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How Strong are the Strong Interactions?

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We report about the recent completion of a research project by the ALPHA collaboration which is managed by the NIC group at DESY, Zeuthen. Nonperturbative running couplings are determined in three-flavour QCD and with their help the strong coupling at high energy is determined to be $\alpha^{(5)}_{\overline{\rm MS}}(m_{\rm Z}) = 0.11852(84)$. This is an essential input parameter for separating known from unknown physics at the Large Hadron Collider (LHC) by approximating the QCD contributions in the form of a series expansion in $\alpha_{\overline{\rm MS}}$.

1 Introduction

QCD is the quantum field theory which gives an accurate description of the strong interactions. No significant deviations of experiments to theory have been observed so far. Of course this holds as far as we have been able to work out the theory at all. At the fundamental level, the basic building blocks of QCD are the quark fields and the gluons. Their analogy in quantum electrodynamics are electrons and photons. However, quarks and gluons have an extra intrinsic degree of freedom called colour which provides more complex interactions and phenomena.

The most intriguing one is confinement: despite the fact that the theory is formulated in terms of quarks and gluon and a number of observables can be computed and tested accurately, there is no experiment that has detected a quark or a gluon as a particle. This is not an experimental deficiency but a property of the theory. The physical particles (the physical eigenstates of the Hamilton and momentum operator) are bound states of the fundamental fields. This is one reason why we speak of the **strong** interactions. They bind the constituents of hadrons, such as proton and neutron, so strongly that it is impossible to "pull them out of the bound states". When protons are broken up by hard scattering with large momentum transfer, the products that come out are either leptons such as electrons and neutrinos or hadrons.[a] In an intuitive way, confinement tells us that there are strong forces at distance scales of the order of the radii of hadrons, i.e. about one fm. Expressed in terms of a dimensionless coupling constant, $\alpha_{\rm s}$, e.g. as a coulomb-like force $F(r) \approx \alpha_s/r^2$, the coupling is of order one. We may look ahead at our result in Fig. 1. Indeed, at a distance $r = 1/200\,\mathrm{MeV} = 1\,\mathrm{fm}$ the gradient flow coupling is around $\alpha_{\rm GF} = 1$. The name "gradient flow coupling" refers to the exact definition, dubbed *scheme*, of the coupling. Depending on the scheme, rather different values can be assumed at distances of order of a fm, but generically one will find large values. We will come back to this.

On the other hand, at short distances, reached in particular by scattering experiments at the LHC with energies of the order of TeV, α_s drops below $1/10$ and one can perform series

[a]Nuclear forces, historically called strong interactions, are believed to be the Van-der-Waals forces of QCD. We write "believed" since theoreticians have not yet been able to derive nuclear forces from QCD in detail, but progress is being made.

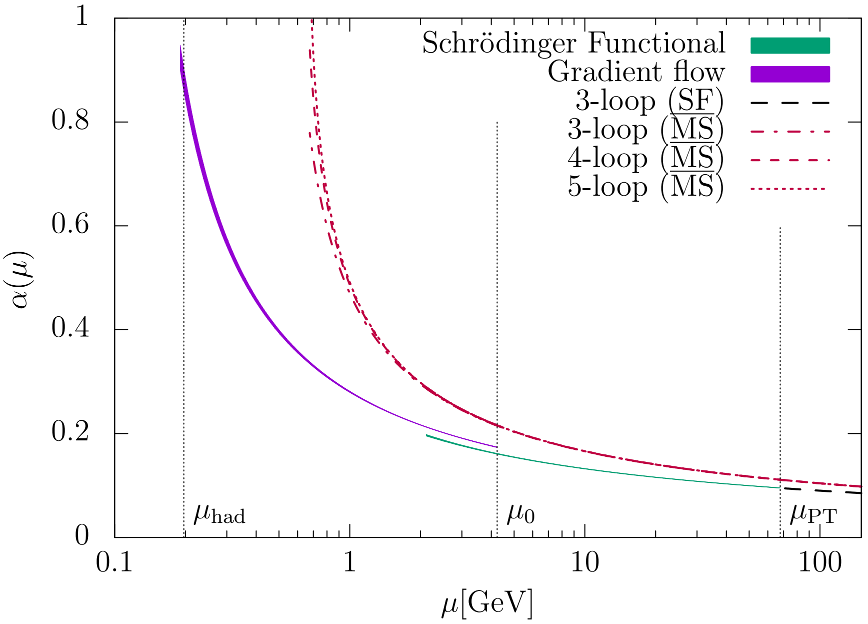

Figure 1. Running couplings of $N_{\mathrm{f}} = 3$ QCD from integrating the nonperturbative β-functions in the SF and GF schemes[1,2]. They are matched nonperturbatively by defining $\bar{g}^2_{\mathrm{SF}}(\mu_0) = 2.012$ and computing $\bar{g}^2_{\mathrm{GF}}(\mu_0/2) = 2.6723(64)$.

expansions in α_s. This is referred to as perturbation theory (PT). For many quantities these series expansions are then trustworthy and remainders can be estimated. Still, $\alpha_{\mathrm{s}} \approx 0.1$ is the largest of the coupling constants in the complete Standard Model of the subatomic world. This holds for all energies accessible in laboratories. So the QCD interactions are strong at basically all relevant distances and are consequently also very important to understand in order to interpret experiments searching for small deviations from the Standard Model. The most important input for the QCD theory predictions through the series expansions is the precise value of α_{s} at a certain, high, energy and a given scheme. It has become customary to choose an energy of $\mu = m_{\mathrm{Z}} = 91.19$ GeV as a reference and the so-called $\overline{\mathrm{MS}}$ scheme.

2 Coupling Definitions, Lattice Field Theory, Asymptotic Freedom

It is important to recall how couplings are defined. First, observe that in addition to Lorenz invariance, the Lagrangian density of quantum field theories is very much restricted by mathematical consistency – the buzz word is renormalisability. A gauge theory, such as QCD[b], has one single dimensionless parameter, g_0. For the classical theory, this uniquely defines the coupling $\alpha_0 = g_0^2/(4\pi)$. The mathematical definition of quantum field theories needs in addition to the classical Lagrangian a so-called regularisation, which achieves

[b]Quark masses, the only other parameters of QCD, are irrelevant for our discussion.

that the mathematical expressions for quantum fluctuations, in particular the pair-creation of field quanta (e.g. quark-antiquark creation), are well defined. The best regularisation of the theory is the lattice regularisation, where fields $\Phi(x)$, defined on a continuum space-time are replaced by fields on a lattice $x_\mu = an_\mu$ with $n_\mu \in \mathbb{Z}$ and a discretisation length or lattice spacing, a. This changes a quantum field theory with infinitely many degrees of freedom in any finite region of space-time to one with finitely many. The mathematical expressions (the Feynman path integral) become well-defined and finite. Still, in the desired limit $a \to 0$, taken at fixed g_0, infinities of the form a^{-n} or $\log(ap_\mu)$ reappear. The renormalisability of the theory, alluded to before, assures that all these infinities (in any observable) disappear when we consider g_0 to be unphysical, unobservable, and take the limit $a \to 0$ at a fixed renormalised coupling $\bar{g}_s(\mu)$. Here μ is an energy scale and the index s labels the details of the definition of the coupling, the scheme. In the process $a \to 0$, the bare coupling, g_0 may run to infinity or a finite value. In QCD, the latter is the case, $\lim_{a\to 0} g_0(a) = 0$.

How are renormalised couplings defined? When one works in terms of the power series in the coupling to a finite order, i.e. one works perturbatively, one can subtract the occurring divergences in the function $g_0(a)$ order by order in g_0 and thus define a finite coupling. Such schemes are called minimal subtraction schemes. An example is the $\overline{\mathrm{MS}}$-scheme mentioned before.

A nonperturbative definition of a coupling, valid beyond the series expansion, is also easily given. Take a QCD observable, depending on fields concentrated within a 4-dimensional region of space-time of linear size $R = 1/\mu$ and with a perturbative expansion[c]

$$\mathcal{O}_s(\mu) = k\,\bar{g}^2_{\overline{\mathrm{MS}}}(\mu)\,[1 + c_1^s \bar{g}^2_{\overline{\mathrm{MS}}}(\mu) + \ldots]\,. \tag{1}$$

Then the nonperturbative coupling,

$$\bar{g}_s^2(\mu) \equiv \mathcal{O}_s(\mu)/k = \bar{g}^2_{\overline{\mathrm{MS}}}(\mu) + c_1^s \bar{g}^4_{\overline{\mathrm{MS}}}(\mu) + \ldots\;, \tag{2}$$

depends on μ, the only scale that enters. Due to Eq. 2, power series in $\bar{g}^2_{\overline{\mathrm{MS}}}$ are equivalent to power series in $\bar{g}_s^2$. How does $\bar{g}_s^2(\mu)$ depend on μ? In the region where $\bar{g}_s$ is small, one can approximate

$$\beta_s(\bar{g}_s) = \mu \frac{\partial \bar{g}_s(\mu)}{\partial \mu} \tag{3}$$

by the power series

$$\beta_s(\bar{g}_s) \underset{\bar{g}\to 0}{\sim} -b_0\bar{g}_s^3 - b_1\bar{g}_s^5 + -b_{2,s}\bar{g}_s^7 + \ldots\;, \tag{4}$$

where the universal coefficients in the asymptotic expansion take the values $b_0 = 9/(16\pi^2)$ and $b_1 = 1/(4\pi^4)$ in $N_\mathrm{f} = 3$ QCD and coefficients $b_{i,s}\,, i > 2$ depend on the scheme s.

The $s = \overline{\mathrm{MS}}$ scheme is very popular because it starts from dimensional regularisation, which facilitates perturbative computations. In this scheme $b_{i,\overline{\mathrm{MS}}}$ are known up to $i = 4$. Solutions of Eq. 3 in the $\overline{\mathrm{MS}}$-scheme, truncated at different orders in $\bar{g}^2_{\overline{\mathrm{MS}}}$ are shown in Fig. 1. n-loop means that the last term taken into account in β_s is $b_{n-1}\bar{g}_s^{1+2n}$. At large μ the

[c] It is rather generic for an observable, made dimensionless by the appropriate power of μ to have such an expansion; if there is a constant, $\bar{g}^2$-independent, piece one just subtracts it.

curves do not differ much. This is because b_0 (and b_1) are positive and the coefficients $b_{i,\overline{\rm MS}}$ are of order $(4\pi)^{-i-1}$. Therefore the perturbative series makes sense when $\alpha = g^2/(4\pi)$ is 0.3 or so and the coupling decreases with large μ and PT becomes more and more precise. Eqs. 3,4 then show $\lim_{\mu\to\infty} \bar{g}_s^2(\mu) = 0$. This property is called *asymptotic freedom*. It is very useful for high energy physics. QCD effects can be predicted with better and better accuracy at high energies.

3 Coupling Determinations

3.1 Experiment and Perturbation Theory

What can't be derived from PT alone is the free constant which is present upon integration of Eq. 3. It can only be determined from experimental input. One way is to measure the observable of Eq. 1, i.e. the left hand side, and extract $\bar{g}_s(\mu)$ from it. A couple of terms of the perturbative expansion on the right hand side have to be known and $\alpha_s(\mu)$ has to be small, thus μ has to be large. This renders such determinations difficult. In particular, a major obstacle is confinement, which we discussed above. It means that experiments register pions and protons, while the theory expressions involve quarks and gluons. The relation from one to the other involves physics at large distances (small μ) where PT fails. It is nonperturbative. Various tricks (momentum smearing, inclusiveness, various jet-definitions) are being employed to suppress nonperturbative effects.

These difficulties can be seen in the results collected by the Particle Data Group[3,4] for $\alpha_{\overline{\rm MS}}(m_{\rm Z})$: the spread of different values goes quite significantly beyond the error band of the average and individual determinations are several (of their estimated) standard deviations away from the central value. Because of these difficulties, the LHC Higgs working group recommends to use an uncertainty which is about twice as large as the one of the Particle Data Group[4].

3.2 Experiment and Nonperturbative Lattice Simulations

So far we have discussed the direct comparison of experiments with the perturbative series. An alternative is to use the lattice regularisation and perform numerical computations of the theory. These are nonperturbative and can relate the bare coupling g_0, or equivalently the lattice spacing a, to intrinsically nonperturbative quantities such as the pion and proton mass or decay rates of charged mesons into electron, neutrino and photon. The lattice is then calibrated though very precise experimental low energy input. Suitably chosen quantities $\mathcal{O}_s(\mu)$ are then computable as well and one can obtain values for couplings $\bar{g}_s^2(\mu)$ with input from low-energy experiments.

3.2.1 Challenge

In addition to the lattice spacing a, numerical computations also involve a finite total size of the system L, that is simulated. For standard observables, control over finite volume effects of order $\exp(-m_\pi L)$ requires L to be several fm. At the same time, one needs to suppress discretisation errors and should extrapolate $(a\mu)^2 \to 0$ at fixed μ. The necessary restrictions

$$L \gg 1/m_\pi\,, \quad 1/a \gg \mu \quad \Rightarrow\; L/a \ggg \mu/m_\pi \tag{5}$$

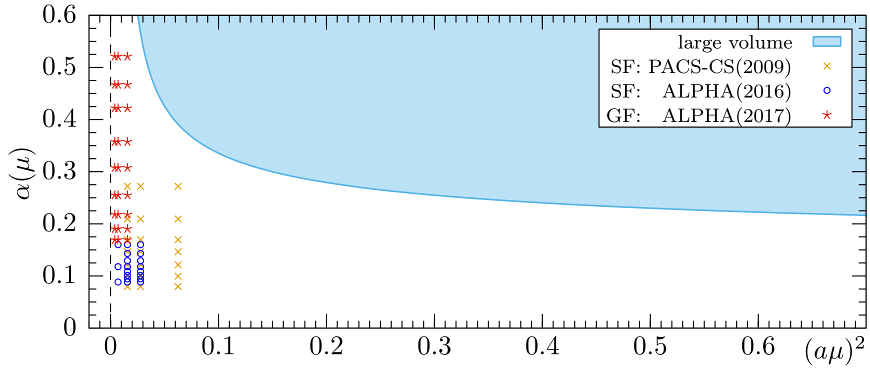

Figure 2. The shaded area shows the $a > 0.04$ fm region of large volume results which dominate the present PDG and FLAG estimates[4,5] of $\alpha_{\overline{\rm MS}}(\mu)$ (curve evaluated to two loops for $N_{\rm f} = 3$ with $\Lambda^{(3)}_{\overline{\rm MS}} = 341$ MeV). The data points on the left are finite-size scaling computations[6,1,2].

translate into very large lattices. Fig. 2 displays the region in $\alpha(\mu)$ vs. $(a\mu)^2$ for the range $a \geq 0.04$ fm which can be realised nowadays in large volumes ($m_\pi L \geq 4$). This shaded region is quite far from small coupling and small $(a\mu)^2$. The region where the series Eq. 2 is trustworthy can therefore not be reached with controlled discretisation errors.

3.2.2 Finite Size Scaling Method

The way out has long been known[7,8] and over the years it has been much improved by the ALPHA collaboration. One may identify $R = L = 1/\mu$ by choosing $\mathcal{O}_s$ to depend only on the scale L, not on any other ones. Finite-size effects become part of the observable rather than one of its uncertainties. Eq. 5 is then relaxed to

$$L/a \gg 1\,, \tag{6}$$

such that $L/a = 10 - 50$ is sufficient.

Different scales μ are then connected by the step scaling function

$$\sigma(u) \equiv \bar{g}^2_s(\mu/2)\Big|_{\bar{g}^2_s(\mu)=u}\,. \tag{7}$$

It describes scale changes by discrete factors of two, in contrast to the infinitesimal changes covered by the β-function Eq. 3. For a chosen value of u, $\sigma(u)$ can be computed by determining $\bar{g}^2$ on lattices of size L/a and $2L/a$ and performing an extrapolation $a \to 0$ at $L = 1/\mu$, fixed through $\bar{g}^2(\mu) = u$, see Fig. 3 for illustration. In fact, in the process also the β-function can be computed as long as $\sigma(u)$ is a smooth function of u. A recent detailed description of step scaling is given in Ref. 9.

4 ALPHA Collaboration Project

The above idea has been implemented in a large scale, multi-year project by the ALPHA collaboration. We sketch here several milestones.

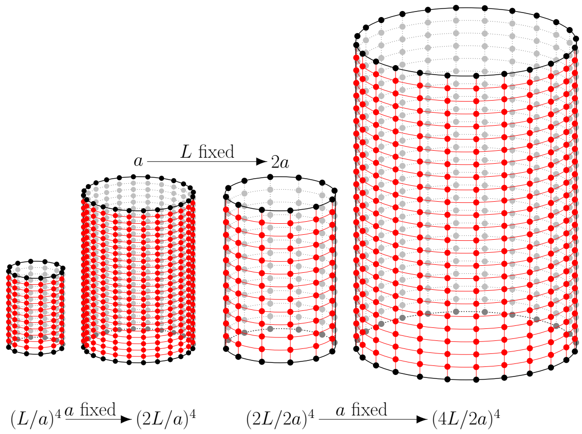

Figure 3. Illustration of the Finite size scaling method[7]. At fixed lattice spacing a one simulates both a lattice with L/a points and $2L/a$ points in each dimension to obtain the change of $g^2(L) \to g^2(2L)$. One may then increase a and thus continue to larger and larger L. Repeating the whole procedure with a different L/a but same L allows to remove the discretisation errors by taking the continuum limit (not shown). We depict a cylinder because boundary conditions are periodic in the three spatial dimensions and Dirichlet in the time direction.

- A practical definition, $\bar{g}_{\rm SF}$ of a finite size coupling with small discretisation errors and a manageable perturbative expansion was developed in the 90's[10–13].
- The function $\beta_{\rm SF}$ was computed to three loops[14,15].
- Extensive numerical investigations were carried out in the pure gauge theory, and QCD with two and four flavours, see a review[9].
- A definition of a new finite-size coupling, $\bar{g}_{\rm GF}$, with various advantages in the region of not-too-large μ, has been given and investigated[16].
- Finite size simulations with $N_{\rm f} = 3$ dynamical quarks have been carried out, at various L and resolutions L/a, computing step scaling functions[1,2] as well as the relation of the two different renormalised couplings in the form $\bar{g}^2_{\rm GF}(\mu/2) = \varphi(\bar{g}_{\rm SF}(\mu))$ at the point $\bar{g}_{\rm SF}(\mu) = 2.012$.
- Large volume simulations with $N_{\rm f} = 3$ dynamical quarks have been carried out in order to calibrate lattices in physical units[17,18].
- The transition from large ($L > 3\,\text{fm}$) to finite volume ($L \approx 1\,\text{fm}$) was covered and all results combined[19].
- Finally, the connection between the three-flavour theory and Nature with five and six active flavours was taken care of[19]. In order to understand that connection, it was also important to study the decoupling of heavy quarks from the low-energy physics in a nonperturbative model close to QCD[20].

The essential steps requiring particular care in all these numerical computations were the removals of the discretisation effects by extrapolations $a \to 0$. We show two examples in Fig. 4.

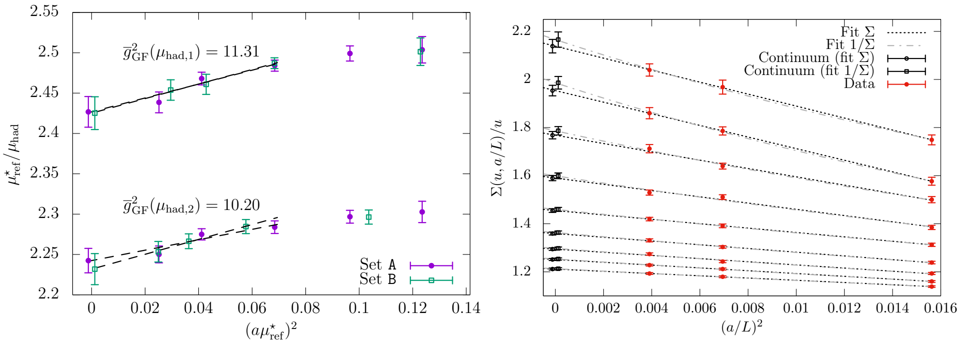

Figure 4. Left: Continuum extrapolation of scales $\mu_{\rm had,i}$ in units of a large volume reference scale $\mu^\star_{\rm ref}$. Extrapolated values are shown in proximity of $a = 0$. Right: Continuum extrapolation of the lattice approximant, Σ to the step scaling function, σ of the GF-coupling.

5 The Result

The central, entirely nonperturbative, results are the curves labelled "Gradient flow" and "Schrödinger functional" in Fig. 1. These give the running couplings $\alpha_{\rm GF}$ and $\alpha_{\rm SF}$, respectively. They are actually bands, with the width of the bands given by the uncertainty of our calculation. Converting perturbatively to the $\overline{\rm MS}$-scheme at asymptotically large μ, and then using the perturbative $\beta_{\overline{\rm MS}}$ yields the $\overline{\rm MS}$-curves (only central values are shown). All these results are in the $N_{\rm f} = 3$ theory.

The heavier quarks, c,b,t, can be added perturbatively and we arrive, e.g., at

$$\alpha^{(5)}_{\overline{\rm MS}}(m_{\rm Z}) = 0.11852(84)\,. \tag{8}$$

Where the label $^{(5)}$ indicates that this is in the 5-flavour theory, appropriate for energies of order $m_{\rm Z} \approx 92\,{\rm GeV}$.

6 Conclusions

We now have a result for $\alpha_{\overline{\rm MS}}(m_{\rm Z})$ with unprecedented control over all theory systematic errors. Still it has a remarkable accuracy, better than the one of the recent world average of the PDG. This is an excellent input for LHC phenomenology.

Acknowledgements

We note that the computationally most demanding simulations (large volume) were performed on JUQUEEN in Jülich and Supermuc in Munich. However, resources granted by HLRN and also locally at DESY were essential to perform the many simulations to compute the step scaling functions. The CPU requirements of these simulations vary by orders of magnitude.

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