{m}}} \begin{pmatrix} I \\ \frac{\vec{\sigma} \cdot \vec{p}}{\tilde{E} + \tilde{m}} \end{pmatrix} \chi_s , \quad (3.1.7)$$

where

$$\tilde{m} \equiv m + U_s, \quad \tilde{E} \equiv \sqrt{\tilde{m}^2 + \vec{p}^2}. \quad (3.1.8)$$

This means that, within this simple model, the effect of the self-energy on the NN potential is taken into account by replacing the nucleon mass m by $\tilde{m}$ everywhere (note that U_0 enters into the single particle energies $E = \tilde{E} + U_0$). This allows to display very clearly how the medium effects modify the bare NN potential.

In the following we will adopt the meson-nucleon vertices of Table 1 of Ref. 4. Moreover, we assume pseudo-vector coupling instead of a pseudo-scalar one. The single-particle energies occurring in the meson propagators as well as in the meson-nucleon vertices are evaluated in the on-mass shell approximation as in the two-body scattering.

With these assumptions taking, for example, only σ - and ω -mesons, eq. (3.1.2) gives

$$\begin{aligned} V(\vec{p}', \vec{p}) = & -4\pi g_\sigma^2 \frac{1}{q^2 + \mu_\sigma^2} \left\{ 1 + \frac{q^2 - Q^2}{8\tilde{m}^2} + \frac{i}{4\tilde{m}^2} \left(\frac{\sigma_1 + \sigma_2}{2} \right) \cdot (\vec{q} \wedge \vec{Q}) + \dots \right\} \\ & + 4\pi g_\omega^2 \frac{1}{q^2 + \mu_\omega^2} \left\{ 1 - \frac{q^2 - 3Q^2}{8\tilde{m}^2} - \frac{1}{12} \frac{q^2}{\tilde{m}^2} (2\sigma_1 \cdot \sigma_2 - S_{12}(\hat{q})) \right. \\ & \left. - \frac{3i}{4\tilde{m}^2} \left(\frac{\sigma_1 + \sigma_2}{2} \right) \cdot (\vec{q} \wedge \vec{Q}) + \dots \right\}. \end{aligned} \quad (3.1.9)$$

The above expression is obtained in the limit $|\vec{p}'| = |\vec{p}|$ and keeping terms up to order $(\vec{p}/\tilde{m})^2$. (Here, the minimal relativity factor $\sqrt{\frac{\tilde{m}}{\tilde{E}_{p'}}}$, $\sqrt{\frac{\tilde{m}}{\tilde{E}_p}}$ has been included.) The momentum transfer $\vec{q}$ is given by $\vec{q} = \vec{p} - \vec{p}'$, while the exchange momentum transfer $\vec{Q}$ is $\vec{Q} = \vec{p}' + \vec{p}$. From eq. (3.1.9) it is clear that in the scalar-isoscalar channel the σ -meson contribution becomes less or more attractive depending

on $q^2 < Q^2$ or $q^2 > Q^2$ compared to the corresponding results of the previous approach. This is because in the previous approach $\tilde{m}$ is equal to m , while in the present relativistic approach $\tilde{m} \sim 0.6 m$. For the ω -meson the situation is just the opposite, i.e., for $q^2 < 3Q^2$ the potential becomes much more repulsive and for $q^2 > 3Q^2$ it becomes much less. Note the factor of 3 in the ω -meson contribution, which amplifies the effect of the self-energy compared to the σ -meson case. The effective interaction is obtained from a calculation which involves a large cancellation between the attractive σ -meson and the repulsive ω -meson contributions. Therefore, in the low momentum transfer region one expects much less attraction in the scalar-isoscalar channel. It is also interesting to note that the spin-orbit term is increased by a factor ~ 2 due to the overall factor $1/\tilde{m}^2$ as shown in eq. (3.1.9).

Similarly, for the π - and the ρ -meson exchange potentials one gets

$$\begin{aligned}
 V(\vec{p}', \vec{p}) = & -\frac{4\pi}{3} g_\pi^2 \frac{1}{4m^2} \frac{q^2}{q^2 + \mu_\pi^2} \{ \sigma_1 \cdot \sigma_2 + S_{12}(\hat{q}) \} \tau_1 \cdot \tau_2 \\
 & + 4\pi g_\rho^2 \frac{1}{q^2 + \mu_\rho^2} \left\{ 1 - \frac{[1 + 4 (\frac{\tilde{m}}{m}) \frac{f_\rho}{g_\rho}] q^2 - 3Q^2}{8\tilde{m}^2} \right. \\
 & - \frac{1}{12} [1 + (\frac{\tilde{m}}{m}) \frac{f_\rho}{g_\rho}]^2 \frac{q^2}{\tilde{m}^2} (2\sigma_1 \cdot \sigma_2 - S_{12}(\hat{q})) \\
 & \left. - \frac{i}{4\tilde{m}^2} [3 + 4 (\frac{\tilde{m}}{m}) \frac{f_\rho}{g_\rho}] (\frac{\sigma_1 + \sigma_2}{2}) \cdot (\vec{q} \wedge \vec{Q}) + \dots \right\} \tau_1 \cdot \tau_2 .
 \end{aligned} \tag{3.1.10}$$

The above results show that the π -meson exchange potential is basically unaffected by the nuclear medium. This is due to our choice of the coupling vertex (pseudo-vector coupling). It is known that in free space the pseudo-scalar and the pseudo-vector vertices are equivalent to each other⁴). However, in the nu-

clear medium this is no longer the case. For instance, if one uses the pseudo-scalar vertex eq. (3.1.2) leads to the result for the π -exchange potential as displayed in eq. (3.1.10) with the important difference that the nucleon mass m is replaced by $\tilde{m}$. This shows that when one uses the pseudo-scalar coupling the potential is strongly modified in the nuclear medium due to the overall factor $(\frac{1}{\tilde{m}^2})$ and has an important consequence in obtaining an effective interaction. The OPEP gives a large attraction in the scalar-isoscalar channel F of the G -matrix through its second-order contribution. Indeed, the G -matrix interaction obtained using the pseudo-scalar vertex is a very large and unrealistic attractive force.

The ρ -meson contribution has a similar structure as the ω -meson exchange potential with additional terms arising from the tensor coupling, which introduces an interesting feature. The ρ -meson exchange potential contributes dominantly to the spin-isospin and the isovector tensor channels due to the large tensor coupling strength f_ρ compared to the corresponding vector coupling strength g_ρ (4,76). Although there appears an overall factor $(\frac{1}{\tilde{m}^2})$ in terms quadratic in g_ρ , the corresponding terms quadratic in f_ρ are unchanged from the previous ($\tilde{m}=m$) to the present approach. Terms linear in f_ρ are affected only by a factor $(\frac{\tilde{m}}{m})$. Therefore, the effect of the self-energy on the spin-isospin and the isovector-tensor channels of the ρ -meson exchange potential is to increase the coupling strength by $\sim 25\%$. Since the effective isovector tensor force is basically given by the sum of the attractive π -meson contribution and the repulsive ρ -meson contribution this channel becomes less attractive at high momentum transfer q compared to the previous approaches.

In Figs. 32 and 33 the exact results for the σ - and ω -meson exchange potential in the scalar-isoscalar channel F_D (direct term only) are displayed as a func-

tion of the momentum transfer q . The HEA version⁷⁶⁾ of the OBEP has been used. Here we restricted ourselves to the case $|\vec{p}'| = |\vec{p}|$ keeping $p = 0.95 \text{ fm}^{-1}$, so that both q and Q are varying according to eq. (1.3.9). There, p is denoted as k_0 . The solid lines are the results within the relativistic treatment using the values of U_S and U_0 given by eq. (3.1.6). The dashed lines are the corresponding results within the conventional approach ($\tilde{m}=m$). The behaviour of these results can easily be understood from eq. (3.1.9). In Fig. 34 the exact results for the spin-isospin (G_D) and the isovector tensor (T_D) components (direct terms) of the ρ -meson exchange are also displayed.

The above analysis show how the different meson-exchange contributions to the bare NN potential are affected by a proper relativistic treatment of the nucleon wave function in nuclear matter.

3.2 Relativistic Effects on the Quasi-Particle G-Matrix Interaction

In section 3.1 we have shown how the nuclear medium can affect the bare NN potential in a relativistic formulation. The use of such a potential may change the characteristics of the effective interaction in an important way.

A fully consistent relativistic many-body theory requires a more general equation than the BG-equation. In the two-body scattering problem one usually starts from the Bethe-Salpeter equation⁵⁰⁾ and makes further approximations such as the Blanckenbecker-Sugar reduction⁹⁶⁾. In principle, in the many-body problem one should also start from a relativistic covariant many-body equation (analogous to the Bethe-Salpeter equation for the two-body case) and then do the corresponding reduction of it. In other words, one needs a consistent

treatment which has a well defined prescription for the transition from the two-body to the many-body system. Such a consistent treatment may also be achieved by starting from a field-theoretic Hamiltonian as proposed by Schütte⁹⁷⁾. However, as far as the effective interaction is concerned, we believe the major effect due to the modification of the Dirac spinors can be still studied within the framework of the BG-equation, as in chapter I.

The quasi-particle G-matrix interaction at $k_F = 1.36 \text{ fm}^{-1}$ has been generated exactly in the same way as in chapter I. We again used the simple model for the nucleon self-energy given by eqs. (3.1.4)-(3.1.6). Here, the HEA⁷⁶⁾ version of the OBEP has been used.

Fig. 35 shows the central components of the quasi-particle G-matrix interaction at normal density. The solid lines correspond to the results within the relativistic treatment, while the dashed lines are the results of the conventional approach (free Dirac spinors in eq. (3.1.1)). The scalar-isoscalar (F) channel is affected mostly by the relativistic effects. The relative difference can be easily understood from the considerations of the previous section. As we expected, this channel becomes less attractive at small momentum transfer q (reduction of a factor ~ 2 at $q=0$) and more attractive at large q from the conventional to the relativistic approach. This is essentially due to the change of the σ - and ω -meson exchange potentials as discussed before and has important consequences in both nuclear structure and reaction physics. For instance, in the previous chapter we have shown that the experimental differential cross section for 2^+ ($E_x = 4.44 \text{ MeV}$, $T=0$) excitation in ^{12}C suggests an effective interaction with a weaker scalar-isoscalar component at low momentum transfers than that of the G-matrix interaction obtained from the conventional approach. At higher momentum transfers it somehow suggests a stronger interac-

tion than the conventional G-matrix interaction (see Fig. 28a and 29a). Inclusion of relativistic effects tends to give exactly this feature in the scalar-isoscalar component. The spin G-channel is also reduced by a factor $\sim 1/2$ from the conventional to the relativistic treatment; however this channel is very weak. The isospin channel becomes more repulsive ($\sim 30\%$) from the conventional to the relativistic approach at small momentum transfer region, while the spin-isospin G' -channel is reduced at higher momentum transfer.

The spin-orbit S - and S' -channels are displayed in Fig. 36. Again the solid lines correspond to the relativistic treatment and the dashed lines to the conventional one. In both channels the results of the relativistic approach are increased by a factor of ~ 2 compared to that of the conventional approach. The reason is that the bare NN potential in the spin-orbit channels has an overall factor $(\frac{1}{\tilde{m}^2})$. Note that, as discussed in section 1.4, these channels are essentially unaffected by correlations. This large enhancement of the spin-orbit force has also important consequences. In nuclear structure, for example, the spin-orbit splitting of single-particle energy levels can be reproduced only in a relativistic formulation⁴⁵⁾ where the spin-orbit coupling is larger than in a conventional approach. Also in nuclear reactions spin observables, such as the analyzing power, are much better reproduced within the relativistic treatment⁹⁸⁾.

In Fig. 37 the tensor components are shown. The solid lines correspond to the results of the relativistic treatment and the dashed lines to the conventional one. The isovector-tensor channel T' is reduced by $\sim 25\%$ from the conventional to the relativistic approach, which is practically caused by the modification in the ρ -meson exchange potential. In the T -channel the difference is due to the ω -meson (see eq. (3.1.9)); however this channel is extremely small. The

differences in the H- and H'-channels are essentially due to the ω - and ρ -meson through their exchange terms. Remember also that tensor forces are unaffected by correlations (see chapter I, section 1.4).

The results for the Landau parameters at $k_F = 1.36 \text{ fm}^{-1}$ are shown in Table 12. In the first line the results of the G-matrix interaction within the conventional treatment are displayed. The second line shows the corresponding results of the relativistic treatment. As we expected, f_0 becomes much less attractive, although it still violates the stability condition $2f_0 > -1$ (17,35). The parameter f'_0 increases by $\sim 30 \%$, while the spin channel g_0 is decreased by $\sim 40 \%$ from the conventional to the present treatment. The spin-isospin channel is practically unaffected. In the third line we also display the results of the relativistic treatment with different values of U_S and U_0 in order to show how sensitive the interaction is to these parameters. Both (second and third lines) give results quite similar to each other except in the scalar-isoscalar channel.

In the last column of Table 12 we also display the results of the Landau parameter $(g'_0)_{\Delta N}$ for the Δ -isobar-nucleon-hole interaction, which in the Landau limit has the form

$$F^{\Delta h} = C_0 (g'_0)_{\Delta N} S_1 \cdot \sigma_2 T_1 \cdot \tau_2 , \quad (3.2.1)$$

where $C_0 = \frac{\pi^2 \hbar^2}{m k_F}$. Since there are practically no effects of short-range correlations and the Pauli blocking for this interaction (see discussion of chapter I, section 1.6), here the difference from the conventional to the relativistic treatment is caused by the modification of the ρ -meson exchange potential. Relativistic effects enhance the Δ -nucleon coupling strength by $\sim 15 \%$.

For the sake of comparison we have also displayed in the last line of Table 12 the G-matrix results of Ref. 49 which account also for the relativistic effects. As one can see, the effects of the self-energy on the nucleon spinors are much larger compared to our results (second and third lines). The methods used in Ref. 49 and in the present treatment to account for the effects of the nucleon self-energy on the Dirac spinors are equivalent within the approximation of a momentum-independent mean field (eq. (3.1.4)). The assumption of a momentum-independent mean field is fully justified, as shown in Refs. 49 and 99. The present work is based on the self-consistent mean field of Ref. 47, while in Ref. 49 the mean field is calculated within the Brueckner-Hartree-Fock approximation performing, however, only one iteration in the self-consistent procedure. The numerical values of U_S and U_0 corresponding to the treatment of Ref. 49, are close to the ones in the present work at normal nuclear matter density (i.e. $k_F = 1.36 \text{ fm}^{-1}$). The major difference between Ref. 49 and the present work appears to be the treatment of the ρ -meson. Since the parameters of the bare NN potential⁷⁶⁾ were determined with a modified Pauli coupling of the ρ -meson (see Table 1 of Ref. 4), we kept this modification throughout this calculation for consistency. In Ref. 49, a different ρ -meson coupling was used, although the same parameters as in the present calculation were taken.

CHAPTER IV

Effective Interaction Including Medium Polarization Effects

In chapter I it has been shown that the G-matrix interaction leads to an instability in nuclear matter due to its strongly attractive scalar-isoscalar component and, consequently, to a value of the compression modulus $K < 0$. Inclusion of relativistic effects, in chapter III, cures a part of this problem but still leads to a value of the compression modulus $K \lesssim 0$, which is far away from that determined empirically ($K \sim 210$ MeV)³⁶⁾. This clearly indicates that correlations of other nature than that accounted for by the Bruckner G-matrix have to be taken into account. Therefore, in this chapter we have investigated medium or core polarization effects on the effective interaction which build up correlations of long-range character. The role of core polarization effects in the lowest order in finite nuclei has also been studied by several authors¹⁰⁰⁾.

In this chapter, we are particularly interested in the effects on the Δ -hole interaction since, as mentioned before, the magnitude of this interaction is of crucial importance to understand the role of the Δ -isobar in nuclei. In Ref. 74 it was pointed out that especially the short-range parts of the Δ -hole interaction, such as the rho-exchange, involve almost exact cancellations between the direct and the exchange terms. The reason is due to the Pauli principle as discussed in section 1.6, chapter I. Because of that argument, Arima et al.⁷⁸⁾ concluded that the isobar probably plays only a minor role in explaining the reduction of magnetic strength in nuclei. However, the G-matrix is only a part of the full effective interaction. In fact, it has been shown by Babu and Brown²⁶⁾ and Sjöberg²⁷⁾ that the inclusion of screening effects in the so-called "crossed channel" (core polarization effects) reduces strongly

the attraction of the NN G-matrix interaction, e.g. the contributions are strongly repulsive.

In chapter I we have shown that Δ -isobars do not feel like short-range correlations as nucleons due to the Pauli principle. Therefore, it is of special interest here to investigate how the correlations of long-range character affect the Δ -nucleon effective interaction. A general feature in many-body systems and one which we shall show applied here, is that when an interaction is strongly repulsive (here that in the $\sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2$ channel), then the exchange term in the particle-hole interaction is strongly screened, whereas the direct term is unaffected.

4.1 Inclusion of Medium Polarization Effects in the Effective Interaction

An easier way to understand how the correlations, built up by virtual collective excitations in the nuclear medium, can be taken into account in the effective interaction is perhaps to invoke the semi-phenomenological Landau Fermi liquid theory^{17,18,35}). In this theory the effective particle-hole interaction is given by the double functional differentiation of the ground state energy E with respect to the quasi-particle occupation number $n(k)$

$$F^{ph} = \frac{\partial^2 E}{\partial n(k_1) \partial n(k_2)} . \quad (4.1.1)$$

Consider now a nucleus, say ^{208}Pb , which has strong collective states, such as the $J^\pi = 3^-$ low-lying state. Then the ground state $J^\pi = 0^+$ of this nucleus may contain a configuration of the type $|(3^- \otimes 3^-)0^+\rangle$, which contributes to the ground state energy as shown in Fig. 38a. There, the shaded blocks indicate

phonons. In terms of graphs the double functional differentiation of the ground state energy with respect to the quasi-particle occupation number means a cut of quasi-particle lines as indicated in Fig. 38b. Now, turning it around one gets the part of the effective particle-hole interaction as shown in Fig. 38c, which is usually called "induced interaction", the lower orders of which are also displayed in Fig. 38c. Therefore the induced interaction is a part of the effective interaction which accounts for the correlations of long-range character arising from the virtual collective excitations of a nucleus.

The above considerations are an intuitive way to understand the physics of the induced interaction. We now want to present briefly a method to incorporate the medium polarization and other correlation effects on the effective interaction from more basic considerations.

The starting point of our considerations is the two-fermion propagator

$$G(34;12) \equiv \langle \psi_0 | T[\psi(3)\psi(4)\psi^+(1)\psi^+(2)] | \psi_0 \rangle, \quad (4.1.3)$$

where $\psi(i)$ denotes the field operator for fermions, T is the Wick time ordering operator and $|\psi_0\rangle$ stands for the exact ground state. The connected part of the two-fermion propagator defined in eq. (4.1.3) can be evaluated through the Bethe-Salpeter equation⁵⁰⁾ which sums all the diagrams which are separated to each other only by the two single-fermion propagators.

In the following we will discuss in terms of the effective interaction Γ . The Bethe-Salpeter equation for the effective interaction Γ is written as

$$\Gamma(34;12) = v(34;12) + i \int d5d6d7d8 v(34;56)g(5,7)g(6,8)\Gamma(78;12), \quad (4.1.4)$$

where the irreducible kernel with respect to the two single-fermion lines cut has already been approximated by the bare NN potential v . $g(i,j)$ stands for the single-fermion propagator. A graphical representation of eq. (4.1.4) is shown in Fig. 39. To be more specific let us consider the particle-particle interaction in eq. (4.1.4). If one restricts only to the single-particle propagations in the intermediate states (ladder approximation), i.e., $t_5, t_6 < t_7, t_8$ and, in addition, if one approximates v by the static NN potential, eq. (4.1.4) reduces to the BG-equation.

In the nuclear medium, however, other correlations than those accounted for by the BG-equation are also important. In order to explore what kind of correlations can be taken into account from a scheme as given by eq. (4.1.4), consider the time dependence of the fermion propagators explicitly. All possible time independent blocks are shown in Fig. 40. Due to the Pauli-principle they are restricted to the nine cases. $\Gamma_{(1)}$, $\Gamma_{(4)}$ and $\Gamma_{(9)}$ are the particle-particle, particle-hole and hole-hole interactions respectively. $\Gamma_{(2)}$ and $\Gamma_{(3)}$ give rise to the particle-phonon couplings, while $\Gamma_{(7)}$ and $\Gamma_{(8)}$ give rise to the hole-phonon couplings. Finally, $\Gamma_{(5)}$ and $\Gamma_{(6)}$ give the ground state correlations. The idea now is to combine these blocks in order to build up what one wants. For example, $\Gamma_{(2)}$ and $\Gamma_{(3)}$ may be combined with v (join the internal hole lines and the particle lines) to get a part of the interaction which contributes to the full particle-particle interaction $\Gamma_{(1)}$. This procedure couples all the nine time independent blocks and allows us to include other kinds of correlations than those accounted for by the Brueckner theory. Therefore, we now replace eq. (4.1.4) by the matrix equation

$$\Gamma_{(i)} = v_{(i)} + i \sum_{j=1}^9 \frac{1}{2} v g g \Gamma_{(j)}]_{(i)} . \quad (4.1.5)$$

In the above equation the bracket $[\]_{(i)}$ is a time-ordering operator which forms all possible internal time-ordering of the product $v g g \Gamma_{(j)}$ in such a way that a contribution to the interaction of the type $\Gamma_{(i)}$ results. For instance, in the case of particle-particle scattering, both $v_{(2)} g g \Gamma_{(2)}$ and $\Gamma_{(1)} g g v_{(1)}$ are included. However, by symmetry, these terms are identical and the factor 1/2 takes care of this double counting. The indices i and j run over all nine time-independent blocks. Note that for a given i only a few j 's can actually contribute.

In the following we will entirely neglect the hole-hole interaction $\Gamma_{(9)}$ and the ground state correlation blocks $\Gamma_{(5)}$ and $\Gamma_{(6)}$. With this assumption all possible contributions $\Gamma_{(j)}$ to the particle-particle interaction $\Gamma_{(1)}$ for example are restricted to $\Gamma_{(1)}$, $\Gamma_{(2)}$ and $\Gamma_{(3)}$. The contributions to the particle-hole interaction $\Gamma_{(4)}$ are $\Gamma_{(2)}$, $\Gamma_{(3)}$, $\Gamma_{(4)}$, $\Gamma_{(7)}$ and $\Gamma_{(8)}$.

Now, since we are interested in obtaining effective particle-particle and particle-hole interactions, we will restrict our attention only to the equations for $\Gamma_{(1)}$ and $\Gamma_{(4)}$. These equations involve also the blocks like $\Gamma_{(2)}$, $\Gamma_{(3)}$, etc. as mentioned above; however, contributions from these blocks ($\Gamma_{(2)}$, $\Gamma_{(3)}$, etc.) to $\Gamma_{(1)}$ and $\Gamma_{(4)}$ may be obtained (within an approximation) from $\Gamma_{(1)}$ or $\Gamma_{(4)}$ by particle-hole and particle-particle transformations and also by applying the Pauli exchange operator $(-P_K P_\sigma P_\tau)$ on $\Gamma_{(4)}$. These are illustrated in Figs. 41 and 42. In Fig. 41 we show the contribution from $\Gamma_{(2)}$ to the particle-particle interaction $\Gamma_{(1)}$, whose lowest order is also displayed. This contribution may be obtained by means of the particle-particle transformation of a second-order diagram in the particle-hole interaction $\Gamma_{(4)}$ as indicated in the r.h.s. of Fig. 42. The lowest order contribution from $\Gamma_{(2)}$ to the particle-hole interaction $\Gamma_{(4)}$ is displayed in Fig. 42. In this case there are

two possibilities as shown in the r.h.s. of Figs. 42a and 42b, which may be obtained by a particle-hole transformation of a second-order contribution to the particle-particle interaction $\Gamma_{(1)}$ (Fig. 42a) and by applying the Pauli exchange operator on a second-order contribution to the particle-hole interaction $\Gamma_{(4)}$ (Fig. 42b).

The above examples illustrate how the contributions from $\Gamma_{(2)}$, $\Gamma_{(3)}$, etc. to the particle-particle $\Gamma_{(1)}$ and particle-hole $\Gamma_{(4)}$ interactions can be obtained from $\Gamma_{(1)}$ and $\Gamma_{(4)}$ themselves in the lowest order. Higher order contributions may also be obtained from the above procedure. In order to do this, it is convenient to write the equations for the particle-particle $\Gamma_{(1)}$ and the particle-hole $\Gamma_{(4)}$ interactions as

$$G_{pp} = K_{pp} + K_{pp}ggG_{pp} \quad (4.1.6)$$

and

$$U_{ph} = K_{ph} + K_{ph}ggU_{ph} . \quad (4.1.7)$$

In the above equations we have denoted the particle-particle $\Gamma_{(1)}$ and the particle-hole $\Gamma_{(4)}$ interaction by G_{pp} and U_{ph} , respectively. Moreover, we have also introduced the notations K_{pp} and K_{ph} to design the particle-particle and the particle-hole irreducible kernels, respectively. These are given by

$$K_{pp} = v_{pp} + T^{PP}(U_{ph}-K_{ph}) \quad (4.1.8)$$

and

$$K_{ph} = v_{ph} + T^{Ph}(G_{pp}-K_{pp}) + K^{ind}(U_{ph}-K_{ph}) . \quad (4.1.9)$$

Here, $v_{pp}(v_{ph})$ denotes the bare particle-particle (particle-hole) interaction (direct + exchange term). $T^{PP}(I)$ ($T^{Ph}(I)$) means the particle-particle (par-

ticle-hole) transformation of I , while $K^{ind}(I)$ denotes the Pauli exchange operation $(-P_K P_\sigma P_\tau)$ on I . The difference between G_{pp} (U_{ph}) and K_{pp} (K_{ph}) appearing in the argument of T^{pp} (T^{ph} and K^{ind}) is to avoid double countings.

The procedure defined through eqs. (4.1.6)-(4.1.9) takes into account approximately the contributions from the blocks $\Gamma_{(2)}$, $\Gamma_{(3)}$, etc. to the particle-particle $\Gamma_{(1)}$ and the particle-hole $\Gamma_{(4)}$ interactions without referring explicitly to these blocks. The particle-particle and particle-hole interactions given by eqs. (4.1.6) and (4.1.7) couple each other through eqs. (4.1.8) and (4.1.9) which form a set of self-consistent non-linear equations. A diagrammatic representation of these equations is displayed in Fig. 43. There we show also a graphical representation of the bare interaction v which is properly antisymmetrized (direct + exchange term).

The second term T^{pp} ($U_{ph}-K_{ph}$) in eq. (4.1.8) accounts for the core-polarization effects (sums all particle-hole bubbles in the crossed channel) on the particle-particle interaction G_{pp} , while these effects are taken into account by K^{ind} ($U_{ph}-K_{ph}$) in eq. (4.1.9) for the particle-hole interaction. Note that U_{ph} given by eq. (4.1.7) is actually the Tamm-Dancoff approximation with the effective particle-hole interaction K_{ph} . A similar equation for the effective particle-hole interaction K_{ph} as given by eq. (4.1.9) has been derived by Babu and Brown²⁶⁾ at vanishing momentum transfer.

In the limit when one approximates the irreducible particle-particle kernel K_{pp} in eq. (4.1.8) by the bare NN interaction v_{pp} , eq. (4.1.6) reduces exactly to the BG-equation of chapter I, while eq. (4.1.9) yields for the particle-hole interaction (with the above approximation K_{ph} will be denoted by F^{ph})

$$F^{ph} = G_{ph} + K^{ind}(U_{ph}-F^{ph}) \quad (4.1.10)$$

with the obvious notation $G_{ph} = T^{ph}(G_{pp})$. A graphical representation of eq. (4.1.10) is shown in Fig. 44. Note that the induced piece contributes only to the exchange term of F^{ph} . A linearized version of eq. (4.1.10) has been studied in Ref. 30. Recently, Dickhoff et al.³¹⁾ have also obtained eq. (4.1.10) by a successive iteration starting from the G-matrix interaction.

The major results of this section are the self-consistent coupled equation given by (4.1.6)-(4.1.9). These equations have been obtained by making convenient approximations which certainly are not consistent with convergence criteria, such as the hole line expansion, but rather guided by physical considerations.

4.2 Role of Medium Polarization on the Effective Particle-Hole and Δ -Hole Interactions

In this section we will analyze medium polarization effects on the effective interaction within the approximation (4.1.10) of the previous section. For the G-matrix interaction G_{ph} we will take the results of chapter III which includes relativistic effects.

With the approximation (4.1.10) for the particle-hole interaction we avoid the coupling between particle-particle and particle-hole interactions, but the self-consistent non-linear equation has still to be solved, which involves a considerable effort (see e.g. Ref. 31). However, the major effects of eq. (4.1.10) can be studied in the Landau limit, where the particle-hole interaction has the form

$$F^{ph} = C_0 (f_0 + f'_0 \tau_1 \cdot \tau_2 + g_0 \sigma_1 \cdot \sigma_2 + g'_0 \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2) \quad (4.2.1)$$

with a constant $C_0 = \hbar^2 \pi^2 / m k_F$. Neglecting in a first step also all possible momentum dependences in the exchange channel, one obtains the following simple model:

$$\begin{aligned} f_0 &= f_0^G - \frac{1}{4} (f_0^I + 3f_0'^I + 3g_0^I + 9g_0'^I) \\ f'_0 &= f_0'^G - \frac{1}{4} (f_0^I - f_0'^I + 3g_0^I - 3g_0'^I) \\ g_0 &= g_0^G - \frac{1}{4} (f_0^I + 3f_0'^I - g_0^I - 3g_0'^I) \\ g'_0 &= g_0'^G - \frac{1}{4} (f_0^I - f_0'^I - g_0^I + g_0'^I) \end{aligned} \quad (4.2.2)$$

Here, within our approximation, f_0^G etc. denote the Landau parameters derived from the G-matrix, while the terms arising from the induced interaction are given by

$$f_0^I = \frac{f_0^2 U(q=0)}{1 - f_0^2 U(q=0)} \quad (4.2.3)$$

with $U(q=0) = -1$ denoting only the forward going contribution of the Lindhard function

$$U(q) = -\frac{1}{2} + \frac{k_F}{2q} \left(1 - \left(\frac{q}{2k_F} \right)^2 \right) \ln \left| \frac{1 - \frac{q}{2k_F}}{1 + \frac{q}{2k_F}} \right| \quad (4.2.4)$$

at zero momentum transfer in nuclear matter. In finite nuclei, the shell model introduces a gap between the energies of the occupied and the unoccupied levels. Consequently the backward going contribution of the Lindhard function is reduced with respect to the forward going one. In order to obtain an estimate as realistic as possible for the Landau parameters in finite nuclei, we decided to omit the backward going contribution in nuclear matter entirely.

The major effect of the induced interaction is to stabilize the parameter f_0 , since all induced contributions add coherently to compensate the attractive f_0^G . Note that the induced interaction screens out the exchange term of the interaction, since it contributes only to this term (see Fig. 44). In all other channels, however, the induced pieces cancel to a large extent. It introduces e.g. in the spin-isospin strength g_0' a correction of about 20 % ²⁹). Using the Landau parameters obtained from the G-matrix calculation including the relativistic effects of the previous chapter based on HEA⁷⁶) (see Table 12) one gets from eq. (4.2.2) $f_0 = -0.05$, $f_0' = 0.20$, $g_0 = -0.07$ and $g_0' = 0.69$ at normal density ($k_F = 1.36 \text{ fm}^{-1}$).

In the isobar sector the situation is quite different from that of the nucleon sector because of two reasons:

- (i) The factor 1/4 (due to the exchange term) which reduces the induced pieces in eq. (4.2.2) does not occur, since $S_1 \cdot \sigma_2 P_\sigma = S_1 \cdot \sigma_2$.
- (ii) The isobar has to be excited via the spin-isospin channel. Therefore the cancellation in eq. (4.2.2) does no longer appear.

In the Landau limit, the force in the isobar sector is given by eq (3.2.1). An estimate of the coupling strength $(g_0')_{\Delta N}$ can be obtained in the same approximation as before by summing only nucleon-nucleon ring diagrams in the crossed channel:

$$(g_0')_{\Delta N} = (g_0')_{\Delta N}^G - \frac{(g_0')_{\Delta N} g_0' U(q=0)}{1 - g_0' U(q=0)} . \quad (4.2.5)$$

Using again the Landau parameter $(g_0')_{\Delta N}^G$ from the G-matrix calculation at $k_F = 1.36 \text{ fm}^{-1}$ (see last column of Table 12) and the results for g_0' obtained

above from eq. (4.2.2), one gets $(g'_0)_{\Delta N} = 1.77$, which means a change by a factor of 1.69. This rough estimate shows that the virtual collective excitation in the nucleon sector has an important effect on the isobar-nucleon hole interaction.

A more reliable calculation should, however, take care of the momentum dependence of F^{ph} and consider the self-consistent coupling between isobars and nucleons.

Since the induced interaction contributes only to the exchange term of F^{ph} , one has to average over the momentum in order to get Landau parameters. This averaging process is not done explicitly in eq. (4.2.3). Therefore one might argue that this dependence modifies considerably the above estimate. In fact, the spin-isospin channel contains the one-pion- and one-rho-exchange interaction, which makes this channel strongly momentum dependent. The reduction factor in the induced interaction due to this momentum dependence may be estimated as follows. Since the isovector tensor force starts to contribute to the spin-isospin channel as soon as the momentum transfer q increases from zero, one has to take into account this contribution in order to estimate the reduction factor. The spin-isospin and the isovector tensor components of the effective G -matrix interaction

$$G_{\sigma\tau}(\vec{q}) = C_0 \{ g'(q) \sigma_1 \cdot \sigma_2 + t'(q) S_{12}(\hat{q}) \} \tau_1 \cdot \tau_2 \quad (4.2.6)$$

may be rewritten in terms of the longitudinal $(\sigma_1 \cdot \hat{q} \sigma_2 \cdot \hat{q})$ and transverse $(\sigma_1 \wedge \hat{q} \cdot \sigma_2 \wedge \hat{q})$ components as

$$G_{\sigma\tau}(\vec{q}) = C_0 \{ Q_L(q) \sigma_1 \cdot \hat{q} \sigma_2 \cdot \hat{q} + Q_T(q) \sigma_1 \wedge \hat{q} \cdot \sigma_2 \wedge \hat{q} \} \tau_1 \cdot \tau_2, \quad (4.2.7)$$

where

$$\begin{aligned} Q_L(q) &\equiv (g'(q) + 2t'(q)) , \\ Q_T(q) &\equiv (g'(q) - t'(q)) . \end{aligned} \quad (4.2.8)$$

In terms of these quantities, the reduction factor in the spin-isospin channel of the induced contribution due to the momentum dependence may be estimated as

$$\lambda = \frac{\frac{1}{2} \int_{-1}^{+1} -\frac{1}{4} \left[\frac{1}{3} \left\{ \frac{Q_L^2(q)U(q)}{1-Q_L(q)U(q)} + 2 \frac{Q_T^2(q)U(q)}{1-Q_T(q)U(q)} \right\} \right] d(\cos\theta)}{-\frac{1}{4} \left[\frac{(g'(q=0))^2 U(q=0)}{1-g'(q=0)U(q=0)} \right]} . \quad (4.2.9)$$

Here the angle θ is related to the momentum q by

$$q = 2k_F \sin \frac{\theta}{2} . \quad (4.2.10)$$

The integrand in eq. (4.2.9) is shown in Fig. 45 as a function of momentum q , using the G-matrix interaction based on the HEA potential. As one can see, the contribution to the induced interaction from the spin-isospin component $g'(q)$ (dashed line) decreases rapidly with increasing momentum, while the isovector-tensor interaction $t'(q)$ gives a contribution (long-dashed line) which increases with momentum. The total contribution (sum of these two contributions) to the induced interaction in the spin-isospin channel is represented by a solid line in Fig. 45. By averaging that result over the momentum q , one gets for the reduction factor λ in eq. (4.2.9) the value of

$$\lambda = 0.4 . \quad (4.2.11)$$

We now improve our model given by eq. (4.2.3) for g_0^I , replacing it by

$$g_0^I = \lambda \frac{g_0'^2 U(q=0)}{1 - g_0' U(q=0)} . \quad (4.2.12)$$

The induced contribution in the isobar sector (second term of eq. (4.2.5)) has also been multiplied by the same reduction factor λ . The results are shown in the third row of Table 13. In the second row we also show the results of the G-matrix interaction of the previous chapter for comparison. As one can see, the large attractive scalar-isoscalar component of the G-matrix interaction is reduced by a factor of $\sim 1/2$ when the core polarization effects are included and leads to a value of the Landau parameter f_0 close to that determined empirically. The influence of the induced interaction in the remaining channels of the particle-hole interaction is relatively small compared to that in the scalar-isoscalar channel. The parameter $(g_0')_{\Delta N}$ in the isobar sector is largely affected by the medium polarization, which gives an additional repulsion of about 40 %, while the NN coupling g_0' is increased by only 10 %, the chief difference being the factor of 4 in the different ratios of exchange to the direct term.

So far, the influence of the isobar on the nucleon sector has been neglected. It is straightforward, however, to generalize our model to include a self-consistent coupling between isobars and nucleons, which leads to a coupled self-consistent equation of the form

$$\begin{aligned} F^{ph} &= K^{ph} + F_{induced}^{ph} (F^{ph} + F^{\Delta h}) , \\ F^{\Delta h} &= K^{\Delta h} + F_{induced}^{\Delta h} (F^{ph} + F^{\Delta h}) . \end{aligned} \quad (4.2.13)$$

Here we entirely neglected the couplings with interactions which involve more than one Δ -isobar in their vertices. A diagrammatic representation of the above equation is shown in Fig. 46. In appendix C, a method to solve eq. (4.2.13) is presented.

In the fourth line of Table 13, the results of the fully coupled self-consistent equation (4.2.13) are displayed. In the nucleon sector it gives a small effect compared to that of the uncoupled equation (third row) while in the isobar sector it reduces the coupling $(g'_0)_{\Delta N}$ by $\sim 16\%$ with respect to the result of the uncoupled equation. This is again due to the factor $1/4$ which is not present in the induced contribution in the isobar sector. Therefore, in the present calculation $(g'_0)_{\Delta N}$ is increased by $\sim 17\%$ with respect to the G-matrix result including relativistic effects.

For the sake of comparison we have also shown in the first row of Table 13 the results for the Landau parameters from the G-matrix interaction obtained as in chapter I with no inclusion of relativistic effects. The Δ -nucleon coupling strength $(g'_0)_{\Delta N}$ is enhanced by $\sim 35\%$ with inclusion of both relativistic and medium polarization effects. Also in the last row we displayed the Landau parameters determined empirically¹⁹⁾ in order to have some feeling about the coupling strength of the effective interaction. A quantitative comparison with the present results should be avoided since our present model is perhaps too simple for such a purpose. In fact, since the induced interaction describes essentially a long-range phenomenon (i.e., collective modes), the extrapolation of a nuclear matter calculation of such effects to a finite system may not be a good procedure. In particular, the magnitude of the induced interaction depends on the multipolarity J^π of the collective mode⁸⁵⁾. Moreover, the empirical values involve relatively large uncertainties. Nevertheless, it is

clear that in a microscopic theory for the effective interaction one has to take into account the medium polarization effects built up by virtual collective excitations of the nucleus.

Conclusion

In chapter I a derivation of the exact operator structure of the Brueckner G-matrix has been presented. The method is quite general and can be applied, in principle, to extract the operator structure of any two-body interaction satisfying the standard symmetry conditions. In the present work we have restricted our considerations to the case of identical particles with spin $1/2$. The method has been applied to the Brueckner G-matrix based on an OBEP model, from which a quasi-particle interaction has been derived. In addition, a prescription for constructing the symmetric part of the interaction has been suggested. Although this interaction is quite useful in studying many aspects of the Brueckner G-matrix, care must be taken when one uses it for other purposes. Of course, any ambiguity in the symmetric part is removed after its antisymmetrization.

Some aspects of the quasi-particle interaction have also been analyzed. It has been verified explicitly that the non-locality of the interaction introduced by correlations is very small, although a strong Q dependence is present in the central (spin and isospin independent) channel due to the use of a highly non-local bare potential. It has also been shown that correlations are confined largely to S-states and these affect primarily the central components of the interaction. The spin-orbit terms are weakly influenced by correlations. Also a very weak effect of correlations is observed in the tensor channels, at least in the region of the density $0.35 \rho_0 \leq \rho \leq \rho_0$. This is due to the momentum mismatch between the defect functions in ${}^3S_1$ - ${}^3D_1$ ($L=L'=0$) and ${}^3S_1$ - ${}^3D_1$ ($L=0, L'=2$) states and the corresponding partial wave contributions of the bare NN potential. Based on these results, a momentum transfer and density dependent quasi-particle interaction has been constructed. The density dependence

has been extracted in the range of $0.35\rho_0 \leq \rho \leq \rho_0$ and also parametrized in terms of a relatively simple expression based on analytical G-matrix results. The usual Landau parameters have been calculated for the momentum-transfer and density-dependent interactions and they reproduce closely the exact values. It has also been verified, by calculating the Landau parameters, that the G-matrix interaction is quite sensitive to the input NN potential. Therefore, a study of the excited states of nuclei provides an excellent area for distinguishing different phase-shift-equivalent potentials. The different G-matrix results were related in a very simple way to the bare NN potential, especially in the spin and isospin channels which are less understood than are the other channels.

Although our method for extracting the operator structure of the G-matrix allows us, in principle, to keep all of its complexity, our aim here has been to derive a quasi-particle interaction which preserves the most essential features of the exact G-operator and yet is simple enough to be manageable in nuclear structure calculations. In this context, the simple model for the G-matrix interaction developed in section 1.5 is of special interest. The success of our simple model clearly shows that the G-matrix has a very simple structure, even though formally it appears to be a highly renormalized object.

The nuclear matter G-matrix interaction seems to work reasonably well in a nuclear structure calculation when supplemented by higher order effects. However, this interaction alone leads to an instability in heavier nuclei such as ^{208}Pb .

In contrast to the nucleon-nucleon case the Δ -nucleon interaction does not feel short-range correlation effects due to the Pauli principle. As a conse-

quence, the Δ -nucleon G-matrix interaction is well approximated by the bare Δ -nucleon transition potential.

In chapter II we have derived an effective G-matrix interaction appropriate for applications to the nucleon-nucleus scattering problem. The energy, density and momentum transfer dependences of this interaction have been analyzed. At low incident energies, the interaction is strongly density dependent, especially in the scalar-isoscalar channel. At higher bombarding energies it approaches the t-matrix interaction. Also at low densities, the effective G-matrix interaction shows a strong energy dependence and becomes weaker with increasing density. At small momentum transfers the interaction is dominated by its scalar-isoscalar component.

We have analyzed in some detail the energy and the density dependence of the effective tensor force in section 2.3, where an apparent contradiction between experimental evidences on the effective isovector tensor force observed in nuclear structure and in nuclear reactions has been explained.

Our effective G-matrix interaction gives encouraging results when applied to the nucleon-nucleus scattering problem at intermediate energies.

Inclusion of relativistic effects as discussed in chapter III may improve the agreement with the experimental data. In fact, a simultaneous description of differential cross sections, analyzing powers and spin rotation probabilities seems to require a relativistic treatment of the nuclei^{46,98}).

In chapter III, using the momentum independent approximation for the nucleon self-energy, which seems to be well justified, we have shown in a very transparent way the medium effects on the bare nucleon-nucleon potential. The effects are density-dependent and depend also on the momentum transfer. More-

over, in the current work, the dominant modifications due to these medium effects have been observed in the ρ -, ω - and σ -meson exchange potentials. The spin-orbit force is increased by a factor of two. The resulting G-matrix interaction shows a number of interesting features which may improve the agreement with the experimental data. Relativistic effects also enhance the Δ -nucleon coupling strength by $\sim 15\%$ with respect to the conventional G-matrix result.

It should be stressed, however, that relativistic effects alone are not sufficient to stabilize nuclear matter.

Since within the OBE model for the nuclear force the σ -meson simulates a part of the two-pion exchange contributions and since the pseudo-vector meson contributions are essentially unaffected by relativistic effects, the present calculation probably overestimates these effects.

Finally, in chapter IV we have investigated the medium polarization effects on the effective interaction within a simple model. In the nucleon sector they reduce the large attraction of the G-matrix interaction in the scalar-isoscalar channel by screening out the exchange term in the interaction. The medium polarization affects the Δ -nucleon interaction much more than the particle-hole interaction due to the Pauli principle and it also enhances the coupling strength $(g'_0)_{\Delta N}$. In the present calculation $(g'_0)_{\Delta N}$ is increased by $\sim 17\%$ with respect to the G-matrix results including relativistic effects. Therefore, including relativistic effects enhances the Δ -nucleon coupling strength by $\sim 35\%$ with respect to the conventional G-matrix result. However, the quenching mechanism of the total Gamow-Teller strength via excitation of Δ -isobars is still an open question. More careful and detailed analyses of the medium polarization in finite nuclei as well as of relativistic effects seem to be required.

Appendix A

In this appendix we present a few details of the derivation of the operator structure of the two-body interaction for identical particles with spin $1/2$.

Choosing the relative momentum $\vec{k}_0$ (see Fig. 4) to be in the positive z-direction, the quantity $[\gamma_L(\hat{k}) \otimes \gamma_L(\hat{k}_0)]^2$ in eq. (1.2.8) can easily be expressed in terms of Legendre and associated Legendre polynomials, $P_L(x)$ and $P_L^M(x)$ respectively. Moreover, the 6-j symbols restrict the sum over L and L' , so that eq. (1.2.8) for the G-operator reduces to

$$\begin{aligned}
 G(K, \vec{k}, \vec{k}_0) = & \left\{ \frac{1}{2\pi^2} \sum_L [L]^2 G_{LL}^{L0T}(K, k, k_0) P_L(\hat{k} \cdot \hat{k}_0) P_{S=0} \right. \\
 & + \frac{1}{2\pi^2} \sum_L [J]^2 G_{LL}^{J1T}(K, k, k_0) P_L(\hat{k} \cdot \hat{k}_0) \frac{1}{3} P_{S=1} \\
 & + \frac{1}{4\pi^2} \sqrt{\frac{3}{2}} \sum_{LJ} (-)^{L+J+1} [J]^2 \frac{[L]}{\sqrt{L(L+1)}} \left\{ \begin{matrix} 1 & L & J \\ L & 1 & 1 \end{matrix} \right\} G_{LL}^{J1T}(K, k, k_0) P_L^1(\hat{k} \cdot \hat{k}_0) \\
 & \times i(\sigma_1 + \sigma_2) \cdot \hat{n} \\
 & + \frac{1}{2\pi^2} \sum_{LL'J} i^{L-L'} (-)^{J+1} [J]^2 [LL'] \left\{ \begin{matrix} 1 & L' & J \\ L & 1 & 2 \end{matrix} \right\} G_{LL'}^{J1T}(K, k, k_0) \\
 & \times \left(\sum_M (L' - ML | 2 - M) \sqrt{\frac{(L'+M)!}{(L'-M)!}} P_{L'}^M(\hat{k} \cdot \hat{k}_0) \frac{(\sigma_1 \otimes \sigma_2)_M^2}{2} \right) \Big\} P_T.
 \end{aligned} \tag{A.1}$$

A comparison with eq. (1.3.2) immediately yields the results given in eq. (1.3.4) for the coefficients $G_{S=0,1}$ and G_{LS} . In order to reduce the tensor term in the above equation to the form given by eq. (1.3.2), one has to evaluate the coefficients δ' and ϵ' given in eq. (1.2.12). Since we are on the energy shell $[|\vec{k}| = |\vec{k}_0|]$ due to our quasi-particle assumption, the term con-

taining the coefficient ω' in eq. (1.2.12) does not contribute. Using eqs. (1.2.11) and (1.2.13) and the definition of the tensor operator S_{12} , the coefficients δ' and ϵ' can be re-expressed as

$$\begin{aligned}\delta' &= \frac{1}{36} \text{Tr}\{G(2S_{12}(\hat{q}) + S_{12}(\hat{Q}))\} , \\ \epsilon' &= \frac{1}{36} \text{Tr}\{G(S_{12}(\hat{q}) + 2S_{12}(\hat{Q}))\} .\end{aligned}\tag{A.2}$$

In this expression only the tensor (last) term of the G-operator in eq. (A.1) can contribute since the other terms vanish when one takes the trace. To calculate the trace in eq. (A.2) it is convenient to use the tensor term of the G-operator in the form given by eq. (1.2.8) (the last term). It is also convenient to use

$$S_{12}(q) = \sqrt{\frac{24\pi}{5}} (\sigma_1 \otimes \sigma_2)^2 \cdot \gamma_2(\hat{q})\tag{A.3}$$

for the tensor operator. Substituting eqs. (1.2.8) and (A.3) into eq. (A.2), and recoupling yields

$$\delta' = \frac{1}{\pi} \sum_{LL',J} i^{L-L'} (-)^{J+1} [J]^2 \left\{ \begin{matrix} 1 & L' & J \\ L & 1 & 2 \end{matrix} \right\} G_{L'L}^{J1T}(K,k,k_0) \frac{1}{36} (2x+y) ,\tag{A.4}$$

$$\epsilon' = \frac{1}{\pi} \sum_{LL',J} i^{L-L'} (-)^{J+1} [J]^2 \left\{ \begin{matrix} 1 & L' & J \\ L & 1 & 2 \end{matrix} \right\} G_{L'L}^{J1T}(K,k,k_0) \frac{1}{36} (x+2y) ,$$

where,

$$x \equiv 4 \sqrt{\frac{24\pi}{5}} [\gamma_{L'}(\hat{k}) \otimes \gamma_L(\hat{k}_0)]^2 \cdot \gamma_2(\hat{q}) ,$$

$$y \equiv 4 \sqrt{\frac{24\pi}{5}} [\gamma_{L'}(\hat{k}) \otimes \gamma_L(\hat{k}_0)]^2 \cdot \gamma_2(\hat{Q}) .$$

Choosing again $\vec{k}_0$ along the positive z-axis and $\vec{k}$ in the xz-plane, one gets

$$\begin{aligned}
 x &= 4 \sqrt{\frac{24\pi}{5}} \frac{[L'L]}{4\pi} \sum_{M \geq 0} (L'MLO|2M) \sqrt{\frac{(L'-M)!}{(L'+M)!}} P_{L'}^M(\hat{k} \cdot \hat{k}_0) \\
 &\times Y_{2M}\left(\frac{\pi-\theta}{2}, 0\right) (2-\delta_{M0}) , \\
 y &= 4 \sqrt{\frac{24\pi}{5}} \frac{[L'L]}{4\pi} \sum_{M \geq 0} (L'MLO|2M) \sqrt{\frac{(L'-M)!}{(L'+M)!}} P_{L'}^M(\hat{k} \cdot \hat{k}_0) (-)^M \\
 &\times Y_{2M}\left(\frac{\theta}{2}, 0\right) (2-\delta_{M0}) ,
 \end{aligned} \tag{A.5}$$

where θ denotes the angle between $\vec{k}$ and $\vec{k}_0$. Finally, substituting eq. (A.5) into eq. (A.4), one obtains the results given in eq. (1.3.4). There, δ' and ε' are denoted by G_t^{1T} and G_t^{1T} respectively.

Appendix B

The Landau limit corresponds to the situation when all of the particles in the initial and final states are on the Fermi surface and the momentum transfer $q > 0$. In this situation our G-matrix interaction given by eq. (1.3.7) (direct + exchange) depends only on the exchange momentum transfer $\vec{Q}$ and the density. Note that in terms of the momenta of the particles, $\vec{q}$ and $\vec{Q}$ are given by (see Fig. 4)

$$\begin{aligned}\vec{q} &= \vec{k}_\alpha - \vec{k}_{\alpha'} , \\ \vec{Q} &= \frac{1}{2} (\vec{k}_\alpha - \vec{k}_\beta) + \frac{1}{2} (\vec{k}_{\alpha'} - \vec{k}_{\beta'}) ,\end{aligned}\tag{B.1}$$

which in the Landau limit become

$$\begin{aligned}\vec{q} &= 0 , \\ \vec{Q} &= \vec{k}_\alpha - \vec{k}_\beta .\end{aligned}\tag{B.2}$$

This implies that F , F' , G , G' , H and H' are functions of k_F and the angle θ between $\vec{k}_\alpha$ and $\vec{k}_\beta$, since $|\vec{k}_\alpha| = |\vec{k}_\beta| = k_F$ (the remaining channels S , S' , T and T' vanish identically in the Landau limit). This velocity dependence is expanded in Legendre polynomials,

$$N_0 F = \sum_{\ell} f_{\ell} P_{\ell}(\cos\theta) ,\tag{B.3}$$

where

$$N_0 = \frac{k_F m^*}{\hbar^2 \pi^2} \equiv C_0^{-1}\tag{B.4}$$

is the density of states per unit energy at the Fermi surface per unit volume and m^* is the effective mass of a nucleon. The tensor channel interactions H and H' are expanded in accordance with the work of Dabrowski and Haensel⁷¹⁾

$$N_0 H = \left(\frac{Q}{k_F}\right)^2 \sum_{\ell} h_{\ell} P_{\ell}(\cos\theta) . \quad (B.5)$$

Using the orthogonality of Legendre polynomials, eqs. (B.3) and (B.5) can be inverted to get

$$f_{\ell} = C_0^{-1} \frac{2\ell+1}{2} \int_{-1}^{+1} d(\cos\theta) P_{\ell}(\cos\theta) F(\cos\theta) , \quad (B.6a)$$

and

$$h_{\ell} = C_0^{-1} \frac{2\ell+1}{2} \int_{-1}^{+1} d(\cos\theta) P_{\ell}(\cos\theta) H(\cos\theta) \left(\frac{k_F}{Q}\right)^2 . \quad (B.6b)$$

From F , F' , G , G' , H and H' , the corresponding Landau parameters can be extracted by means of eq. (B.6). In particular, if the interaction has the form

$$F_D = A \frac{m^2}{q^2 + m^2} , \quad (B.7)$$

one obtains

$$f_{\ell, \text{DIRECT}} = C_0^{-1} A (2\ell+1) \delta_{\ell 0} , \quad (B.8)$$

and

$$f_{\ell, \text{exchange}} = \left(-\frac{1}{4}\right) C_0^{-1} A \frac{2\ell+1}{2} \left(\frac{m^2}{k_F^2}\right) Q_{\ell} \left(1 + \frac{m^2}{2k_F^2}\right) , \quad (B.9)$$

for the direct and exchange contributions, respectively. The factor $(-1/4)$ in the last equation is due to the Pauli exchange operator $(-P_K P_{\sigma} P_{\tau})$. $Q_{\ell}(x)$ is a Legendre function of the second kind of order ℓ . Similarly, for the tensor interaction of the form

$$T_D = A \frac{q^2}{q^2 + m^2}, \quad (B.10)$$

one has

$$h_{\ell, \text{exchange}} = (-1/2) C_0^{-1} A \frac{2\ell+1}{2} Q_\ell \left(1 + \frac{m^2}{2k_F^2}\right). \quad (B.11)$$

Note that P_K in the Pauli exchange operator $(-P_K P_\sigma P_\tau)$ converts direct T-type to H-type operators and vice versa. Also P_τ in $(-P_K P_\sigma P_\tau)$ mixes the primed and unprimed types of operators.

Appendix C

In this appendix we will present a simple method to solve the fully coupled self-consistent equation (4.2.13).

Let us start from an interaction which has the form

$$F = G_0 + G_\tau \tau_1 \cdot \tau_2 + G_\sigma \sigma_1 \cdot \sigma_2 + G_{\sigma\tau} \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 \quad (C.1)$$

$$+ G_{t\tau} S_{12}(\hat{q}) \tau_1 \cdot \tau_2 + G_{\Delta\sigma} S_1 \cdot \sigma_2 T_1 \cdot \tau_1 + G_{\Delta t} S_{12}^\Delta(\hat{q}) T_1 \cdot \tau_2 ,$$

where

$$S_{12}^\Delta(\hat{q}) \equiv 3S_1 \cdot \hat{q} \sigma_2 \cdot \hat{q} - S_1 \cdot \sigma_2 . \quad (C.1')$$

Defining the operator A_i as

$$\begin{aligned} A_1 &= 1 , \\ A_2 &= \tau_1 \cdot \sigma_2 , \\ A_3 &= \sigma_1 \cdot \sigma_2 , \end{aligned} \quad (C.2)$$

$$A_7 = S_{12}^\Delta(\hat{q}) T_1 \cdot \tau_2 ,$$

eq. (C.1) can be rewritten as

$$F = \sum_{i=1}^7 G_i A_i . \quad (C.3)$$

Now, one has to note that the product of two operators A_i and A_j can always be written as the linear combination of A_k

$$A_i A_j = \sum_{k=1}^7 C_{ij}^k A_k, \quad (C.4)$$

where

$$C_{ij}^k = C_{jk}^i. \quad (C.5)$$

The effective interaction U_{eff} which sums all the ring diagrams is obtained by summing the series (the Lindhard function is not written down explicitly)

$$U_{\text{eff}} = \sum_{v=1}^{\infty} F^v. \quad (C.6)$$

Due to the closure relation (C.4), F^v can be expressed as

$$F = \sum_{k=1}^7 \gamma_v^k A_k. \quad (C.7)$$

In particular, for $v=1$ one has

$$\gamma_1^k = G_k. \quad (C.8)$$

To determine γ_v^k in any order v , let us consider the term F^{v+1} , which is given by

$$F^{v+1} = \sum_{k=1}^7 \gamma_{v+1}^k A_k. \quad (C.9)$$

This term, however, can also be expressed as

$$F^{v+1} = F^v F = \left(\sum_{k=1}^7 \gamma_v^k A_k \right) \sum_{j=1}^7 \gamma_1^j A_j. \quad (C.10)$$

Inserting eqs. (C.4) and (C.8) into eq. (C.10), and comparing with eq. (C.9), one gets

$$\gamma_{\nu+1}^{\ell} = \sum_{k=1}^7 \left\{ \sum_{j=1}^7 c_{kj}^{\ell} G_j \right\} \gamma_{\nu}^k . \quad (C.11)$$

With the definition

$$b_{\ell k} \equiv \sum_{j=1}^7 c_{kj}^{\ell} G_j , \quad (C.12)$$

eq. (C.11) can be written as

$$\gamma_{\nu+1}^{\ell} = \sum_{k=1}^7 b_{\ell k} \gamma_{\nu}^k . \quad (C.13)$$

Now, in the above equation γ_{ν}^k can be replaced by the corresponding expansion in terms of $\gamma_{\nu-1}^n$. This process can be continued up to γ_1^m . The result is

$$\gamma_{\nu+1}^{\ell} = \sum_{m=1}^7 (b_{\ell m})^{\nu} \gamma_1^m , \quad (C.14)$$

which can be written in the matrix form

$$\Gamma_{(\nu+1)} = B^{\nu} \Gamma_{(1)} . \quad (C.15)$$

$\Gamma_{(\nu)} = \{\gamma_{\nu}^{\ell}\}$ is a column vector and $B = \{b_{\ell m}\}$ a matrix 7×7 . Using eqs. (C.7) and (C.14), eq. (C.6) becomes

$$U_{\text{eff}} = \sum_{\nu=0}^{\infty} \sum_{k=1}^7 \sum_{m=1}^7 (b_{km})^{\nu} \gamma_1^m A_k , \quad (C.16)$$

which can also be expressed in the matrix form

$$U_{\text{eff}} = A \left\{ \sum_{\nu=0}^{\infty} B^{\nu} \right\} \Gamma_{(1)} . \quad (C.17)$$

Here, A is a row vector the components of which are A_k .

Now the matrix B can be diagonalized to yield

$$S^{-1}BS = D \equiv \begin{pmatrix} d_1 & & 0 \\ & \ddots & \\ 0 & & d_7 \end{pmatrix}, \quad (C.18)$$

where

$$S^{-1}S = 1. \quad (C.19)$$

Inverting the relation (C.18) and inserting into eq. (C.17) and using eq. (C.19), one obtains

$$U_{\text{eff}} = A S \left\{ \sum_{v=0}^{\infty} D^v \right\} S^{-1} \Gamma_{(1)}. \quad (C.20)$$

Now, if the eigenvalues d_i satisfy the condition $|d_i| < 1$ ($i=1, \dots, 7$), then

$$U_{\text{eff}} = A S \begin{pmatrix} \frac{1}{1-d_1} & & \\ & \ddots & \\ & & \frac{1}{1-d_7} \end{pmatrix} S^{-1} \Gamma_{(1)}. \quad (C.21)$$

The diagonalization in eq. (C.18) may be performed numerically which determines the matrix S and the eigenvalues $\{d_i\}$.

Finally, the induced interaction can be obtained by applying the Pauli exchange operator on $(U_{\text{eff}} - F)$

$$U_{\text{IND}} = - P_K P_{\sigma} P_{\tau} (U_{\text{eff}} - F). \quad (C.22)$$

In section 4.2, U_{IND} is denoted by F_{induced} (see eq. (4.2.13)).

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Table Captions

Table 1: Contribution of various terms to the binding energy per particle (in units of MeV) at $k_F = 1.4 \text{ fm}^{-1}$. The results are from Ref. 11. The G-matrix result has also been added.

Table 2: Phenomenological Landau parameters in units of $C_0 = 302 \text{ MeV fm}^3$.
+ Tensor force and finite range of the interaction are considered.

Table 3: Partial-wave contributions of the wound integral K_{LL}^{JST} (as defined by eq. (1.1.18)) at normal density and initial relative momentum $k_0 = 0.95 \text{ fm}^{-1}$ and center-of-mass momentum $K = 0.97 \text{ fm}^{-1}$. The HM3A version of the OBEP has been used.

Table 4: Results of the parametrization (as defined by eq. (1.4b.2)) of the quasi-particle G-matrix interaction $\mathcal{F}$ obtained at normal density and at initial relative momentum $k_0 = 0.95 \text{ fm}^{-1}$ and center-of-mass momentum $K = 0.97 \text{ fm}^{-1}$. The HM3A version of the OBEP has been used. The coupling strengths A_i of the effective mesons are in MeV fm^3 . The OPEP with contact term removed (the column corresponding to $m_3 = 0.71 \text{ fm}^{-1}$) has been included in the central channels and in the tensor terms the regularized-OPEP (which includes the form factor) has been added. The parameters γ_i and β_i of the density dependence (as defined by eq. (1.4b.9)) are in units of fm and fm^2 , respectively.

+: The spin-orbit and tensor channels have been parametrized with functional forms which differ from those given in Ref. 42.

Table 5: Parameters of the density dependence (as defined by eq. (1.4b.11)) of the quasi-particle G-matrix interaction $\mathcal{F}$. The HM3A version of the OBEP has been used. $\gamma_i^!$ is in units of fm and $\beta_i^!$ in fm².

Table 6: Landau parameters, in units of $C_0 = 302 \text{ MeV fm}^3$, at normal density. The first column corresponds to the results using the HM3A version of the OBEP. The results in parentheses are obtained from our quasi-particle G-matrix interaction $\mathcal{F}$ as constructed in section 1.3. The second and the third columns correspond to the results obtained using the HM3B and the HM3C versions of the OBEP, respectively.

Table 7: Difference of the Landau parameters between the HM3C and the HM3A versions of the OBEP (third and first columns of Table 4) estimated with the simple model given by eq. (1.4c.2).

Table 8: Second-order contributions to the Landau parameters (in units of $C_0 = 302 \text{ MeV fm}^3$) for the nucleon-nucleon ($g_0^!$) and Δ -nucleon ($g_0^!$) _{Δ N} interactions. Exchange graph contributions were also included.

Table 9: Results of the parametrization of the quasi-particle G-matrix interaction $\mathcal{F}$, in the spin-isospin transfer representation (see eq. (1.3.7)) at $k_F = 1.36 \text{ fm}^{-1}$ (Aⁱⁿ) and at $k_F = 0.95 \text{ fm}^{-1}$ (A^{ex}). The starting energy was chosen to be $\omega = -40 \text{ MeV}$. The HEA version of the OBEP has been used. For further details see the explanation of Table 4.

Table 10: Results for the lowest excited $T=0$, $J^\pi = 0^+$, 2^+ and 3^- states in ^{160}O . The first column are the experimental values (see Ref. 82). In the case of the quadrupole 2^+ state the width Γ is given. The second column are the results using the harmonic oscillator single-particle basis with $b = 1.726$ fm. The G-matrix interaction with the fixed density ($k_F = 1.36 \text{ fm}^{-1}$) and a starting energy $\omega = -40$ MeV was used. In the third column are the results using the Woods-Saxon basis and the same G-matrix interaction as in the second column. The fourth column are the results corresponding to the density-dependent G-matrix interaction. For the breathing mode (0^+) $k_F = 1.36 \text{ fm}^{-1}$ is used, while for the surface modes (2^+ and 3^-) $k_F = 0.95 \text{ fm}^{-1}$ has been used. The single-particle basis is the same as in the third column.

Table 11: Results for the lowest excited $T=0$, $J^\pi = 0^+$, 2^+ , 3^- and 5^- states in ^{208}Pb . The first column are the experimental data (see Ref. 83). The second column corresponds to the results using the G-matrix interaction with $k_F = 1.36 \text{ fm}^{-1}$ and the starting energy $\omega = -40$ MeV. The third column are the results with the density-dependent G-matrix interaction. For the 0^+ state $k_F = 1.36 \text{ fm}^{-1}$ is used, while for the remaining states $k_F = 0.95 \text{ fm}^{-1}$ has been used. In the fourth column the results correspond to the density-dependent G-matrix interaction (as in the third column) multiplied by the reduction factor $Z^2 = 0.6$ (see eq. (1.7.5)). In all the cases the Woods-Saxon single-particle basis was used.

Table 12: Landau parameters (in units of $C_0 = 302 \text{ MeV fm}^3$) in both nucleon (f_0, f'_0, g_0, g'_0) and isobar $(g'_0)_{\Delta N}$ sectors based on the HEA potential. The experimental value $(f^*/f)=2$ was used. The row G^{NR} denotes the results derived from a G-matrix within the conventional approach, while G^{R} denotes the G-matrix results from the relativistic treatment, using $U_S = -371.5 \text{ MeV}$ and $U_0 = 289.8 \text{ MeV}$ given by eq. (3.1.6). $G^{\text{R}*}$ is the result for $U_S = -400 \text{ MeV}$ and $U_0 = 300 \text{ MeV}$. The Landau parameters obtained in Ref. 49 are displayed in the row $G^{\text{R**}}$.

Table 13: Landau parameters (in units of $C_0 = 302 \text{ MeV fm}^3$) in both nucleon (f_0, f'_0, g_0, g'_0) and isobar $(g'_0)_{\Delta N}$ sectors based on the HEA potential. The experimental value $(f^*/f)=2$ was used. The rows G^{R} and $G^{\text{R}*}$ denote the G-matrix results from the relativistic treatment (see Table 12). $F^{\text{R}}(N)$ denotes the results including medium polarization effects considering only the nucleon ring diagrams and using the results of the row G^{R} . $F^{\text{R}}(N+\Delta)$ corresponds to those with self-consistent couplings between isobars and nucleons. $F^{\text{R}*}(N)$ and $F^{\text{R}*}(N+\Delta)$ correspond to the results obtained using $G^{\text{R}*}$. In the first row the results from the G-matrix interaction within the conventional approach G^{NR} (no inclusion of relativistic effects) is also displayed for sake of comparison. The last row corresponds to the empirical values of Ref. 19.

⁺ More recent analyses favour a much smaller value of g_0 .

Table 1

G-matrix	-33
three-body clusters	-5
total potential energy	-38
kinetic energy	23
binding energy	-15

Table 2

f_0^{in}	f_0^{ex}	$f_0^{\text{'in}}$	$f_0^{\text{'ex}}$	g_0	$g_0^{\text{'}}$	Reference
-0.11	-2.92	1.08	0.08	0.91	1.15	19
0.11	-3.40	0.52	0.71	0.91	1.15	19
0.40	-4.90	3.0	3.0	1.1	1.4	20
-0.26	-2.61	1.04	1.04			21 ⁺
				1.4	$1.5 < g_0^{\text{'}} < 1.8$	22 ⁺

Table 3

SINGLET-EVEN	TRIPLET-EVEN	SINGLET-ODD	TRIPLET-ODD
$^1S_0 : 0.0107$			
	$^3S_1$ - $^3D_1$ $(L=0, L'=0): 0.0197$ $(L=0, L'=2): 0.0407$ $(L=2, L'=0): 1.0 \times 10^{-5}$ $(L=2, L'=2): 3.0 \times 10^{-5}$		$^3P_0 = 1.4 \times 10^{-3}$
$^1D_2 : 1.6 \times 10^{-4}$	$^3D_2 : 6.0 \times 10^{-4}$	$^1P_1 : 3.9 \times 10^{-3}$	$^3P_1 : 4.6 \times 10^{-3}$
	$^3D_3$ - $^3G_3$ $(L=2, L'=2): 3.0 \times 10^{-5}$ $(L=2, L'=4): 1.3 \times 10^{-3}$ $(L=4, L'=2): 1.0 \times 10^{-6}$ $(L=4, L'=4): 1.0 \times 10^{-6}$		$^3P_2$ - $^3F_2$ $(L=1, L'=1): 2.7 \times 10^{-3}$ $(L=1, L'=3): 2.3 \times 10^{-3}$ $(L=3, L'=1): 1.0 \times 10^{-6}$ $(L=3, L'=3): 7.0 \times 10^{-6}$
$^1G_4 : 1.0 \times 10^{-6}$	$^3G_4 : 6.0 \times 10^{-6}$	$^1F_3 : 7.0 \times 10^{-6}$	$^3F_3 : 1.4 \times 10^{-5}$

Table 4

CHANNEL	$m_1 = 4.0 \text{ fm}^{-1}$			$m_2 = 2.5 \text{ fm}^{-1}$			$m_3 = 0.71 \text{ fm}^{-1}$		
SL	A	γ	β	A	γ	β	A	$\Lambda \text{ (MeV)}$	
S0	3489.8578	-	-	-1072.6901	0.40738	-	1195.3360	-	
SE	1332.2011	-0.21537	-0.61972	-1809.5155	-0.15600	-0.65536	-398.4453	-	
T0	1253.1275	0.35298	-	-914.8167	0.45308	-	132.8151	-	
TE	2006.6900	0.82068	0.13303	-2627.8311	0.82708	0.06749	-398.4453	-	
+TNO	355.0412	-	-	79.9680	-	-	-132.8151	960	
+TNE	356.9312	-	-	-475.3378	-	-	398.4453	960	
+LS0	-181.4617	-	-	-135.5124	-	-	-	-	
+LSE	+852.9600	-	-	-206.6745	-	-	-	-	

TABLE 5

CHANNEL	$m_1 = 4.0 \text{ fm}^{-1}$		$m_2 = 2.5 \text{ fm}^{-1}$	
	γ'	β'	γ'	β'
SO	0.06415	-	-0.36892	-
SE	-0.43169	-1.26363	-0.47568	-1.17548
TO	-0.15332	-	-0.36518	-
TE	-1.63484	-1.79617	-1.61828	-1.56478

Table 6

	HM3A		HM3B	HM3C
f_0	(-1.08)	-1.12	-1.22	-1.39
f_1	(-0.65)	-0.77	-0.73	-0.69
f_2	(-0.23)	-0.31	-0.32	-0.30
f'_0	(0.27)	0.28	0.32	0.41
f'_1	(0.32)	0.35	0.37	0.37
f'_2	(0.15)	0.17	0.18	0.19
g_0	(0.17)	0.17	0.15	0.12
g_1	(0.24)	0.32	0.33	0.33
g_2	(0.11)	0.14	0.15	0.14
g'_0	(0.72)	0.73	0.73	0.74
g'_1	(0.04)	0.13	0.15	0.16
g'_2	(0.04)	0.04	0.04	0.05
h_0	(0.32)	0.32	0.31	0.29
h_1	(0.51)	0.52	0.52	0.51
h_2	(0.44)	0.45	0.45	0.45
h'_0	(-0.11)	-0.11	-0.10	-0.09
h'_1	(-0.17)	-0.17	-0.17	-0.17
h'_2	(-0.14)	-0.15	-0.15	-0.15

Table 7

(HM3C)-(HM3A)	ESTIMATED	EXACT (G-matrix)
Δf_0	-0.14	-0.27
$\Delta f'_0$	+0.14	+0.13
Δg_0	+0.003	-0.05
$\Delta g'_0$	+0.04	+0.01

Table 8

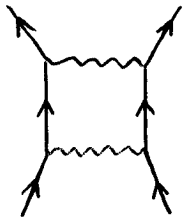
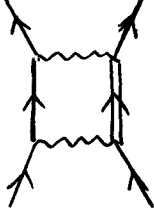
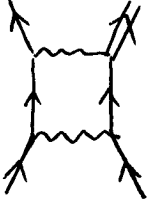
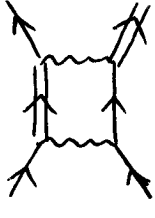
(g_0')		$(g_0')_{\Delta N}$	
			
0.138	0.012	0.010	0.004

Table 9

CHANNEL	$m_1 = 4.0 \text{ fm}^{-1}$	$m_2 = 2.5 \text{ fm}^{-1}$	$m_3 = 0.71 \text{ fm}^{-1}$			
	A^{in}	A^{ex}	A^{in}	A^{ex}	$A^{in}=A^{ex}$	$\Lambda \text{ (MeV)}$
1	3084.745	3224.994	-2195.210	-2365.745	-	-
$\tau_1 \cdot \tau_2$	-469.734	-552.469	391.106	493.572	-	-
$\sigma_1 \cdot \sigma_2$	-286.598	-254.897	171.321	131.515	-	-
$\sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2$	810.983	798.676	-285.104	-260.746	132.815	-
$i(\sigma_1 + \sigma_2) \cdot n$	146.627	95.757	-192.669	-132.574	-	-
$i(\sigma_1 + \sigma_2) \cdot n \tau_1 \cdot \tau_2$	-330.450	-316.568	1.203	38.582	-	-
$S_{12}(q)$	356.750	822.556	-60.430	-142.357	-	960
$S_{12}(q) \tau_1 \cdot \tau_2$	-1119.104	-1362.150	376.490	416.969	-132.815	960

Table 10

$^{16}_0$		Exp.	$G(k_F = 1.36)$ (Harm. Oscill.)	$G(k_F = 1.36)$ (Woods-Saxon)	$G(k_F)$ (Woods-Saxon)
0^+	E (MeV)		14.04	22.30	22.30
	$B(E,0)$ ($e^2\text{fm}^2$)		135	45	45
2^+	E (MeV)	21.	22.95	26.93	26.16
	$B(E,2)$ $e^2\text{fm}^4$	$\Gamma \sim 6 \text{ MeV}$	70	30	31
3^-	E (MeV)	6.13	5.74	7.11	6.25
	$B(E,3)$ ($e^2\text{fm}^6$)	1490	1635	1214	1434

Table 11

^{208}Pb		Exp.	$G(k_F = 1.36)$ (Woods-Saxon)	$G(k_F)$ (Woods-Saxon)	$Z^2G(k_F)$ (Woods-Saxon)
0^+	E (MeV)	12.5-15			6.11
	$B(E,0)$ ($e^2\text{fm}^2$)		unstable	unstable	8271
2^+	E (MeV)	4.086			4.74
	$B(E,2)$ ($e^2\text{fm}^6$)	2965	unstable	unstable	428
3^-	E (MeV)	2.615			1.82
	$B(E,3)$ ($e^2\text{fm}^6$)	$(540 \pm 30) \times 10^3$	unstable	unstable	1063×10^3
5^-	E (MeV)	3.197	1.49		3.37
	$B(E,5)$ ($e^2\text{fm}^{10}$)	$(462 \pm 55) \times 10^6$	4512×10^6	unstable	93×10^6

Table 12

	f_0	f'_0	g_0	g'_0	$(g'_0)_{\Delta N}$
G^{NR}	-1.17	0.32	0.20	0.63	0.91
G^R	-0.72	0.42	0.12	0.63	1.05
G^{R*}	-0.58	0.39	0.12	0.61	1.07
G^{R**}	-0.45	0.32	0.19	0.53	

Table 13

	f_0	f'_0	g_0	g'_0	$(g'_0)_{\Delta N}$
G^{NR}	-1.17	0.32	0.20	0.63	0.91
G^R	-0.72	0.42	0.12	0.63	1.05
$F^R(N)$	-0.34	0.37	0.15	0.67	1.47
$F^R(N+\Delta)$	-0.32	0.35	0.13	0.67	1.23
G^{R*}	-0.58	0.39	0.12	0.61	1.07
$F^{R*}(N)$	-0.26	0.33	0.13	0.64	1.48
$F^{R*}(N+\Delta)$	-0.23	0.31	0.10	0.64	1.24
Exp.	0.11	0.52	0.91^+	1.15	

Figure Captions

Fig. 1: Effective interaction between electrons in an electron gas. The dashed lines denote the bare Coulomb interaction. The series of ring diagrams on the r.h.s. renormalizes the bare interaction.

Fig. 2: Diagrammatic representation of the Brueckner G-matrix.

Fig. 3: Three-body cluster diagrams.

Fig. 4: Diagrammatic representation of the G-matrix in the particle-particle channel.

Fig. 5: Selfconsistent single-particle potential $U(k_\mu)$, at $k_\mu=k_F$, as a function of the Fermi-momentum k_F . The HM3A version of the OBEP has been used.

Fig. 6: Exchange momentum transfer (Q) dependence of the quasi-particle interaction $\mathcal{F}_D$ (symmetric part as defined in section 1.3) (solid lines) obtained at normal density and at $q=0$, and using the HM3A version of the OBEP. Figures a, b, c and d are the central F_D , iso-spin F_D^I , spin G_D and spin-isospin G_D^I channels, respectively. The dashed lines are the bare NN potential (HM3A) in the corresponding channels and normalized at $Q=0$ for comparison.

Fig. 7: Central components of the quasi-particle interaction $\mathcal{F}$ (solid lines), at normal density at and initial relative momentum $k_0 = 0.95$

fm^{-1} and center-of-mass momentum $K = 0.97 \text{ fm}^{-1}$, as a function of the momentum transfer q . The HM3A version of the OBEP has been used. The dashed lines are the bare NN potential (HM3A) in the corresponding channels.

Fig. 8: Defect function $F_{LL}^{JST}(K, k, k_0)$ (solid lines), at normal density and at initial relative momentum $k_0 = 0.95$ and center-of-mass momentum $K = 0.97 \text{ fm}^{-1}$, as a function of the relative momentum k . Figures a, b, c, d and e correspond to the $F_{00}^{001}(^1S_0)$, $F_{00}^{110}(^3S_1-^3D_1; L=L'=0)$, $F_{02}^{110}(^3S_1-^3D_1; L=0, L'=2)$, $F_{00}^{110}(^3S_1-^3D_1; L=L'=0)$ and $F_{02}^{110}(^3S_1-^3D_1; L=0, L'=2)$ contributions respectively. The dashed lines are the partial-wave contributions kV_{LL}^{JST} of the bare NN potential (HM3A). Figures a, b, c, d and e correspond to the kV_{00}^{001} , kV_{00}^{110} , kV_{02}^{110} , kV_{20}^{110} and kV_{22}^{110} respectively. Figures a, b and c give contributions to the central channels of the G-matrix interaction, while d and e contribute to the tensor forces. kV_{LL}^{JST} is dimensionless, while F_{LL}^{JST} is in units of fm^2 .

Fig. 9: Spin-orbit components of the quasi-particle interaction $\mathcal{F}$ (solid lines). The dashed lines are the bare NN potential (HM3A) in the corresponding channels. The Fermi momentum is $k_F = 1.36 \text{ fm}^{-1}$; the initial relative momentum is $k_0 = 0.95 \text{ fm}^{-1}$, while the center-of-mass momentum is $K = 0.97 \text{ fm}^{-1}$.

Fig. 10: Tensor components of the quasi-particle interaction $\mathcal{F}$ (solid lines). The dashed lines are the bare NN potential V in the corresponding channels. Essentially no difference between $\mathcal{F}$ and V is found. The

Fermi momentum is $k_F = 1.36 \text{ fm}^{-1}$; the initial relative momentum is $k_0 = 0.95 \text{ fm}^{-1}$, the center-of-mass momentum is $K = 0.97 \text{ fm}^{-1}$.

Fig. 11: Comparison between the interaction which includes correlations from only the $^1S_0$ and $^3S_1$ - $^3D_1$ states (dashed lines) and the exact interaction (solid lines) of Fig. 7. Note that in the G' -channel no difference is found in this scale.

Fig. 12: Density dependence of the quasi-particle interaction $\mathcal{F}$, at initial relative momentum $k_0 = 0.95$ and center-of-mass momentum $K = 0.97 \text{ fm}^{-1}$, as a function of the momentum transfer q . The HM3A version of the OBEP has been used. Figures a, b, c and d correspond to the central F , isospin F' , spin G and spin-isospin G' channels, respectively. The solid lines correspond to $k_F = 1.36 \text{ fm}^{-1}$; the dashed lines to $k_F = 1.25 \text{ fm}^{-1}$; the dashed-dotted lines to $k_F = 1.10 \text{ fm}^{-1}$ and the long-dashed lines to $k_F = 0.95 \text{ fm}^{-1}$.

Fig. 13: Density dependence of the integral $I(K, k_0; k_F)$ (as defined by eq. (1.4b.6)) at initial relative momentum $k_0 = 0.95$ and center-of-mass momentum $K = 0.97 \text{ fm}^{-1}$. α and $a(\text{fm}^{-1})$ are the strength and the range of the interaction, respectively.

Fig. 14: The σ -coupling strength $g_{NN\sigma}^2$ (square) and the δ -coupling strength $g_{NN\delta}^2$ (cycle) versus the pion cut-off parameter Λ_π , corresponding to the HM3A, HM3B and HM3C versions of the OBEP given in Ref. 59.

Fig. 15: Central components of the exact G-matrix interaction (solid lines) of sub-section 1.4a and the corresponding results from the simple model as described in section 1.5 (dashed lines) at $k_F = 1.36 \text{ fm}^{-1}$.

Fig. 16: Comparison between the exact G-matrix interaction (solid lines) and the effective interaction used by the Jülich-Stony Brook group⁷⁵⁾ in the spin-isospin (G') and isovector tensor (T') channels.

Fig. 17: Diagrammatic representation of the Δ -nucleon G-matrix interaction as given by eq. (1.6.1).

Fig. 18: Defect function $F_{LL}^{JST}(K, k, k_0)$ for $J=S=0$, $T=1$ and $L=L'=0$ (solid line) at normal density and at initial relative momentum $k_0 = 0.95$ and center-of-mass momentum $K = 0.97 \text{ fm}^{-1}$, as a function of the relative momentum k . The dashed line is the corresponding partial-wave contribution $k(V_{LL'})_{\Delta N}$ of the bare Δ -nucleon transition potential for $L=2$, $L'=0$. The defect function is in units of fm^2 , while $k(V_{LL'})_{\Delta N}$ is dimensionless.

Fig. 19: Diagrammatic representation of the lowest-order induced contribution $[I^{ph}(E)]$ and the self-energy insertion contribution in both the particle $[P^p(E)]$ and the hole $[P^h(E)]$ lines.

Fig. 20: Moduli of the central components of the G-matrix interaction at $E_p = 50 \text{ MeV}$ for densities $\rho = 0.0$ (dashed lines), $0.34 \rho_0$ (dash-dotted lines), $0.5 \rho_0$ (long-dashed lines) and ρ_0 (double-dashed lines), where ρ_0 denotes normal nuclear matter density ($\rho_0 \approx 0.17$ particles per fm^3).

Fig. 21: Same quantities as in Fig. 20 at $E_p = 210$ MeV for densities $\rho = 0.0$ (dashed lines) and ρ_0 (double dashed lines).

Fig. 22: Same quantities as in Fig. 20 at $\rho=0$ for $E_p = 50$ (dashed lines) 100 (dash-dotted lines), 140 (long-dashed lines) and 210 MeV (double dashed lines).

Fig. 23: Upper part:

The product of the intermediate momentum k' , the bare interaction $V_{02}(k_0, k')$ and the D-state defect function $F_{22}(k_0; k', k_0)$ describing the correlated contribution of the transition from a D- to an S-state via a D-state is shown as a function of k' . Two different incident energies of $E_p = 50$ MeV (curve (a)) and $E_p = 210$ MeV (curve (b)), which correspond to the initial relative momentum of $k_0 = 0.78$ and 1.59 fm^{-1} , respectively, are displayed. The final relative momentum k_f has been chosen to be equal to k_0 .

Middle part:

The same quantity as in the upper part is shown now for three different densities ρ : $\rho=0$ (curve 1), $\rho = 0.53 \rho_0$ (curve 4) and $\rho = \rho_0$ (curve 5), where ρ_0 stands for the normal density of nuclear matter. Here, the values of k_0 , as well as of the center-of-mass momentum K , have been determined from the local momentum of a particle with an asymptotic energy of $E_p = 50$ MeV that interacts with a nucleon having momentum $3/4 k_F$ inside the nucleus at a density corresponding to the Fermi momentum $k_F = (\frac{3\pi^2}{2} \rho)^{1/3}$.

Lower part:

The same quantity as in the middle part is shown for densities $\rho=0$ (curve 1), $\rho = 0.34 \rho_0$ (curve 3) and $\rho=\rho_0$ (curve 5) at the incident energy $E_p = 20$ MeV.

Fig. 24: The real part of the isovector tensor t-matrix interaction is shown as a function of momentum transfer q . The incident energies are $E_p = 20$ MeV (1), 50 MeV (2), 100 MeV (3), 140 MeV (4) and 210 MeV (5). The isovector tensor t-matrix interaction extracted from the SP84 nucleon-nucleon amplitudes of Arndt and Roper⁸⁷⁾ corresponding to these last four energies are given by the dotted curves. The isovector tensor interaction arising from the bare one-pion and one-rho exchange processes is represented by the solid line, (6).

Fig. 25: Upper part:

The real part of the isovector tensor interaction in the medium is shown as a function of the momentum transfer for five different densities. The densities are $\rho = 0$ (1), $0.11 \rho_0$ (2), $0.34 \rho_0$ (3), $0.53 \rho_0$ (4) and ρ_0 (5). The energy of the incident nucleon is $E_p = 20$ MeV.

Middle part:

Same quantity as in the upper part for incident energy of $E_p = 50$ MeV.

Lower part:

Same quantity as in the upper part for an incident nucleon energy of 210 MeV. The densities are $\rho = 0$ (1) and $\rho = \rho_0$ (5). For comparison, the isovector tensor interaction arising from bare one-pion and one-rho exchange is displayed by the solid line, (6).

Fig. 26: Moduli of the tensor components of the G-matrix interaction at $E_p = 50$ MeV for densities $\rho = 0.0$ (dashed lines), $0.34 \rho_0$ (dash-dotted lines), $0.5 \rho_0$ (long-dashed lines) and ρ_0 (double dashed lines). ρ_0 denotes the normal nuclear matter density.

Fig. 27: Comparison of calculated differential elastic cross section with experimental data for $^{12}\text{C}(p,p)$ at $E_p = 200$ MeV, which indicates how well our G-matrix interaction generates distorted waves.

Fig. 28: Comparison of DWIA results using the 210 MeV G-matrix with experimental data for excitation of the 2^+ ($E_x = 4.44$ MeV, $T=0$) state in ^{12}C at $E_p = 200$ MeV. The distorted waves have been generated consistently using the optical potential from a folding model.

Fig. 29: Same quantities as in Fig. 28, using the phenomenological optical potential. The solid line corresponds to the results using our G-matrix interaction of chapter II; the dash-dotted line corresponds to the results using the G-matrix interaction of Ref. 91, while the dashed line corresponds to those using the t-matrix of Ref. 92.

Fig. 30: Comparison of DWIA results using the 140 MeV G-matrix with experimental data for excitation of the isobaric analog of the ^{90}Zr ground state at $E_p = 120$ MeV. The distorted waves have been generated consistently using the optical potential from a folding model. The normalization factor is $N = 1.54$.

Fig. 31: Same quantities as in Fig. 30, using the phenomenological optical potential. The solid line denotes the results using the G-matrix interaction of chapter II. The dash-dotted line corresponds to the results using the G-matrix of Ref. 91, while the dashed line corresponds to those using the t-matrix interaction of Ref. 92.

Fig. 32: Scalar-isoscalar component (direct term) of the σ -meson exchange potential including relativistic effects (solid line) as a function of momentum transfer q . The dashed line corresponds to that of the conventional approach (no inclusion of relativistic effects).

Fig. 33: Same quantities as in Fig. 32 from the ω -meson exchange contribution.

Fig. 34: Spin-isospin G_0^1 and isovector tensor T_0^1 components (direct term) of the ρ -meson exchange potential as a function of momentum transfer. The solid lines denote the results including relativistic effects, while the dashed lines are those from the conventional approach.

Fig. 35: Central components of the antisymmetrized G-matrix interaction, based on the HEA potential, at normal density. The initial (k_0) and

final (k) relative momenta are $k = k_0 = 0.95 \text{ fm}^{-1}$, while the center-of-mass momentum is $K = 0.97 \text{ fm}^{-1}$. The solid curves are the results including relativistic effects, while the dashed curves are those of the conventional approach.

Fig. 36: Spin-orbit components of the antisymmetrized G-matrix interaction. See Fig. 35 for details.

Fig. 37: Tensor components of the antisymmetrized G-matrix interaction. See Fig. 35 for details.

Fig. 38: a) Contribution to the ground state energy arising from the virtual nuclear collective excitations. The shaded blocks denote the virtual collective excitations.
b) Graphical representation of the double differentiation of the contribution to the ground state energy (as in Fig. 38a) with respect to the quasi-particle occupation number.
c) Induced contribution to the effective particle-hole interaction which accounts for the correlations built up by virtual collective excitations.

Fig. 39: Diagrammatic representation of the Bethe-Salpeter equation (4.1.4).

Fig. 40: All possible time-independent blocks which may be obtained from the effective interaction Γ as given by eq. (4.1.4).

Fig. 41: Contribution of $\Gamma_{(2)}$ to the particle-particle interaction $\Gamma_{(1)}$. The corresponding lowest order contribution is also shown, which may be obtained by the particle-particle transformation of a second order particle-hole interaction $\Gamma_{(4)}$ (r.h.s.). The bracket denotes the particle-particle transformation.

Fig. 42: a) A lowest order contribution of $\Gamma_{(2)}$ to the particle-hole interaction $\Gamma_{(4)}$ (r.h.s.), which may be obtained by the particle-hole transformation of a second order particle-particle interaction $\Gamma_{(1)}$ (l.h.s.). The bracket denotes the particle-hole transformation.

b) Another lowest order contribution of $\Gamma_{(2)}$ to the particle-hole interaction $\Gamma_{(4)}$ (r.h.s.), which may be obtained by applying the Pauli exchange operator ($\langle P_K P_\sigma P_\tau \rangle$) on a second order particle-hole interaction $\Gamma_{(4)}$. Here, the bracket denotes the Pauli exchange operation.

Fig. 43: Diagrammatic representation of the bare NN interaction v (direct + exchange) (a), equations (4.1.8) (b), (4.1.9) (c), (4.1.6) (d) and (4.1.7) (e).

Fig. 44: Diagrammatic representation of eq. (4.1.10).

Fig. 45: Induced contribution (integrand of eq. (4.2.9)) to the spin-isospin channel of the effective particle-hole interaction as a function of momentum transfer. The dashed line denotes the contribution arising only from the spin-isospin component of the G-matrix interaction

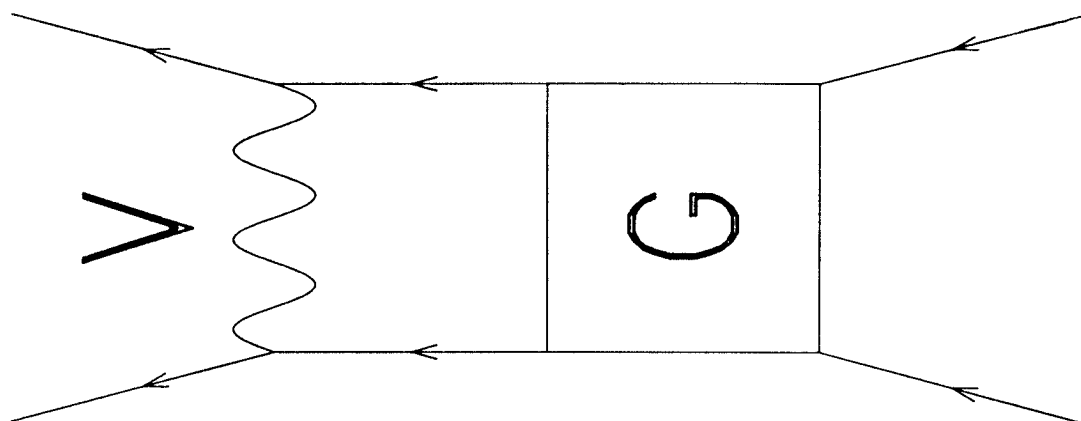
based on the HEA potential, while the long-dashed line corresponds to that arising only from the isovector tensor component. The solid line corresponds to the sum of them.

Fig. 46: Diagrammatic representation of the self-consistent coupled equations (4.2.13) for the particle-hole and Δ -hole interactions.

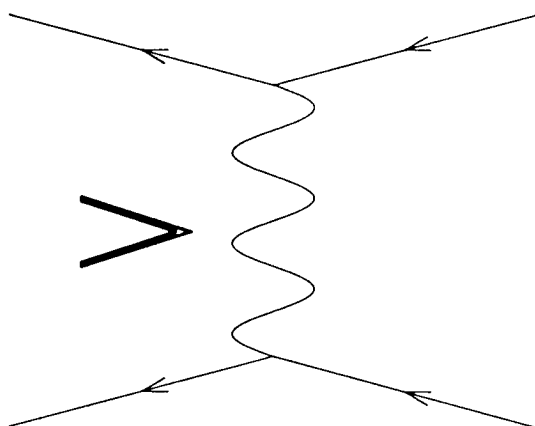


$$\begin{array}{c}
 \vdots \\
 + \\
 \begin{array}{c} \text{Diagram 1: Two vertices connected by a two-loop bubble diagram.} \end{array} \\
 + \\
 \begin{array}{c} \text{Diagram 2: Two vertices connected by a one-loop bubble diagram.} \end{array} \\
 + \\
 \begin{array}{c} \text{Diagram 3: Two vertices connected by a vertical dashed line.} \end{array} \\
 = \\
 \begin{array}{c} \text{Diagram 4: Two vertices connected by a vertical wavy line.} \end{array}
 \end{array}$$

Fig. 1



+



=

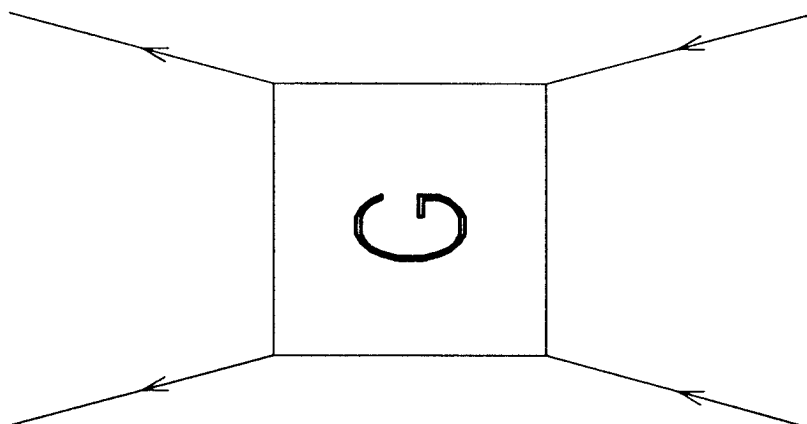


Fig. 2

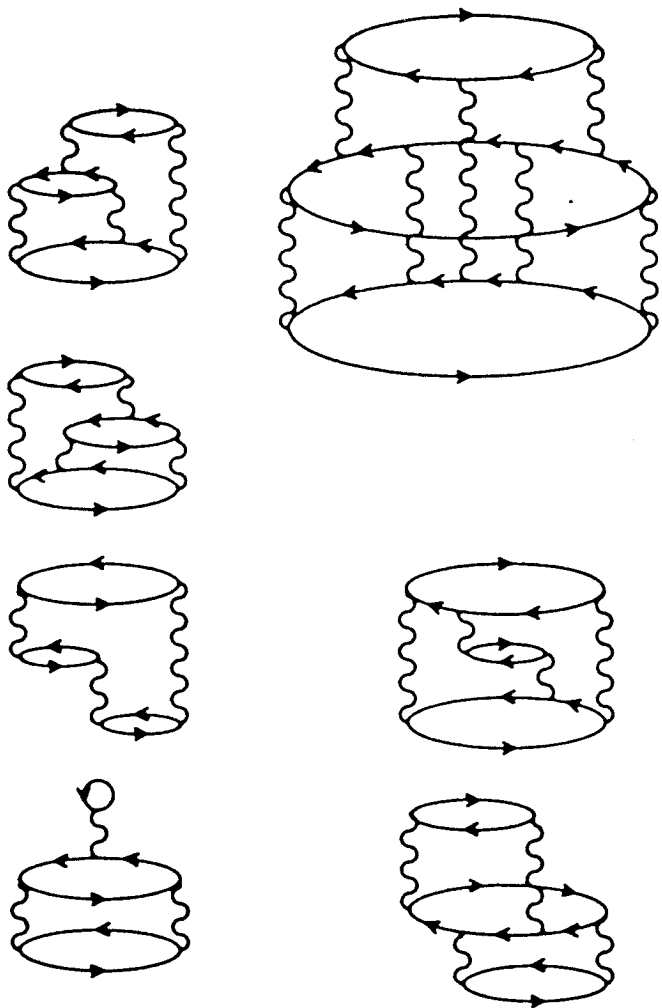
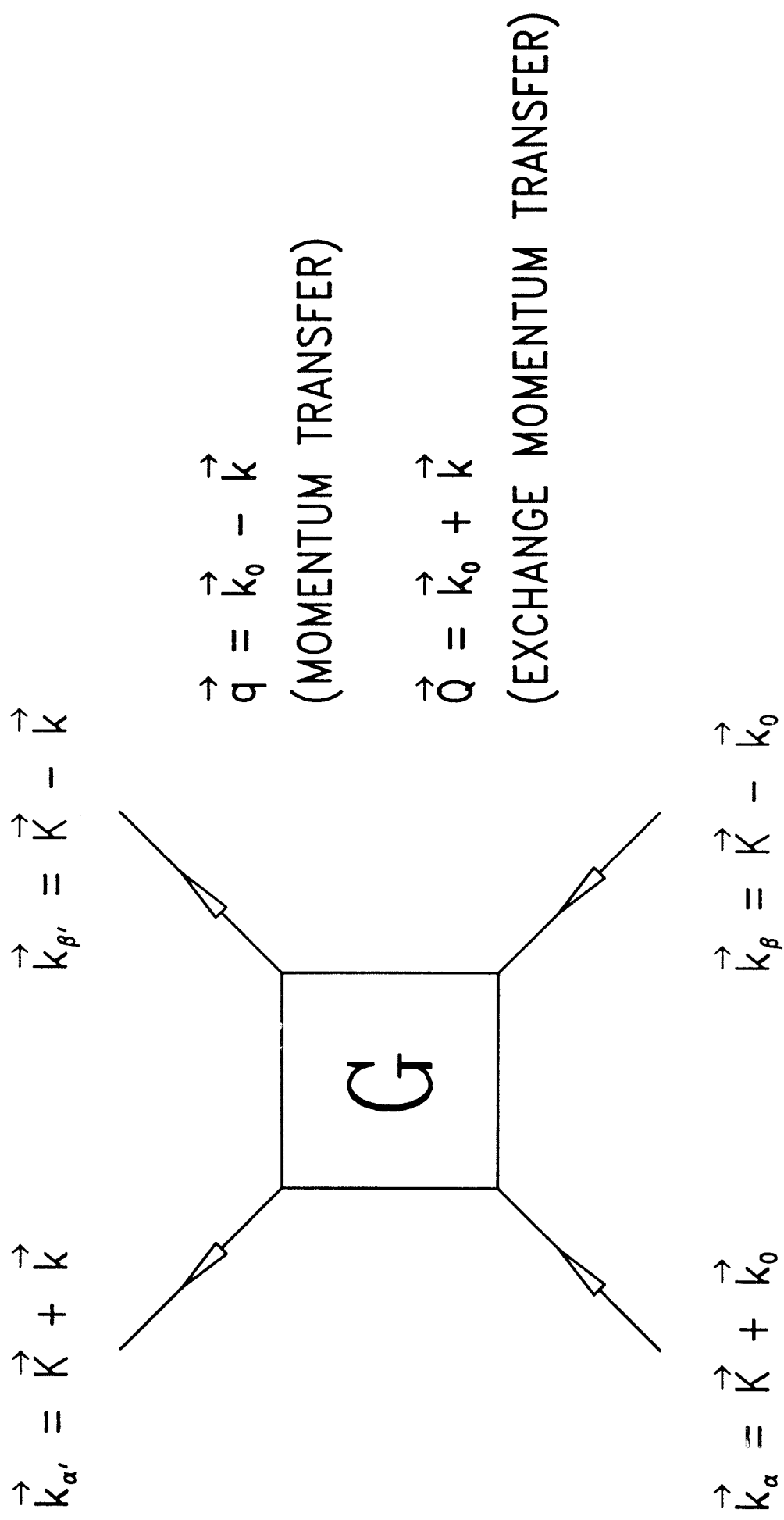


Fig. 3



PP - CHANNEL

Fig. 4

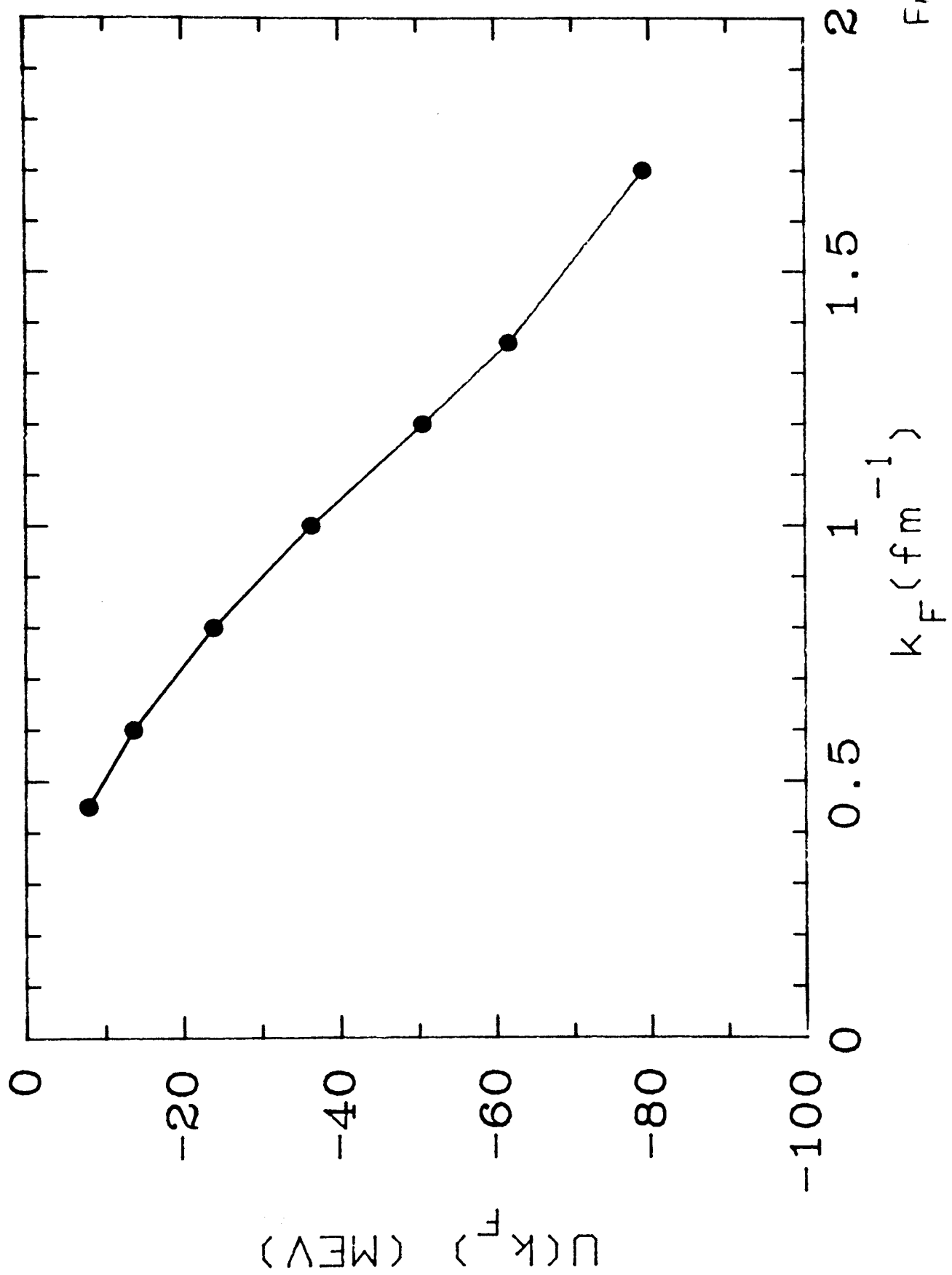


Fig. 5

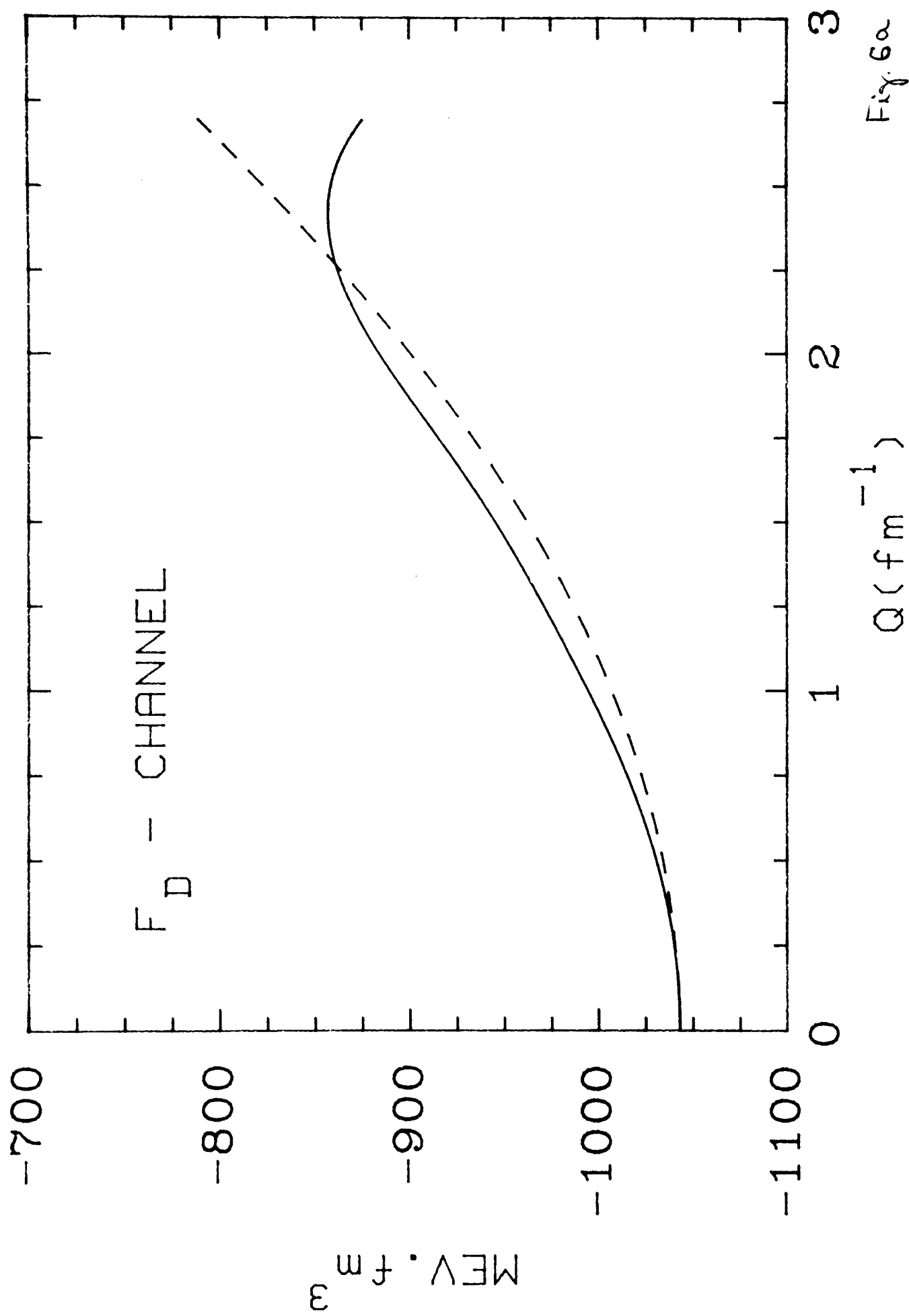


Fig. 6a

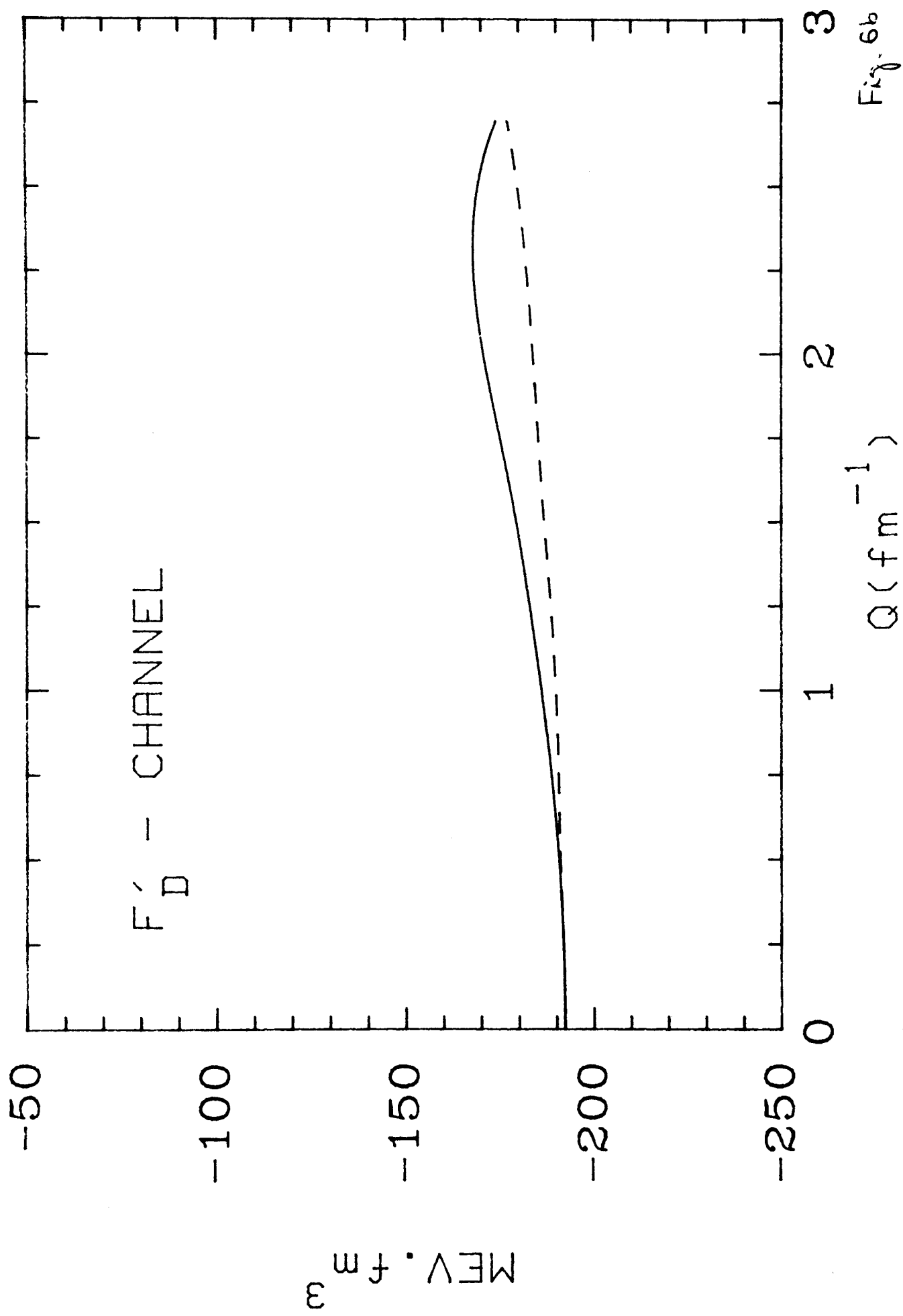


Fig. 6b

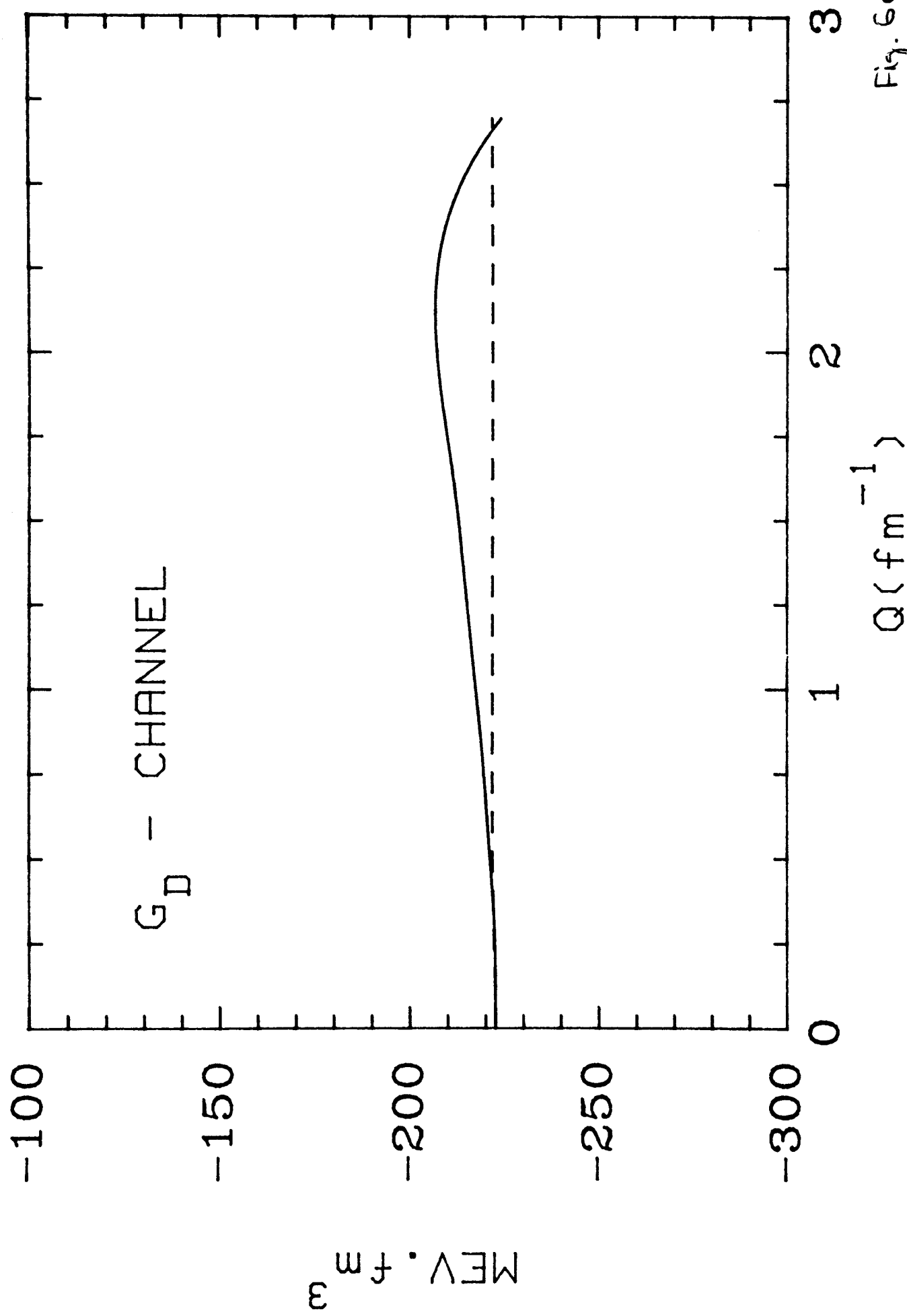


Fig. 6c

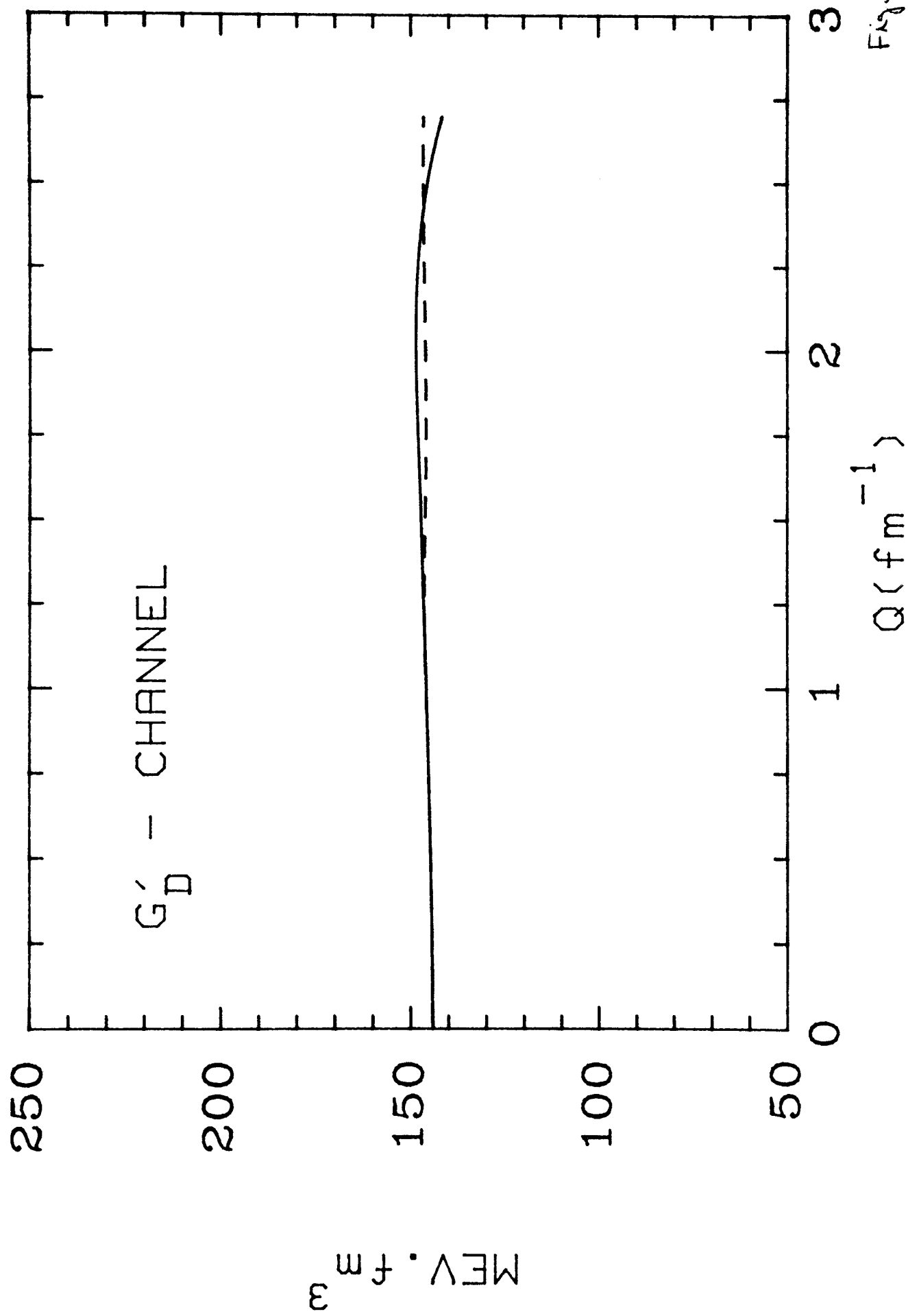


Fig. 6d

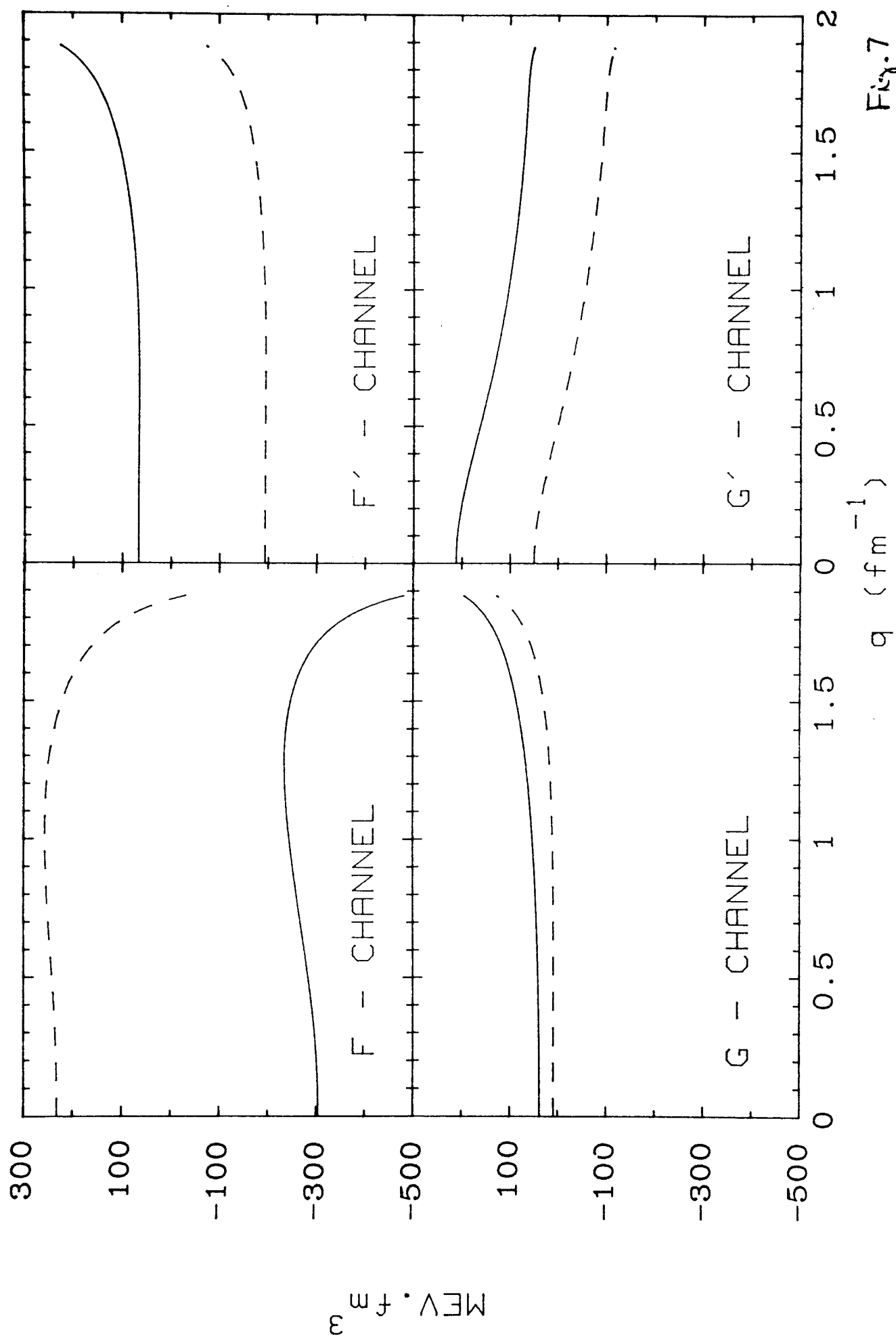


Fig. 7

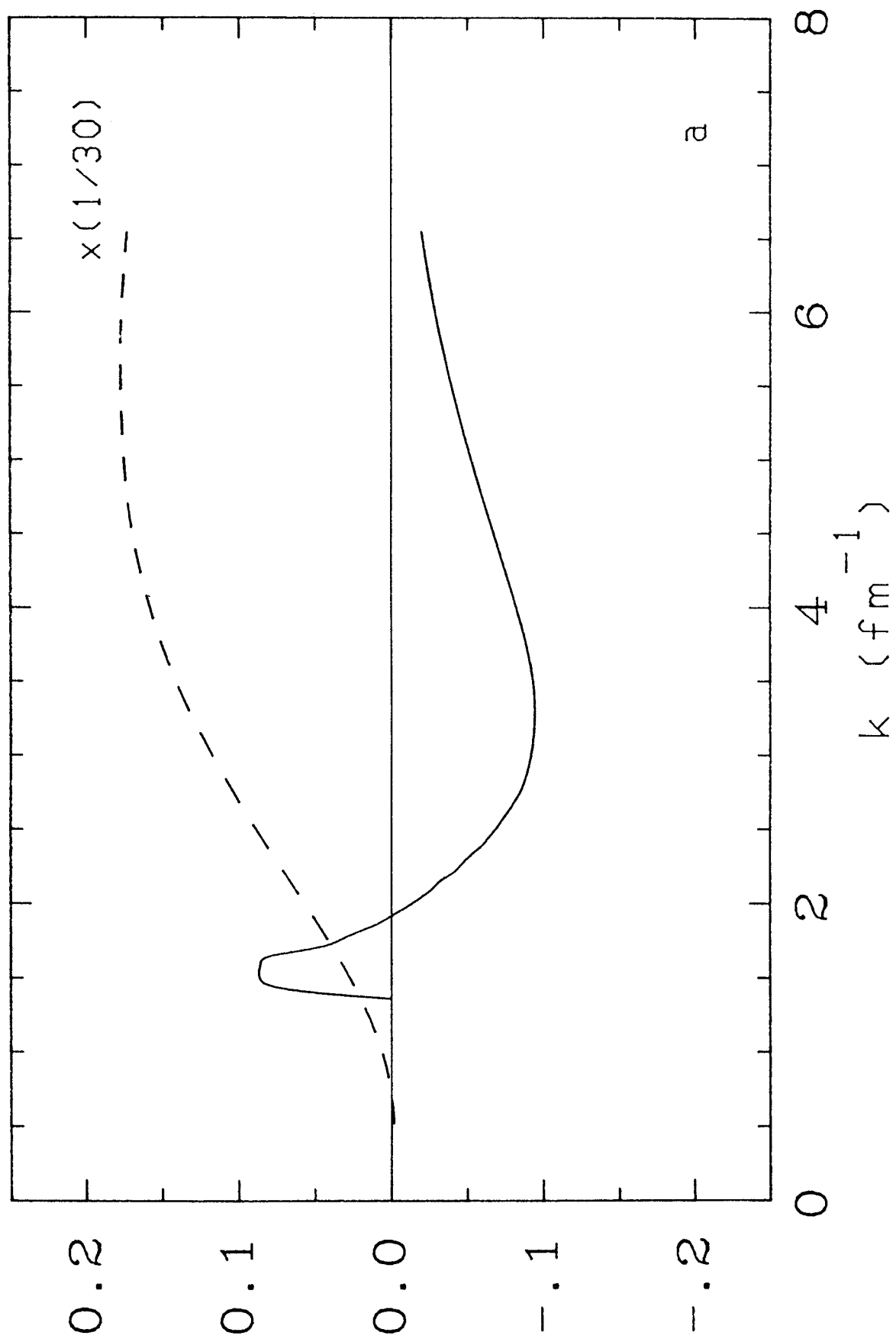


Fig. 8

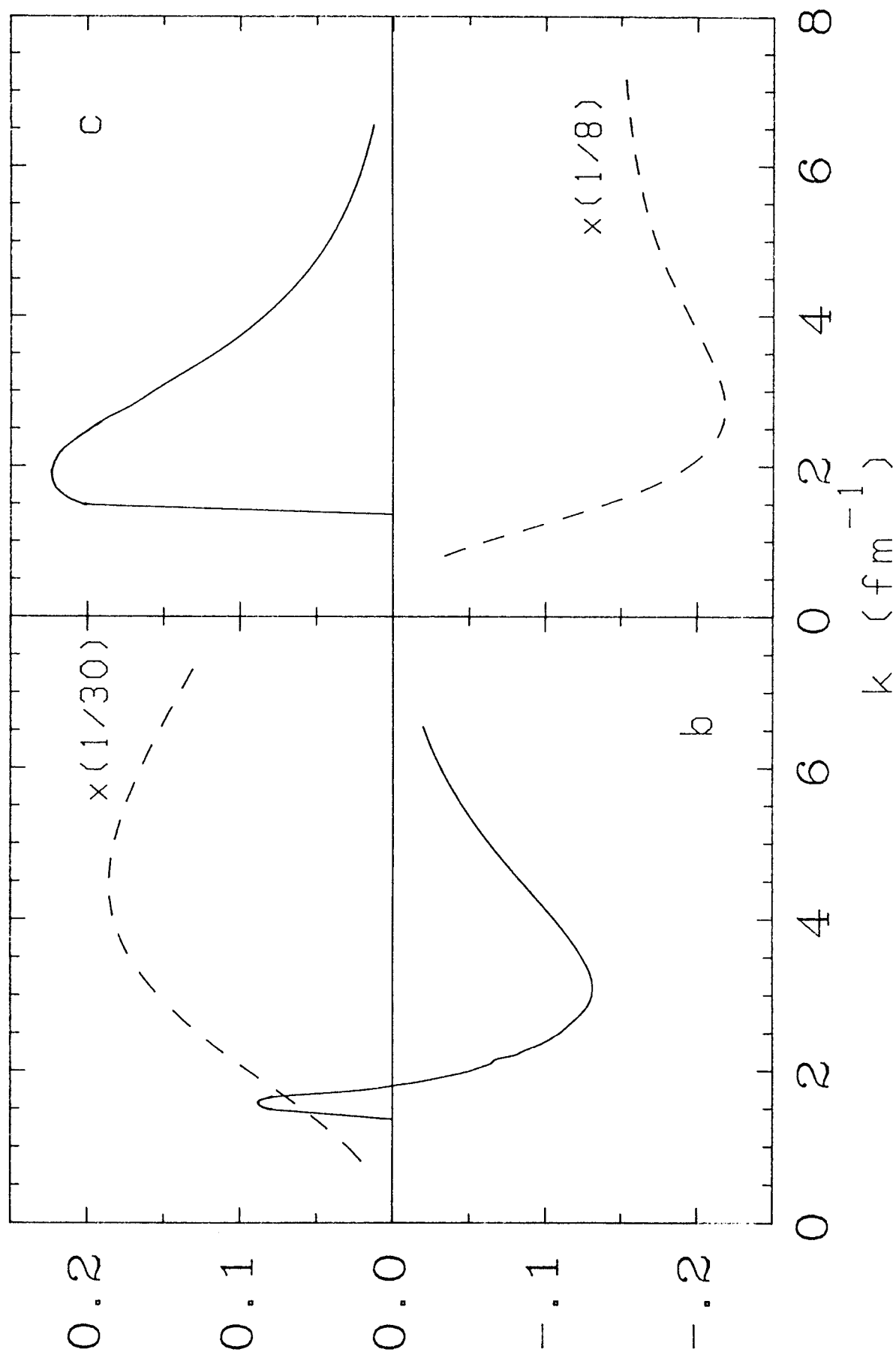


Fig. 8

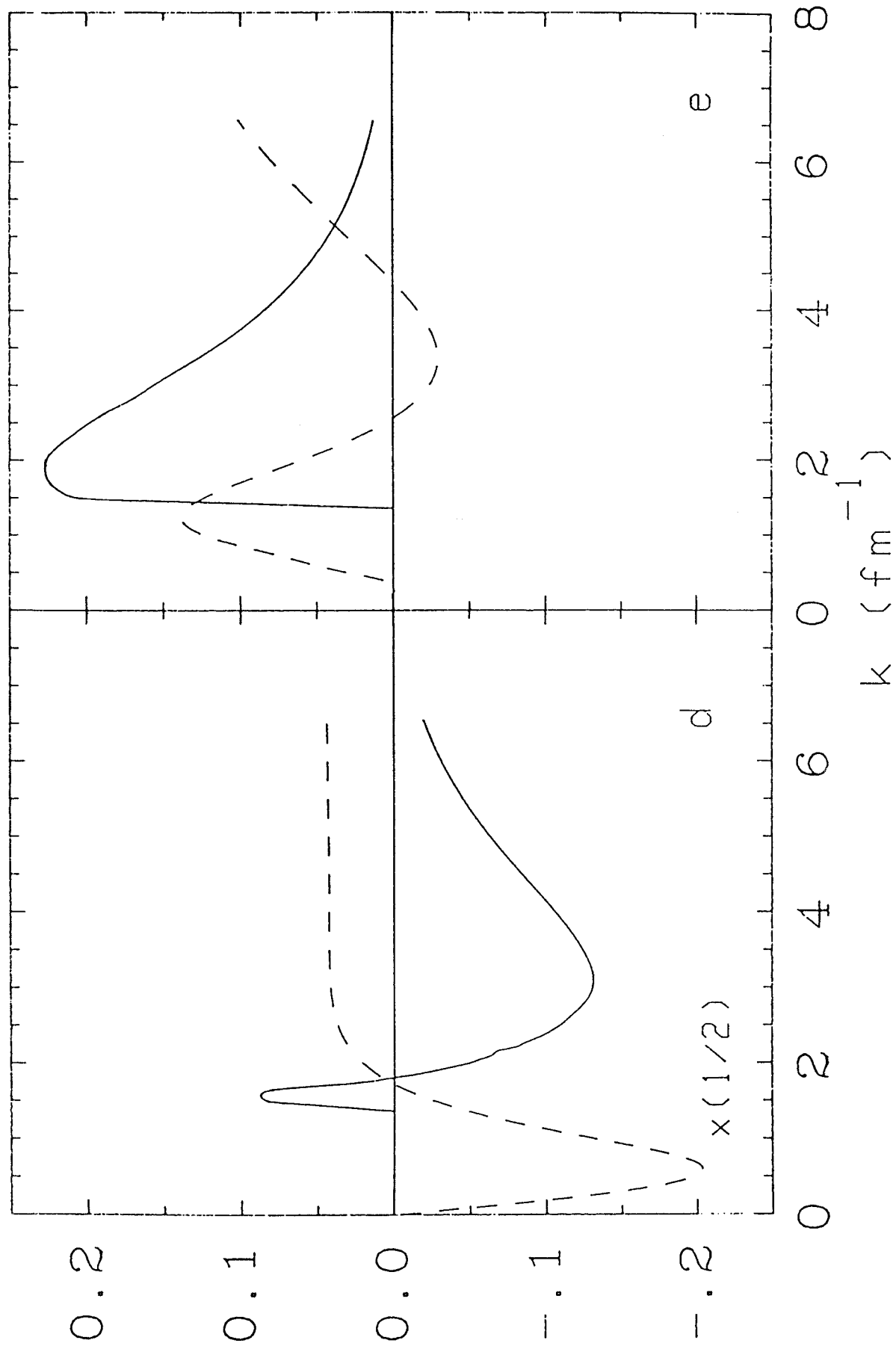


Fig. 8

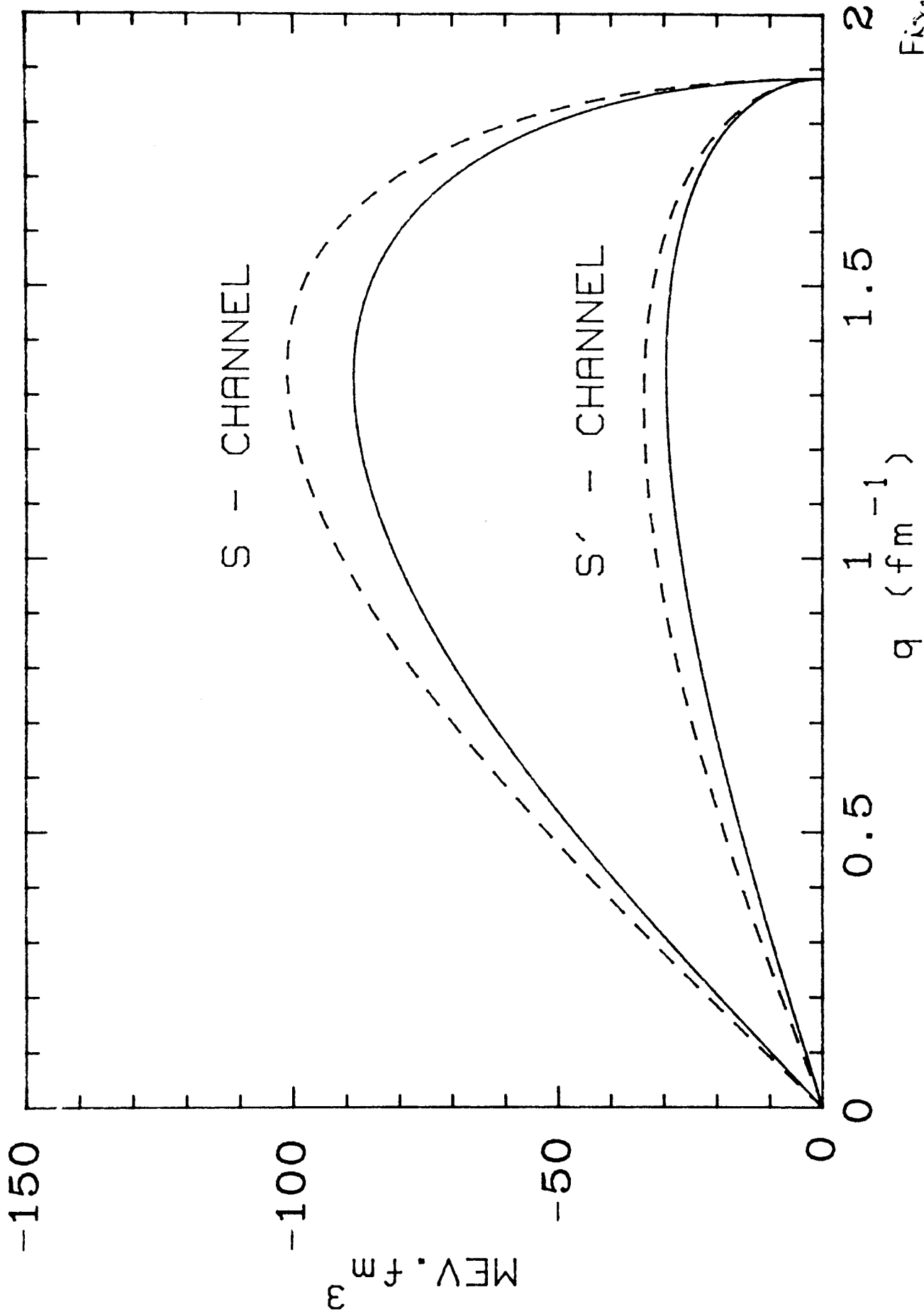


Fig. 9

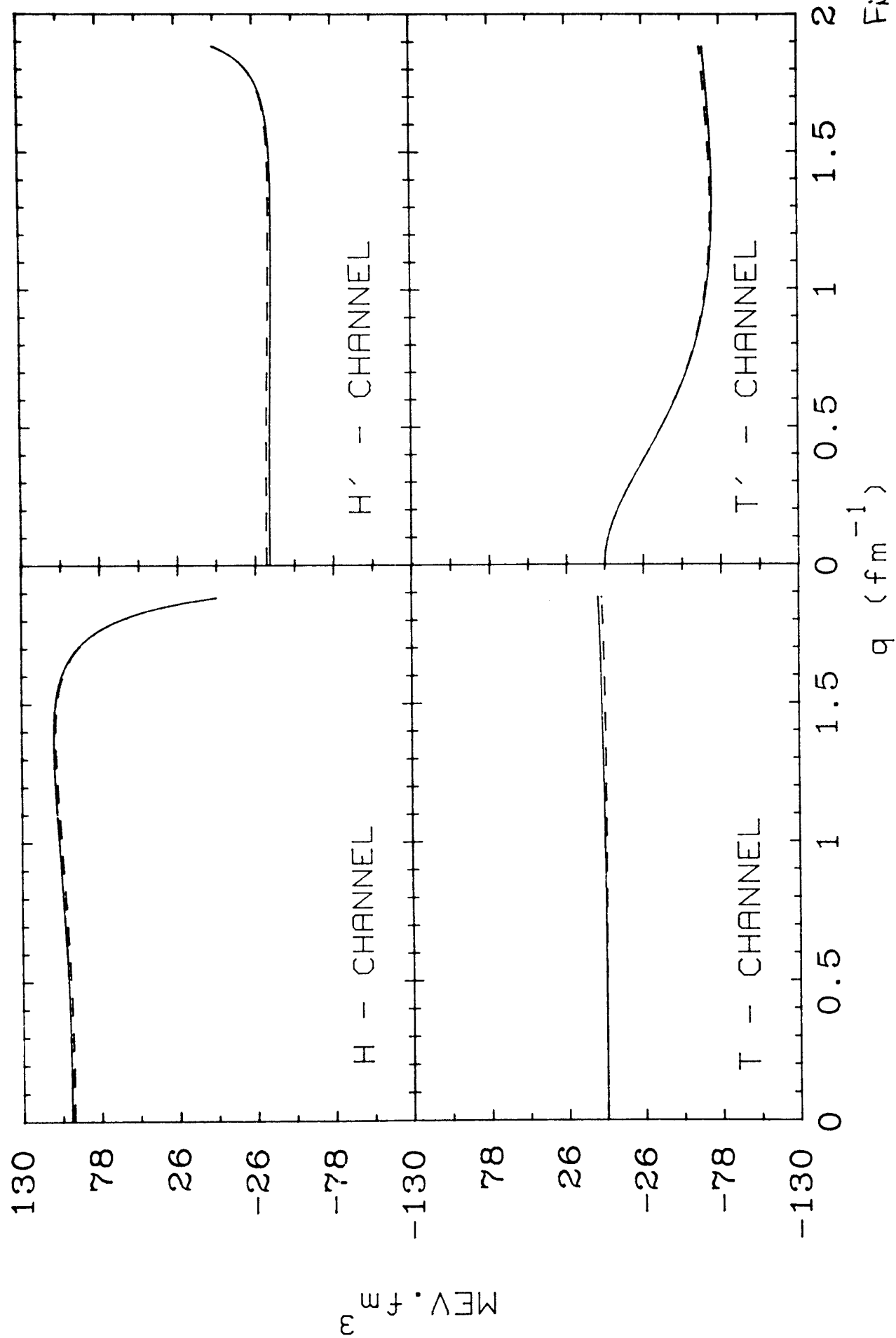


Fig. 10

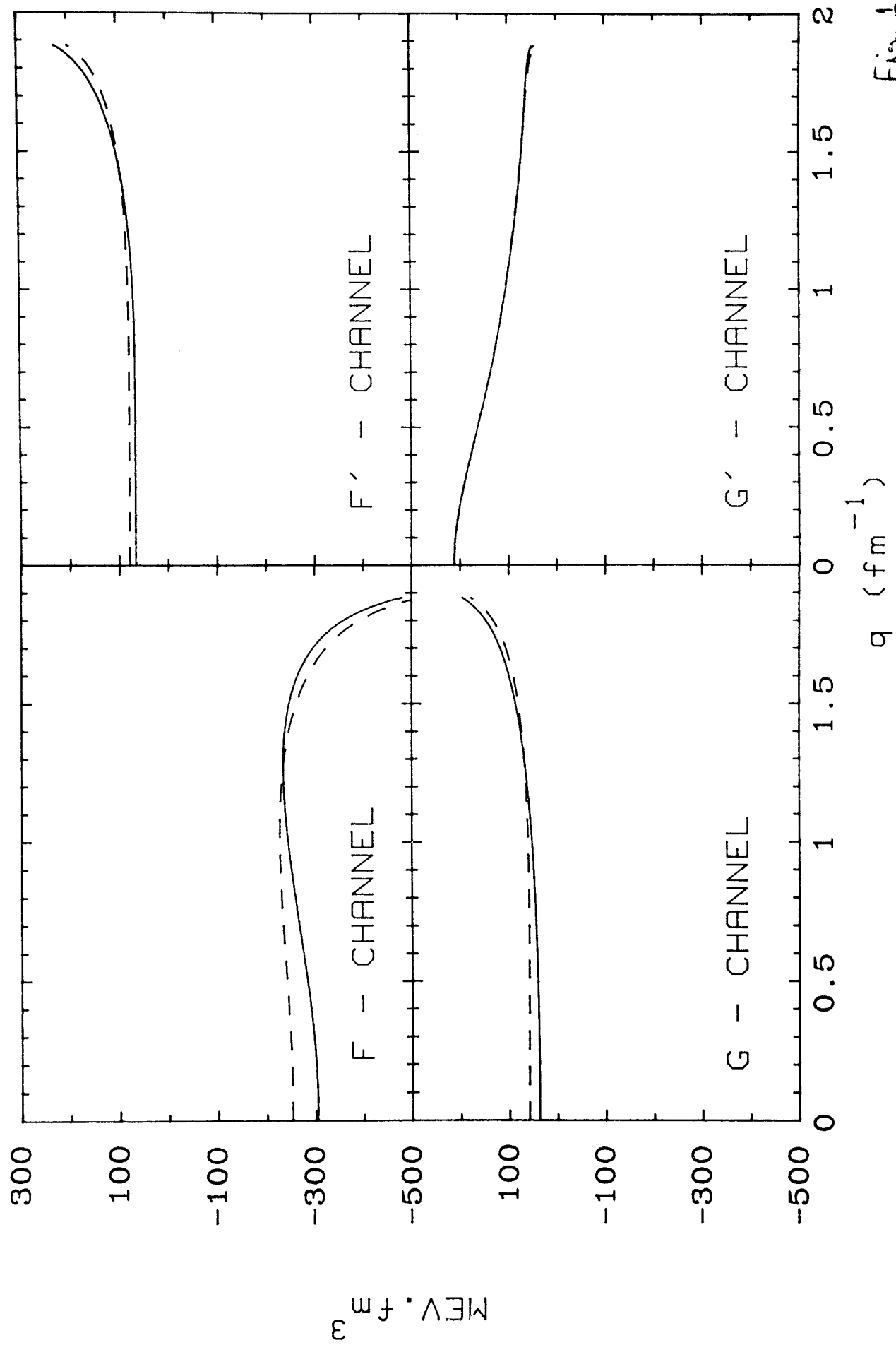


Fig. 11

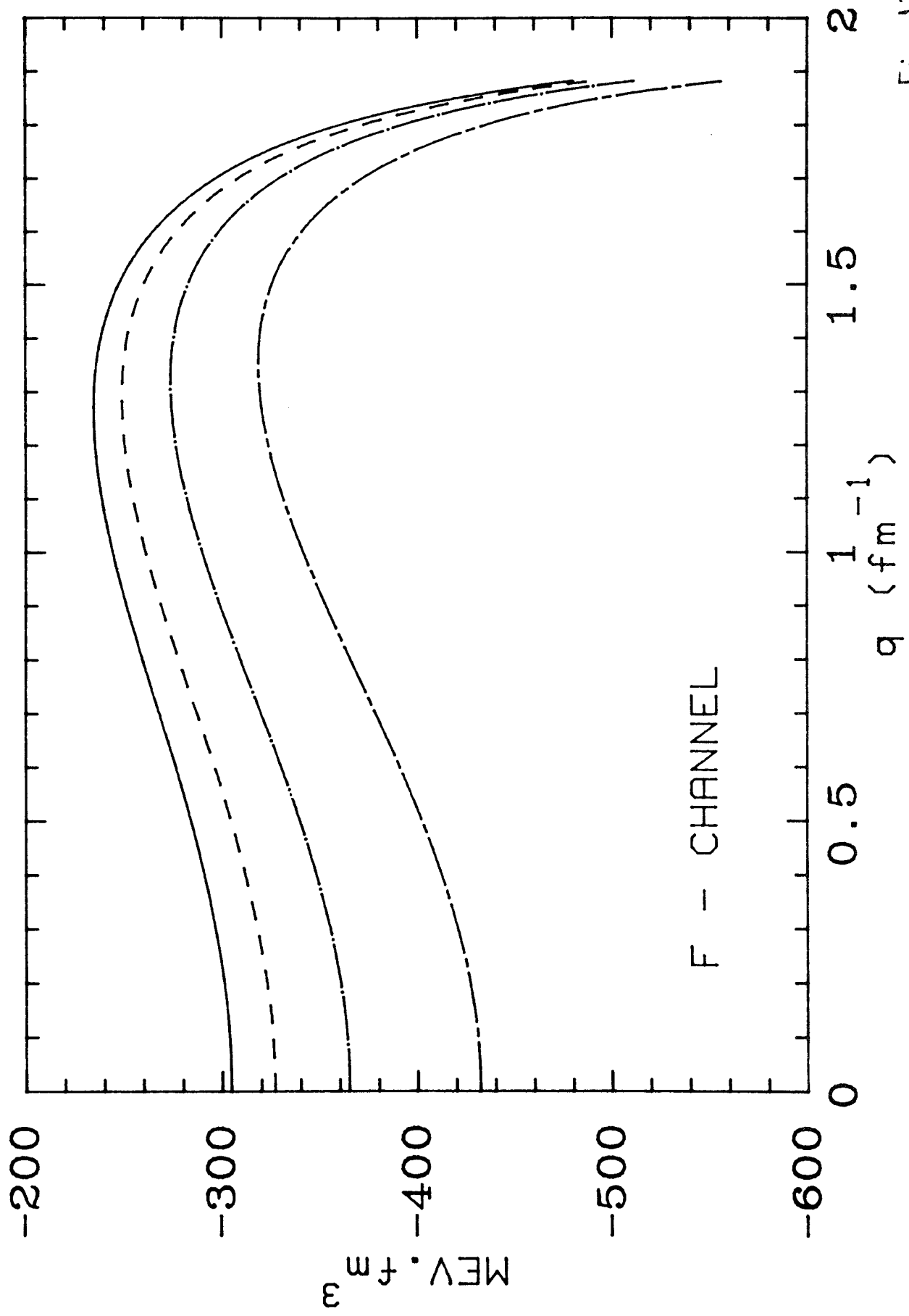


Fig. 12a

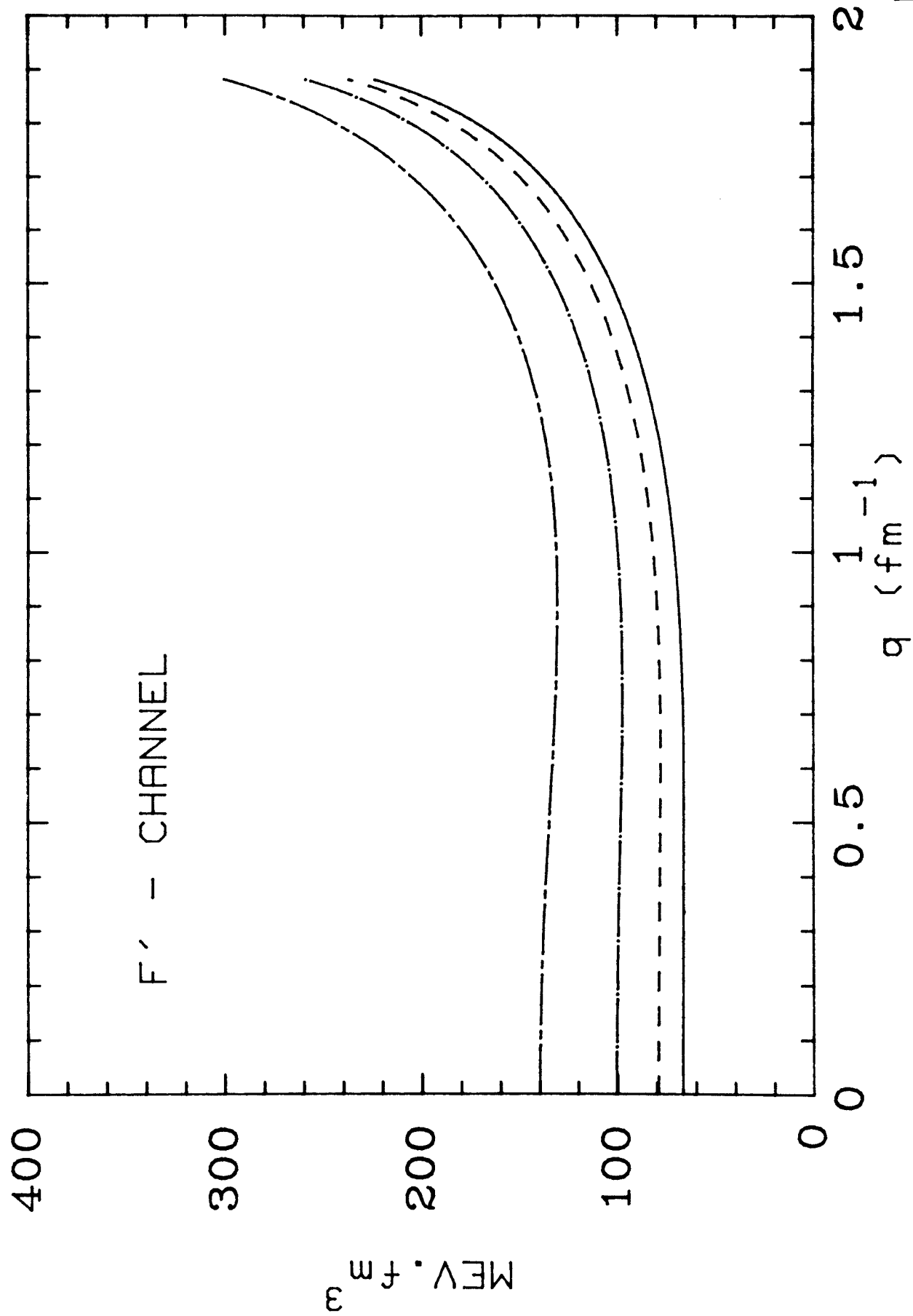


Fig. 12b

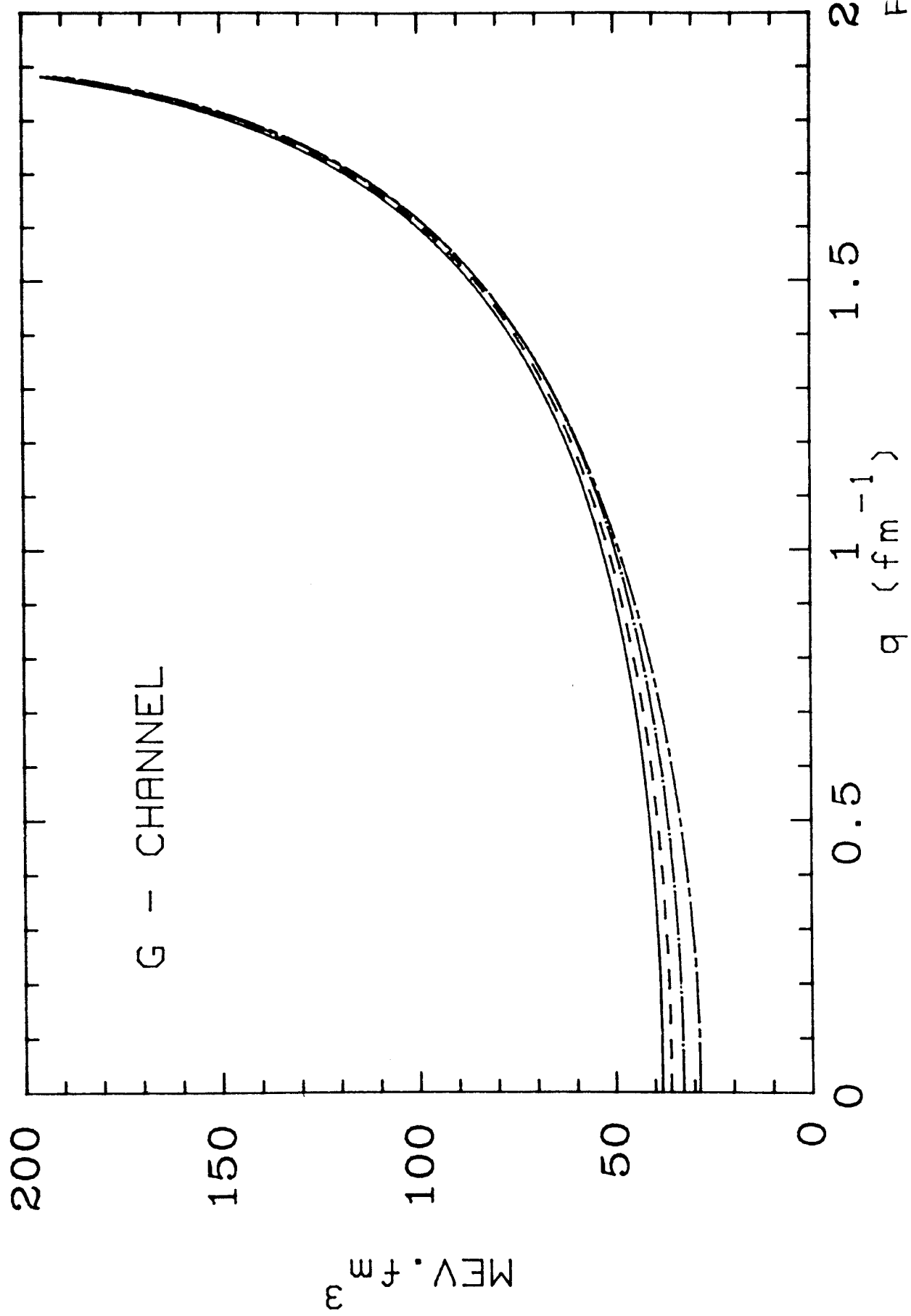


Fig. 12c

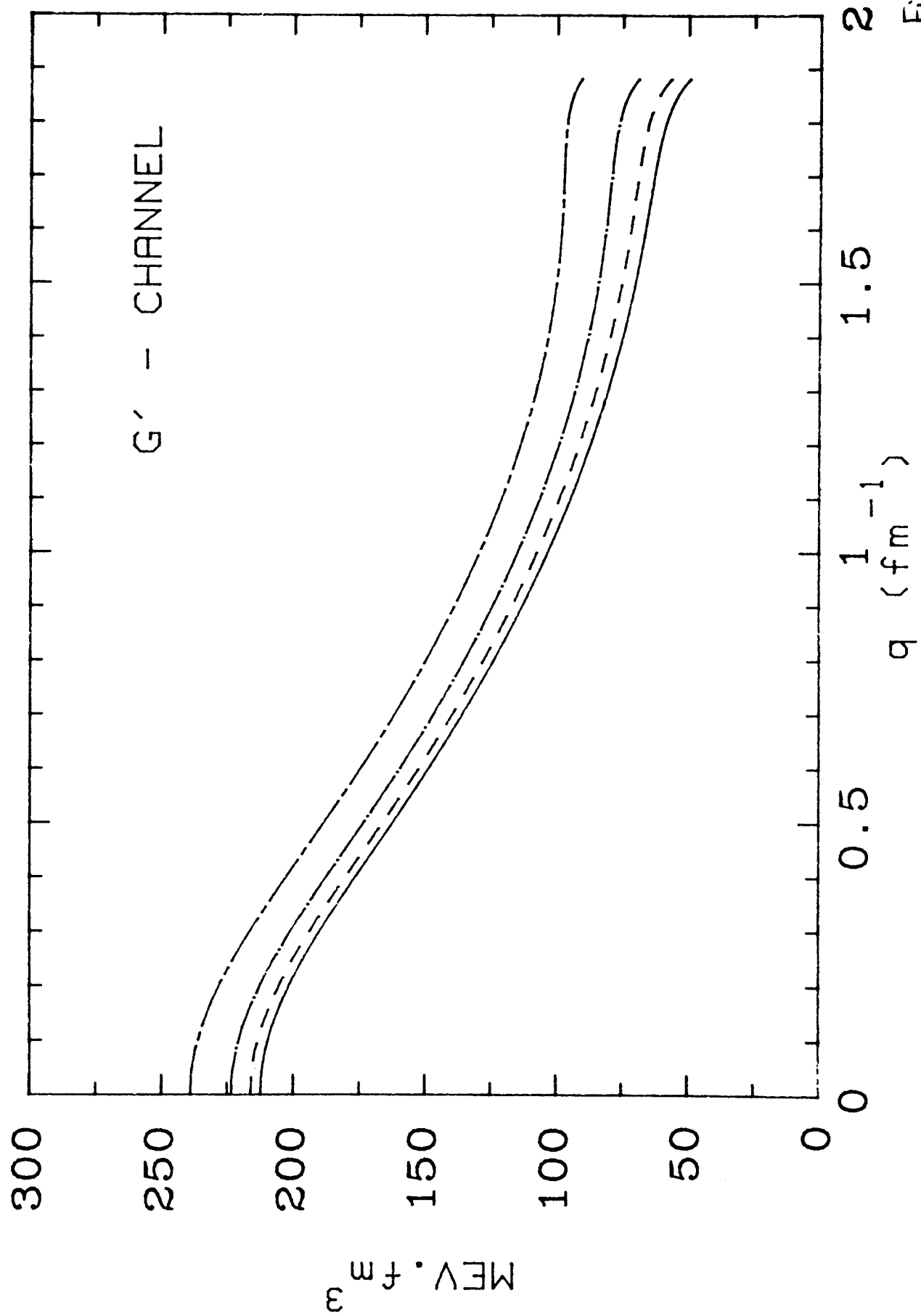


Fig. 12d

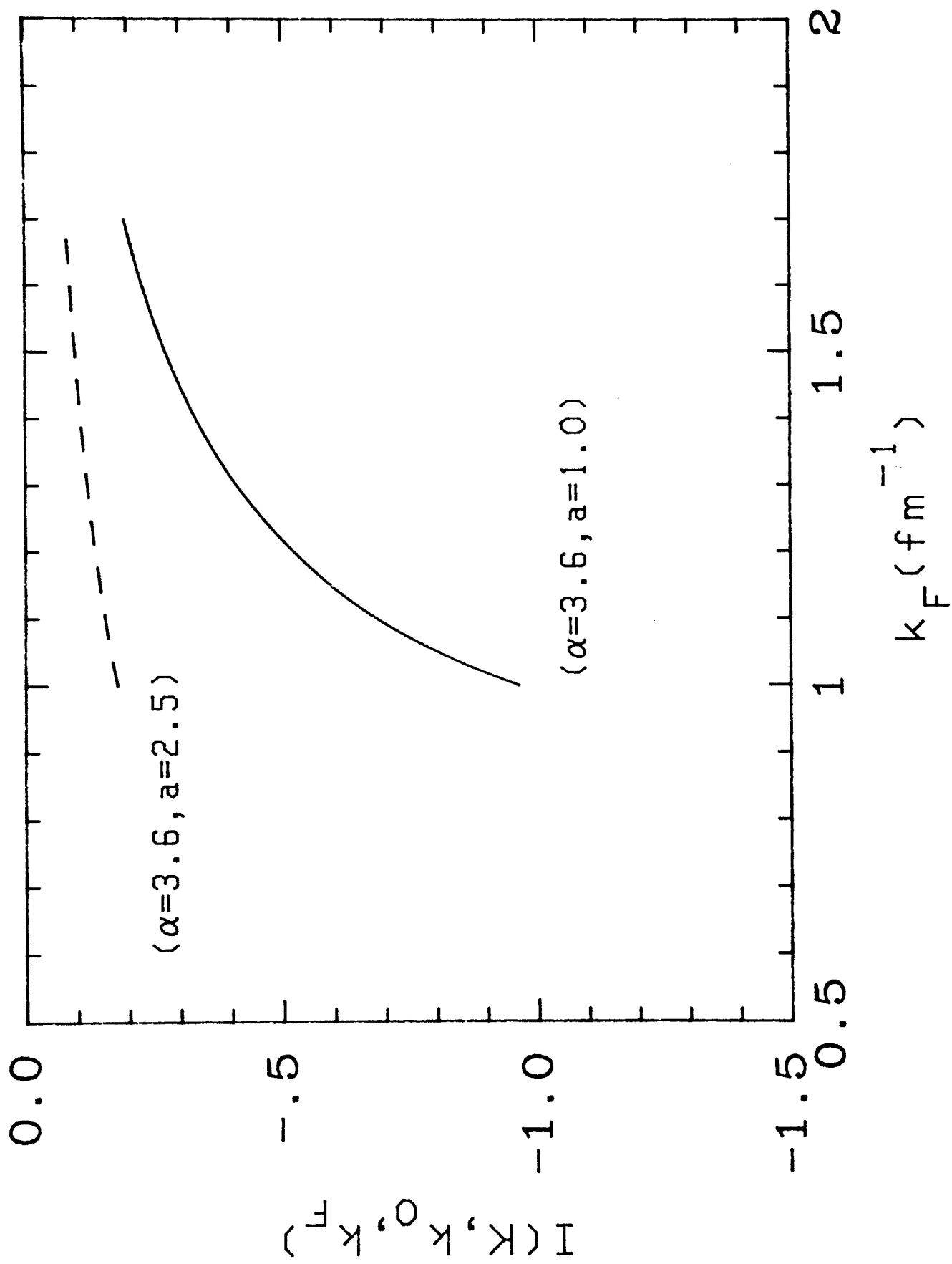


Fig. 13

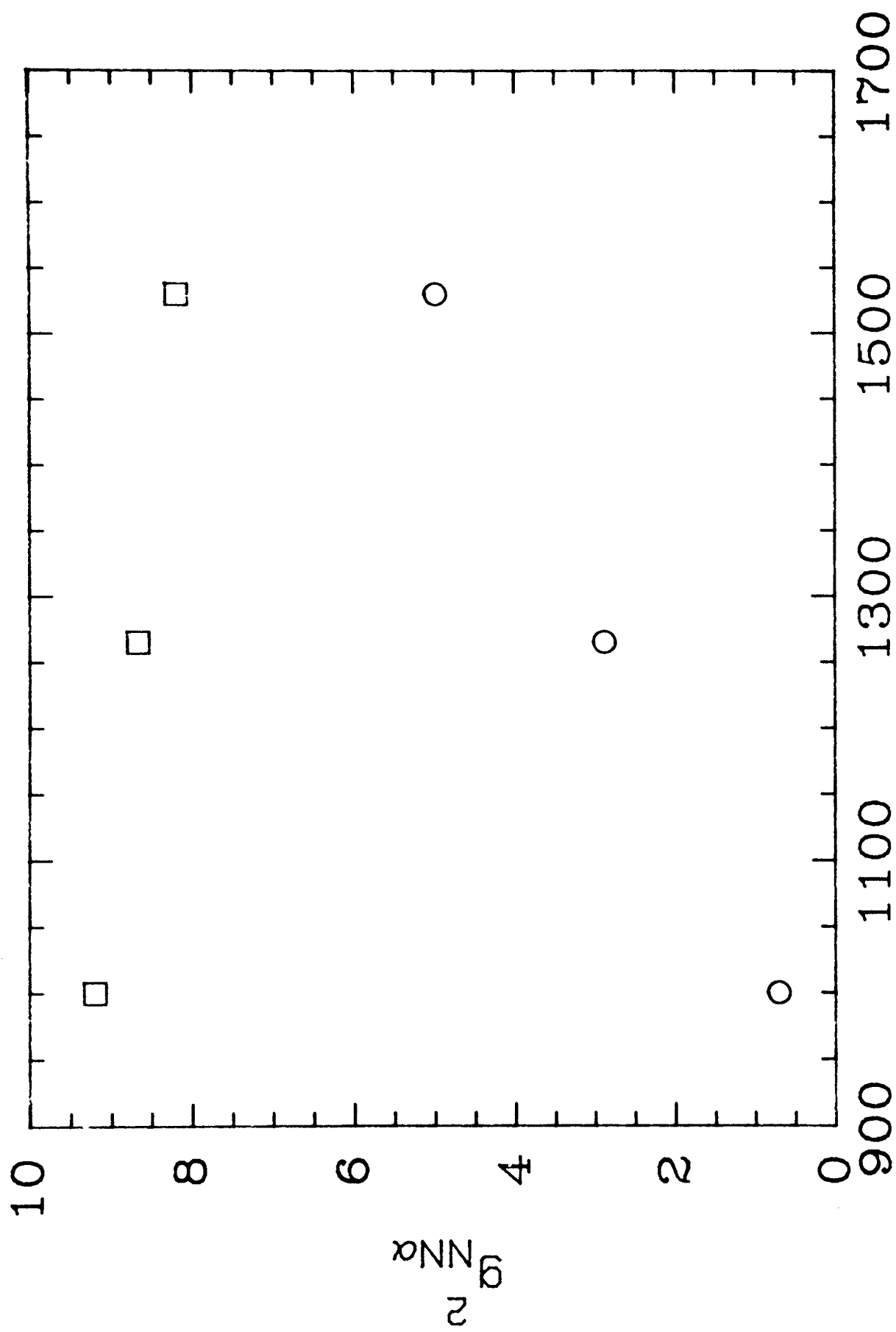


Fig. 14

PION CUTOFF (MEV)

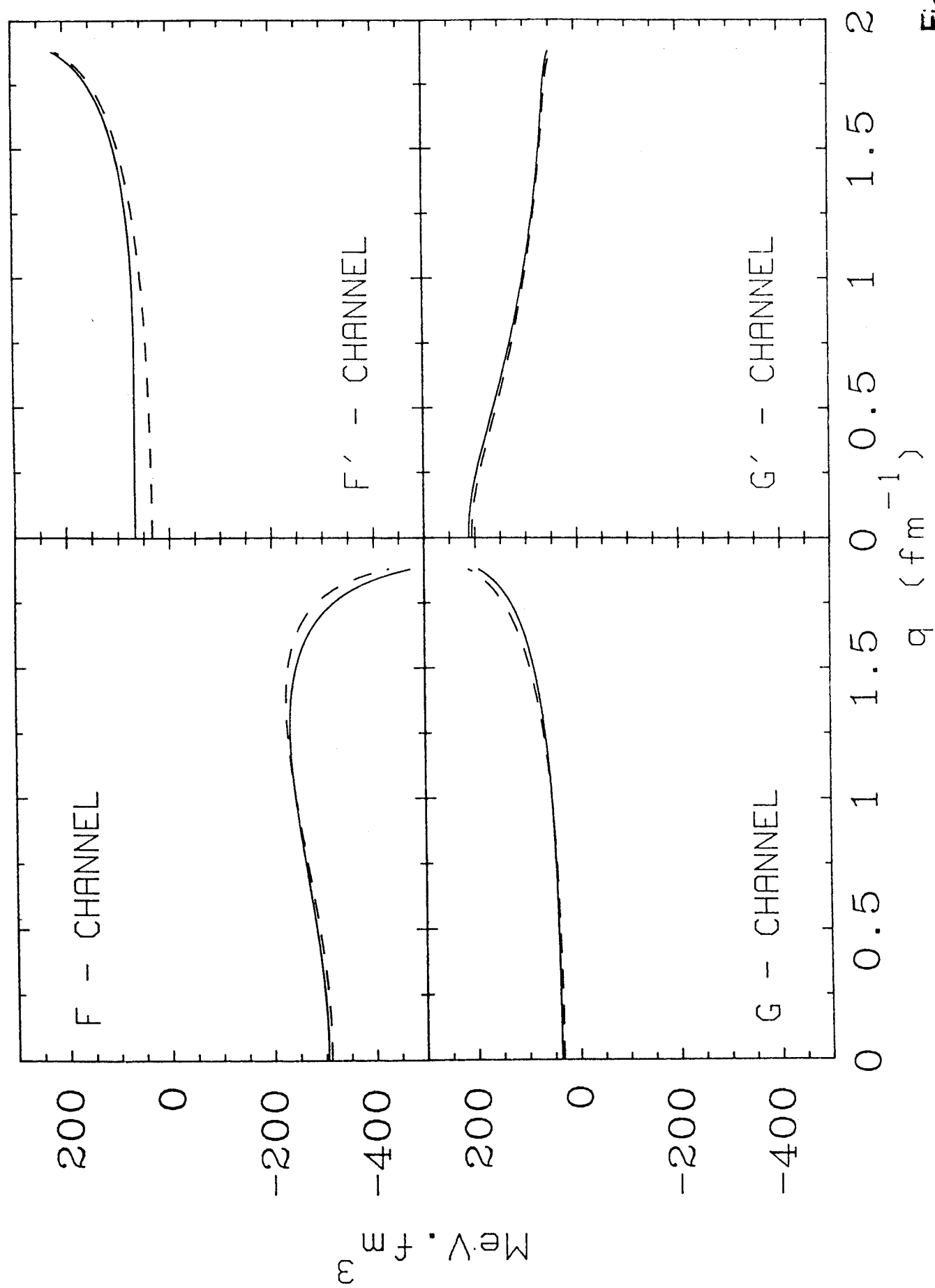


Fig. 15

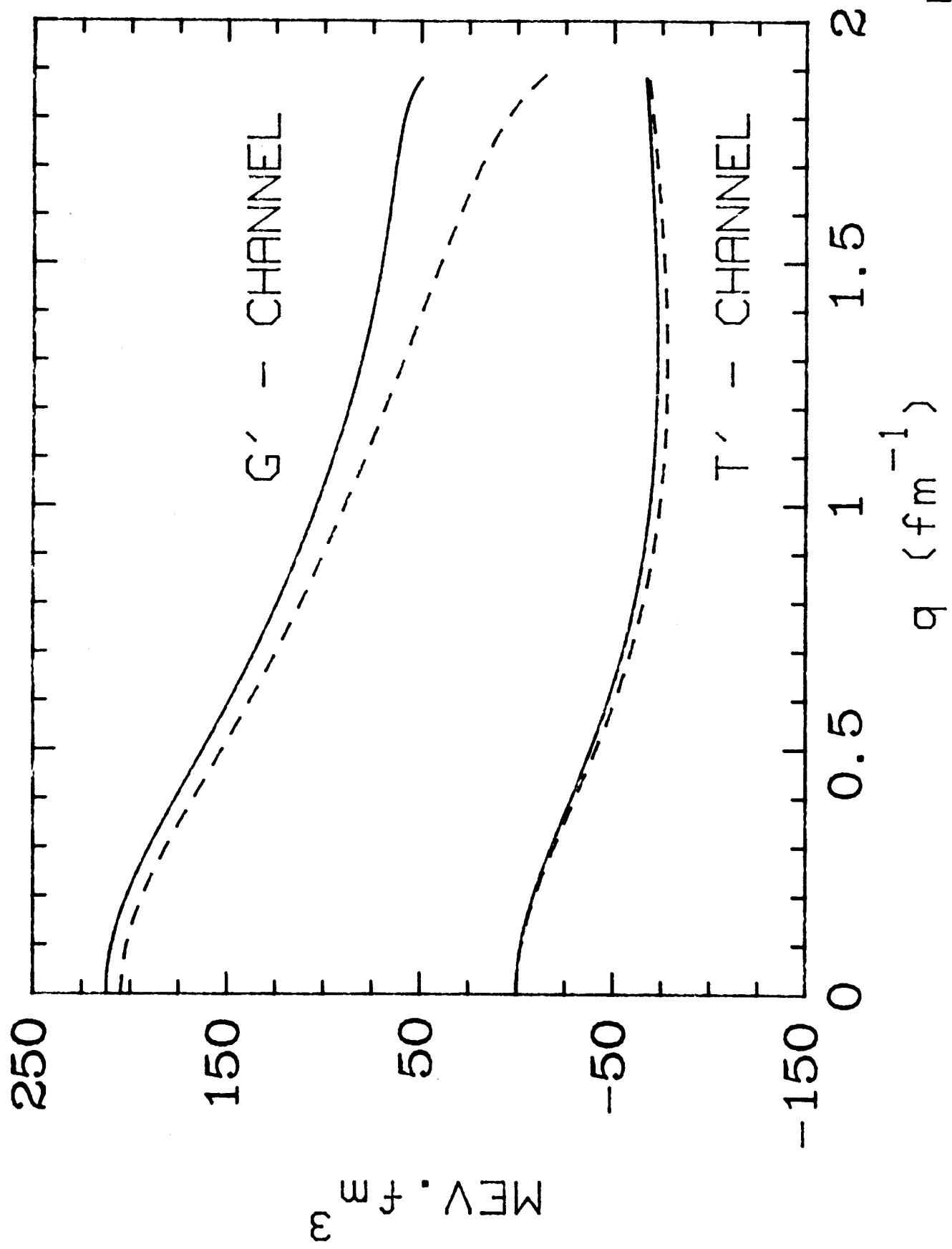
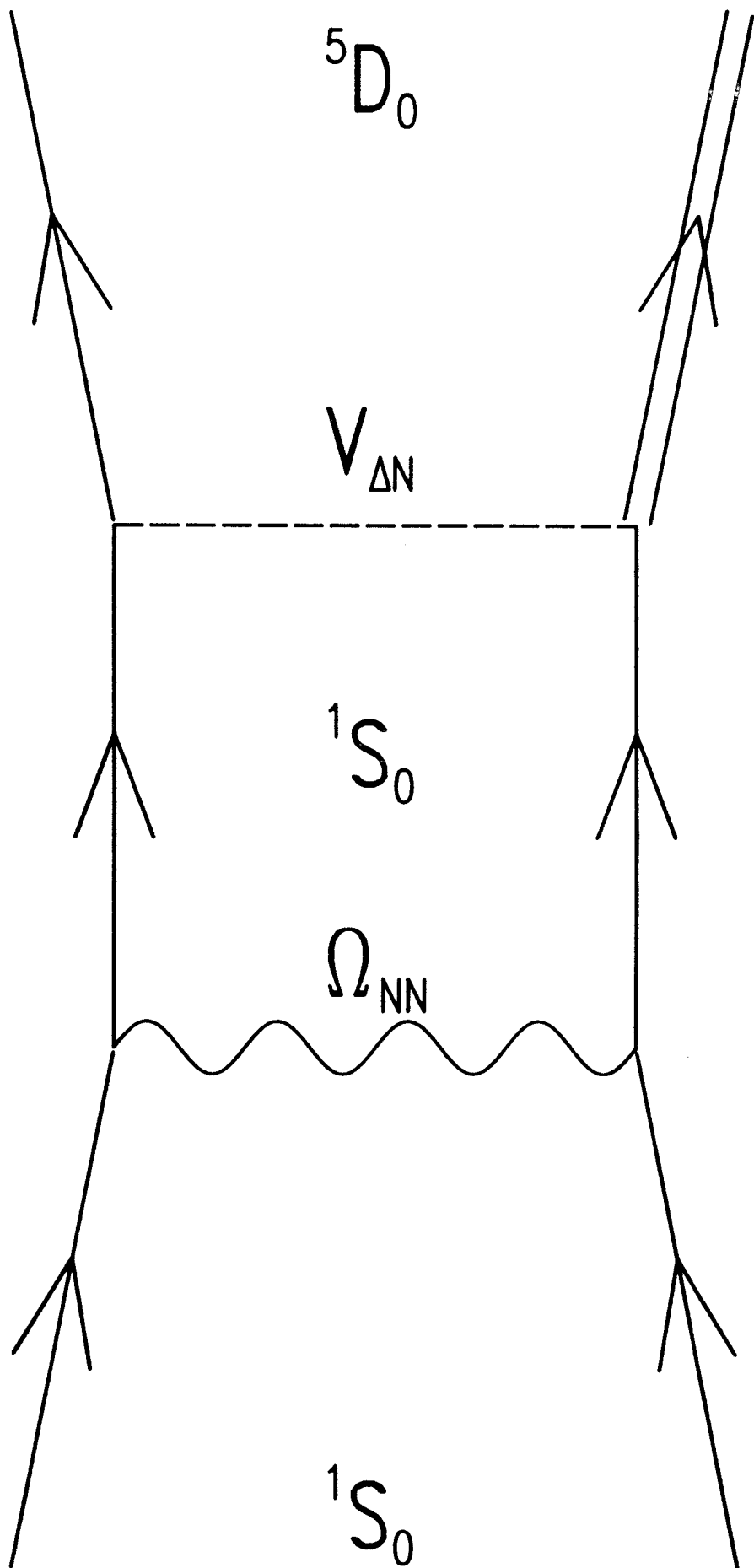


Fig. 16

Fig. 17



$$T = 1$$

$$T = 1$$

$$T = 1$$

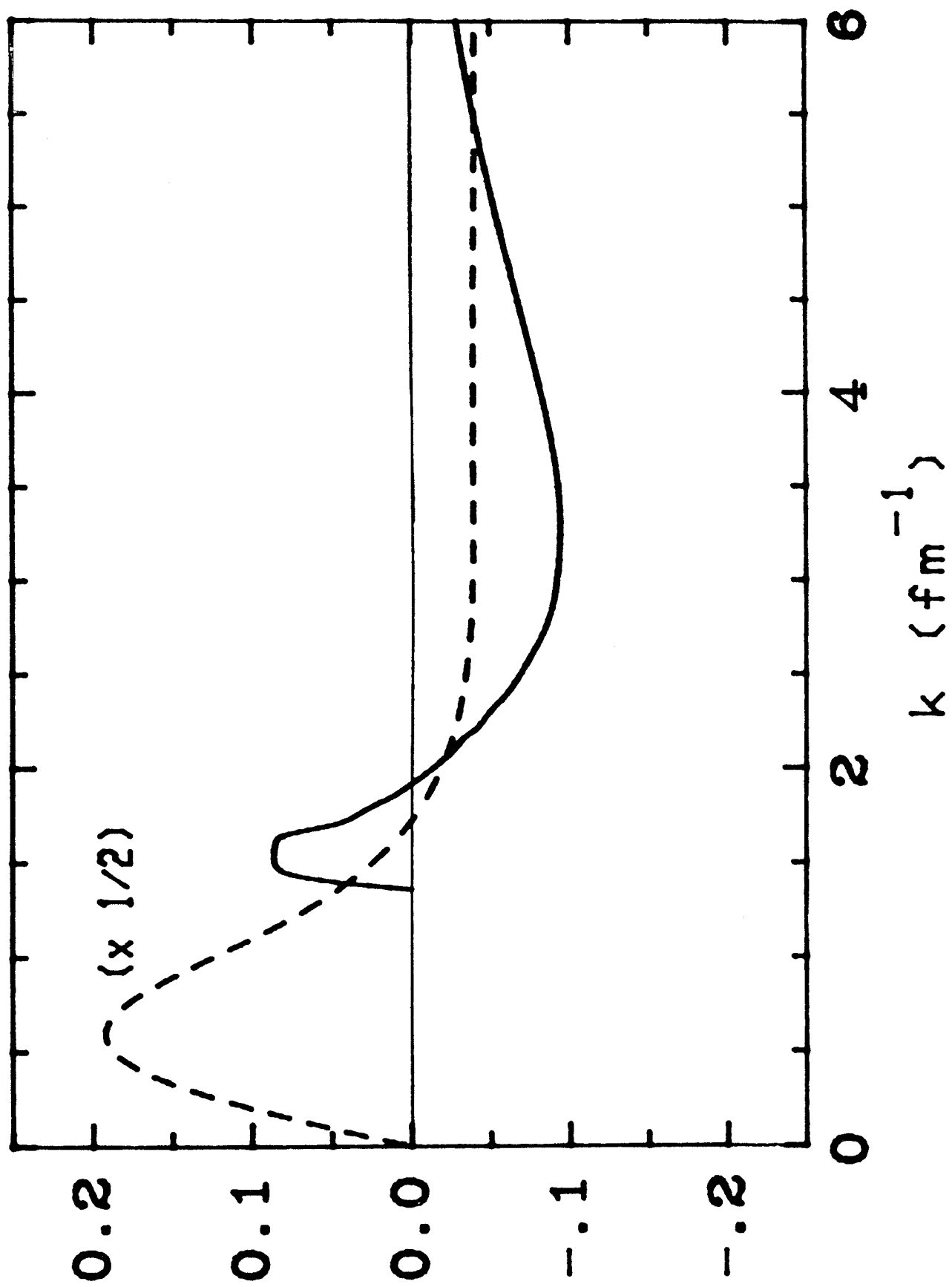


Fig. 18

$$I^{\text{ph}}(E) =$$

E

E

$$P^{\text{p}}(E) =$$

E

$$P^{\text{h}}(E) =$$

E

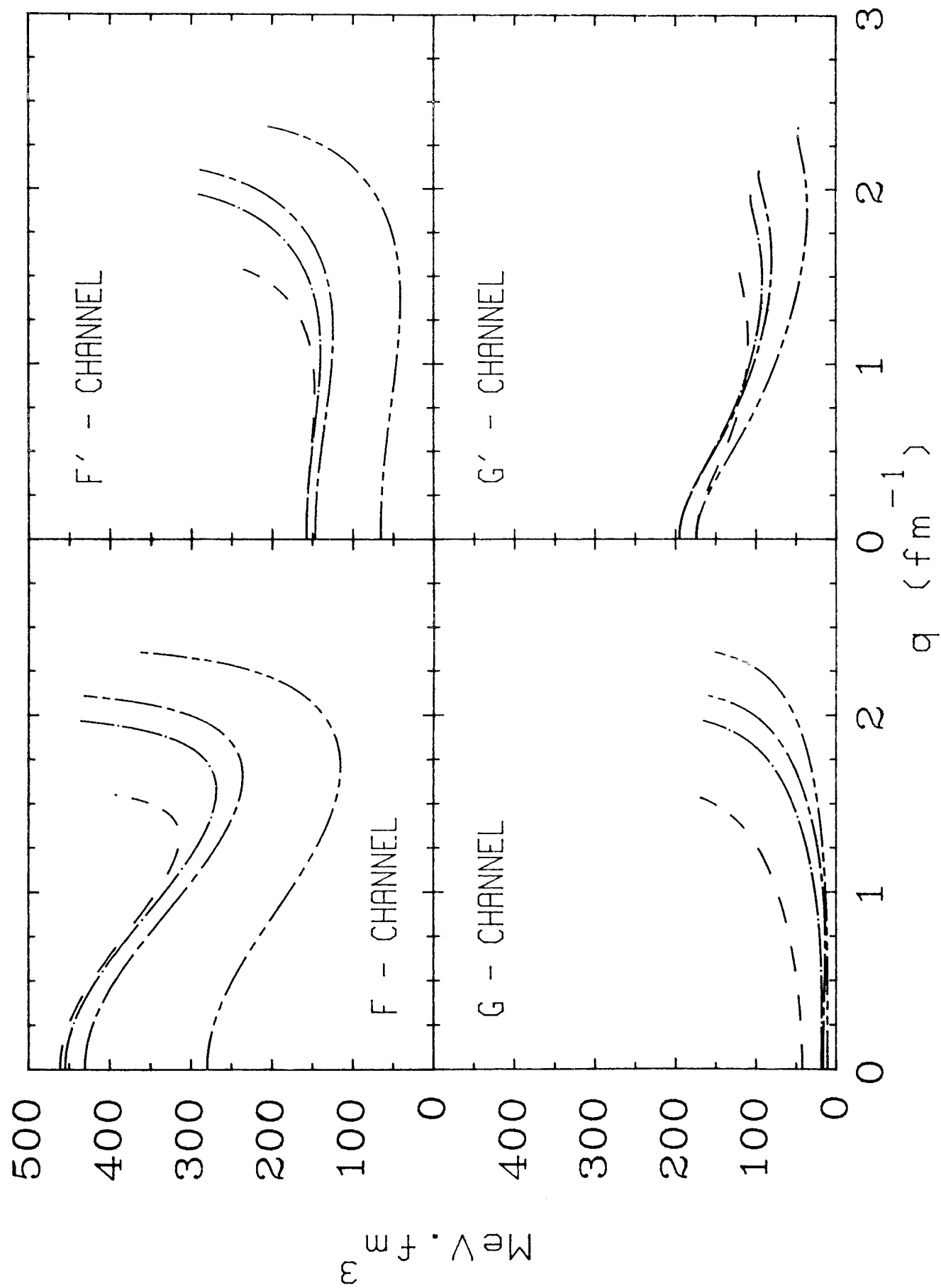


Fig. 20

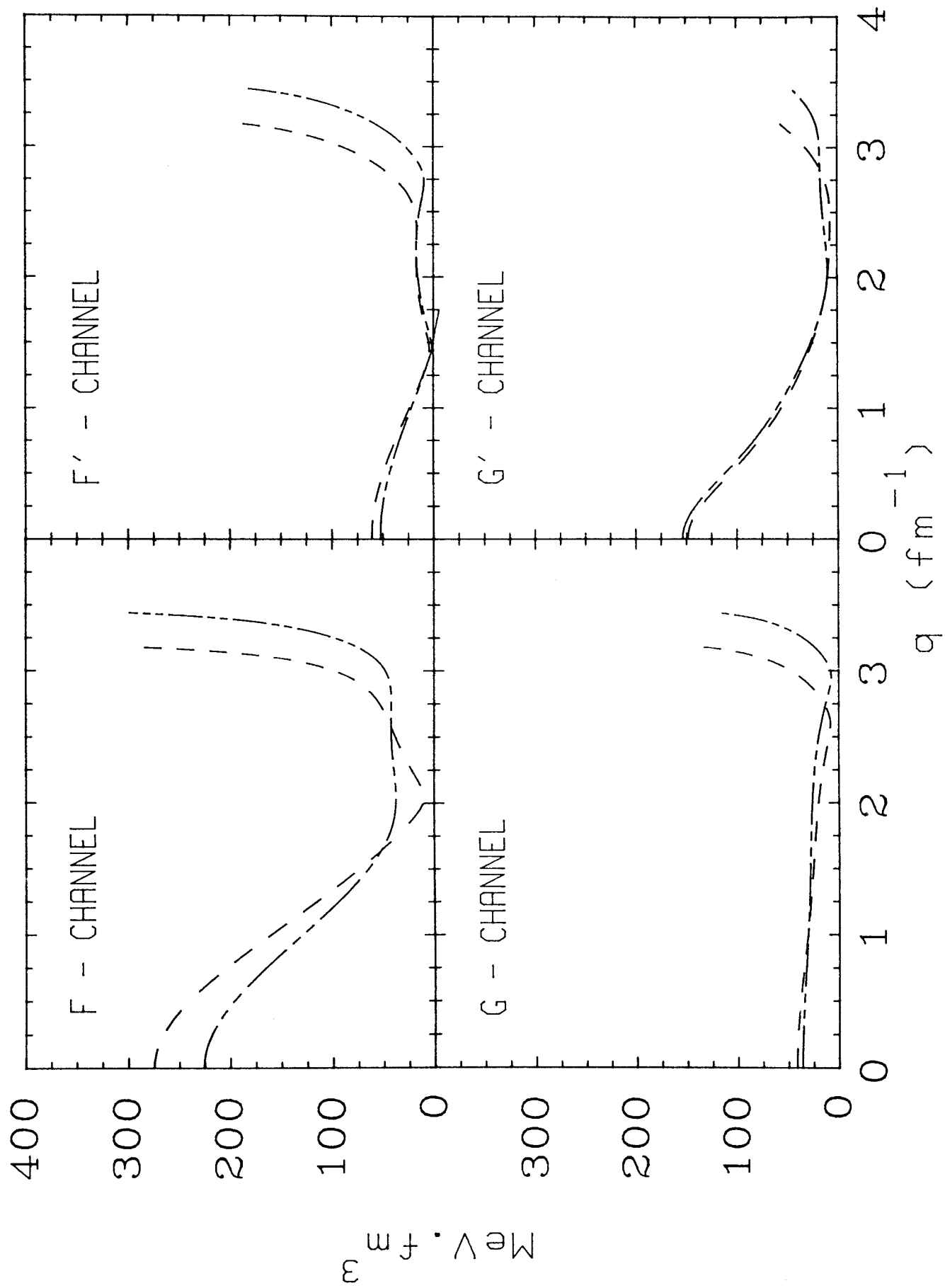


Fig. 21

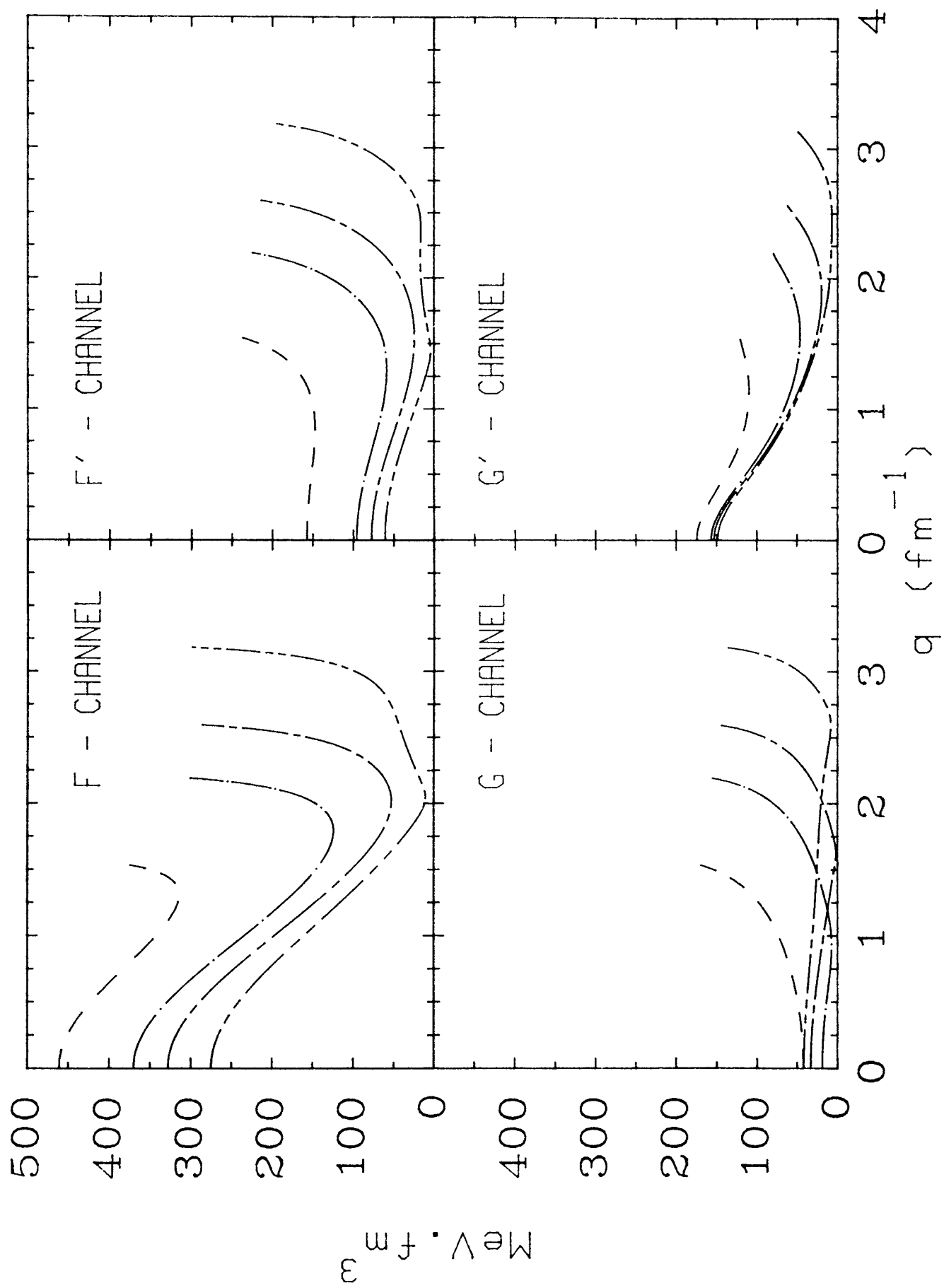


Fig. 22

$$k'V_{02}(k_0, k')F_{22}(k_0; k', k_0) \text{ [fm}^2\text{]}$$

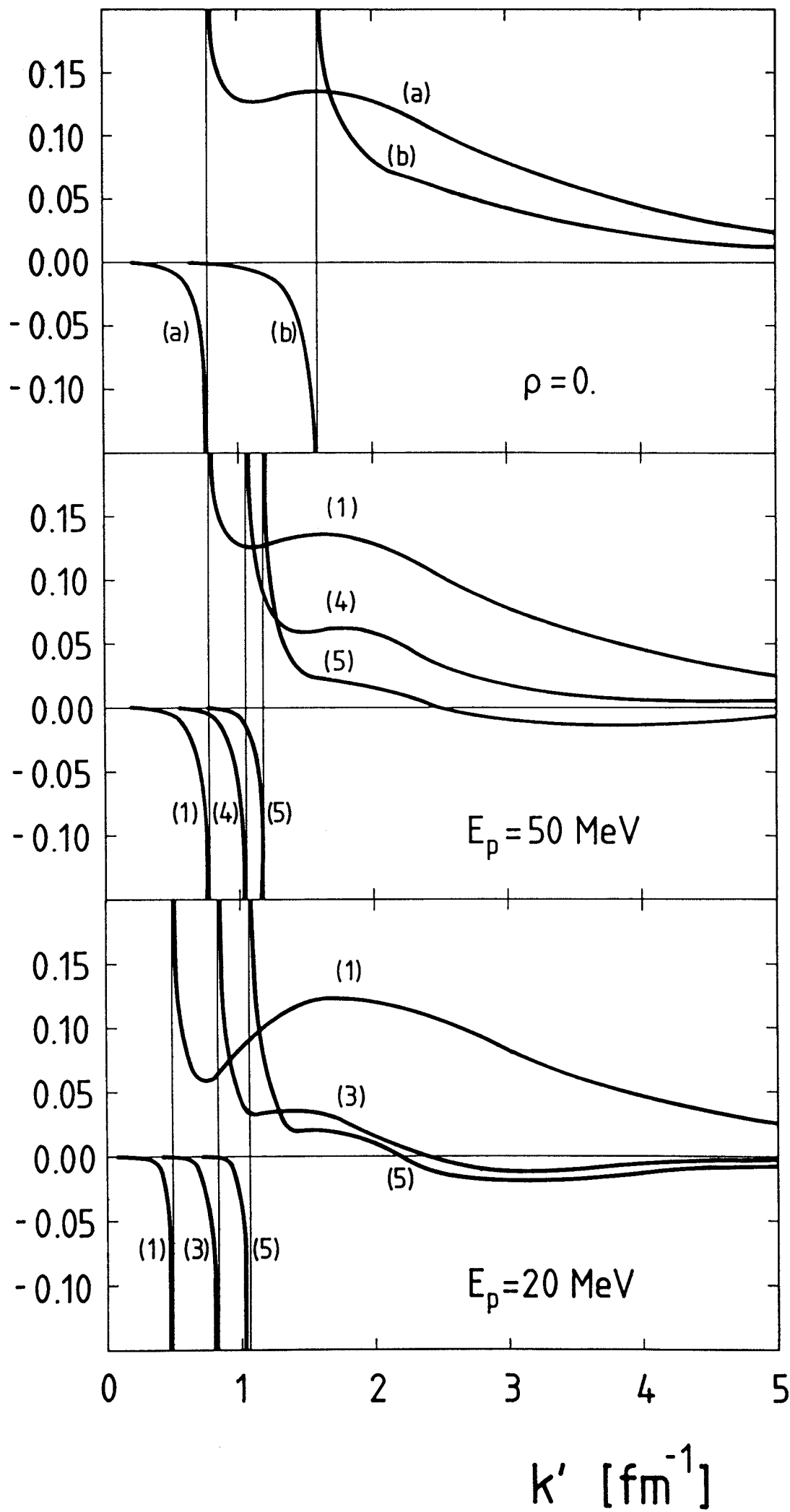


Fig. 23

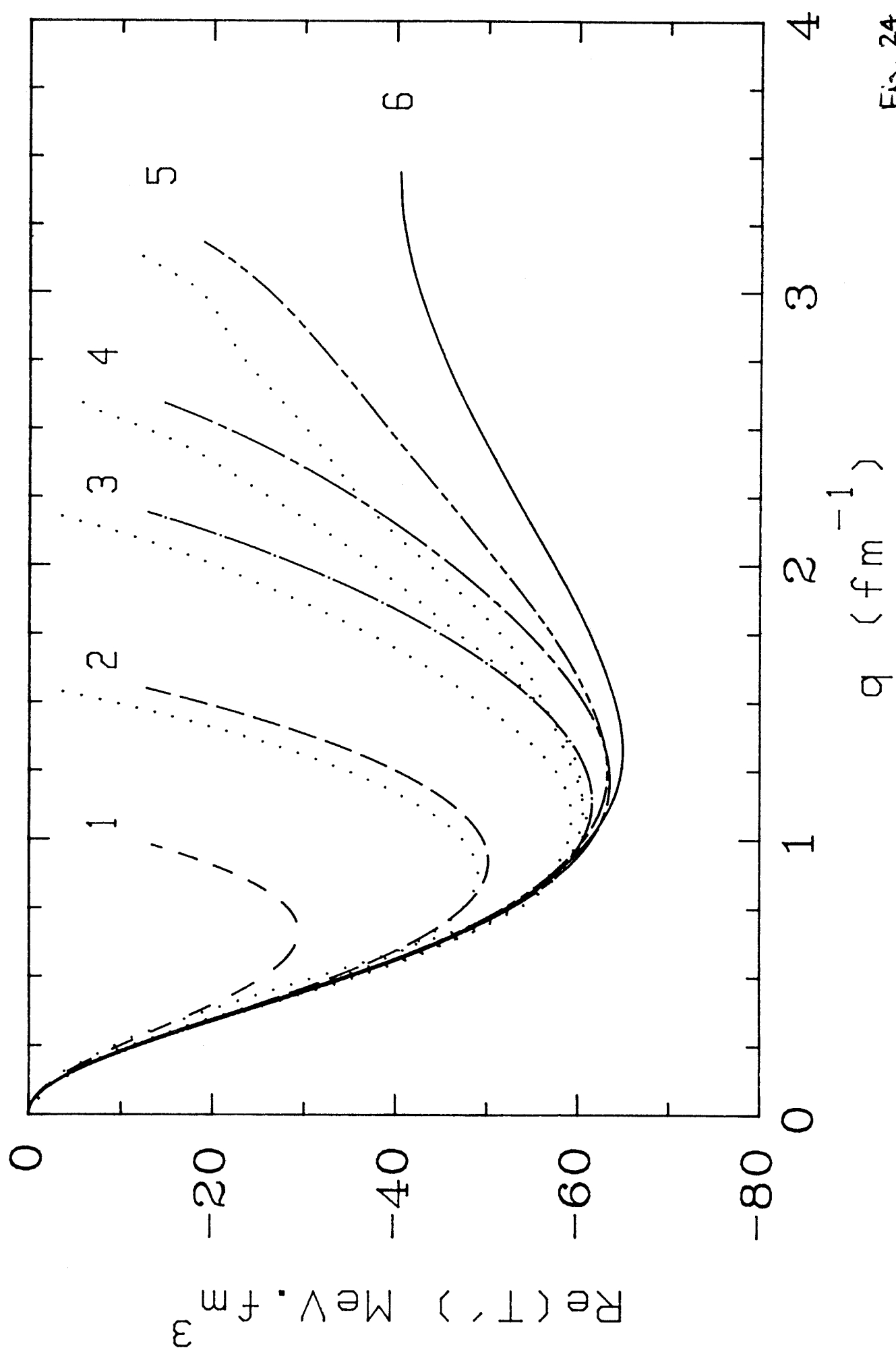
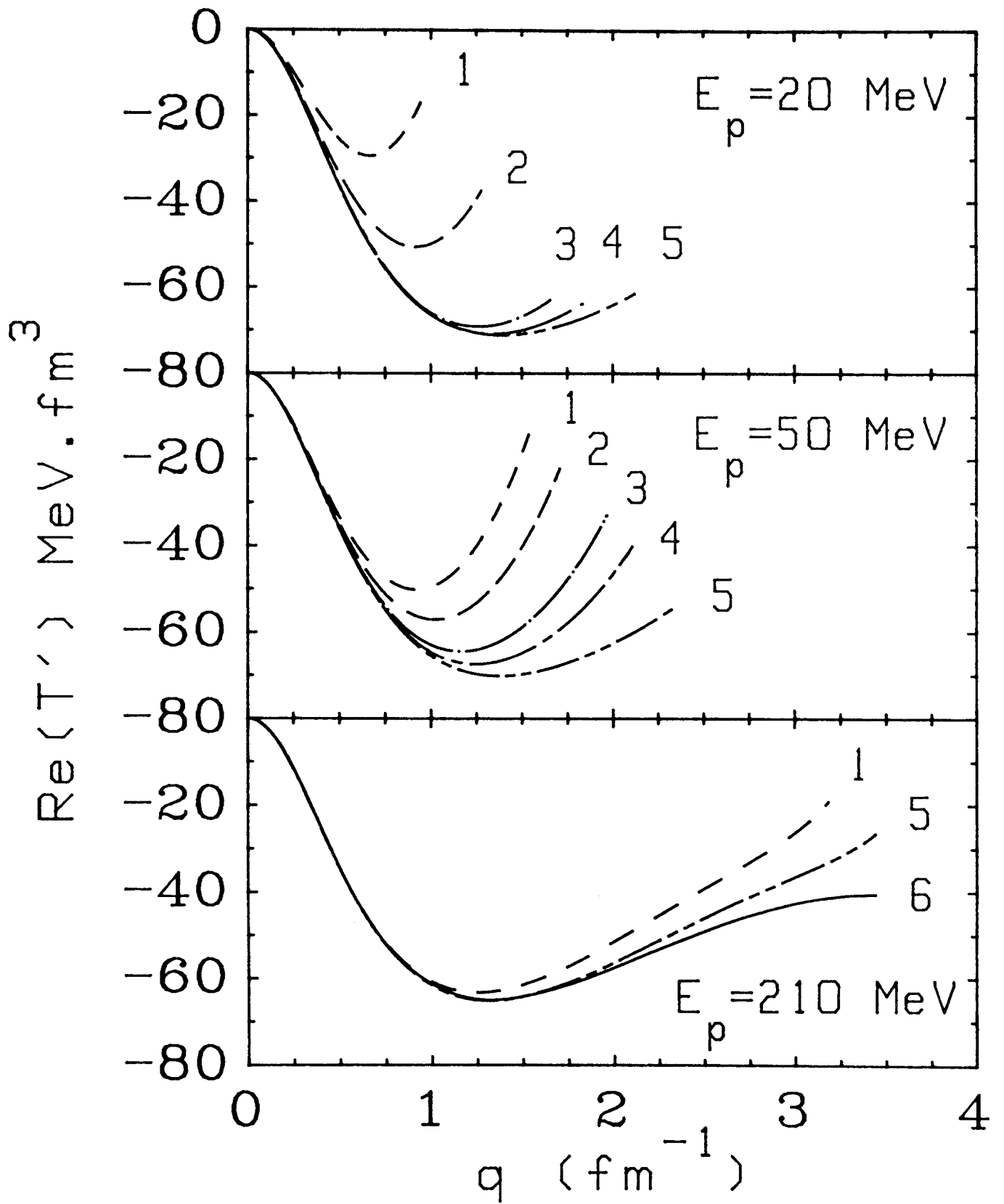


Fig. 24

Fig. 25



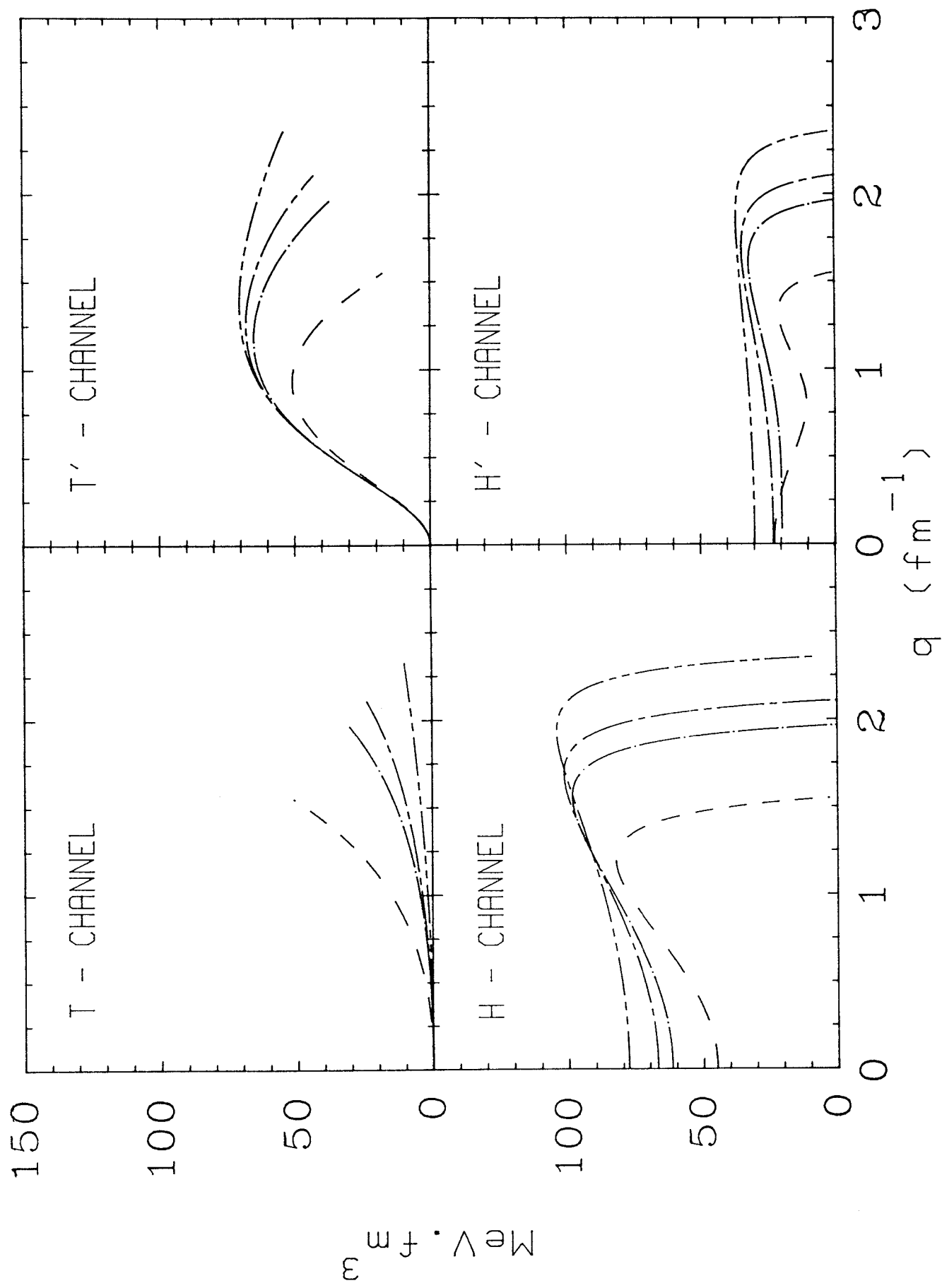


Fig. 26

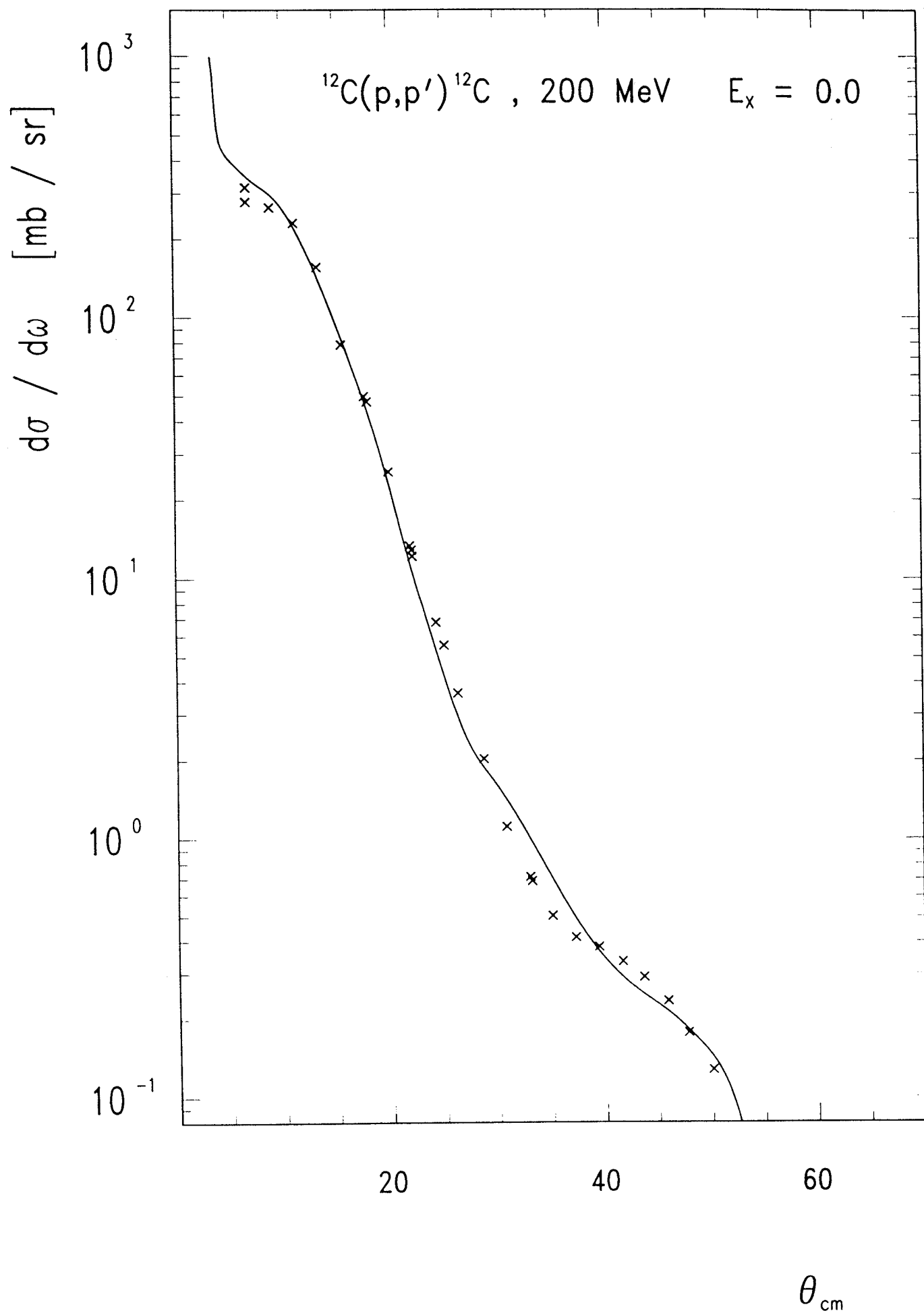


Fig. 27

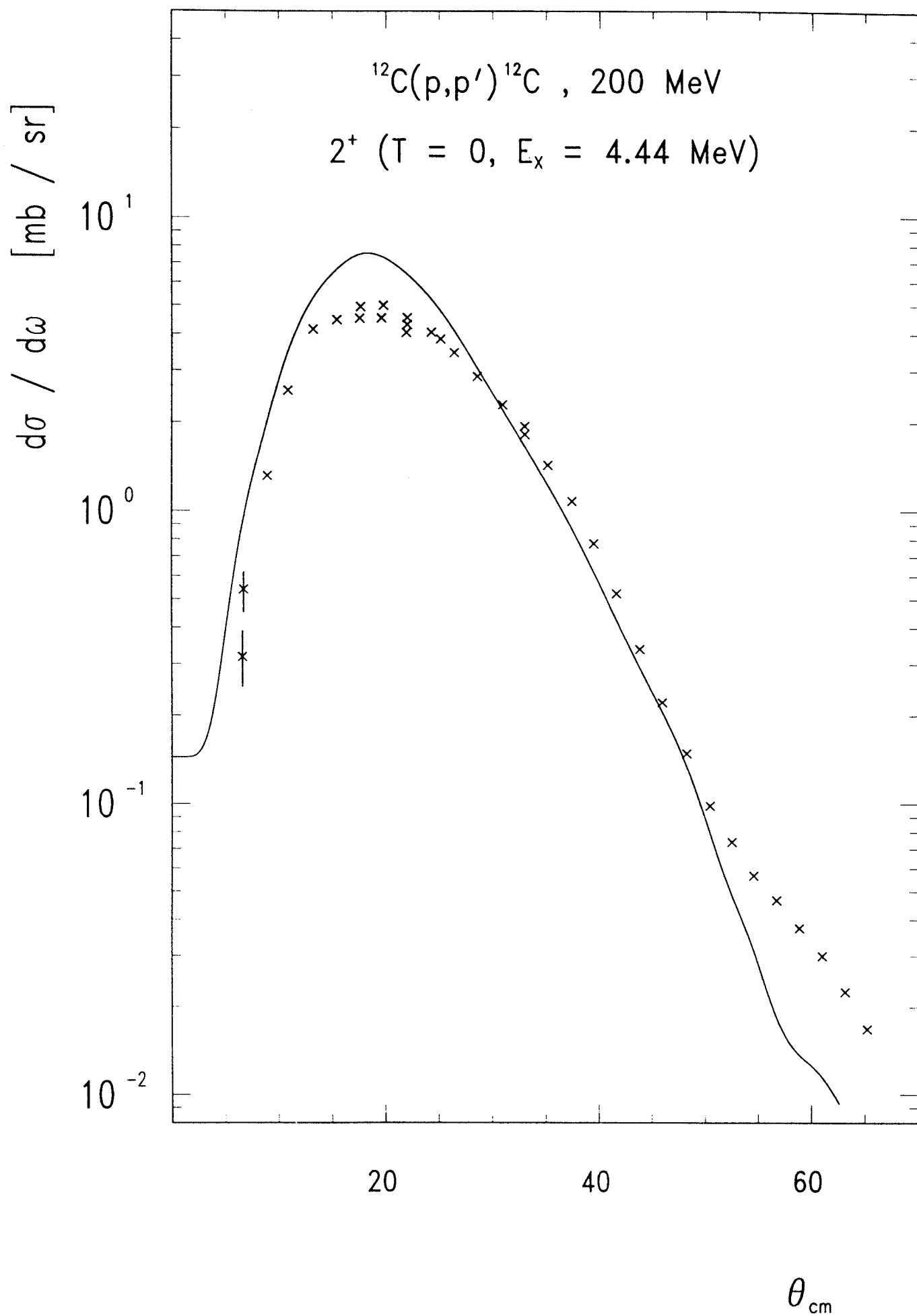


Fig. 28

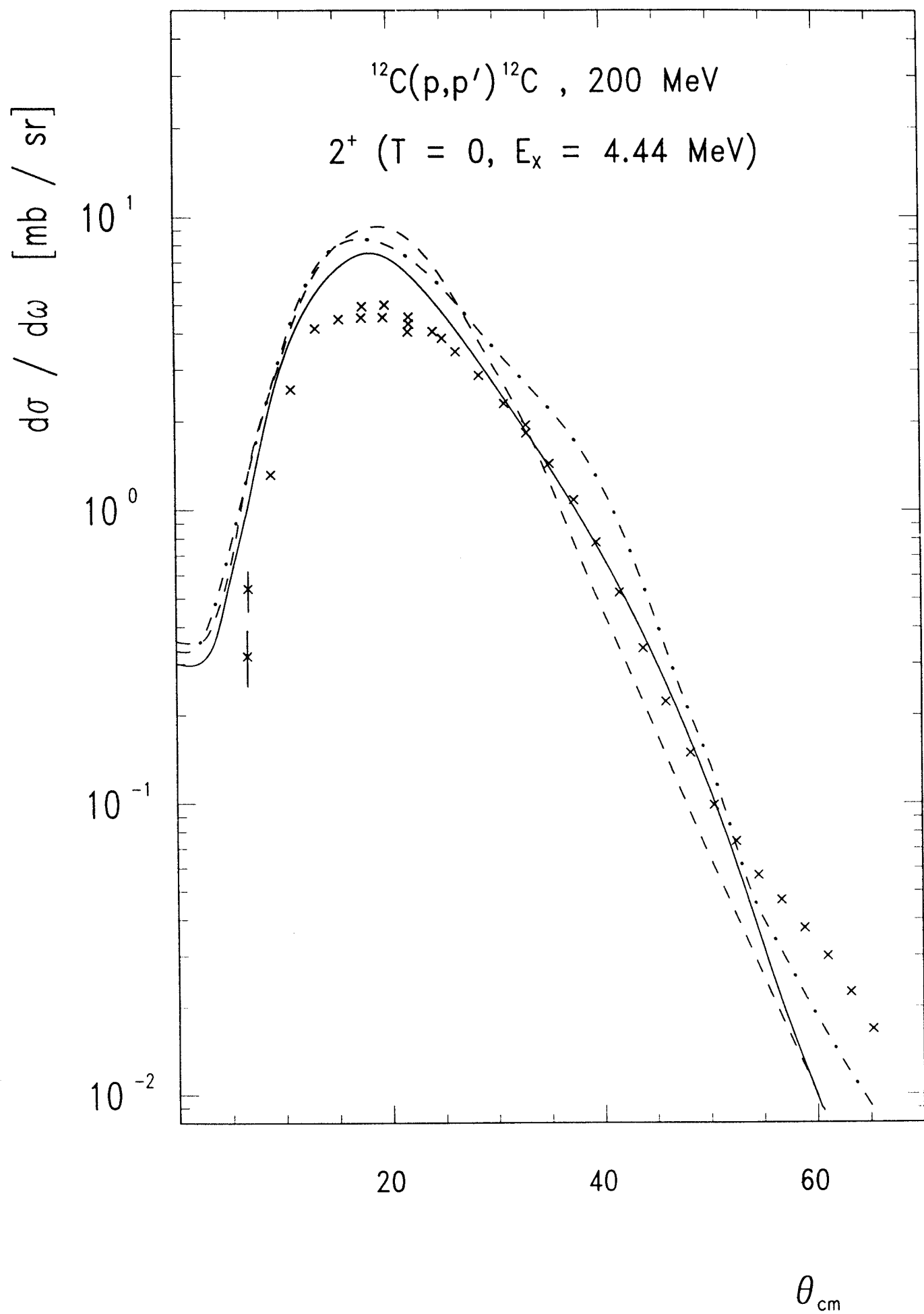


Fig. 29

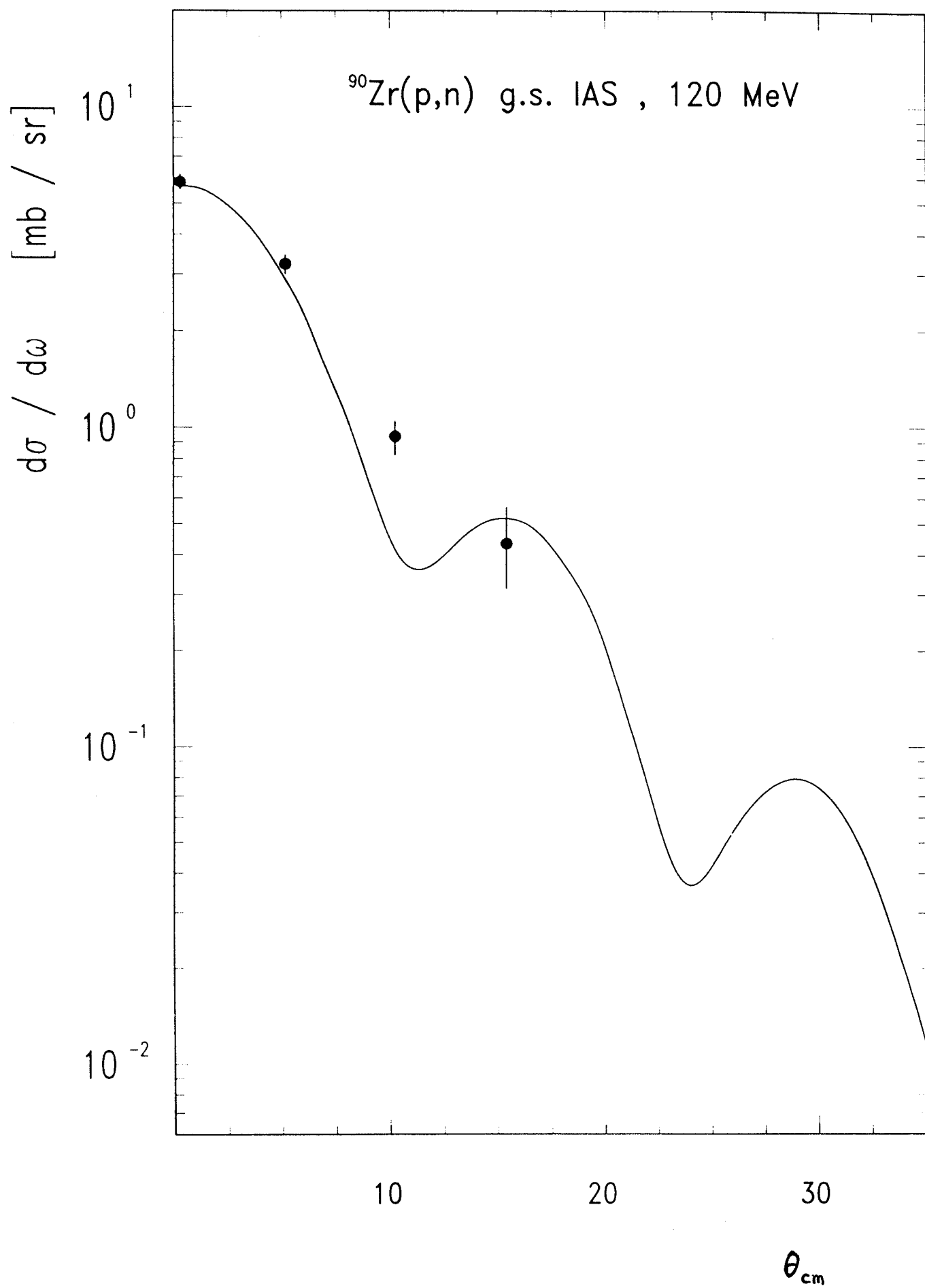


Fig. 30

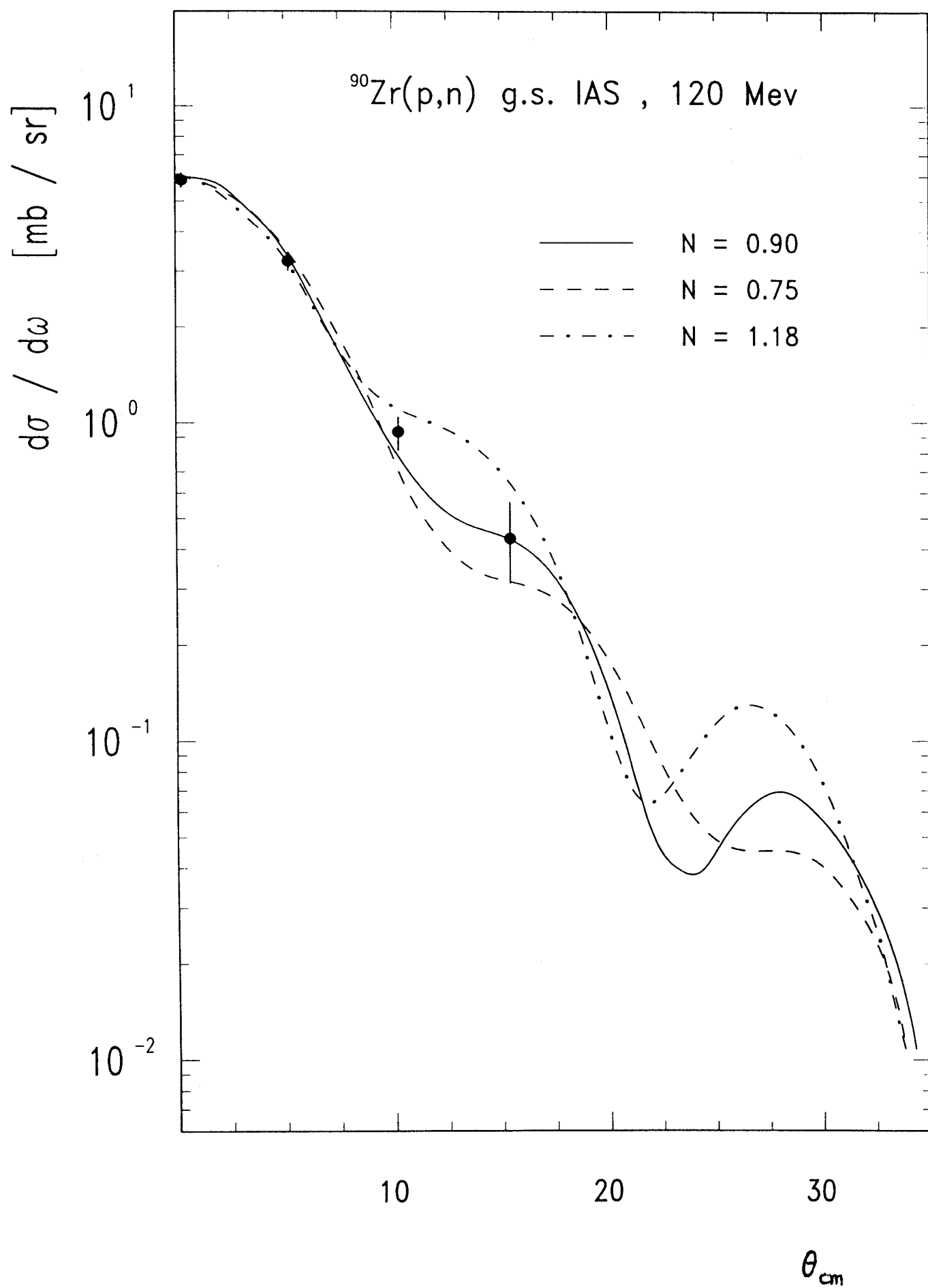


Fig. 31

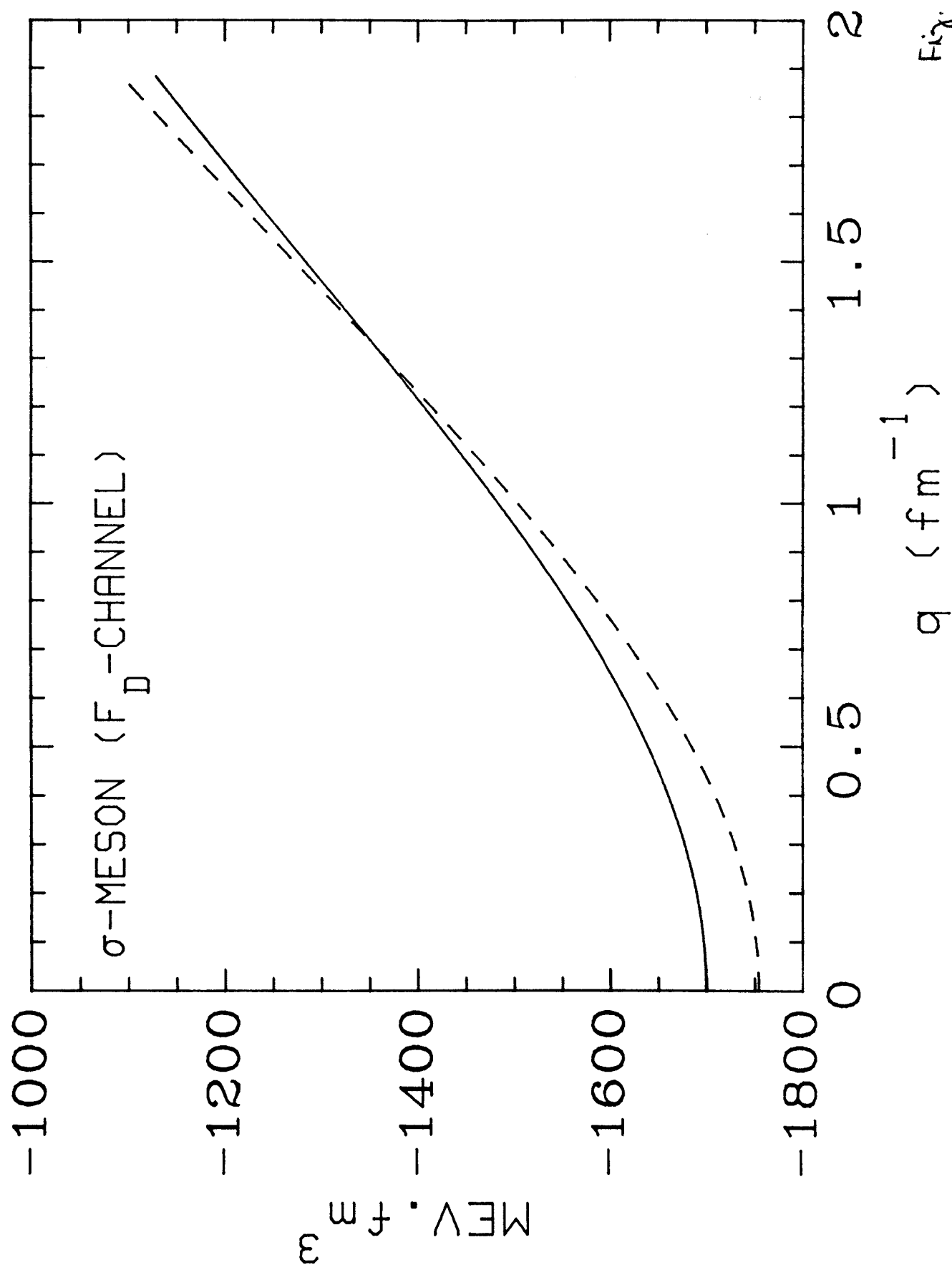


Fig. 32

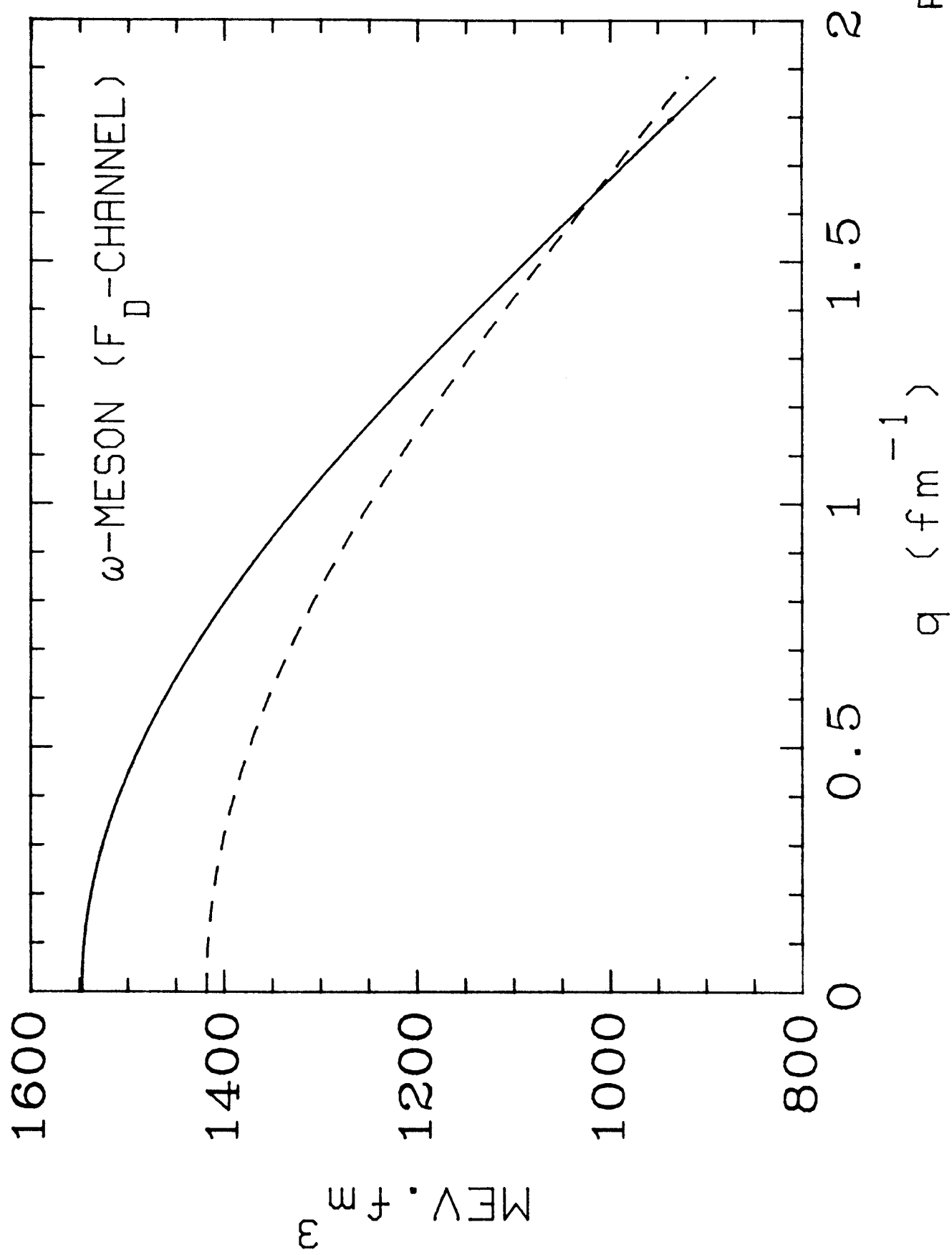


Fig. 33

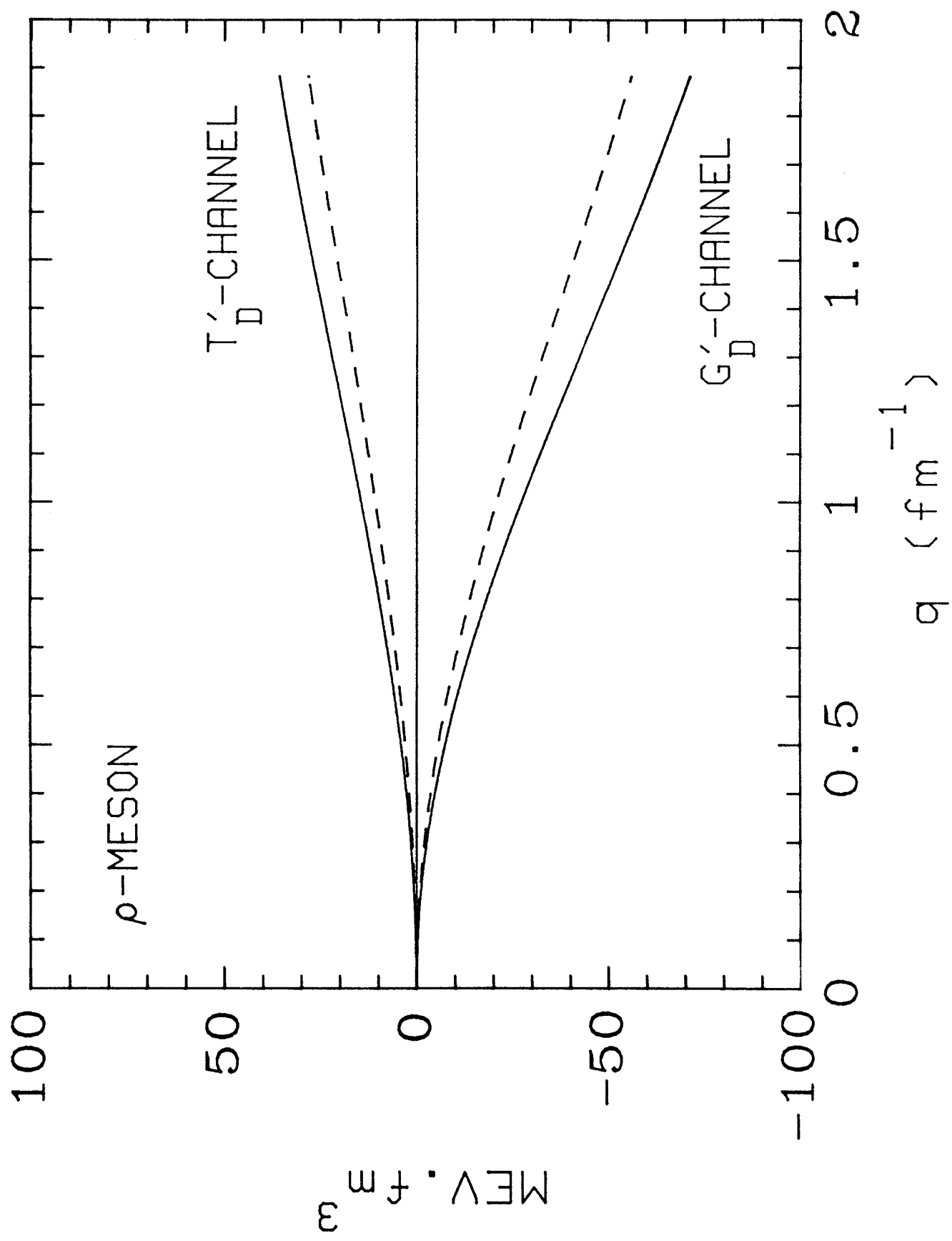


Fig. 34

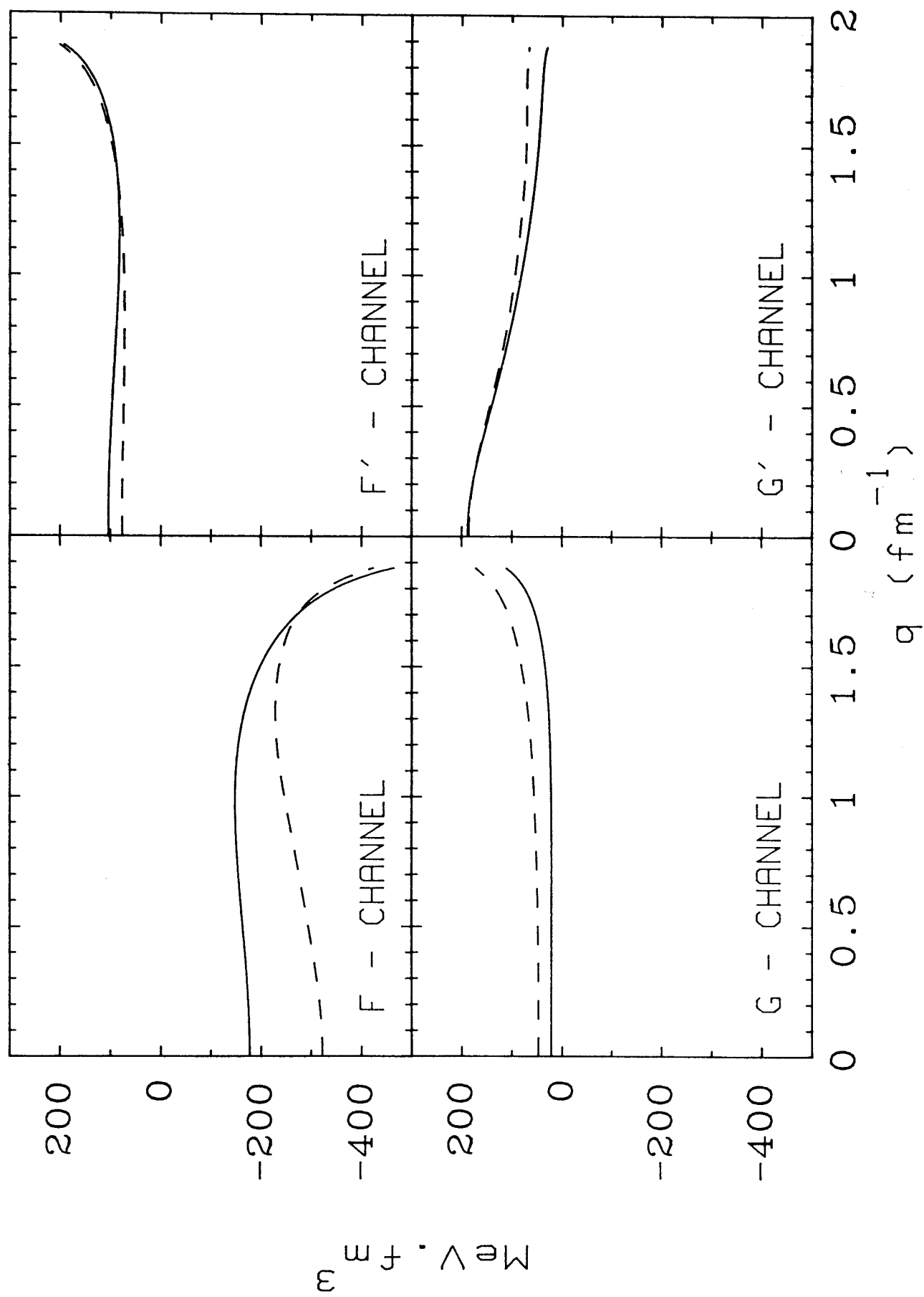


Fig. 35

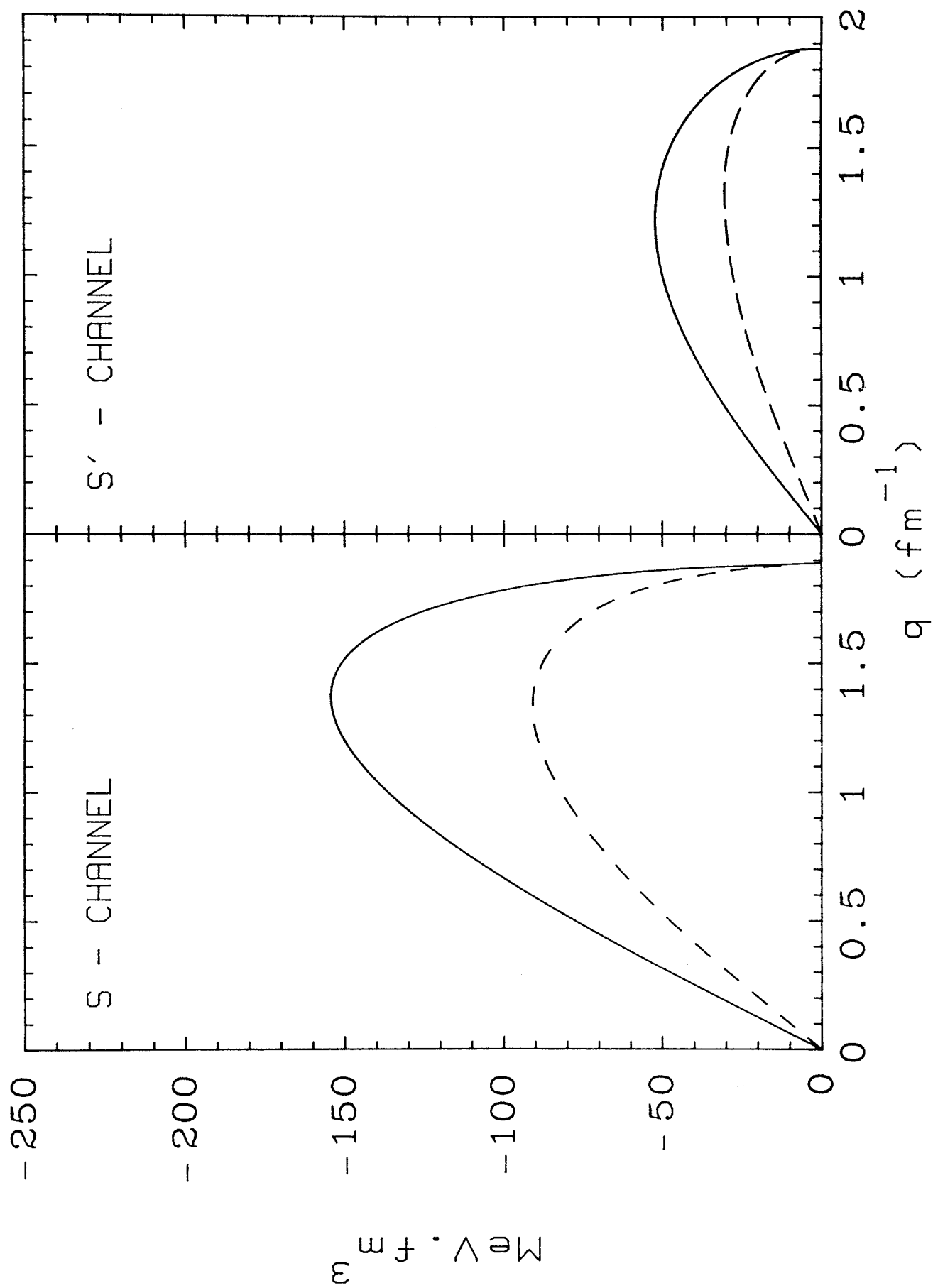


Fig. 36

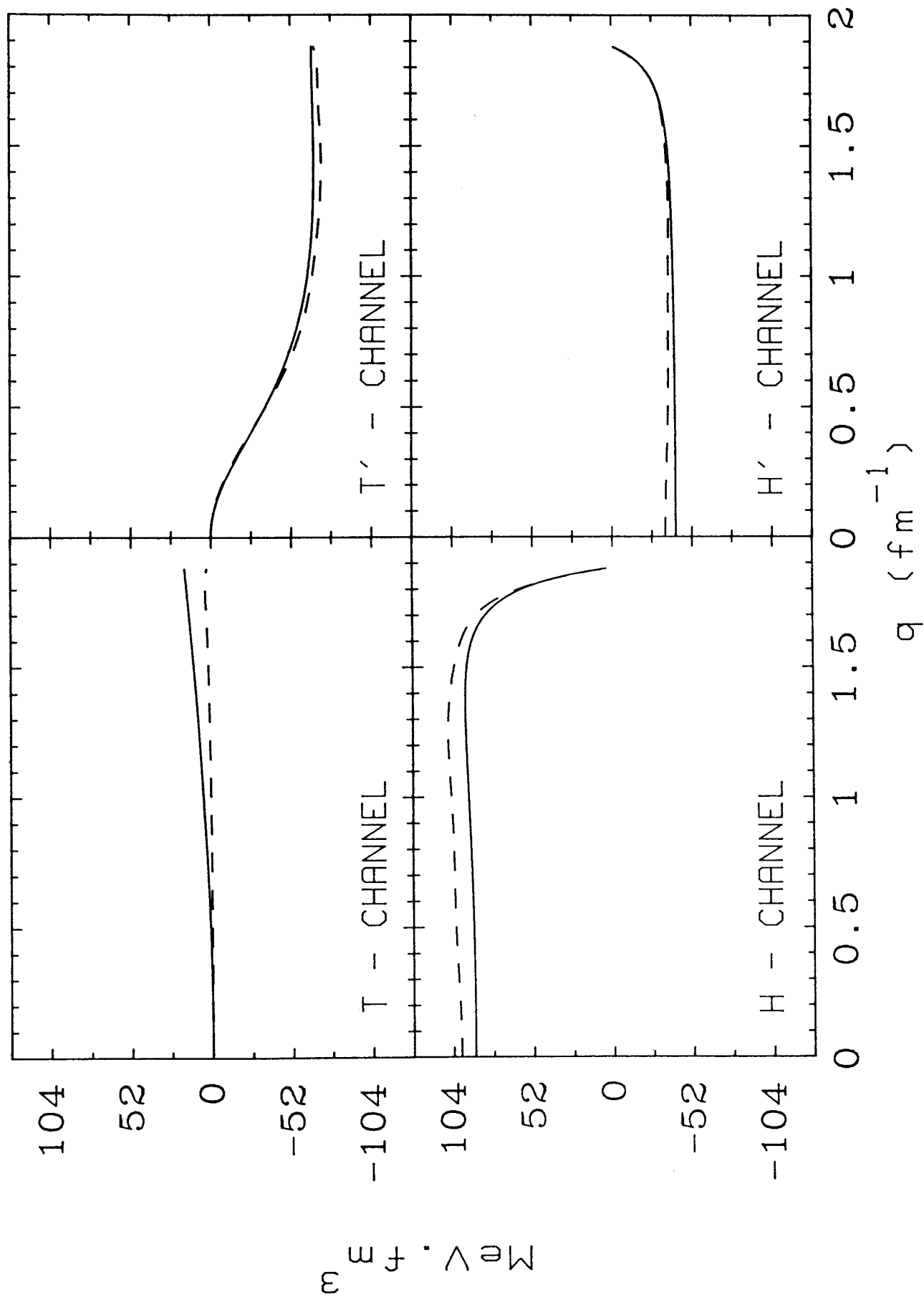
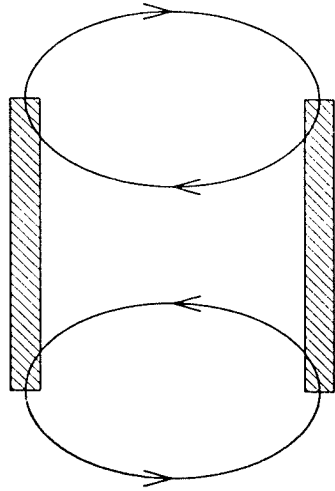
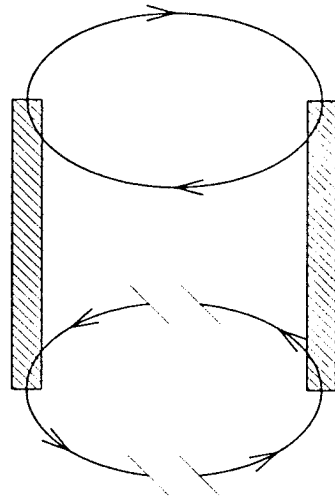


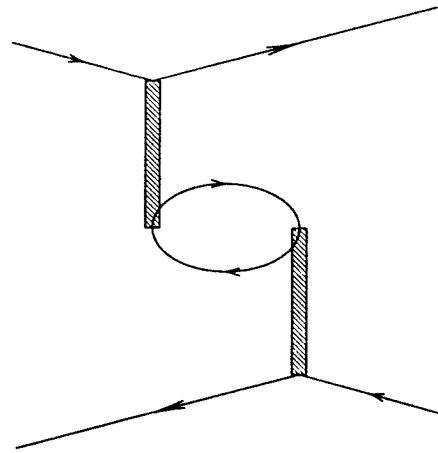
Fig. 37



a)

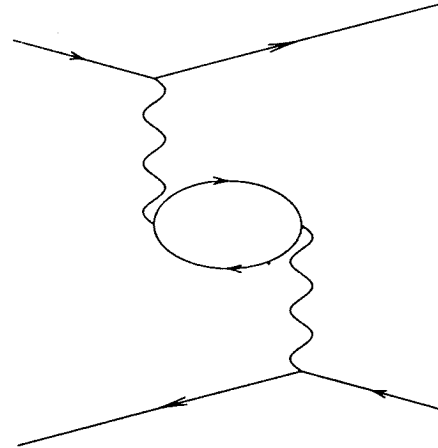


b)

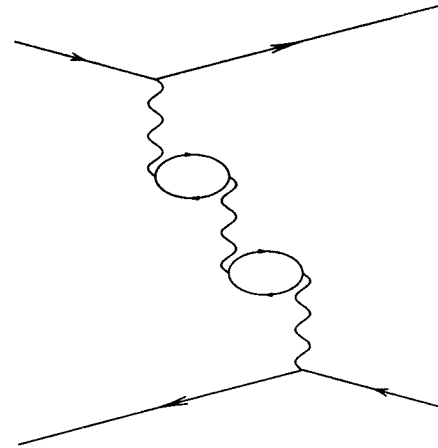


c)

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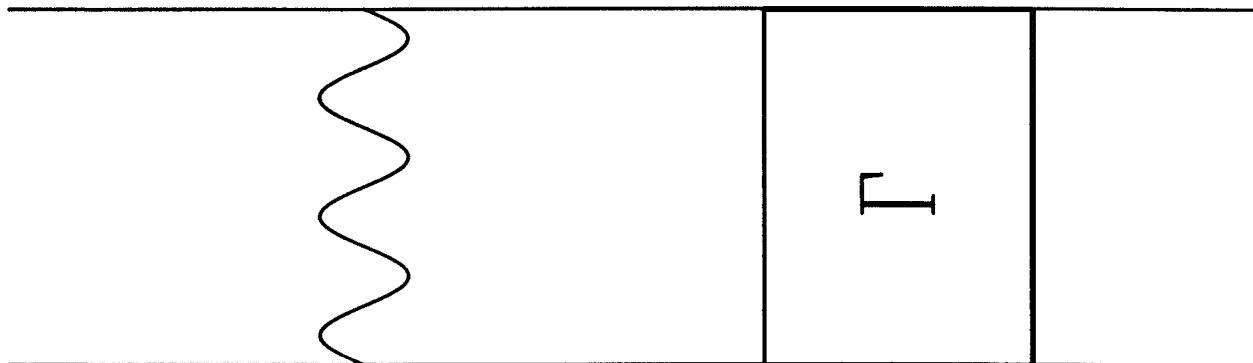
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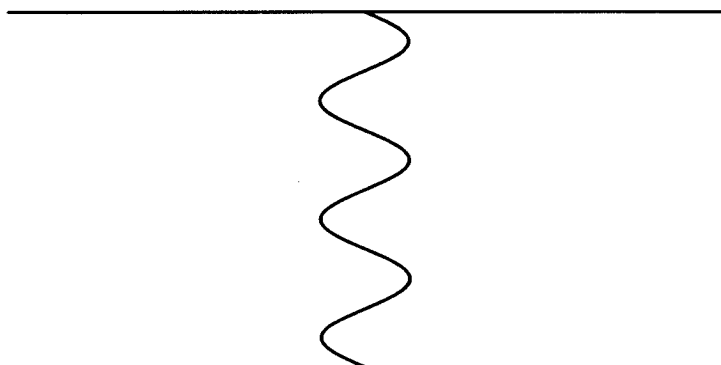
+

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Fig. 38



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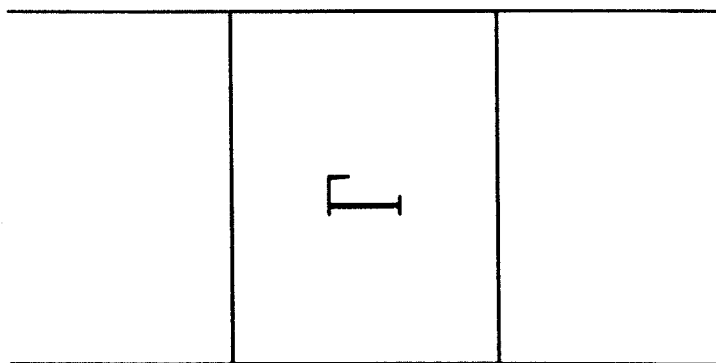


Fig. 39

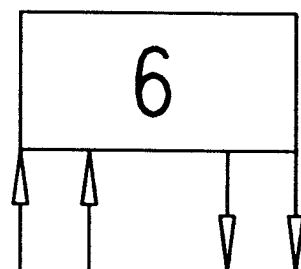
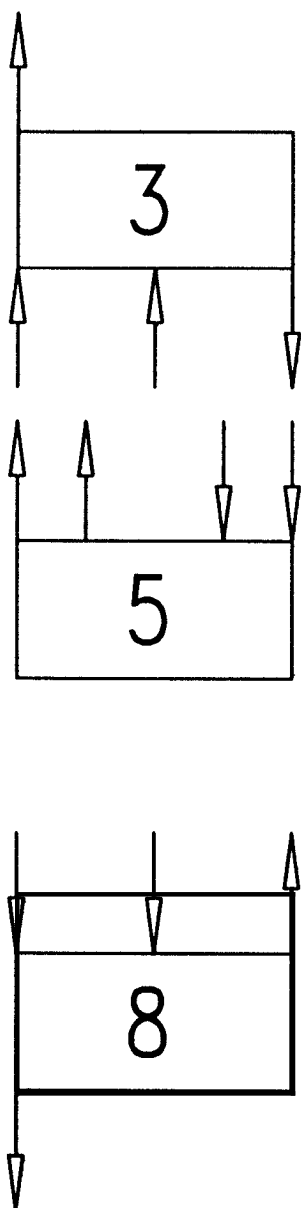
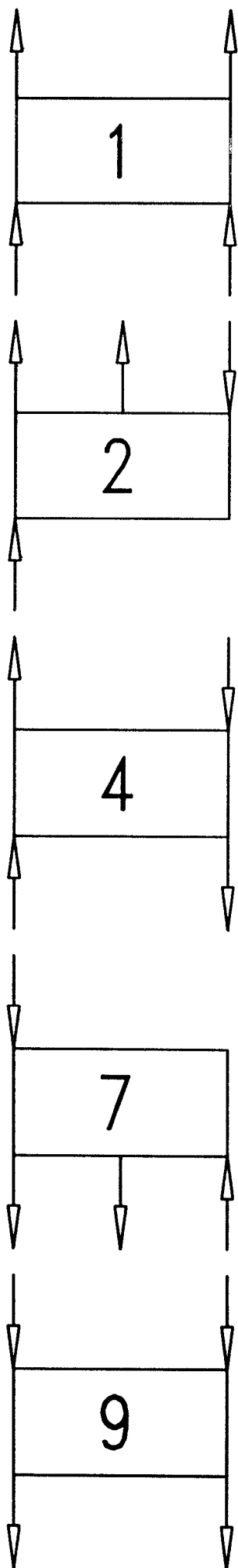
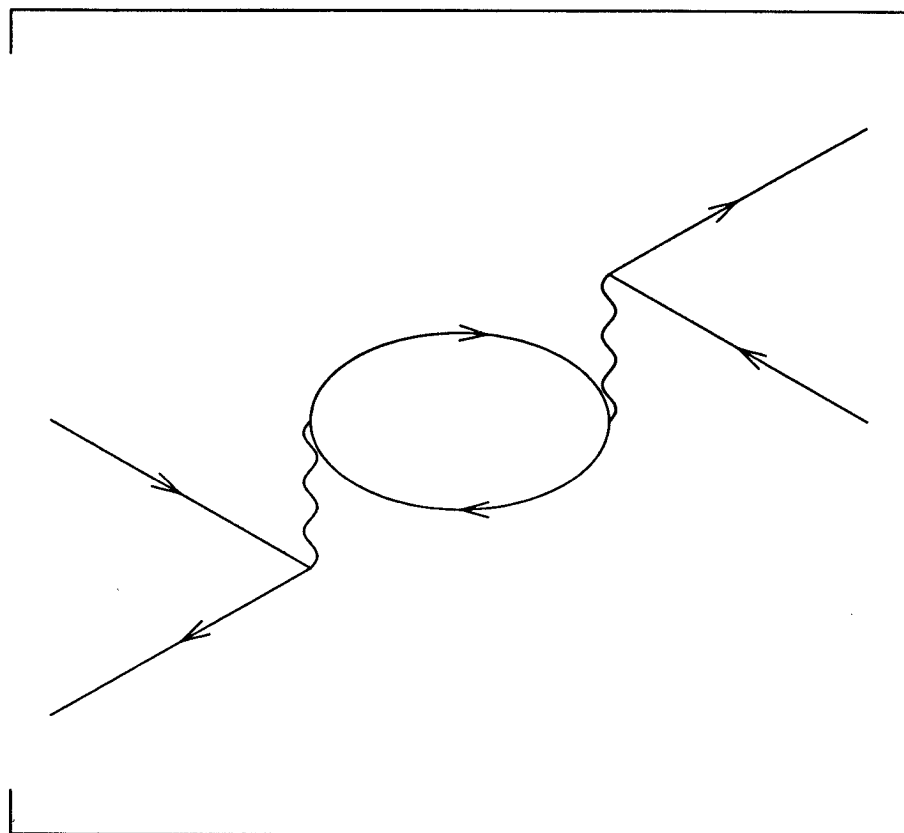


Fig. 40

Fig. 41



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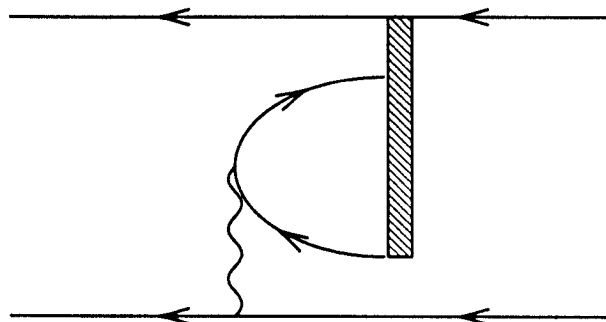
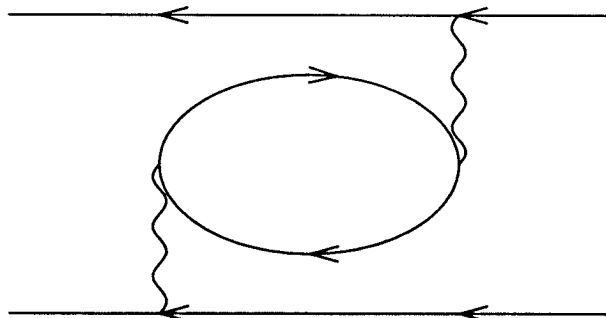


Fig. 42a

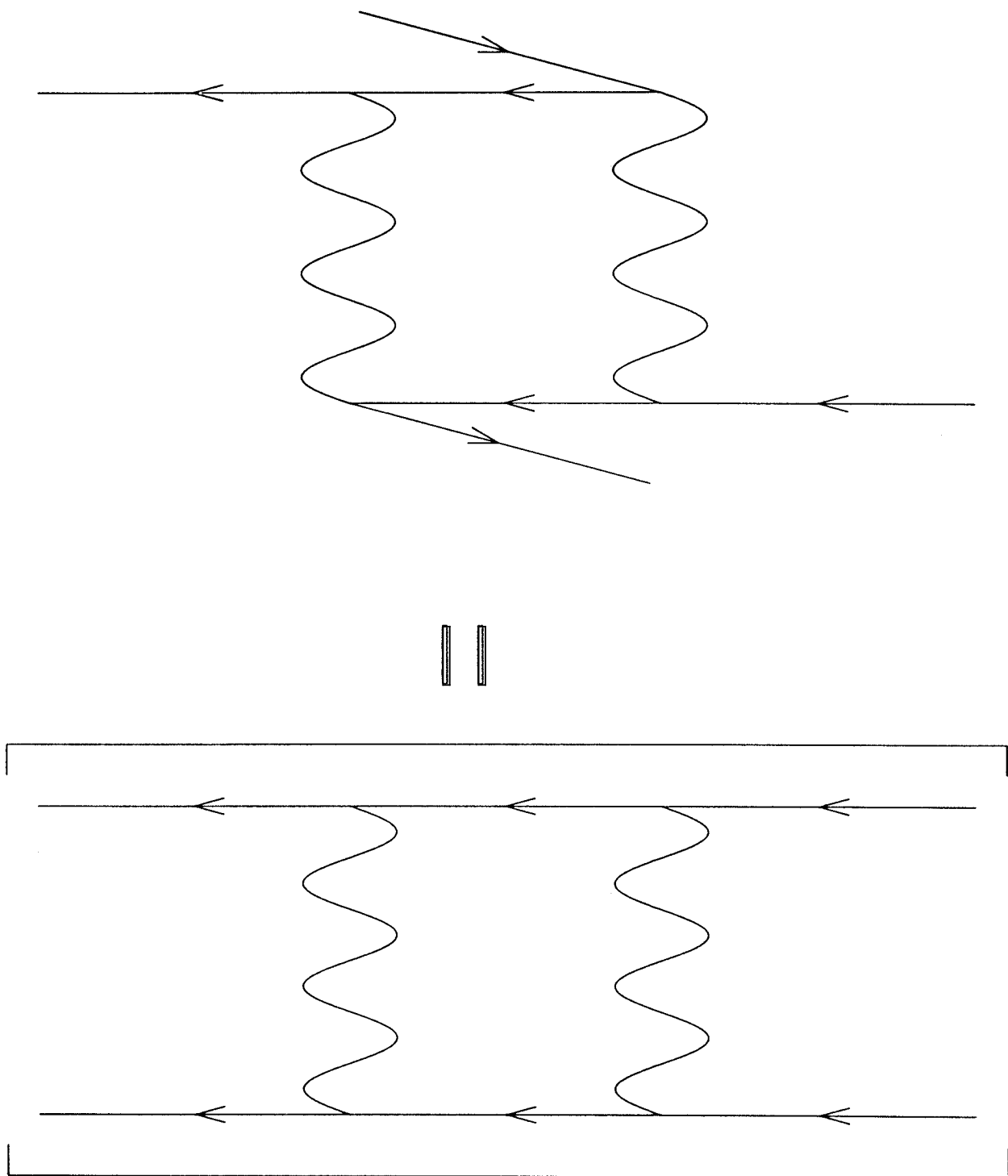
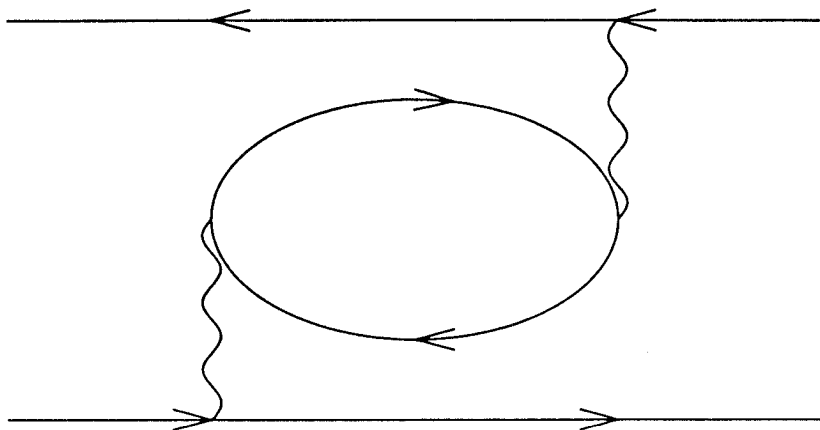
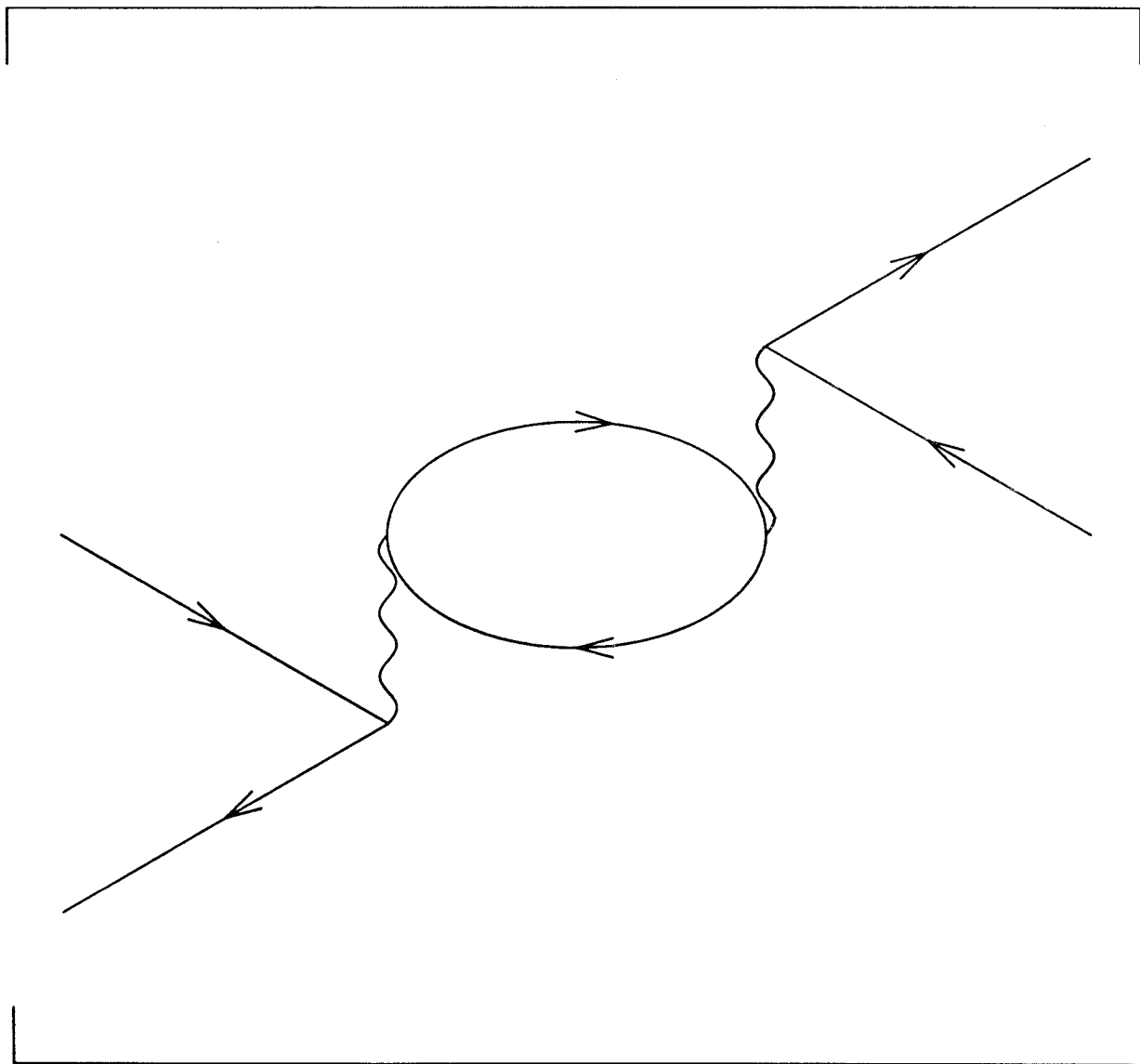
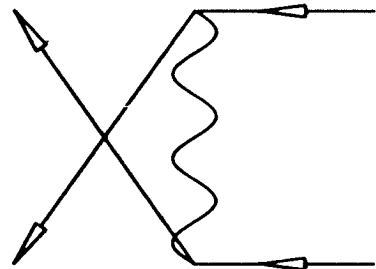


Fig. 42b

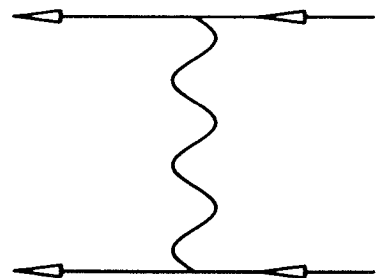


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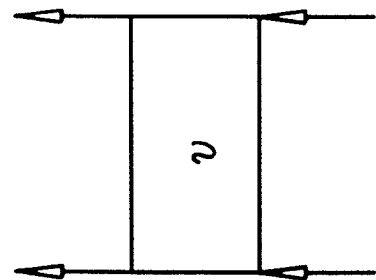




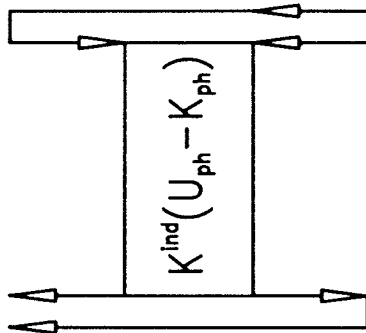
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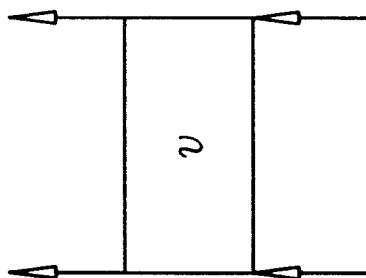
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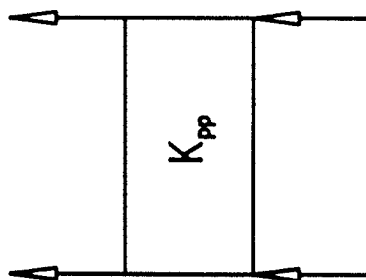
a)



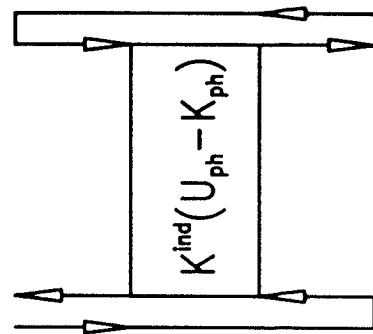
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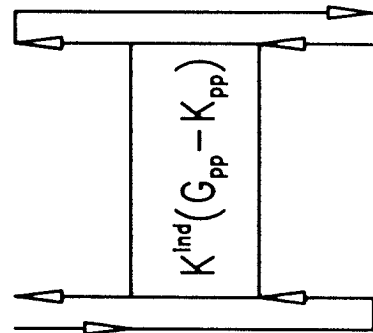
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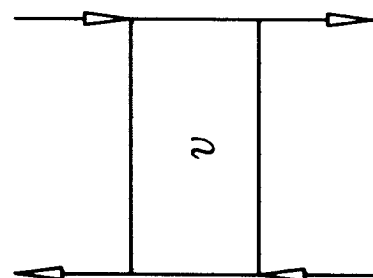
b)



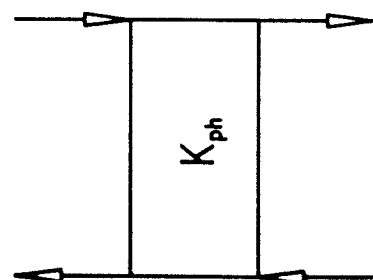
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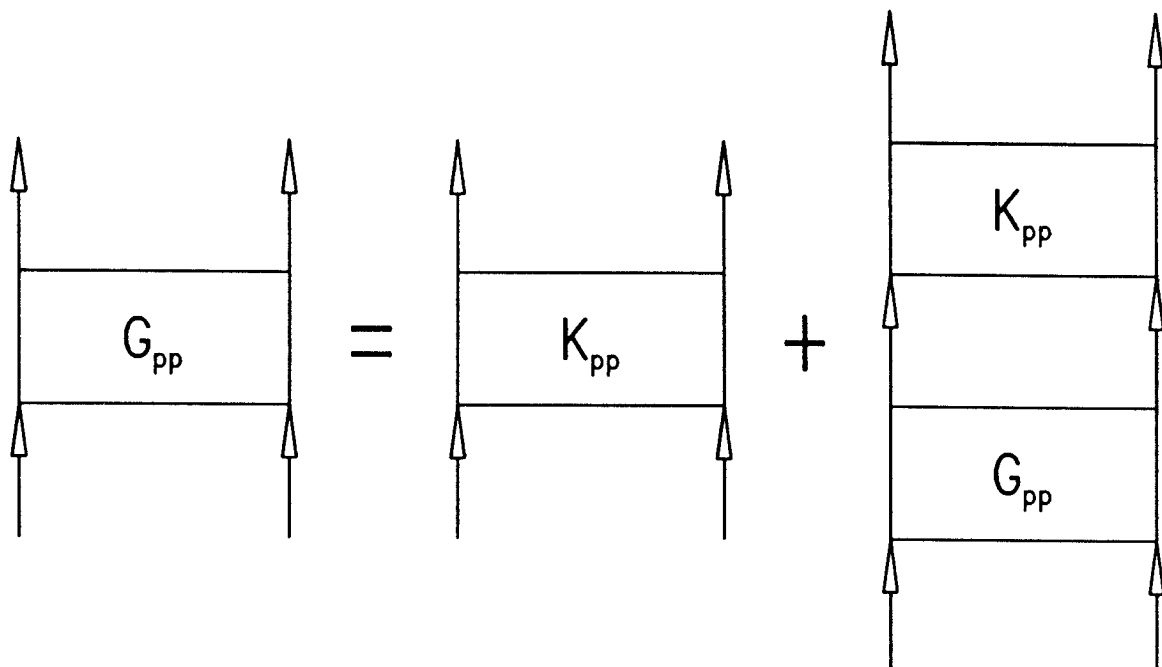


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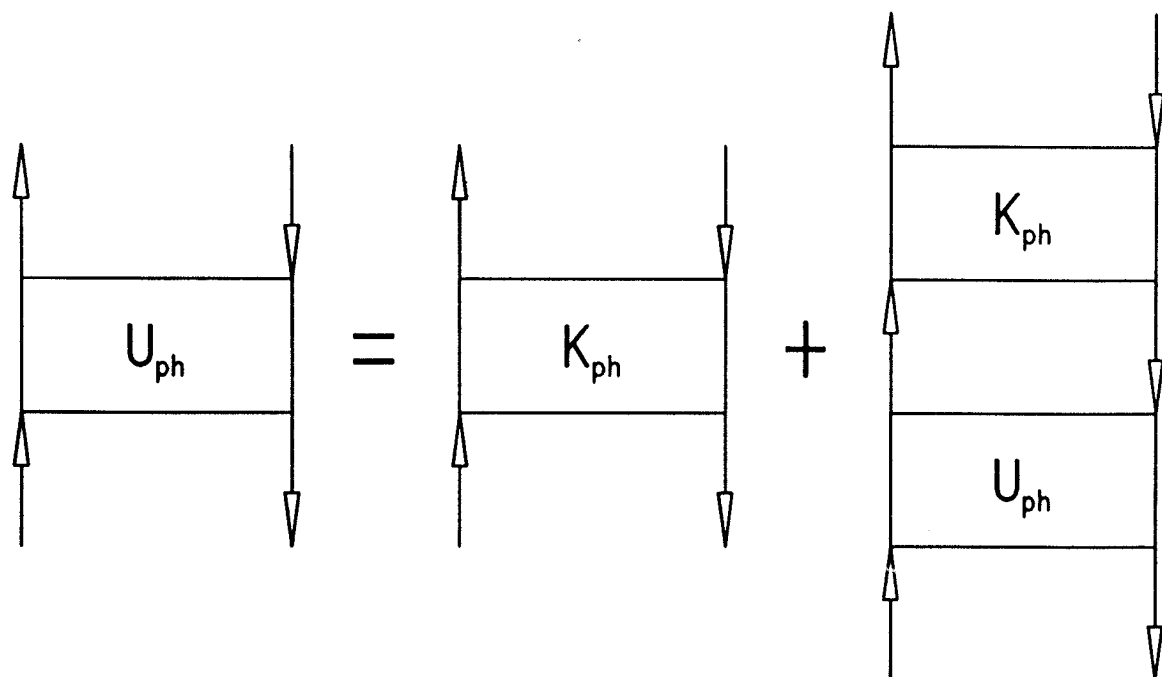


c)

d)



e)



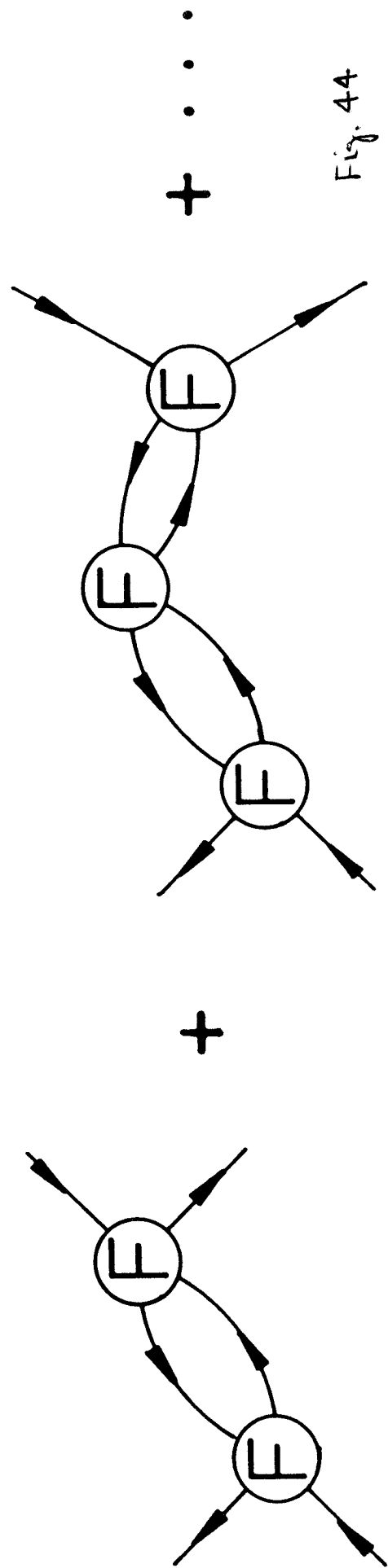
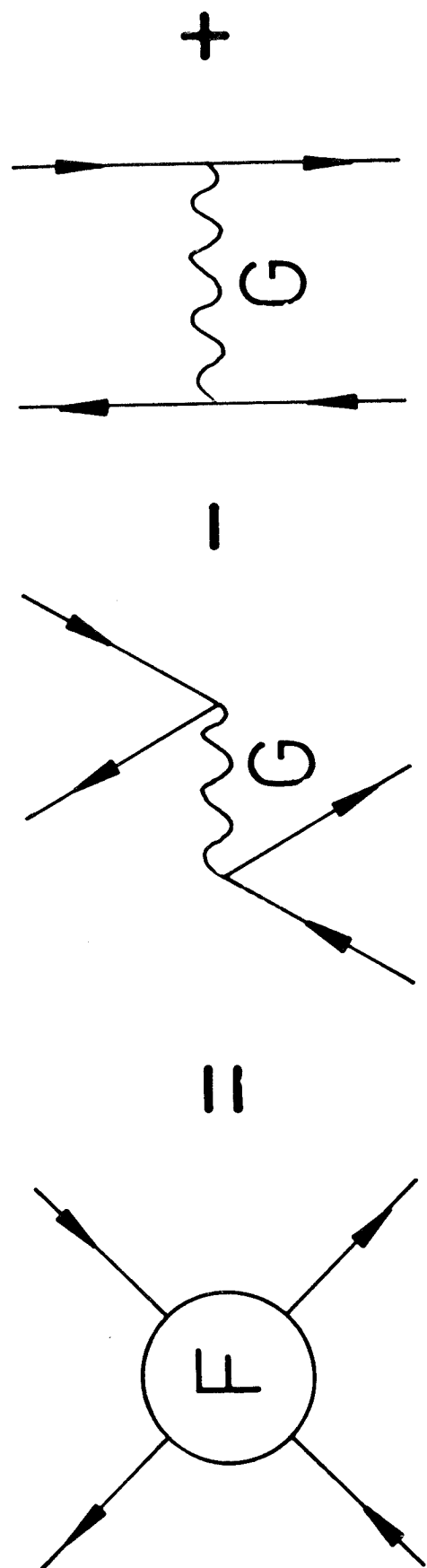


Fig. 44

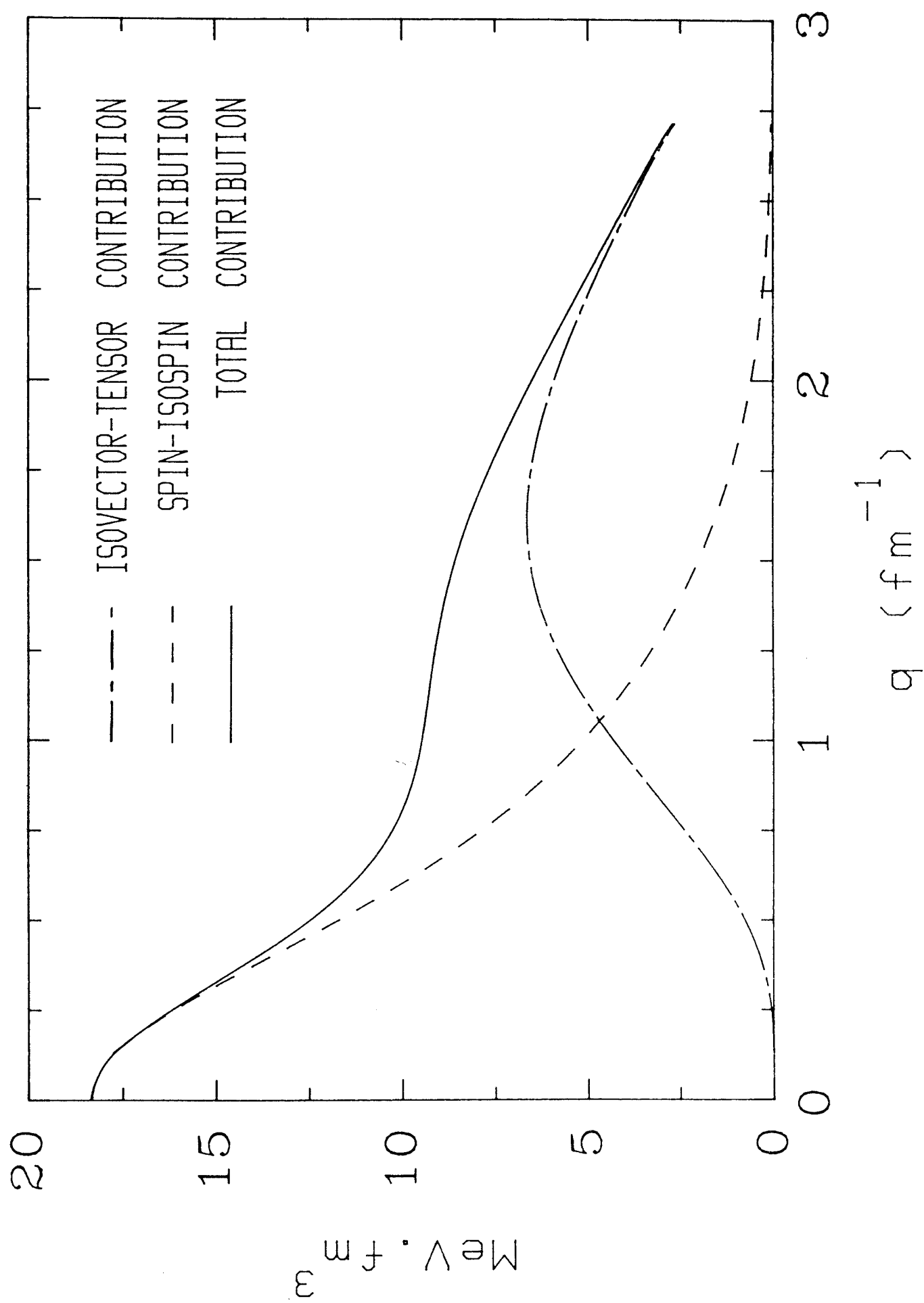


Fig. 45

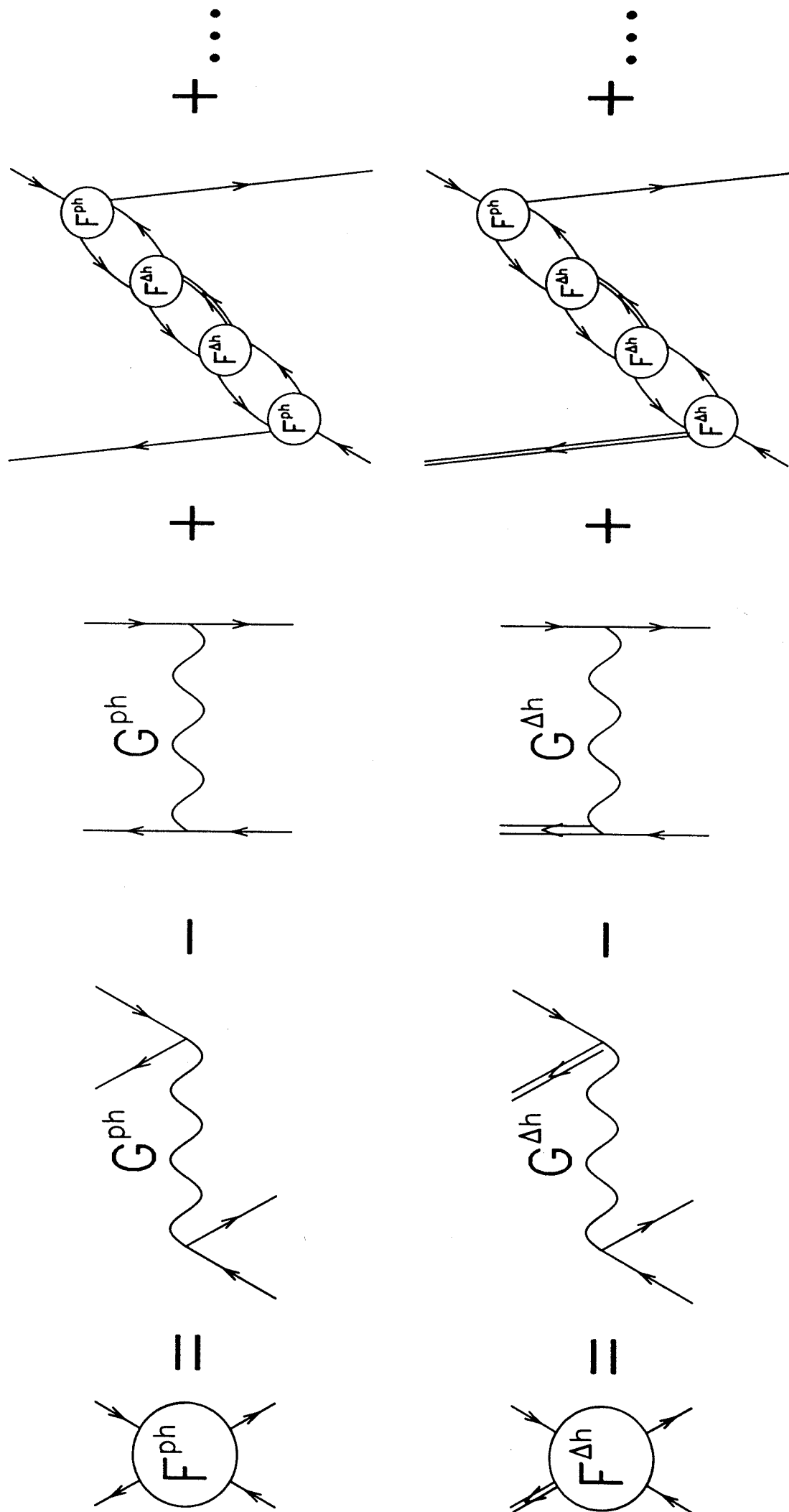


Fig. 46

