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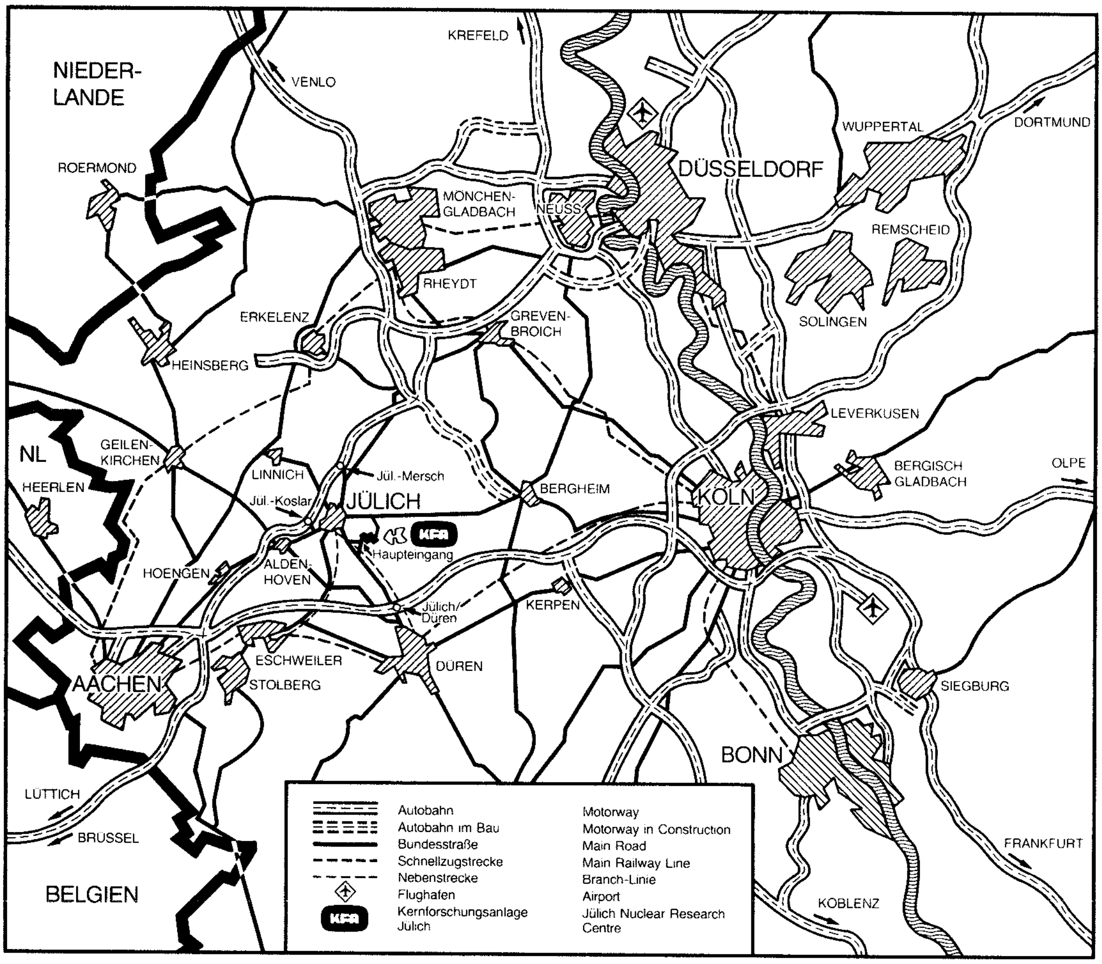

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ABSTRACT

It is shown that the parallel electron viscosity leads to an enhanced pinching effect in the collisional regime. The inward neoclassical convection velocity is $\langle U_r \rangle = - \langle cB_\chi E_\phi / B^2 \rangle \left[1 + 2.15\, q^2 (\lambda_{mfp}/qR)^2\right]$ - assuming cylindrical cross-sections - where λ_{mfp} is the electron mean-free path. It is also shown that experimental profiles in the plasma edge are inconsistent, from the point of view of power balance, with solely neoclassical heat losses by many orders of magnitude. Finally the poloidal rotation velocity obtains from ambipolarity

I. INTRODUCTION

Collisional transport in axisymmetric plasmas has been extensively studied in a large number of specialized papers (Pfirsch and Schlüter, 1962; Galeev and Sagdeev, 1967; Johnson and von Goeler, 1969; Kovrizhnikk, 1969; Rutherford, 1970; Ware, 1970; Rutherford et al., 1970; Grad and Hogan, 1970; Stringer, 1971 and 1972; Samain, 1972; Engelmann and Nocentini, 1976; Bolton and Ware, 1981; Chang and Hinton, 1981) and of review articles (Hinton and Hazeltine, 1976; Hirschmann and Sigmar, 1981; Kovrizknikk, 1984). One interesting result, first discovered by Ware (1970), is the enhanced pinching effect which appears at low and intermediate collision frequencies (respectively Banana and Plateau regimes). We show here that an enhanced pinch, driven by electron viscosity, also appears in the high collisionality regime. The poloidal rotation velocity is obtained consistently from the ambipolarity constraint.

The enhancement factor is at most $2.15q^2$ in the limits of validity of the fluid equations (mean free path $\leq$ connection length). This is still orders of magnitude too small to explain particle balance, especially during gas puffing. It is found similarily that the neoclassical heat transport is orders of magnitude too small to balance the ohmic power deposition.

The analysis thus shows that both an anomalous pinch effect and anomalous heat losses are necessary to explain particle and heat balance in the outer plasma layers. This conclusion is consistent with the observation of a well developed microturbulence in these regions (e.g. Gondhalekar et al., 1978; Gentle et al., 1984). A theory has actually been proposed recently to unfold the nature of the underlying instability (Rogister, 1985 a, b). We expect these fluctuations will give rise both to enhanced particle convection and to enhanced heat losses partly because of their small radial wavelengths. This will be the subject of another research.

The paper is organized as follows. The basic equations and the scaling procedure are described in Section II. The macroscopic equations are solved in Section III. The pinching effect is discussed in Section IV along with the neoclassical losses and the poloidal rotation velocity. Section V concludes the paper by a brief summary.

II. BASIC EQUATIONS AND SCALING

We first order the characteristic lengths, velocities and times in powers of an appropriate expansion parameter in order to be able to solve the macroscopic plasma equations. We also introduce a mass scaling to account easily for the different behaviour of electrons and ions across and along the magnetic field. We assume

$$\frac{\nu_{ei}}{\Omega_i} \sim \frac{a_s}{a} \quad \frac{m_e}{m_i} \sim \mu^2, \tag{1}$$

$$\frac{U_{\|,e}}{c_s} \sim \frac{eV}{T_e} \sim 1 \tag{2}$$

and seek solutions of the form

$$N = N^{(o)} + N^{(1)} + \ldots, \tag{3a}$$

$$T_J = T_J^{(o)} + T_J^{(1)} + \ldots, \tag{3b}$$

$$\vec{B} = \vec{B}^{(o)} + \vec{B}^{(1)} + \ldots, \tag{3c}$$

$$\vec{U}_J = \vec{U}_J^{(o)} + \vec{U}_J^{(1)} + \ldots \qquad \text{with } \vec{U}_J^{(o)} \sim c_s \tag{3d}$$

where $X^{(k)}/X^{(o)} \sim \mu^k$. The notations are standard with $\|$, respectively $\perp$, referring to the direction of the magnetic field; V is the electrostatic self-consistent potential; $c_s = (T_e/m_i)^{1/2}$ in the sound speed; $a_s = c_s/\Omega_i$ is the corresponding Larmor radius; J is the species index, = i for ions, = e for electrons; a is the minor plasma radius. We choose also to differentiate between the equilibrium length-scales across and along the magnetic field:

$$a/qR \sim \mu \tag{4}$$

where R is the major radius and qR the connection length. Accordingly $B_\chi/B_\phi \sim \mu$ where B_χ, respectively B_ϕ, is the poloidal, respectively the toroidal component of the magnetic field.

The scaling defined by Eqs. (1), (2), and (4) is chosen to fit as closely as possible to the ordering of the experimental parameters relevant to the plasma edge - where the fluid description applies - in a tokamak device. For the purpose of an illustration, we may assume B = 4 Tesla, a = 20 cm, R = 100 cm, $T_e = T_i = 10^2$ eV, $q_a = 3$ and $N = 10^{13}$ cm^{-3}. We find $\tau_e = 2 \times 10^{-6}$ sec (τ_e is the electron collision time defined by Braginskii, 1965), $\Omega_i = 4 \times 10^8$ sec^{-1} (assuming the gas is hydrogen), $a_s = 2.5\ 10^{-2}$ cm. Hence $\nu_{ei}/\Omega_i = 1/800$ and $(a/q_aR)^2 = 1/225$ whereas $U_{e,\parallel}/c_s = 1.1$ if the loop voltage $V_L = 2$ volts. We note that (1), (2), and (4) actually imply

$$\frac{eV_L}{T_e} \sim \mu \ . \tag{5}$$

We proceed by writing the macroscopic equations; the different terms are ordered according to the scaling defined in Eqs. (1-5). The reference time-scale will be the transit time a/c_s; hence $t_o c_s/a \sim 1$; $t_1\ c_s/a \sim \mu$, etc. The equations of continuity are

$$\partial N/\partial t + \vec{\nabla} \cdot (N\vec{U}) = 0 \tag{6}$$

$$1; \qquad\qquad 1.$$

since $N_i = N_e = N$ in the limit of vanishingly small Debye length ($\lambda_D^2/a^2 \sim \mu^6$ if $\omega_{pe} \sim \Omega_e$ as is the case in the experiments; ω_{pe} is the electron plasma frequency). The components of the momentum equations are

$$m_J\, N\, \hat{p} \cdot \underset{(\mu^4,\ \mu^2)}{\left(\frac{\partial}{\partial t} + \vec{U}_J \cdot \vec{\nabla}\right) \vec{U}_J} = -\, \hat{p}\vec{\nabla} : \left(\underset{\mu^2;}{P_J \overleftrightarrow{I}} + \underset{(\mu^2,\mu);}{\overleftrightarrow{\P}_J}\right) + e_J N \hat{p} \cdot \Big(\underset{\mu^2;}{-\vec{\nabla}V} + \underset{\mu^4;}{\vec{E}}$$

$$+ \underset{1;}{\frac{\vec{U}_J}{c} \times \vec{B}}\Big) + \underset{\mu^4}{\hat{p}\cdot\vec{R}_J} \tag{7}$$

$$m_J N \hat{b} \cdot \underset{(\mu^4,\mu^2);}{\left(\frac{\partial}{\partial t} + \vec{U}_J \cdot \vec{\nabla}\right)\vec{U}_J} = -\hat{b}\vec{\nabla} : \underset{\mu^2;(\mu^2,\mu);}{\left(P_J\overleftrightarrow{I} + \overleftrightarrow{\P}_J\right)} + e_J N \hat{b} \cdot \Big(\underset{\mu^2;}{-\vec{\nabla}V} + \underset{\mu^4;}{\vec{E}} + \underset{1;}{\frac{\vec{U}_J}{c} \times \vec{B}}\Big) + \underset{\mu^4}{\hat{b}\cdot\vec{R}_J} \qquad ($$

$$m_J N \hat{n} \cdot \underset{(\mu^2,1);}{\left(\frac{\partial}{\partial t} + \vec{U}_J \cdot \vec{\nabla}\right)\vec{U}_J} = -\, \hat{n}\vec{\nabla} : \left(\underset{\mu;}{P_J\overleftrightarrow{I}} + \underset{(\mu,1)}{\overleftrightarrow{\P}_J}\right) + e_J N \hat{n} \cdot \Big(\underset{\mu;}{-\vec{\nabla}V} + \underset{\mu^2;}{\vec{E}}\Big) + \underset{\mu.}{\hat{n}\cdot\vec{R}_J} \tag{9}$$

where $\overleftrightarrow{\P}$ is the stress tensor of which the leading part is

$$\overleftrightarrow{\P}_{o,J} = \left(\hat{n}\hat{n} - \frac{1}{3}\overleftrightarrow{I}\right) A_{o,J} \tag{10a}$$

$$A_{o,J} = -3\eta_{o,J} \left(\hat{n}\hat{n} - \frac{1}{3}\overleftrightarrow{I}\right) : \vec{\nabla}\vec{U}_J\ , \tag{10b}$$

$\eta_{o,j} = 0.96\ N T_i \tau_i$, $\eta_{o,e} = 0.73\ P_e \tau_e$. There follows that

$$\vec{\nabla}\cdot\overleftrightarrow{\P}_{o,J} = \Big(\underset{\mu^2;}{-\frac{1}{B}\,\hat{n}\cdot\vec{\nabla}B\hat{n}} + \underset{\mu;}{\hat{n}\cdot\vec{\nabla}\hat{n}}\Big) A_{o,J} + \underset{\mu;}{\hat{n}\ \hat{n}\cdot\vec{\nabla}\ A_{o,J}} - \underset{1.}{\frac{1}{3}\,\vec{\nabla}\ A_{o,J}} \tag{11}$$

and we note that

$$\hat{n}\cdot\vec{\nabla}\hat{n} = \frac{\vec{\nabla}B}{B} - \frac{\hat{n}\cdot\vec{\nabla}B}{B}\,\hat{n} + 4\pi\,\frac{\vec{J}x\vec{B}}{cB^2}\,. \tag{12}$$

$\vec{E}$ is the inductive electric field which obtains from the Maxwell's equations

$$\frac{\partial}{\partial\chi}\,(h_\phi E_\phi) = 0 \tag{13a}$$

$$\underset{\mu^3;}{\frac{\partial}{\partial\psi}\,(h_\phi E_\phi)} = \underset{1.}{\frac{1}{c}\,\frac{1}{B_\chi}\,\frac{\partial B_\chi}{\partial t}} \tag{13b}$$

$$\underset{\mu^4;}{\frac{\partial}{\partial\psi}\,(h_\chi E_\chi)} - \underset{\mu^4;}{\frac{\partial}{\partial\chi}\,(h_\psi E_\psi)} = \underset{1}{-\,\frac{1}{c}\,\frac{\nu}{B_\phi}\,\frac{\partial B_\phi}{\partial t}} \tag{13c}$$

where we have scaled $E_\psi \sim E_\chi \sim E_\phi \sim V_L/R$;

$$\nu = h_\chi B_\phi / h_\phi B_\chi \tag{14}$$

is related to the differential rotation of the field lines and $\oint d\chi\nu/\oint d\chi = q(\psi)$. Furthermore $\hat{n} = \vec{B}/B$ is the unit vector along the lines of force; $\hat{p} = -\vec{\nabla}\psi/|\vec{\nabla}\psi|$ is the unit vector orthogonal to the flux surface $\psi(\vec{r}) = \mathrm{Cte}$; $\hat{b} = \hat{n} \times \hat{p}$ is the binormal to $\vec{B}$. We have finally introduced the coordinate system $\hat{e}_\psi \equiv \hat{p}$, $\hat{e}_\chi \equiv \hat{e}_\phi \times \hat{e}_\psi$, and $\hat{e}_\phi \equiv$ unit vector in the toroidal direction. The functions $h_\psi(\vec{r})$, $h_\chi(\vec{r})$ and $h_\phi(\vec{r})$ are defined by the expression for the elementary length, squared:

$$ds^2 = (h_\psi d\psi)^2 + (h_\chi d\chi)^2 + (h_\phi d\phi)^2$$

$$= (d\psi/RB_\chi)^2 + (JB_\chi d\chi)^2 + (Rd\phi)^2 \tag{15}$$

with the convention introduced by Mercier (1960). J is the Jacobian of the transformation $\vec{r} \to \psi, \chi, \phi$. Further $\vec{R}_J$ is the friction force given by

$$\vec{R}_e = -\vec{R}_i = -\frac{m_e N}{\tau_e}\left[0.51\ \underset{\mu;}{(U_{\parallel,e}-U_{\parallel,i})\,\hat{n}} + \underset{\mu;}{(\vec{U}_{e,\perp}-\vec{U}_{i,\perp})}\right] - \underset{1}{0.71\, N\, \hat{n}\, \hat{n}\cdot\vec{\nabla}T_e}$$

$$+ \underset{\mu^3}{\frac{3}{2}\frac{N}{\Omega_e \tau_e}\,\hat{n} \times \vec{\nabla}T_e}$$

($\Omega_e = -\ |e|B/m_e c$). It is also noted that the first scaling in Eqs. (7-9) and (11) applies to electrons; the second to ions. Where only one ordering is given, it applies to both.

Ampere's equations couple the magnetic field and the flows:

$$\frac{\partial}{\partial\chi}(h_\phi B_\phi) = 0 \tag{16a}$$

$$\underset{1;}{\frac{\partial}{\partial\psi}(h_\phi B_\phi)} = -\underset{\mu}{\frac{4\pi}{c}\frac{1}{B_\chi}J_\chi} \tag{16b}$$

$$\underset{1;}{\frac{\partial}{\partial\psi}(h_\chi B_\chi)} = \underset{1;}{\frac{4\pi}{c}\frac{\nu}{B_\phi}J_\phi} \tag{16c}$$

We have assumed here that β_P, the ratio of the kinetic to the magnetic pressure exerted by the poloidal field, is of order μ; the ordering is adequate for the cold outer plasma layers.

We have finally to consider the electron and ion heat equations:

$$\underset{1;}{\frac{3}{3}N\left(\frac{\partial}{\partial t} + \vec{U}_J\cdot\vec{\nabla}\right)T_J} + \underset{1;}{P_J\vec{\nabla}\cdot\vec{U}_J} + \frac{5}{2}\frac{c}{e_J}\vec{\nabla}T_J\cdot\left(\underset{\mu^3;}{\frac{P_J}{B^2}\frac{4\pi}{c}\vec{J}} + \underset{\mu^2;}{\vec{\nabla}\frac{P_J}{B^2}\times\vec{B}}\right)$$

$$= -\underset{(1,\mu);}{\vec{\nabla}\cdot\vec{q}_J} - \underset{(1,\mu^{-1});}{\overleftrightarrow{\pi}_J\cdot\vec{\nabla}\vec{U}_J} + \underset{(\mu,\mu^2)}{Q_J} \quad , \tag{17}$$

where q_J is the heat flux

$$\vec{q}_e = \underset{\mu;}{0.71\, P_e\, (U_{\parallel,e}-U_{\parallel,i})\, \hat{n}} - \underset{\mu^5;}{\frac{3}{2}\,\frac{P_e}{\Omega_e \tau_e}\, \hat{n} \times (\vec{U}_e-\vec{U}_i)} - \underset{1}{\chi_{\parallel,e}\, \hat{n}\cdot\vec{\nabla}T_e \hat{n}} - \underset{\mu^7}{\chi_{\perp,e} \vec{\nabla}_{\perp} T_e} \qquad (18a)$$

$$\vec{q}_i = - \underset{1;}{\chi_{\parallel,i}\, \hat{n}\cdot\vec{\nabla}T_i\, \hat{n}} - \chi_{\perp,i}\, \vec{\nabla}_{\perp} T_i \qquad (18b)$$

We note that the terms $(5\, cP_J/2\, e_J B)\, \hat{n} \times \vec{\nabla}T_J$ appearing in Braginskii's definition of the heat fluxes has been incorporated in the left-hand side of Eq. (17). We recall that

$$\chi_{\parallel,J} = \alpha_J\, P_J \tau_J / m_J \qquad (19a)$$

$$\chi_{\perp,J} = \beta_J\, P_J / m_J \Omega_J^2 \tau_J \qquad (19b)$$

where $\alpha_e = 3.16$, $\alpha_i = 3.9$, $\beta_e = 4.66$, and $\beta_i = 2$. The source and sink terms are

$$Q_e = - \underset{1;}{\vec{R}_e \cdot (\vec{U}_e - \vec{U}_i)} - \underset{\mu.}{\frac{3m_e}{m_i}\,\frac{N}{\tau_e}\,(T_e - T_i)} \qquad (20a)$$

$$Q_i = \underset{\mu.}{\frac{3m_e}{m_i}\,\frac{N}{\tau_e}}\,(T_e - T_i) \qquad (20b)$$

if there is no auxiliary heating.

Practically one is interested in knowing the losses through the magnetic surfaces. We thus integrate Eqs. (6) and (17) over the volume $d\psi\oint J d\chi d\phi$ and obtain

$$\frac{\partial}{\partial t}\underset{1}{\oint J d\chi N} + \frac{\partial}{\partial \psi}\underset{1}{\oint J d\chi\; h_\psi^{-1}\; N\; U_{\psi,J}} = 0 \tag{21}$$

$$\frac{3}{2}\frac{\partial}{\partial t}\underset{1;}{\oint J d\chi\; NT} + \frac{3}{2}\frac{\partial}{\partial \psi}\underset{1;}{\oint J d\chi\; h_\psi^{-1}\; NT_J U_\psi} + \underset{1;}{\oint J d\chi P_J \left[\frac{1}{J}\frac{\partial}{\partial \psi}\left(J\, h_\psi^{-1}\; U_{\psi,J}\right)\right.}$$

$$\underset{1;}{\left. + \frac{1}{J}\frac{\partial}{\partial \psi}\left(J\, h_\chi^{-1}\; U_{\chi,J}\right)\right]} - \frac{5}{2}\frac{c}{e_J}\frac{\partial}{\partial \psi}\underset{\mu^2;}{\oint J d\chi\; h_\psi^{-1}\; \left(\vec{\nabla} T_J \times \frac{P_J}{B^2}\vec{B}\right)_\psi}$$

$$= -\frac{\partial}{\partial \psi}\underset{(\mu^4;\mu^5)}{\oint J d\chi\; h_\psi^{-1}\; q_\psi} - \underset{(1;\mu^{-1})}{\oint J d\chi\; \overleftrightarrow{\pi}_J : \vec{\nabla}\vec{U}_J} + \underset{(\mu;\mu^2)}{\oint J d\chi Q_J} \tag{22}$$

since the variable ϕ is ignorable.

The time derivatives in Eqs. (6), (17), (21) and (22) may include particle and heat sources and sinks. These will be chosen in such a way that the macroscopic evolution of the plasma occurs on time-scales comparable to the diffusion processes.

III. SOLUTION OF MACROSCOPIC EQUATIONS

Our aim is to solve Eqs. (6-22) without introducing any ad hoc assumption. Equations (7) and (8), taken at orders μ^o and μ, imply that

$$\hat{n} \times \vec{U}_J^{(0,1)} = 0.$$

Equation (21) then shows that $\partial N^{(o)}/\partial t_o = 0$: We assume, in order to avoid secular behaviour of the other or the higher order variables, that this implies that $\partial/\partial t_o \equiv 0$ whatever the function to which the derivative applies. Accordingly we have also $\partial N^{(o)}/\partial t_1 = 0$ (and $\partial/\partial t_1 \equiv 0$). It follows from the continuity equation (6) that $(\vec{\nabla}\cdot\vec{U}_J)^{(o)} = -\left[(\vec{U}_J\cdot\vec{\nabla})^{(o)}\, N^{(o)}\right]/N^{(o)} = 0$ and $(\vec{\nabla}\cdot\vec{U}_J)^{(1)} = \hat{n}\cdot\vec{\nabla}U^{(o)}_{J,\parallel} = -U^{(o)}_{J,\parallel}\,(\hat{n}\cdot\vec{\nabla}\, N^{(o)})/N^{(o)}$. Indeed the divergence of any vector $\vec{f}$ is

$$\vec{\nabla}\cdot\vec{f} = \underset{1;}{\hat{p}\cdot\vec{\nabla}\ \hat{p}\cdot\vec{f}} + \underset{1;}{\hat{b}\cdot\vec{\nabla}\ \hat{b}\cdot\vec{f}} + \underset{\mu;}{\hat{n}\cdot\vec{\nabla}\ \hat{n}\cdot\vec{f}} + \underset{1;}{\hat{p}\cdot\vec{f}\ \hat{b}\cdot\vec{\nabla}\hat{p}\cdot\hat{b}} + \underset{\mu;}{\hat{b}\cdot\vec{f}\ \hat{p}\cdot\vec{\nabla}\hat{b}\cdot\hat{p}}$$

$$- \vec{f}\cdot\Big(\underset{\mu;}{\frac{\vec{\nabla}B}{B}} + \underset{\mu.}{\frac{4\pi}{c}\frac{\vec{J}\times\vec{B}}{B^2}}\Big)$$

since $\hat{n}\cdot\vec{\nabla} \equiv (B_\chi/B)h_\chi^{-1}\ \partial/\partial\chi \sim \mu/a$ and $\hat{p}\cdot\vec{\nabla}\ B_{\phi,\chi}/B_{\phi,\chi} \sim \hat{b}\cdot\vec{\nabla}\ B_{\phi,\chi}/B_{\phi,\chi} \sim 1/R \sim \mu/a$. Equations (10a,b) yield accordingly $\overleftrightarrow{\Pi}^{(o)}_{o,J} = 0$ and $\overleftrightarrow{\Pi}^{(1)}_{o,J} = -2\,\eta_{o,J}\,(\hat{n}\cdot\vec{\nabla}U^{(o)}_{\parallel,J})$ $(\hat{n}\hat{n} - \overleftrightarrow{I}/3)$.

The electron heat equation writes now

$$\hat{n}\cdot\vec{\nabla}\,(\chi^{(o)}_{\parallel,e}\ \hat{n}\cdot\vec{\nabla}\,T^{(o)}_e) = 0$$

which implies, because of periodicity requirements,

$$\hat{n}\cdot\vec{\nabla}\,T^{(o)}_e = 0 \tag{23}$$

The ion heat equation yields in turn:

$$\frac{3}{2} N^{(o)} U_{\|,i}^{(o)} \hat{n}\cdot\vec{\nabla}T_i^{(o)} + P_i^{(o)} \hat{n}\cdot\vec{\nabla}U_{\|,i}^{(o)} = \hat{n}\cdot\vec{\nabla} (\chi_{\|,i}^{(o)} \hat{n}\cdot\vec{\nabla}T_i^{(o)})$$

$$+ \frac{4}{3} \eta_{o,i}^{(o)} (\hat{n}\cdot\vec{\nabla}U_{\|,i}^{(o)})^2 \tag{24}$$

We sum up Eqs. (9) at order μ over electrons and ions to obtain

$$N^{(o)}\hat{n}\cdot\vec{\nabla}\left[\frac{1}{2} m_i (U_{\|,i}^{(o)})^2 + T_i^{(o)}\right] = - (T_e^{(o)} + T_i^{(o)}) \hat{n}\cdot\vec{\nabla}N^{(o)} +$$

$$+ \frac{4}{3} \hat{n}\cdot\vec{\nabla} (\eta_{o,i}^{(o)} \hat{n}\cdot\vec{\nabla}U_{\|,i}^{(o)}) \tag{25}$$

Equations (24) and (25) together with

$$\hat{n}\cdot\vec{\nabla} (NU_{\|,i}^{(o)}) = 0 \tag{26}$$

(continuity equation) can be solved to obtain $\hat{n}\cdot\vec{\nabla}N^{(o)}$, $\hat{n}\cdot\vec{\nabla}T_i^{(o)}$ and $\hat{n}\cdot\vec{\nabla}U_{\|,i}^{(o)}$. The solution that we shall retain is

$$\hat{n}\cdot\vec{\nabla}N^{(o)} = \hat{n}\cdot\vec{\nabla}T_i^{(o)} = \hat{n}\cdot\vec{\nabla}U_{\|,i}^{(o)} = 0 \tag{27}$$

Equation (9) and the electron continuity equation then yield

$$\hat{n}\cdot\vec{\nabla}V^{(o)} = \hat{n}\cdot\vec{\nabla}U_{\|,e}^{(o)} = 0 \tag{28}$$

[Toroidal axisymmetry of course implies $\hat{b}\cdot\vec{\nabla}(N^{(o)}, T_J^{(o)}, V^{(o)}, U_{\|,J}^{(o)}) = 0$]. Therefore $\overleftrightarrow{\Pi}_{J,o}^{(1)} = o$, $\vec{R}_e^{(o)} = - 0.71\ N\ \hat{n}\ \hat{n}\cdot\vec{\nabla}T_e^{(o)} = 0$, and hence $Q_e^{(o)} = 0$.

Equations (7) and (8), taken at order μ^2, imply that

$$\hat{p}\cdot\vec{U}_J^{(2)} = 0 \tag{29}$$

$$\hat{b}\cdot\vec{U}_J^{(2)} = \frac{c}{B}\left(\frac{\hat{p}\cdot\vec{\nabla}P_J^{(o)}}{e_J N^{(0)}} + \hat{p}\cdot\vec{\nabla}V^{(o)}\right) \tag{30}$$

Equation (21) then shows that $\partial N^{(o)}/\partial t_2 = 0$ (and hence $\partial/\partial t_2 \equiv 0$). It follows from the continuity equation (6) that

$$\hat{n}\cdot\vec{\nabla}U_{\|,J}^{(1)} = U_{\|,J}^{(o)}\left(\frac{\hat{n}\cdot\vec{\nabla}B}{B} - \frac{\hat{n}\cdot\vec{\nabla}N^{(1)}}{N^{(o)}}\right) \tag{31}$$

and from Eqs. (10a, b):

$$\overleftrightarrow{\Pi}_{o,J}^{(2)} = -\eta_{o,J}^{(o)}\, U_{J,\|}^{(o)}\left(3\,\frac{\hat{n}\cdot\vec{\nabla}B}{B} - 2\,\frac{\hat{n}\cdot\vec{\nabla}N^{(1)}}{N^{(o)}}\right)\left(\hat{n}\hat{n} - \frac{1}{3}\overleftrightarrow{I}\right)$$

The electron heat equation, taken at order μ, implies

$$\hat{n}\cdot\vec{\nabla}T_e^{(1)} = 0 \tag{32}$$

and the ion heat equation at order μ^2:

$$\frac{3}{2}\,N^{(o)}U_{\|,i}^{(o)}\,\hat{n}\cdot\vec{\nabla}T_i^{(1)} - T_i^{(o)}U_{\|,i}^{(o)}\,\hat{n}\cdot\vec{\nabla}N^{(1)} = \hat{n}\cdot\vec{\nabla}(\chi_{\|,i}^{(o)}\,\hat{n}\cdot\vec{\nabla}T_i^{(1)}) + Q_i^{(1)} \tag{33}$$

Periodicity constraint obviously requires that

$$Q_i^{(1)} = 0 \tag{34}$$

and hence $T_e^{(o)} = T_i^{(o)}$ (which is not surprising in this collisionality regime) unless the heat transferred from the electrons to the ions is balanced by a radiative sink term $-\,Q_{RAD,i}^{(1)}$. In the following we shall therefore continue to distinguish between the electron and ion temperatures. The sum of Eqs. (9) at order μ^2 over electrons and ions yields:

$$N^{(o)}\ \hat{n}\cdot\vec{\nabla}\ (m_i U^{(o)}_{\parallel,i}\ U^{(1)}_{\parallel,i} + T^{(1)}_i) = -\ (T^{(o)}_e + T^{(o)}_i)\ \hat{n}\cdot\vec{\nabla}N^{(1)} +$$

$$+ \frac{2}{3}\ \eta^{(o)}_{o,i}\ U^{(o)}_{\parallel,i}\ \hat{n}\cdot\vec{\nabla}\ (\hat{3}\ \frac{\hat{n}\cdot\vec{\nabla}B}{B} - 2\ \frac{\hat{n}\cdot\vec{\nabla}N^{(1)}}{N^{(o)}}) \qquad (35)$$

As in the lower order, the solution of Eqs. (31), (33-34), and (35) that we shall retain is

$$\hat{n}\cdot\vec{\nabla}N^{(1)} = \hat{n}\cdot\vec{\nabla}T^{(1)}_i = \hat{n}\cdot\vec{\nabla}U^{(1)}_{\parallel,i} = U^{(o)}_{\parallel,i} = 0 \qquad (36)$$

According to Eqs. (13a) and (16a), we set

$$E_\phi/B_\phi = \langle E_\phi\rangle/\langle B_\phi\rangle \qquad (37)$$

where the quantities within angular brackets are flux-surface averages. Noting that $(\hat{n}\cdot\vec{E})^{(o)} = \langle E_\phi\rangle$, what remains of the electron equation (9) implies that

$$\hat{n}\cdot\vec{\nabla}V^{(1)} = 0 \qquad (38)$$

and

$$U^{(o)}_{\parallel,e} = \frac{e_e \tau^{(o)}_e}{0.51\ m_e}\ \langle E_\phi\rangle \qquad (39)$$

Returning to Eqs. (7) and (8), we obtain, noting that $\overleftrightarrow{\Pi}^{(2)}_{o,i} = 0$:

$$\hat{p}\ \cdot\ \vec{U}^{(3)}_J = 0 \qquad (40)$$

$$\hat{b}\ \cdot\ \vec{U}^{(3)}_J = 0 \qquad (41)$$

We are indeed entitled to absorb $N^{(1)}$ in $N^{(o)}$, $V^{(1)}$ in $V^{(o)}$ and $T_J^{(1)}$ in $T_J^{(o)}$ without loss of generality in view of (23, 27, 28) and (32, 36, 38). Equation (21) shows that $\partial N^{(o)}/\partial t_3 = 0$ and we assume as before that $\partial/\partial t_3 \equiv 0$. It follows from Eq. (6) that

$$\hat{n}\cdot\vec{\nabla}U_{\parallel,J}^{(2)} = U_{\parallel,J}^{(1)} \frac{\hat{n}\cdot\vec{\nabla}B}{B} - U_{\parallel,J}^{(o)} \frac{\hat{n}\cdot\vec{\nabla}N^{(2)}}{N^{(o)}} + 2\ \hat{b}\cdot\vec{U}_J^{(2)} \frac{\hat{b}\cdot\vec{\nabla}B}{B} \tag{42}$$

where the last term results from $\vec{\nabla}\cdot(\hat{b}\ \hat{b}\cdot\vec{U}^{(2)})$ and from Eqs. (10a,b):

$$\overleftrightarrow{\Pi}_{o,J}^{(3)} = -\eta_{o,J}^{(o)} \left[3\ U_{\parallel,J}^{(1)} \frac{\hat{n}\cdot\vec{\nabla}B}{B} + 3\ \hat{b}\cdot\vec{U}_J^{(2)} \frac{\hat{b}\cdot\vec{\nabla}B}{B} - 2\ U_{\parallel,J}^{(o)} \frac{\hat{n}\cdot\vec{\nabla}N^{(2)}}{N^{(o)}}\right] (\hat{n}\hat{n} - \frac{1}{3}\overleftrightarrow{I}).$$

We note that

$$U_{\parallel,J}^{(1)} \frac{\hat{n}\cdot\vec{\nabla}B}{B} + \hat{b}\cdot\vec{U}_J^{(2)} \frac{\hat{b}\cdot\vec{\nabla}B}{B} = U_\theta^{(2)} \frac{\hat{b}\cdot\vec{\nabla}B}{B_\phi}$$

Therefore the viscosity tensor does not operate on the toroidal component of the velocity. This should indeed be the case since the toroidal momentum is an invariant of the motion.

The electron heat equation at order μ^2 writes

$$\chi_{\parallel,e}^{(o)} (\hat{n}\cdot\vec{\nabla})^2\ T_e^{(2)} + Q_e^{(1)} = 0 \tag{43}$$

where

$$Q_e^{(1)} = \frac{m_e N^{(o)}}{\tau_e} \left[0.51\ (U_{\parallel,e}^{(o)})^2 - 3\ \frac{T_e^{(o)} - T_i^{(o)}}{m_i}\right]$$

Periodicity constraints imply that $Q_e^{(1)} = 0$. In view of experimental evidence that $U_{\parallel,e} \sim c_s$, i.e. $U_{\parallel,e}^{(o)} \neq 0$, we are forced to conclude either that $T_e^{(o)} \neq T_i^{(o)}$ or that ohmic heating is balanced by electron radiation losses; the sole requirement which follows from our analysis is in fact:

$$\frac{m_e N^{(o)}}{\tau_e} 0.51 \ (U_{\parallel,e}^{(o)})^2 = \langle Q_{RAD,i} + Q_{RAD,e} + Q_{cx} + Q_{AN} \rangle$$

where Q_{AN} represents the eventual anomalous losses through the electron and ion channels. There now follows from (43) that

$$\hat{n}\cdot\vec{\nabla}T_e^{(2)} = 0$$

although the conclusion, strictly speaking, requires that radiation and anomalous losses be independent from the poloidal angle. As for the ions (Eq. 34) this will be assumed to be the case in the lowest order considered so far, but not in the first order considered now.

The ion heat equation, at order μ^3, writes accordingly:

$$\chi_{\parallel,i}^{(o)} \ (\hat{n}\cdot\vec{\nabla})^2 \ T_i^{(2)} + Q_{\Delta,i}^{(2)} = 0 \qquad (45)$$

where $Q_{\Delta}^{(2)}$ represents the first order radiation, charge exchange, or anomalous power losses. We are obviously entitled to consider $Q_{\Delta,i}^{(2)}$ as a periodic function of the poloidal variable χ, i.e.

$$\oint JBd\chi \ Q_{\Delta,i}^{(2)} = 0 \quad , \qquad (46)$$

the eventual aperiodic part being absorbed in $Q_{\Delta,i}^{(1)}$. The sum of Eqs. (9) at order μ^3 over electrons and ions yields:

$$(T_e^{(o)} + T_i^{(o)})\ \hat{n}\cdot\vec{\nabla}N^{(2)} + N^{(o)}\hat{n}\cdot\vec{\nabla}T_i^{(2)} = 2\ \eta_{o,e}^{(o)}\ \hat{n}\cdot\vec{\nabla}\ (U_{\|,e}^{(o)}\ \frac{n\cdot\vec{\nabla}B}{B})$$

$$+ 2\ \eta_{o,i}^{(o)}\ \hat{n}\cdot\vec{\nabla}\ (U_{\|,i}^{(1)}\ \frac{\hat{n}\cdot\vec{\nabla}B}{B} + \hat{b}\cdot\vec{U}_i^{(2)}\ \frac{\hat{b}\cdot\vec{\nabla}B}{B}) \tag{47}$$

which equation, as already noted, is consistent with the conservation of angular momentum in the toroidal direction.

According to Eqs. (13a), (16a), (31), and (39),

$$\hat{n}\cdot\vec{\nabla}U_{\|,e}^{(1)} = \frac{e_e\tau_e^{(o)}}{0.51\ m_e}\ \hat{n}\cdot\vec{\nabla}\ (E_\phi - \langle E_\phi\rangle) \tag{48}$$

It can be shown from Eqs. (13c) and (16b) on the one hand, and (13b) and (16c) on the other hand that $E_\chi/E_\phi \sim J_\chi/J_\phi \sim \mu$. Therefore $(\hat{n}\cdot\vec{E})^{(1)} = E_\phi - \langle E_\phi\rangle$. Periodicity considerations applied to the electron equation (9) at order μ^3 determine the constant of integration in Eq. (48), namely

$$U_{\|,e}^{(1)} - U_{\|,i}^{(1)} = \frac{e_e\tau_e^{(o)}}{0.51\ m_e}\ (E_\phi - \langle E_\phi\rangle) \tag{49}$$

where we note that indeed $\hat{n}\cdot\vec{\nabla}U_{\|,i}^{(1)} = 0$ in view of (36). We have also

$$\hat{n}\cdot\vec{\nabla}\ \left[T_e^{(o)}\ N^{(2)} + e_e\ (N^{(o)}V^{(2)} + N^{(2)}V^{(o)}) - 2\ \eta_{o,e}^{(o)}U_{\|,e}^{(o)}\ \frac{\hat{n}\cdot\vec{\nabla}B}{B}\right] = 0 \tag{50}$$

We return once more to Eqs. (7) and (8). We find with the help of (50):

$$N\ \hat{p}\cdot\vec{U}_e^{(4)} = -\frac{c}{e_eB}\ 3\ \eta_{o,e}\ \hat{b}\cdot\vec{\nabla}(U_{\|,e}^{(o)}\ \frac{\hat{n}\cdot\vec{\nabla}B}{B}) \tag{51}$$

Similarly, with the help of Eqs. (47) and (50), we obtain

$$N\ \hat{p}\cdot\vec{U}_i^{(4)} = -\frac{c}{e_iB}\ 3\ \eta_{o,i}\ \hat{b}\cdot\vec{\nabla}\ (U_{\|,i}^{(1)}\ \frac{\hat{n}\cdot\vec{\nabla}B}{B} + \hat{b}.U_i^{(2)}\ \frac{\hat{b}.\vec{\nabla}B}{B}) \tag{52}$$

Inserting these results in Eq. (21) shows that $\partial N^{(o)}/\partial t_4 = 0$ by virtue of the a/R expansion, Eq. (4). The radial velocities (51) and (52) however lead to a variation of the density on the next time-scale t_5. To obtain the full variation we also require $N\,\hat{p}\cdot\vec{U}_J^{(5)}$. We obtain

$$N\,\hat{p}\cdot\vec{U}_e^{(5)} = -\frac{c}{e_e B}\Big[\hat{b}\cdot\vec{\nabla}\,(P_e^{(3)} + e_e N^{(o)} V^{(3)} + e_e N^{(3)} V^{(o)}$$

$$+ e_e N^{(o)}\Big(\frac{B_\chi}{B}\,\langle E_\phi\rangle - \frac{B_\phi}{B}\,E_\chi^{(1)}\Big) - 3\,\eta_{o,e}^{(o)}\,U_{\|,e}^{(o)}\,\frac{\hat{b}\cdot\vec{\nabla}B}{B}\,\frac{\hat{n}\cdot\vec{\nabla}B}{B}$$

$$+ \eta_{o,e}^{(o)}\,\hat{b}\cdot\vec{\nabla}\,\Big(U_{\|,e}^{(1)}\,\frac{\hat{n}\cdot\vec{\nabla}B}{B} - \frac{2}{3}\,U_{\|,e}^{(o)}\,\frac{\hat{n}\cdot\vec{\nabla}N^{(2)}}{N^{(o)}} + \hat{b}\cdot\vec{U}_e^{(2)}\,\frac{\hat{b}\cdot\vec{\nabla}B}{B}\Big)\Big] \tag{53}$$

We note that in practice the discharge is in quasi-steady state, owing e.g. to recycling at the walls or gas puffing, for many confinement times. Hence $\partial B_\phi/\partial t \propto E_\chi \to 0$ whereas the toroidal electric field E_ϕ is maintained by the flux swing in the transformer. In the next section, we calculate at last the particle and heat fluxes across the magnetic surfaces.

IV. PARTICLE AND HEAT FLUXES ACROSS THE MAGNETIC SURFACES

A. Particle Flux

We introduce the results (51) and (53) in the flux surface average of the electron continuity equation (21). We obtain, up to the relevant order:

$$\frac{\partial N^{(o)}}{\partial t_5} = \frac{\partial}{\partial\psi}\oint J d\chi\, h_\psi^{-1}\,\frac{c}{e_e B}\,\frac{B_\chi}{B}\Big[\,e_e N^{(o)}\,\Big(\langle E_\phi\rangle - \frac{B}{B_\chi}\,E_\chi^{(1)}\Big)$$

$$+ 3\,\eta_{o,e}^{(o)}\,U_{\|,e}^{(o)}\,\Big(\frac{\hat{b}\cdot\vec{\nabla}B}{B}\Big)^2\Big] \times \Big(\oint J d\chi\Big)^{-1} \tag{54}$$

The problem of ambipolarity naturally arises. Performing a similar calculation for the ions yields

$$\frac{\partial N^{(o)}}{\partial t_5} = \frac{\partial}{\partial \psi} \oint J d\chi \; h_\psi^{-1} \frac{c}{e_i B} \frac{B_\chi}{B} \Big[e_i N^{(o)} \left(\langle E_\phi \rangle - \frac{B}{B_\chi} E_\chi^{(1)} \right)$$

$$+ 3\, \eta_{o,i}^{(o)} \left(U_{\parallel,i}^{(1)} + \frac{B_\phi}{B_\chi} \hat{b}\cdot\vec{U}_i^{(2)} \right) \left(\frac{\hat{b}\cdot\vec{\nabla}B}{B} \right)^2 \Big] \left(\oint J d\chi \right)^{-1} \tag{55}$$

Comparison of Eqs. (54) and (55) permits to calculate the ion streaming velocity along the line, which is then given by the amipolarity condition.

$$U_{\parallel,i}^{(1)} = - \frac{\eta_{o,e}^{(o)}}{\eta_{o,i}^{(o)}} U_{\parallel,e}^{(o)} - \frac{c}{B} \left(\frac{\hat{p}\cdot\vec{\nabla}P_i^{(o)}}{e_i N} + \hat{p}\cdot\vec{\nabla}V^{(o)} \right) \tag{56a}$$

which equation can also be read as

$$U_{\chi,i}^{(2)} = - \frac{\eta_{o,e}^{(o)}}{\eta_{o,e}^{(o)}} U_{\chi,e}^{(o)} \tag{56b}$$

The first term on the right-hand side of Eqs. (54, 55) represents the usual pinch effect, directed inwards in the quasi-steady state where $E_\chi^{(1)} \to 0$ [see the comment below Eq. (53)]. The second term, also directed inwards, results from the viscous forces $\vec{F} = -\vec{\nabla}\cdot\overleftrightarrow{\Pi}$. The latter give rise to a radial drift which does not average out over the magnetic surfaces because of the inhomogenity of the field.

B. Heat Losses

The lowest order heat balance equations have already been obtained, namely

$$Q_J^{(o)} \equiv 0$$

$$Q_e^{(1)} = \frac{m_e N^{(o)}}{\tau_e^{(o)}} \left[0.51 \, (U_{\|,e}^{(o)})^2 - 3 \, \frac{T_e^{(o)} - T_i^{(o)}}{m_i} \right] - Q_{RAD,e} = 0$$

$$Q_i^{(1)} = \frac{m_e N^{(o)}}{\tau_e^{(o)}} \, 3 \, \frac{T_e^{(o)} - T_i^{(o)}}{m_i} - Q_{RAD,i} - Q_{cx} = 0$$

$$\int JBd\chi \; Q_{\Delta,i}^{(2)} = 0,$$

see Eqs. (43), (34), (46), and the comment below Eq. (28). There is therefore not much interest in calculating the neoclassical conduction and convection losses. We shall merely determine here the order to which they contribute.

The first and the second terms of Eq. (22) are actually of order μ^5. There are therefore three orders of magnitude too small (since $Q_e^{(o)} \equiv 0$) to balance the ohmic power deposited in the plasma. The third term is also of order μ^5 for the following reason. We have

$$\oint d\chi \; NT_e \left[\frac{\partial}{\partial\psi} (Jh_\psi^{-1} U_{\psi,e}) + \frac{\partial}{\partial\chi} (Jh_\chi^{-1} U_{\chi,e}) \right]$$

$$= \oint Jd\chi \; NT_e \; \vec{\nabla}\cdot\vec{U}_e$$

$$= - \oint Jd\chi \; T_e \left[\frac{\partial N}{\partial t} + U_{\psi,e} \, h_\psi^{-1} \frac{\partial N}{\partial\psi} + U_\chi h_\chi^{-1} \frac{\partial N}{\partial\chi} \right]$$

The two first contributions are clearly of order μ^5. The third term can be rewritten as

$$- \oint Jd\chi \; T_e^{(o)} \left(U_{\|,e}^{(o)} + U_{\|,e}^{(1)} + \frac{B_\phi}{B_\chi} \hat{b}\cdot\vec{U}_e^{(2)} \right) \hat{n}\cdot\vec{\nabla}N^{(2)}$$

up to order μ^4 included. This expression vanishes because: $U_{\|,e}^{(o)} + U_{\|,e}^{(1)} \propto E_\phi \propto B$ apart from the constant of integration $U_{\|,i}^{(1)}$ [Eqs. (39) and (49)]; $\hat{n}\cdot\vec{\nabla} = (1/JB)\ \partial/\partial\chi$; and $B_\phi\ \hat{b}\cdot\vec{U}_e^{(2)}/B_\chi$ is independent from χ in lowest order (by virtue of the a/R expansion). The fourth term of Eq. (22) is of order μ^6 for the electron and μ^5 for the ions [because of $\partial T_i^{(2)}/\partial\chi \neq 0$ see (45)]. The first term on the right-hand side is really of order μ^6 for the electrons, because $\hat{n} \times (\vec{U}_e^{(o,1)} - \vec{U}_i^{(o,1)}) = 0$, see (18a) and or order μ^5 for the ions. Finally the second term on the right-hand side is also of order μ^5 for the electrons and ions.

In conclusion the convection and conduction losses are three orders too small, in the μ expansion, to balance the ohmic power input. This result has prompted us to discuss potential anomalous losses in a forthcoming publication.

V. CONCLUSION

The results of this research can be summarized as follows.

(1) The plasma parameters typically measured in the outer layers of tokamaks, and ordered according to Eqs. (1), (2), and (4), are inconsistent with the assumption that the dominant heat losses are neoclassical, and this by three orders of magnitude in the expansion parameter μ, i.e. by an extremely large factor! Although another ordering could lead to a lesser discrepancy, it is obvious that the neoclassical model is fully inadequate to explain the power balance. Ohmic power input has thus to be either radiated away, or balanced by charge exchange, or convected away by a turbulent mechanism (anomalous transport).

(2) Electron viscosity leads to an increased pinch effect, which, compared to the usual pinch in the collisional regime, is, in a plasma with circular cross-sections:

$$\langle U_r \rangle_{pinch} = - \left\langle \frac{c\, B_\chi E_\phi}{B^2} \right\rangle \left[1 + 2.15\, q^2 \left(\frac{\lambda_{mfp}}{qR}\right)^2 \right] \tag{57}$$

where the mean free path $\lambda_{mfp} = c_e \tau_e$. The fluid theory is expected to hold as long as $\lambda_{mfp} \lesssim qR$, the connection length. Therefore the enhancement factor is at most $2.15\, q^2$ which is typically or order 20. This is not sufficient to explain experimental results. There again an anomalous effect thus seems necessary. It is worth noting that the neoclassical pinching effect enters the power balance equation on the same time-scale as the neoclassical convection and conduction transport enter the heat balance equations.

(3) The ion streaming velocity along the lines - or, to formulate it another way - the poloidal rotation velocity, is determined through ambipolarity according to

$$U_{\chi,i} = - \frac{\eta_{o,e}}{\eta_{o,i}} U_{\chi,e} \; .$$

It is noted that the relation which links $U_{\chi,i}$ and $U_{\parallel,i}$, Eq. (56a), contains the electric drift frequency $- c\, p \cdot \vec{\nabla} V / B$ which is still undetermined.

After this work was completed, it was brought to our knowledge that W. Engelhardt had derived a result *qualitatively* similar to Eq. (57) from physical considerations. This work has not been published /W. Engelhardt, (1985), private communication/.

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