Abstract

One of the possible arguments for the breaking of CP invariance is the existence of non-vanishing electric dipole moments (EDM) of elementary particles. Currently, the Jülich Electric Dipole Moment Investigations (JEDI) collaboration is developing the conceptual design of a specialized ring for the search of the deuteron electrical dipole moment (dEDM). The proposed quasi-frozen spin (QFS) concept differs from the frozen spin (FS) concept in that the spin of the reference particle is alternately deflected by a few degrees in different directions relative to the momentum in the electric and magnetic parts of the ring. The QFS concept will enable to use the existing COSY ring as a pilot facility. The paper presents a conceptual approach to ring design, based on the results of our study of spin decoherence and systematic errors, as well as a sensitivity estimate of the EDM determination method.

INTRODUCTION

Currently, the Jülich Electric Dipole Moment Investigations (JEDI) collaboration is performing the deuteron ring design in two directions: firstly, the conceptual design of a specialized ring for the search of the deuteron electrical dipole moment (dEDM) [1], and secondly, a quasi-frozen spin (QFS) concept, which can be adapted to the existing COSY ring. This article is devoted mainly to the dEDM ring for a deuteron beam. For the design of such a ring, we need to address three major challenges:
- the lattice should meet the conditions of stability of motion and minimization of beam loss, and it should incorporate straight sections to accommodate an accelerating station, beam injection and extraction equipment, a polarimeter, and sextupoles;
- the beam polarization lifetime must reach approximately ~1000 seconds, by utilizing an RF cavity and a certain number of sextupole families.;
- systematic errors have to be minimized to eliminate the induced fake EDM signal.

FROZEN AND QUASI-FROZEN SPIN CONCEPTS

In this paper, we will mainly analyze the quasi-frozen spin (QFS) type of structure described in [2].

The concept of a “frozen spin” (FS) lattice had been suggested by BNL [3] and is based on elements that incorporate electric and magnetic fields in order for the spin of the reference particle to be always orientated along the momentum.

In the COSY ring, implementation of the FS concept would require a complete optical upgrade. However, taking into account that the deuteron’s anomalous magnetic moment $G = -0.142$ has a small value and the fact that the spin oscillates around the momentum direction within half value of the advanced spin phase $\pi \cdot G/2$ in the magnetic arc, each time returning to the same orientation in the electric-field elements on the straight sections, it is obvious that the effective contribution to the expected EDM effect is reduced only by a few percent. This allows us to proceed to the QFS concept [2], where the spin is not frozen with respect to the momentum vector but continuously oscillates around the momentum with small amplitude of a few degrees.

In case of the QFS lattice, we have two options. In the first option, the electric and magnetic fields are fully spatially separated in arcs and straight section elements [1]. However, this concept inherits the drawback of cylindrical electrodes, namely, the whole set of high-order nonlinearities. Therefore, in second option of the QFS lattice, we introduced a small magnetic field of ~100 mT, compensating the Lorentz force of the electric field on the arcs (see Fig.1).

Figure 1: QFS lattice with TWISS functions.

Both QFS lattices consist of two arcs and two straight sections with approximately similar circumferences to that of the FS lattice. In both cases, the lattice includes
straight sections with zero dispersion in the middle of the magnetic arcs for installation of the polarimeter, beam extraction and injection systems, and the RF cavity.

Since the second option is the most appropriate for the COSY ring, we will only discuss this option here. In the magnetic arc, the particles are rotated by angle $\Phi_{arc} = \pi$, with simultaneous MDM spin rotation in horizontal plane relative to the momentum by angle $\Phi_{arc} = \gamma \cdot \Phi_{arc}^E$. On the straight section, the straight elements with E and B fields provide MDM spin rotation in the horizontal plane in the opposite direction relative to the momentum in E field by angle $\Phi_{ss}^E = \left(\gamma + 1\right) \beta^2 \cdot \Phi_{ss}^E$, where $\Phi_{ss}^E$ is the momentum rotation in electrical field, and in B field by angle $\Phi_{ss}^B = \left(\gamma + 1\right) \beta^2 \cdot \Phi_{ss}^B$, where $\Phi_{ss}^B$ is the momentum rotation in magnetic field. Since the Lorenz force is zero, the angles $\Phi_{ss}^E = \Phi_{ss}^B$ are equal each to other. Therefore, they could be defined through one of them, for instance, through the magnetic field as $\Phi_{ss}^B = \frac{eB_{ss}}{m_B \gamma} L_{ss}$, where $B_{ss}, L_{ss}$ are the magnetic field and the length of the straight element, respectively. To realize the QFS concept, we have to fulfil the condition $\Phi_{ss}^B - \Phi_{ss}^E = \Phi_{arc}^E$, i.e.

$$\left(\gamma + 1\right) \Phi_{ss}^B - \left(\gamma + 1\right) \beta^2 \cdot \Phi_{ss}^E = \gamma \cdot \Phi_{arc}^E. \quad (1)$$

Carrying out simple transformations, we obtain the basic relation for the straight element parameters:

$$L_{ss} E_{ss} = \frac{G}{\gamma + 1} \frac{m_e \beta^2}{e} \pi \gamma \beta^2 \gamma^3 \text{ and } B_{ss} = - \frac{E_{ss}}{c B}, \quad (2)$$

where $L_{ss}$ is the total length of the straight elements in one straight section.

Thus, taking maximum electric field at the level of 120 kV/cm requires a magnetic field below 80 mT. It opens the prospects of simplifying the general construction. In particular, a permanent magnet or an air core electric coil may be used.

**SPIN TUNE DECOHERENCE**

Now, let us briefly mention the main causes of decoherence. Expanding in Taylor series the well-known expression for the spin tune in electric field $\nu_s^E = \left[1/(\gamma^2 - 1) - G\right] \gamma \beta^2$ and in magnetic field $\nu_s^B = \gamma G$ in the vicinity of an arbitrary point $\gamma_0$,

$$\Delta \nu_s^E = \Delta \gamma \cdot G,$$

$$\Delta \nu_s^E = \Delta \gamma \cdot \left[G - (1 + G)/\gamma_0^2\right] + \Delta \gamma^2 \cdot (1 + G)/\gamma_0^3 + \ldots \quad (3)$$

we see that the spin tune spread $\Delta \nu_s^E$ in an electric field has all orders of non-linearity. Obviously, the linear term $\Delta \gamma \cdot G$ in both fields gives the maximum contribution to the spin tune decoherence, and a simple estimate shows that the spin coherence time is limited to a few milliseconds.

Introduction of the RF cavity allows averaging and practically reducing the linear term contribution to zero. Despite the linear term in (3) being practically reduced to zero with RF, the term proportional to $\Delta \gamma^2$ restricts the spin coherence time to a few hundred seconds. The final step, to reduce the spin tune decoherence, is based on sextupoles, which change the orbit length dependent on momentum deviation and dispersion [4]. Detailed numerical consideration of decoherence effects has been performed using codes COSY INFINITY [5] and MODE [6].

**SYSTEMATIC ERRORS**

In the EDM ring experiment, systematic errors arise from misalignments of electric and magnetic elements and cause a “fake” EDM signal. The nature of origin being random errors, misalignments create conditions for systematic errors in EDM experiments. Installation errors (misalignments) are associated with limited capabilities of geodetic instruments. As is known, the bending magnet (or the electric deflector) can be rotated in three planes. We consider only the rotation around the longitudinal and the transverse axes, as the rotation around the vertical axis does not introduce a systematic error. Firstly, let us consider the case of the magnet rotated relative to the longitudinal axis. Due to such rotation, the horizontal component of magnetic field $B_x$ arises and causes the spin rotation $\Omega_x = \Omega_{Bx}$ in the same plane where we expect the EDM rotation. To illustrate, let us write solutions of the T-BMT equations with the initial conditions as $S_x = 0$, $S_y = 0$, $S_z = 1$, $\Omega_x = 0$ and $\Omega_y \neq 0$ in simplest form:

$$S_x(t) = \frac{\Omega_y \sin(\sqrt{\Omega_x^2 + \Omega_y^2} \cdot t)}{\sqrt{\Omega_x^2 + \Omega_y^2}}; S_y(t) = - \frac{\Omega_x \sin(\sqrt{\Omega_x^2 + \Omega_y^2} \cdot t)}{\sqrt{\Omega_x^2 + \Omega_y^2}} \quad (4)$$

We take the following designation of coordinates: $z$ longitudinal direction, $x$ horizontal and $y$ vertical direction. Taking into account the above, we can present the components $\Omega_x = \Omega_{coh} + \Delta \Omega_{incoh}$, $\Omega_x = 1/\Omega_{EDM} + \Omega_{Bx}$, where $\Omega_{EDM}$ is the frequency of spin rotation due to the presence of an EDM, $B_x$ is the horizontal component induced by the magnet rotation (misalignment), and $\Omega_{coh}, \Delta \Omega_{incoh}$ are the coherent and incoherent components of the spin tune in the horizontal plane. The incoherent component is responsible for the spin decoherence due to which we observe the attenuation of
the total signal on the polarimeter. We consider the sextupole correction to be successful when the total spin vector of all particles in the bunch doesn’t fall below half of its initial value in the course of 1000 seconds. The decoherence is allowed to reach the rms value $\langle \delta \Omega_{\text{incoh}} \rangle$ of 1 rad for spin coherence time $t_{\text{CCT}}>1000$ sec. The coherent component reflects the oscillation of the total signal in QFS optics in respect to the momentum with a frequency of two times per turn, that is, $f_{\text{coh}} \approx 2$MHz, and a small amplitude $a_{\text{coh}} \approx 0.2$ rad.

The magnets are supposed to be installed at the technically feasible accuracy of $10 \pm 100$ μm, which corresponds to the rotation angle of the magnet around the axis of about $\alpha_{\text{max}} = 10^{-4} \div 10^{-5}$ rad. Using COSY INFINITY and MODE, we calculated the MDM spin rotation due to $B_x$, which is $\Omega_{Bx} \approx 3 \div 30$ rad/sec. At the same time, at the presumeable EDM value of $10^{-29}$ e·cm, the EDM rotation should be $\Omega_{EDM} = 10^{-9}$ rad/sec, that is, $\Omega_{EDM} \approx \Omega_{Bx} \approx 10^{-9}$, and the expression (4) can be simplified without the loss of measurement accuracy of a possible EDM signal at the level of $10^{-9}$:

$$ S_y(t) = -\sin(\Omega_{Bx} + \Omega_{EDM}) t \quad \text{and} \quad |S_x(t)| < a_{\text{coh}}. \quad (5) $$

We can see that the spin decoherence in the horizontal plane is not growing and is stabilized at the level of $S_x \approx a_{\text{coh}}$. This is a significant positive feature. But, to be fair, we should understand that, since $\Omega_{Bx} = e/m_y (\gamma G +1)B_x$, we will now get, due to $\gamma = \gamma_0 + \Delta \gamma$, the spin frequency decoherence $\Omega_{Bx} = \Omega_{x,\gamma} = \gamma_0 + \Delta \Omega_{x,\Delta \gamma}$ in the vertical plane around horizontal axis, which we can minimize using the same methods (sextupoles, RF) as in the horizontal plane. In addition, we are deprived of the ability to measure the accumulated EDM signal by growth of the vertical spin-vector component suggested in [3], since spin rotation due to magnet errors is much faster than due to possibly-existing EDM, i.e. $\Omega_{Bx} >> \Omega_{EDM}$. That is, $S_y$ reaches a maximum for a very short time, whereas the signal EDM does not have sufficient time to be accumulated.

Therefore, the only solution is to measure the total frequency $\Omega_{Bx} + \Omega_{EDM}$ but, in order to isolate the EDM signal from the total signal, we need an additional condition. Such a condition is to measure the total spin frequency in the experiment with the counter clock-wise (CCW) direction of the beam $\Omega_{CCW} = -\Omega_{Bx} - \Omega_{EDM}$ and to compare with the clock-wise (CW) measurements $\Omega_{CW} = \Omega_{Bx} + \Omega_{EDM}$. Simultaneously, we must understand that the frequency measurement accuracy of $\Omega_{CW}$, $\Omega_{CCW}$ determines the precision of the EDM measurement. For the statistical error of a spin oscillation measurement, one can use $\sigma_{\Omega} = \delta \epsilon \sqrt{24/N/T}$ [7], where $N$ is the total number of recorded events, $\delta \epsilon \approx 0.03$ is the relative error of asymmetry measurement, and $T \approx 1000$ sec is the measurement duration. If we assume that we have a beam of $10^{11}$ particles per fill and the polarimeter efficiency of one percent, then we have the frequency error of $\sigma_{\Omega} \approx 1.5 \cdot 10^{-7}$ rad/sec. Taking into account the average accelerator beam time of 6000 hours per year, we can reach $\sigma_{\Omega} \approx 1.0 \cdot 10^{-9}$ rad/sec with one-year statistics. Taking into account that an EDM value of $d_d = 10^{-29}$ e·cm results in the spin precession frequency of $\Omega_{edm} = 1.0 \cdot 10^{-9}$, such accuracy of frequency determination is quite satisfactory and can provide to reach $d_d = 10^{-29} + 10^{-30}$ e·cm.

However, we need to be sure that when the sign of the driven magnetic field $B_y$ for the CW-CCW is changed, the magnetic field component $B_x$ is restored with the required relative precision of no less than $10^{-10}$. Therefore, we suggest calibrating the field in the magnets using the relation between the beam energy and the spin precession frequency in the horizontal plane, that is, determined by the vertical component $B_y$. Since the magnet orientation remains unchanged, and the magnets are fed from one power supply, the calibration of $B_y$ will restore the component $B_x$ with the same relative accuracy $10^{-9}$, which applies to the difference $\Omega_{Bx} - \Omega_{Bx}^{CW}$ as well. Unfortunately, we are limited by the allowable size of the article, and other details of this method are outside the scope of the article.

REFERENCES