HISKP-TH-09/04

Gauge invariance in the presence of a cutoff

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(Dated: April 27, 2009)

Abstract

We use the method of gauging equations to construct the electromagnetic current operator for the two-nucleon system in a theory with a finite cutoff. The employed formulation ensures that the two-nucleon T-matrix and corresponding five-point function, in the cutoff theory, are identical to the ones formally defined by a reference theory without a cutoff. A feature of our approach is that it effectively introduces a cutoff into the reference theory in a way that maintains the long-range part of the exchange current operator; for applications to effective field theory (EFT), this property is usually sufficient to guarantee the predictive power of the resulting cutoff theory. In addition, our approach leads to Ward-Takahashi (WT) identities that are linear in the interactions. From the point of view of EFT's where such a WT identity is satisfied in the reference theory, this ensures that gauge invariance in the cutoff theory is maintained order by order in the expansion.

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I. INTRODUCTION

Nuclear forces have been extensively studied over the past decade within chiral effective field theory (EFT), see Refs. [1, 2, 3] for recent review articles. The chiral potential derived in this framework can generally be split into its long- and short-range parts. The long-range contributions are due to the exchange of one or more Goldstone bosons [pions in the formulation based on the SU(2) chiral symmetry of quantum chromodynamics (QCD)] and are strongly constrained by the spontaneously and explicitly broken chiral symmetry of QCD. The long-range behavior of the two-nucleon potential is furthermore independent of the regularization procedure employed to evaluate the corresponding loop integrals. The short-range contributions are usually parametrized in terms of contact interactions in a most general way in order to ensure that the results are model independent. It should be emphasized that such contact interactions are not constrained by the chiral symmetry. The a-priori unknown low-energy constants accompanying short-range contact interactions have to be determined from two-nucleon scattering data.

The potentials derived in chiral EFT, in general, are not well defined at large momenta/short distances and cannot be used in a Schrödinger equation without regularization. This is usually accounted for by introducing a high-momentum/short-distance cutoff.[1] For applications to electromagnetic reactions it is desirable to employ a mass-independent regularization scheme such as, for example, dimensional regularization (DR), in order to maintain gauge invariance at every stage of the calculation. Given the nonperturbative nature of the nucleon-nucleon interaction at low energy, it is not yet fully clear how to implement DR in this context in the presence of long-range interactions (see however Ref. [8] for the first attempt along this line). It is therefore important to clarify how cutoff regularization can be carried out in the framework of chiral effective field theory without destroying gauge invariance.

Such a procedure cannot be unique: the way to maintain gauge invariance will, in general, depend on how the interaction without photons is regularized. A lot of literature exists where formalisms are presented to render the meson–baryon system gauge invariant [11]. An alternative method to regularize an interaction while preserving all symmetries (including electromagnetic gauge invariance) was presented in Ref. [12] where a finite cutoff is introduced at the level of the effective Lagrangian. For a different but related approach the reader is referred to Ref. [13]. In Refs. [14, 15] a different, quite general construction for the NN interaction is presented that leads to gauge invariant currents; however, it is unclear how this recipe can be combined with the power counting of an effective field theory. In this work we present a way to introduce a cutoff while imposing the Ward-Takahashi (WT) identities for various building blocks of the full reaction matrix elements. The main advantages of our approach to the introduction of a cutoff are that (i) it preserves the long-range part of the exchange current operator, and (ii) it leads to the usual (linear in the interaction) WT identity for regularized quantities, provided these identities are fulfilled by the same quantities without regularization. The first feature is a necessary requirement for any formulation based on effective field theory while the second enables one to maintain *exact* gauge invariance for observables calculated at any fixed finite order in the EFT expansion.

At the same time, the predictive power of the scheme with cutoff is expected to be the same as that of the corresponding theory without cutoff. To see this, simply observe that the

[1] See Refs. [4, 5, 6, 7, 8, 9, 10] for alternative renormalization schemes.

long-range parts of both theories are identical by construction. In both theories the short-range physics is parametrized by a series of contact interactions. Their number is fixed by the symmetries of the theories and thus is expected to be the same in both schemes, although the actual values of the corresponding strength parameters will change.

In this paper we will introduce a new formalism to construct gauge invariant amplitudes in the presence of non–perturbative interactions using a cutoff which is not restricted to applications based on effective field theories. Consequently we will postpone all detailed discussions intimately linked to effective field theories, like power counting issues, to a subsequent paper.

The paper is organized as follows: in Sect. II we formulate the problem, present our central results, and explain how to calculate the amplitude for various electromagnetic transitions in the two-nucleon sector. In Sect. III, the central results of our work are derived. A brief summary is presented in Sect. IV.

II. FORMALISM AND IMPORTANT RESULTS

In this work we consider non-relativistic nucleons. It is, therefore, preferable to define the reference operators in three-dimensional (3D) momentum space in such a way that gauge invariance of the initial $\pi N\gamma$ Lagrangian is not destroyed. One possibility to do this is discussed in Ref. [16] where a 3D gauge-invariant reduction in the light front formalism is carried out (the difference between the light-front and usual time is not essential for this purpose). A similar formalism is presented in Refs. [17]. The basic idea can be summarized as follows: gauge invariance is manifest for the usual Green functions (vacuum expectation of the time ordered products of the field operators) which fulfill the Ward-Takahashi identities but correspond to operators acting in the space of four-momenta. A gauge invariant 3D reduction can be achieved by equating the two-nucleon times which is equivalent to integrating over the relative energy of the nucleons. This procedure does not destroy the WT identity. There are also other possibilities of a gauge invariant 3D reduction such as e. g. on-mass-shell spectator reduction [15, 18, 19]. For more details on this topic the reader is referred to the original publications. We also emphasize that there exist various three-dimensional approaches to derive nuclear potentials and current operators from meson-nucleon Lagrangians which lead to a different form of the WT identity, see e.g. [20] and Refs. [21, 22, 23] for more discussion on that issue.

Consider the reference nucleon-nucleon T-matrix defined through the formal expression

$$T_{\rm ref} = V_{\rm ref} + V_{\rm ref} G_0(E) T_{\rm ref}\,. \tag{1}$$

In this equation, the kernel $V_{\rm ref}$ refers to the potential reduced to its three dimensional analog along the lines of Ref. [16], and

$$\langle \mathbf{p}'|G_0(E)|\mathbf{p}\rangle = \delta(\mathbf{p}'-\mathbf{p})\,\frac{M}{ME-\mathbf{p}^2+i\eta}\,, \tag{2}$$

where E, $\mathbf{p}$ and $\mathbf{p}'$ denote the center-of-mass energy and the relative initial and final momenta of the NN pair, respectively, and M is the nucleon mass. Of central interest to this paper is the five-point function for two nucleons interacting with a photon, $T^\mu_{\rm ref}$. In terms of diagrams, we shall take $T^\mu_{\rm ref}$ to be the result of attaching the photon line everywhere inside of $T_{\rm ref}$ (but not to the external nucleon lines).[2] It can then be shown that [24]

$$T^\mu_{\rm ref} = (1 + T_{\rm ref} G_0) V^\mu_{\rm ref} (1 + G_0 T_{\rm ref}) + T_{\rm ref} G_0^\mu T_{\rm ref}\,, \tag{3}$$

where $V^\mu_{\rm ref}$ and G_0^μ denote the reference interaction current (gauged potential) and the gauged two-nucleon propagator. In order to proceed, we need to assume that $V^\mu_{\rm ref}$, and hence $T^\mu_{\rm ref}$, obey the usual WT identities

$$q_\mu V^\mu_{\rm ref} = [\Gamma_0, V_{\rm ref}]\,, \tag{4a}$$

$$q_\mu T^\mu_{\rm ref} = [\Gamma_0, T_{\rm ref}]\,, \tag{4b}$$

where q^μ denotes the four–momentum of an incoming photon and where we introduced Γ_0, defined in Eq. (22), to allow for a compact representation of the WT identity (see Sect. III A for more details).

We define the T-matrix and the potential V in the cutoff theory via

$$T = V + V\,(G_0 \Theta)\,T, \tag{5a}$$

$$V = V_{\rm ref} + V_{\rm ref}\,(G_0 \bar\Theta)\,V, \tag{5b}$$

where Θ ($\bar\Theta$) denotes the projector operator onto the space of low (high) relative momenta with the usual properties $\Theta\bar\Theta = \bar\Theta\Theta = 0$ and $\Theta + \bar\Theta = 1$. In particular,

$$\langle \mathbf{p}'|\Theta|\mathbf{p}\rangle = \delta(\mathbf{p}' - \mathbf{p})\theta(\Lambda - |\mathbf{p}|) \tag{6}$$

where Λ is the cutoff momentum. Notice further that $[G_0, \Theta] = 0$. In distinction to previous approaches to the same problem, we regard the cutoff as part of the two-nucleon propagator. As a consequence, both T and V have an inverse. It is easy to see that T equals $T_{\rm ref}$ exactly. Furthermore, it is important to emphasize that the long-range parts of V and $V_{\rm ref}$ are identical by construction, since the term $V_{\rm ref}\,(G_0\bar\Theta)\,V$ is of a short range.

We now need to gauge the above equations. One finds in full analogy to Eq. (3):

$$T^\mu = (1 + T G_0 \Theta) V^\mu (1 + G_0 \Theta T) + T (G_0\Theta)^\mu T\,, \tag{7}$$

with the interaction current operator in the cutoff theory V^μ being defined via

$$V^\mu = (1 + V G_0 \bar\Theta) V^\mu_{\rm ref} (1 + G_0 \bar\Theta V) + V\,(G_0\bar\Theta)^\mu\,V\,. \tag{8}$$

We will show in Sect. III B that T^μ equals $T^\mu_{\rm ref}$ exactly, and for properly choosen $(G_0\Theta)^\mu$ (see discussion in Sec. III C), the current operators V^μ and $V^\mu_{\rm ref}$ have the same long-range parts. Moreover, if the current operator $V^\mu_{\rm ref}$ and the potential $V_{\rm ref}$ are related to each other via the usual WT identity, the same holds true for the corresponding quantities V^μ and V in the cutoff theory provided the completeness relation $(G_0\Theta)^\mu + (G_0\bar\Theta)^\mu = G_0^\mu$ is satisfied.

Thus, the actual problem reduces to constructing $(G_0\Theta)^\mu$ which will be carried out in Sect. III C. As will be shown in this section, one possible choice is

$$\langle \mathbf{p}'|G_0^{-1}(G_0\Theta)^\mu G_0^{-1}|\mathbf{p}\rangle = \frac{1}{2}\delta(\mathbf{p}_2' - \mathbf{p}_2)\,\mathcal{A}_1^\mu + (1 \leftrightarrow 2) \tag{9}$$

[2] As discussed in Ref. [24], any contributions to $T^\mu_{\rm ref}$ that cannot be obtained by attaching photons to $T_{\rm ref}$ are gauge invariant on their own, and can be added separately, as needed.

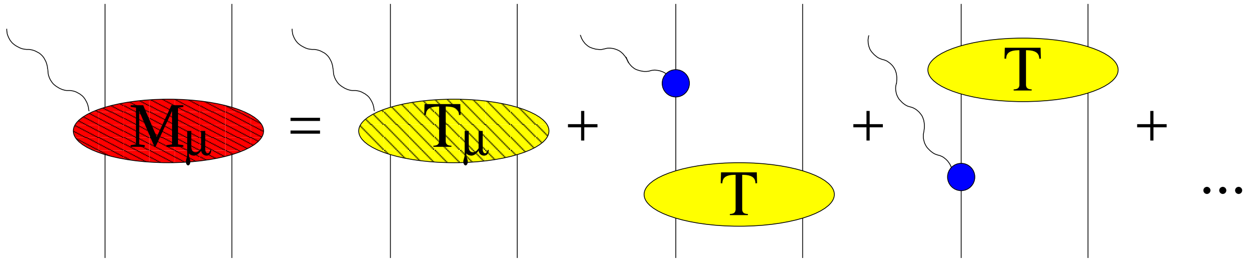

FIG. 1: Graphical illustration for the full bremsstrahlungs amplitude. Straight lines represent nucleons, wavy lines photons. The diagrams where the photon is attached to the other external nucleons are not shown.

where

$$\begin{aligned}\mathcal{A}_1^\mu &= \Gamma_1^\mu(\mathbf{p}_1', \mathbf{p}_1)\,[\theta(\Lambda - p') + \theta(\Lambda - p)] \\ &\quad + (0, \boldsymbol{\Gamma}_1(\mathbf{p}', \mathbf{p}))\,\frac{\theta(\Lambda - p') - \theta(\Lambda - p)}{p'^2 - p^2}(E'_{cm}M - \mathbf{p}'^2 + E_{cm}M - \mathbf{p}^2).\end{aligned} \tag{10}$$

In the above expression, $\Gamma_1^\mu = (\Gamma_1^0, \boldsymbol{\Gamma}_1)$ is the electromagnetic vertex of nucleon 1 [see Eq. (20)], $\mathbf{p}_i$ ($\mathbf{p}_i'$) denotes the initial (final) momentum of particle i, $p = |\mathbf{p}|$, $p' = |\mathbf{p}'|$, and E_{cm} (E'_{cm}) is the energy of the two initial (final) nucleons in their own centre of mass system. The modification of the free two-body current in the cutoff theory due to the second term is crucial in order to maintain the linear form of the resulting WT identity which leads to *exact* gauge invariance even in the case of approximate results for V and V^μ corresponding to a truncated iterative solution of Eqs. (5) and (8).

Note that in this work we only consider undressed nucleons and the corresponding one-body currents (10) in order to explain the main ideas of our formalism. Additional issues connected with the gauge invariant dressing of the nucleons due to meson-nucleon interactions and the proper treatment of relativistic corrections will be discussed in a subsequent publication.

Before proving the results quoted above, we would like to briefly remind the reader of the relations between T^μ and the amplitudes for various electromagnetic reactions. Clearly, the gauged NN scattering amplitude T^μ contains the complete information needed to describe such reactions (in the one-photon approximation). For instance, in the case of the bremsstrahlung process one obtains

$$\begin{aligned}M^\mu(NN \to NN\gamma) &= G_0^{-1}(G_0TG_0)^\mu G_0^{-1} \\ &= T^\mu + G_0^{-1}G_0^\mu T + TG_0^\mu G_0^{-1}\,,\end{aligned} \tag{11}$$

with T^μ defined in Eq. (7). The three terms in the last line of Eq. (11) can be interpreted diagrammatically in a straightforward manner (c.f. Fig. 1): while the first term represents the coupling of the photon to the scattering matrix, the final two give the coupling to the external legs. By construction, T^μ and G_0^μ obey

$$q_\mu T^\mu = [\Gamma_0, T]\,, \tag{12a}$$

$$q_\mu G_0^\mu = [\Gamma_0, G_0]\,, \tag{12b}$$

where Γ_0 will be defined in Eq. (21). With this one immediately finds

$$q_\mu M^\mu(NN \to NN\gamma) = 0 \; , \tag{13}$$

since all terms where G_0^{-1} acts on an external state vanish. Thus the formalism automatically produces conserved currents, which should not come as a surprise since the conditions imposed by the WT identities are stronger than just current conservation.

Eq. (11) can be straightforwardly extended to describe processes involving bound states. To see this we introduce the vertex function ϕ via

$$\langle \mathbf{p}'|T(E)|\mathbf{p}\rangle \stackrel{E\to E_B}{\longrightarrow} \frac{\phi(\mathbf{p}')\,\phi^\dagger(\mathbf{p})}{E-E_B} \tag{14}$$

where E_B is the bound-state energy. Note, by construction ϕ fulfills the homogenous Lippmann-Schwinger equation $\phi = VG_0\Theta\phi$ and is related to the standard wave-function via $\psi_B(\mathbf{p}) = G_0\Theta\phi(\mathbf{p})$. Here we have already used that in the case of non–relativistic kinematics, the bound state vertex functions do not depend on the total momentum P. With this the amplitude for the breakup reaction may be written as $G_0^{-1}(G_0T)^\mu$ with the initial T–matrix evaluated at the bound state pole. We thus have

$$\begin{aligned} M^\mu(B\gamma \to NN) &= G_0^{-1}[G_0^\mu + G_0(1+TG_0\Theta)V^\mu G_0\Theta + G_0T(G_0\Theta)^\mu]\phi \\ &= [G_0^{-1}G_0^\mu + (1+TG_0\Theta)V^\mu G_0\Theta + T(G_0\Theta)^\mu]\phi \, . \end{aligned} \tag{15}$$

Similarly, the expression for bound state form factors/transitions $B+\gamma \to B'$ reads

$$\langle P'|J^\mu(0)|P\rangle = \phi^\dagger_{P'}[G_0\Theta V^\mu G_0\Theta + (G_0\Theta)^\mu]\phi_P \, . \tag{16}$$

We could have also derived this amplitude straightforwardly from Eq. (7) by computing the residue of $T^\mu(E',E)$ at $E'=E'_B$ and $E=E_B$, where E'_B and E_B refer to the binding energies of the final and inital states, respectively.

It may be useful to illustrate the numerical form of our operator equations by explicitly writing out Eq. (16) in terms of momentum-dependent variables. Using the explicit form for the single-nucleon current given in Eq. (9), and, for the sake of brevity, considering just the zeroth component of the current, we have

$$\begin{aligned} \langle P'|J^0(0)|P\rangle &= \int \phi^\dagger_{P'}(\mathbf{p}')\,\frac{d\mathbf{p}'\,\theta(\Lambda-p')}{E'_B-\mathbf{p}'^2/M}\,V^0(P',\mathbf{p}',P,\mathbf{p})\,\frac{d\mathbf{p}\,\theta(\Lambda-p)}{E_B-\mathbf{p}^2/M}\,\phi_P(\mathbf{p}) \\ &+ \frac{ie_1}{2}\int \phi^\dagger_{P'}(\mathbf{p}')\,\frac{d\mathbf{p}\,[\theta(\Lambda-p')+\theta(\Lambda-p)]}{(E'_B-\mathbf{p}'^2/M)(E_B-\mathbf{p}^2/M)}\,\phi_P(\mathbf{p}) + (1\leftrightarrow 2)\; . \end{aligned} \tag{17}$$

Clearly, the first term on the right-hand side of this equation describes the contribution from the exchange currents while the second and the third terms correspond to the impulse approximation (IA) current (in which case $\mathbf{p}'=\mathbf{p}+\mathbf{q}/2$). We further emphasize that contrary to the more traditional approach in which the cutoff is implemented in the potential rather than in the two-nucleon propagator, the vertex function $\phi(\mathbf{p})$ extends up to infinite momenta. The correct normalization of the vertex function $\phi(\mathbf{p})$ can be read off from Eq. (14) or, in case of energy-independent potentials, from the usual wave function normalization condition

$$\int d\mathbf{p}\,\psi^\dagger_B(\mathbf{p})\psi_B(\mathbf{p}) = \int d\mathbf{p}\,\frac{\phi^\dagger(\mathbf{p})\theta(\Lambda-p)}{E_B-\mathbf{p}^2/M}\,\frac{\phi(\mathbf{p})\theta(\Lambda-p)}{E_B-\mathbf{p}^2/M} = 1\, . \tag{18}$$

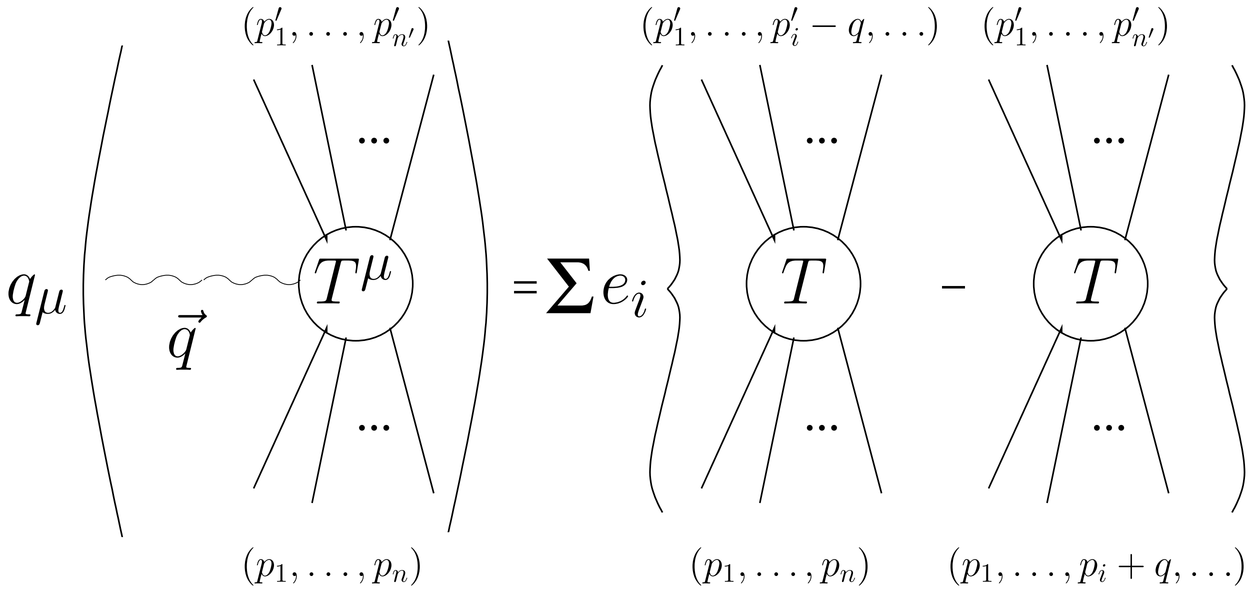

FIG. 2: Graphical illustration for the WT identity for N–point functions. In our case we focus on $n = n' = 2$ and therefore $N = 5$.

III. DERIVATION OF THE CENTRAL RESULTS

We now derive the results already quoted in the previous section. Our derivation is based on the WT identity for the 5-point functions. We therefore begin with a brief discussion where the WT identity, and our notation for it, are specified.

A. The WT identity for the 5-point function

A derivation of the WT identity for N-point functions can be found in most modern textbooks on quantum field theory, see e.g. Ref. [25]. The resulting expression reads

$$\begin{aligned} &q_\mu T^\mu(q; p_1 \ldots p_n; p'_1 \ldots p'_{n'}) \\ &\quad = \sum_i ie_i \left[T\left(p_1 \ldots p_n; p'_1 \ldots, p'_i - q, \ldots\right) - T\left(p_1 \ldots, p_i + q, \ldots; p'_1 \ldots p'_{n'}\right)\right] . \end{aligned} \tag{19}$$

In the case at hand, we have $n = n' = 2$. It is more convenient for our purpose to rewrite the WT identity in operator form. Let Γ_i^μ denote the single-nucleon current operator of nucleon i, whose matrix element is extracted from the initial Lagrangian:[3]

$$\Gamma_i^\mu(\mathbf{p}', \mathbf{p}) = ie_i \left(1,\, \frac{\mathbf{p}' + \mathbf{p}}{2M}\right) . \tag{20}$$

[3] Here one could have equally well used the nonrelativistic limit of the Dirac-fermion current:

$$\Gamma_i^\mu(\mathbf{p}', \mathbf{p}) = ie_i \left(1,\, \frac{\mathbf{p}' + \mathbf{p} + i\boldsymbol{\sigma}_i \times \mathbf{q}}{2M}\right) .$$

Then the matrix element of the free two-nucleon current operator, $\Gamma_0^\mu \equiv G_0^{-1} G_0^\mu G_0^{-1}$, is given by

$$\langle \mathbf{p}' | \Gamma_0^\mu | \mathbf{p} \rangle = \delta(\mathbf{p}_2' - \mathbf{p}_2)\Gamma_1^\mu(\mathbf{p}_1', \mathbf{p}_1) + \delta(\mathbf{p}_1' - \mathbf{p}_1)\Gamma_2^\mu(\mathbf{p}_2', \mathbf{p}_2). \tag{21}$$

The zeroth component of the free two-nucleon current is then (for simplicity of notation, we drop the zero superscript)

$$\begin{aligned} \langle \mathbf{p}' | \Gamma_0 | \mathbf{p} \rangle &= ie_1 \delta(\mathbf{p}_2' - \mathbf{p}_2) + ie_2 \delta(\mathbf{p}_1' - \mathbf{p}_1) \\ &= ie_1 \delta(\mathbf{p}' - \mathbf{p} - \mathbf{q}/2) + ie_2 \delta(\mathbf{p}' - \mathbf{p} + \mathbf{q}/2) \end{aligned} \tag{22}$$

where $\mathbf{q} = \mathbf{P}' - \mathbf{P}$. Then the operator form of Eq. (19) in the two nucleon case simply reads

$$q_\mu T^\mu = [\Gamma_0, T] \ . \tag{23}$$

This representation of the WT identity will prove very useful in subsequent derivations.

B. Transition current in the presence of a cutoff

It is straightforward to show that T^μ agrees identically with T^μ_{ref} provided the following two conditions hold: (i) V is a solution of Eq. (5b), and (ii) the free two-body current $(G_0\Theta)^\mu$ fulfills the completeness relation

$$(G_0\Theta)^\mu + (G_0\bar{\Theta})^\mu = G_0^\mu \, . \tag{24}$$

Here, $G_0^\mu = G_0 \Gamma_0^\mu G_0$ denotes the free two-body current before introducing the cutoff. The explicit form of $(G_0\Theta)^\mu$ is specified in Eqs. (9) and (10) and derived in Sect. III C. With this one finds

$$\begin{aligned} T^\mu &= (1 + TG_0\Theta)V^\mu(1 + G_0\Theta T) + T(G_0\Theta)^\mu T \\ &= (1 + TG_0\Theta)\left[(1 + VG_0\bar{\Theta})V^\mu_{\text{ref}}(1 + G_0\bar{\Theta}V) + V(G_0\bar{\Theta})^\mu V\right](1 + G_0\Theta T) + T(G_0\Theta)^\mu T \\ &= (1 + TG_0\Theta + TG_0\bar{\Theta})V^\mu_{\text{ref}}(1 + G_0\Theta T + G_0\bar{\Theta}T) + T(G_0\bar{\Theta})^\mu T + T(G_0\Theta)^\mu T \\ &= (1 + TG_0)V^\mu_{\text{ref}}(1 + G_0 T) + TG_0^\mu T \\ &= T^\mu_{\text{ref}} \ , \end{aligned} \tag{25}$$

where we have made use of Eq. (5a), Eq. (8), and Eq. (24). In practice, the equation defining V, Eq. (5b), cannot usually be solved exactly. However, what is important for applications based on effective field theory, is that the long-range part of V agree with the one of V_{ref}. In our formalism this holds by construction — see also the discussion before Eq. (29). In effective field theory the transition potential V_{ref} and the corresponding current operator V^μ_{ref} are derived directly from an underlying Lagrangian. We assume that this derivation is carried out in the formulation which preserves the usual form of the WT identity:

$$q_\mu V^\mu_{\text{ref}} = [\Gamma_0, V_{\text{ref}}] \ .$$

We also need to demand that the free two-body current $(G_0\Theta)^\mu$ obey the WT identity

$$q_\mu (G_0\Theta)^\mu = [\Gamma_0, G_0\Theta] \ . \tag{26}$$

It is then straightforward to see that V^μ also obeys the usual WT identity; indeed,

$$\begin{aligned} q_\mu V^\mu &= (1+VG_0\bar{\Theta})\, q_\mu V^\mu_{\rm ref}(1+G_0\bar{\Theta}V) + Vq_\mu(G_0\bar{\Theta})^\mu V \\ &= (1+VG_0\bar{\Theta})[\Gamma_0, V_{\rm ref}](1+G_0\bar{\Theta}V) + V[\Gamma_0, G_0\bar{\Theta}]V \\ &= (1+VG_0\bar{\Theta})\Gamma_0 V - V\Gamma_0(1+G_0\bar{\Theta}V) + V[\Gamma_0, G_0\bar{\Theta}]V \\ &= [\Gamma_0, V]\,. \end{aligned} \tag{27}$$

As a direct consequence, the five-point function T^μ will likewise satisfy the WT identity:

$$q_\mu T^\mu = [\Gamma_0, T]\,. \tag{28}$$

Eqs. (7) and (27) are all we need to compute the five-point function in EFT with maximal predictive power maintained.

C. Construction of the free two-body currents

So far we have not specified the explicit form of the operator $(G_0\Theta)^\mu$, apart from requiring it to satisfy the WT identity, Eq. (26). In order to proceed, further properties of $(G_0\Theta)^\mu$ need to be specified. In particular, we shall demand that:

1. The WT identity of Eq. (26) must hold.

2. The integral given by the term $T(G_0\Theta)^\mu T$ in Eq. (7) must involve only low relative momenta, such that the initial and final scattering amplitudes in Eq. (7) are calculable in low-energy EFT. Because of the completeness relation of Eq. (24), this condition also implies that the term $V(G_0\bar{\Theta})^\mu V$ in Eq. (8) is of short-range, i.e., V^μ has the same long-range/short-range decomposition as $V^\mu_{\rm ref}$. This can be achieved by demanding that $(G_0\Theta)^\mu$ provide a cutoff in a similar way to $G_0\Theta$, i.e., $(G_0\Theta)^\mu = 0$ if both initial and final relative momenta are above the cutoff parameter Λ; in particular, we shall demand that

$$\langle \mathbf{p}'|G_0^{-1}(G_0\Theta)^\mu G_0^{-1}|\mathbf{p}\rangle = \delta(\mathbf{p}'_2-\mathbf{p}_2)[N^\mu\theta(\Lambda-p') + M^\mu\theta(\Lambda-p)] + (1\leftrightarrow 2) \tag{29}$$

   where N^μ and M^μ are yet to be determined.

3. In the cutoff scheme, the leading-order (LO) physical current in the *impulse approximation* [which is determined solely by the $(G_0\Theta)^\mu$ part of the current vertex] can differ from the one in the reference scheme only by terms of higher order. This restriction has to do with the naturalness condition and ensures that the effective interaction current V^μ does not violate power counting, as will be discussed in a subsequent paper.

4. The IA current $\langle \mathbf{p}'|G_0^{-1}(G_0\Theta)^\mu G_0^{-1}|\mathbf{p}\rangle$ must be regular.

5. In the limit $\Lambda=\infty$, the IA current reduces to the usual expression of Eq. (21),

$$\langle p'|G_0^{-1}(G_0\Theta)^\mu G_0^{-1}|p\rangle \;\rightarrow\; \delta(\mathbf{p}'_2-\mathbf{p}_2)\Gamma^\mu_1(\mathbf{p}'_1,\mathbf{p}_1) + \delta(\mathbf{p}'_1-\mathbf{p}_1)\Gamma^\mu_2(\mathbf{p}'_2,\mathbf{p}_2). \tag{30}$$

6. Time reversal invariance must hold:

$$\langle \mathbf{p}'|G_0^{-1}(G_0\Theta)^\mu G_0^{-1}|\mathbf{p}\rangle = \langle \mathbf{p}|G_0^{-1}(G_0\Theta)^\mu G_0^{-1}|\mathbf{p}'\rangle\,. \tag{31}$$

The third restriction is imposed only on the IA part of the LO current to help also the (WT identity based) relation between V^μ and V to be the same as the relation between $V^\mu_{\rm ref}$ and $V_{\rm ref}$. Here, the point is an ambiguity in solving WT identity for V^μ (WT identity can only be resolved for the longitudinal part of the current). To see this, suppose that the interaction charge density in the reference scheme, $V^0_{\rm ref}$, were zero. If the relation between V^μ and V is the same, V^0 should be zero as well. This would not be achieved, if the LO physical current in the *impulse approximation* in the cutoff scheme is different from the one in the renormalization scheme in LO, because this difference should be compensated by the interaction current, V^0. The third restriction can also be formulated as follows: V^μ should depend on the coupling constants composing V in the same way as $V^\mu_{\rm ref}$ depends on the coupling constants composing $V_{\rm ref}$. So we have six restrictions for $(G_0\Theta)^\mu$.

It is interesting that the straightforward expression for

$$\langle \mathbf{p}'|G_0^{-1}(G_0\Theta)^\mu G_0^{-1}|\mathbf{p}\rangle = \delta(\mathbf{p}_2'-\mathbf{p}_2)\Gamma_1^\mu(\mathbf{p}_1',\mathbf{p}_1)\frac{G_0^{-1}(p)\theta(\Lambda-p)-G_0^{-1}(p')\theta(\Lambda-p')}{E'-{p_1'}^2/2M-E+p_1^2/2M}+(1\leftrightarrow 2) \tag{32}$$

is consistent with only five of the six restrictions. For example, it cuts off high relative momenta and ensures that the term $V(G_0\bar\Theta)^\mu V$ in Eq. (8) is of short-range so that V^μ has the same long-range/short-range decomposition as $V^\mu_{\rm ref}$ (in this case they have identical long-range parts). However, the vertex of Eq. (32) is singular in the case of inelastic scattering, where

$$E'_{cm}=E'-\mathbf{P}'^2/4M\neq E-\mathbf{P}^2/4M=E_{cm}\,, \tag{33}$$

and is therefore not consistent with restriction number 4.

Two examples of regular vertices are discussed in the Appendix, see Eqs. (A.4) and (A.5). To satisfy restriction number 6 (as well as the other five restrictions), we choose the current vertex which is symmetric with respect to the interchange of the initial and final states as given previously in Eq. (9). The derivation of this expression is carried out in the Appendix. It provides useful insights into the gauge-invariant treatment of theories where a cutoff is a necessary attribute in actual calculations (such as, for example, the NJL model of Ref. [26]). Gauge invariance can be maintained by a specific regularization, analogous to Eq. (9), of the integrals corresponding to loops with an attached photon, in a close analogy to what is done in the present work. This regularization necessarily (and naturally) depends on the way the loops without photons are regularized.

It is also worth noting that the only terms which violate Galilean invariance in Eq. (9), namely, the combinations $\mathbf{p}_i'+\mathbf{p}_i=\mathbf{P}'/2+\mathbf{P}/2\pm(\mathbf{p}'+\mathbf{p})$ contained in the single nucleon vertex functions Γ_i^μ, are the ones which enter the IA in the underlying theory. The rest depend only on the Galilei-invariant variables $\mathbf{p}',\mathbf{p},E_{cm}$ and E'_{cm}.

Note finally that $\int d^3p\,[\theta(\Lambda-p')-\theta(\Lambda-p)]$ is of order q, and therefore one could think that the most unusual part of the free two-nucleon current of Eq. (9), proportional to $\theta(\Lambda-p')-\theta(\Lambda-p)$, may not contribute at lowest order, since $q\ll\Lambda$. However, there is a compensating enhancement from the denominator, since $\mathbf{p}'^2-\mathbf{p}^2$ is also of order q.

It is important to note that the form of Eq. (9) applies also to the practical case of a smooth cutoff; that is, even if we replace the sharp cutoff θ-functions by smooth regulators, the WT identity will still be satisfied.

IV. SUMMARY

In this paper we have shown how a finite cutoff can be implemented in two-nucleon calculations without destroying the linear form of the WT identity and without loosing any predictive power as compared to a mass independent regularization like dimensional regularisation. In the latter case, we have an NN potential $V_{\rm ref}$ with long- and short-range parts, whose short-range couplings are determined from a fit to data. We also have an interaction current $V^\mu_{\rm ref}$ which is given by the initial Lagrangian and is restricted by gauge invariance. We further assume that $V_{\rm ref}$ and $V^\mu_{\rm ref}$ fulfill the usual WT identity $q_\mu V^\mu_{\rm ref} = [\Gamma_0, V_{\rm ref}]$ (see Sec. III A). The physical NN scattering amplitude and NN currents are then derived via Eq. (1) and Eq. (3) respectively.

In the cutoff scheme we have an NN potential V with long- and short-range parts, whose short-range couplings are also determined from a fit to the data. We also have an interaction current V^μ which, by construction, fulfills the corresponding WT identity $q_\mu V^\mu = [\Gamma_0, V]$. The physical NN scattering amplitude and NN currents can be derived via Eq. (5a) and Eq. (7), which represent the cutoff versions of Eq. (1) and Eq. (3), respectively. The currents in our cutoff scheme are conserved.

The difference between the approaches with the interactions $V_{\rm ref}$ and V (and correspondingly with the currents $V^\mu_{\rm ref}$ and V^μ) is in that $V_{\rm ref}$ is given directly by the initial Lagrangian, whereas V is related to $V_{\rm ref}$ in a complicated way, see Eq. (5). This difference is not important in the philosophy of EFT, because $V_{\rm ref}$ and V have the same long-range parts, and the short-range parts of $V_{\rm ref}$ and V are anyway determined from experimental data.

The new technical element of our formulation is the single-nucleon current, $(G_0\Theta)^\mu$, which is constructed to satisfy the WT identity in Eq. (26) and is explicitly given by Eq. (9). It depends on the cutoff in a specific way which is consistent with gauge invariance.

Finally, we would like to comment on the relation of our interaction current to the ones derived in Refs. [27, 28]. Although the current operators constructed in [27, 28] are sufficient to reproduce the off-shell 5-point Green function (not only the physical processes listed in Sect. II), they are related to the NN potential V via a *modified* WT identity which is *nonlinear* in V [28]. This is in strong contrast to the present formulation where the usual linear WT identity is obtained. However, only on the basis of a linear WT identity can gauge invariance be maintained order by order in some perturbative expansion.

Acknowledgments

We are greatful for discussions with U.-G. Meißner, K. Nakayama, D.R. Phillips, and D.O. Riska. The work of E.E., M.P.V. and A.N.K. was supported in parts by funds provided from the Helmholtz Association to the young investigator group "Few-Nucleon Systems in Chiral Effective Field Theory" (grant VH-NG-222) and through the virtual institute "Spin and strong QCD" (grant VH-VI-231). This work was further supported by the DFG (SFB/TR 16 "Subnuclear Structure of Matter"), by the EU HadronPhysics2 project "Study of strongly interacting matter" and by grant number GNSF/ST08/4-400 of the Georgian National Foundation.

APPENDIX A: IA CURRENT VERTEX

We shall define the momentum space matrix elements of the Θ operator as

$$\langle \mathbf{p}'|\Theta(P',P)|\mathbf{p}\rangle \equiv \Theta(\mathbf{p}_1'\mathbf{p}_2',\mathbf{p}_1\mathbf{p}_2) = \delta(\mathbf{p}'-\mathbf{p})\theta(\Lambda - p) \tag{A.1a}$$
$$= \delta(\mathbf{p}_1'-\mathbf{p}_1)\theta(\Lambda - p) = \delta(\mathbf{p}_2'-\mathbf{p}_2)\theta(\Lambda - p) \tag{A.1b}$$

where $\mathbf{p}_1'+\mathbf{p}_2' = \mathbf{p}_1+\mathbf{p}_2$. The regular solution of the WT identity of Eq. (26) for the free two-nucleon current $(G_0\Theta)^\mu$, corresponds to the following gauged theta operator, Θ^μ:

$$\langle \mathbf{p}'|\Theta^0(P',P)|\mathbf{p}\rangle = 0 \tag{A.2a}$$
$$\langle \mathbf{p}'|\boldsymbol{\Theta}(P',P)|\mathbf{p}\rangle = M\left[\delta(\mathbf{p}_2'-\mathbf{p}_2)\boldsymbol{\Gamma}_1(\mathbf{p}',\mathbf{p}) - \delta(\mathbf{p}_1'-\mathbf{p}_1)\boldsymbol{\Gamma}_2(\mathbf{p}',\mathbf{p})\right] \times \frac{\theta(\Lambda-p')-\theta(\Lambda-p)}{p'^2-p^2}. \tag{A.2b}$$

It is then straightforward to check that Θ^μ satisfies the usual WT identity:

$$\begin{aligned}
q_\mu\langle \mathbf{p}'|\Theta^\mu(P',P)|\mathbf{p}\rangle &= -(\mathbf{p}_1'+\mathbf{p}_2'-\mathbf{p}_1-\mathbf{p}_2)\cdot\langle \mathbf{p}'|\boldsymbol{\Theta}(P',P)|\mathbf{p}\rangle \\
&= -(\mathbf{p}_1'-\mathbf{p}_1)\cdot ie_1\delta(\mathbf{p}_2'-\mathbf{p}_2)\frac{1}{2}(\mathbf{p}'+\mathbf{p})\frac{\theta(\Lambda-p')-\theta(\Lambda-p)}{p'^2-p^2} \\
&\quad + (\mathbf{p}_2'-\mathbf{p}_2)\cdot ie_2\delta(\mathbf{p}_1'-\mathbf{p}_1)\frac{1}{2}(\mathbf{p}'+\mathbf{p})\frac{\theta(\Lambda-p')-\theta(\Lambda-p)}{p'^2-p^2} \\
&= -2(\mathbf{p}'-\mathbf{p})\cdot ie_1\delta(\mathbf{p}_2'-\mathbf{p}_2)\frac{1}{2}(\mathbf{p}'+\mathbf{p})\frac{\theta(\Lambda-p')-\theta(\Lambda-p)}{p'^2-p^2} \\
&\quad - 2(\mathbf{p}'-\mathbf{p})\cdot ie_2\delta(\mathbf{p}_1'-\mathbf{p}_1)\frac{1}{2}(\mathbf{p}'+\mathbf{p})\frac{\theta(\Lambda-p')-\theta(\Lambda-p)}{p'^2-p^2} \\
&= -ie_1\delta(\mathbf{p}_2'-\mathbf{p}_2)[\theta(\Lambda-p')-\theta(\Lambda-p)] - ie_2\delta(\mathbf{p}_1'-\mathbf{p}_1)[\theta(\Lambda-p')-\theta(\Lambda-p)] \\
&= \quad ie_1\Theta(\mathbf{p}_1'-\mathbf{q},\mathbf{p}_2',\mathbf{p}_1,\mathbf{p}_2) - \Theta(\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_1+\mathbf{q},\mathbf{p}_2)ie_1 \\
&\quad + ie_2\Theta(\mathbf{p}_1',\mathbf{p}_2'-\mathbf{q},\mathbf{p}_1,\mathbf{p}_2) - \Theta(\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_1,\mathbf{p}_2+\mathbf{q})ie_2\,.
\end{aligned} \tag{A.3}$$

Before using Eqs. (A.2) to specify the IA current vertex, it is important to note that the use of the "product rule" for gauging [24] gives $[G_0\Theta]^\mu \equiv G_0^\mu\Theta + G_0\Theta^\mu$ and $[\Theta G_0]^\mu \equiv \Theta^\mu G_0 + \Theta G_0^\mu$, and therefore $[G_0\Theta]^\mu \neq [\Theta G_0]^\mu$ even though the operators Θ and G_0 commute, $G_0\Theta = \Theta G_0$. It is, however, easy to check the obvious transversality of the difference, $q_\mu\left\{[\Theta G_0]^\mu - [G_0\Theta]^\mu\right\} = 0$. As expected, gauging alone can only determine the longitudinal part of the free two-nucleon current $(G_0\Theta)^\mu$ uniquely.

Indeed, we can use either form to calculate the IA vertex current:

$$
\begin{aligned}
\langle \mathbf{p}'|G_0^{-1}[G_0\Theta]^\mu G_0^{-1}|\mathbf{p}\rangle &= \langle \mathbf{p}'|\left(\Gamma_0^\mu\Theta + \Theta^\mu G_0^{-1}\right)|\mathbf{p}\rangle \\
&= \int d\mathbf{p}''\left[\delta(\mathbf{p}_2'-\mathbf{p}_2'')\Gamma_1^\mu(\mathbf{p}_1',\mathbf{p}_1'') + \delta(\mathbf{p}_1'-\mathbf{p}_1'')\Gamma_2^\mu(\mathbf{p}_2',\mathbf{p}_2'')\right]\delta(\mathbf{p}''-\mathbf{p})\theta(\Lambda-p) \\
&+ \left\{0,\ \left[\delta(\mathbf{p}_2'-\mathbf{p}_2)\mathbf{\Gamma}_1(\mathbf{p}',\mathbf{p}) - \delta(\mathbf{p}_1'-\mathbf{p}_1)\mathbf{\Gamma}_2(\mathbf{p}',\mathbf{p})\right]\frac{\theta(\Lambda-p')-\theta(\Lambda-p)}{p'^2-p^2}(E_{cm}M-\mathbf{p}^2)\right\} \\
&= \left[\delta(\mathbf{p}_2'-\mathbf{p}_2)\Gamma_1^\mu(\mathbf{p}_1',\mathbf{p}_1) + \delta(\mathbf{p}_1'-\mathbf{p}_1)\Gamma_2^\mu(\mathbf{p}_2',\mathbf{p}_2)\right]\theta(\Lambda-p) \\
&+ \left\{0,\ \left[\delta(\mathbf{p}_2'-\mathbf{p}_2)\mathbf{\Gamma}_1(\mathbf{p}',\mathbf{p}) - \delta(\mathbf{p}_1'-\mathbf{p}_1)\mathbf{\Gamma}_2(\mathbf{p}',\mathbf{p})\right]\frac{\theta(\Lambda-p')-\theta(\Lambda-p)}{p'^2-p^2}(E_{cm}M-\mathbf{p}^2)\right\} \\
&= \delta(\mathbf{p}_2'-\mathbf{p}_2)\left\{\Gamma_1^\mu(\mathbf{p}_1',\mathbf{p}_1)\theta(\Lambda-p) + [0,\ \mathbf{\Gamma}_1(\mathbf{p}',\mathbf{p})]\frac{\theta(\Lambda-p')-\theta(\Lambda-p)}{p'^2-p^2}(E_{cm}M-\mathbf{p}^2)\right\} \\
&\quad + \ (1\leftrightarrow 2),
\end{aligned}
\tag{A.4}
$$

or

$$
\begin{aligned}
\langle \mathbf{p}'|G_0^{-1}[\Theta G_0]^\mu G_0^{-1}|\mathbf{p}\rangle &= \langle \mathbf{p}'|\left(\Theta\Gamma_0^\mu + G_0^{-1}\Theta^\mu\right)|\mathbf{p}\rangle \\
&= \int d\mathbf{p}''\delta(\mathbf{p}'-\mathbf{p}'')\theta(\Lambda-p'')\left[\delta(\mathbf{p}_2''-\mathbf{p}_2)\Gamma_1^\mu(\mathbf{p}_1'',\mathbf{p}_1) + \delta(\mathbf{p}_1''-\mathbf{p}_1)\Gamma_2^\mu(\mathbf{p}_2'',\mathbf{p}_2)\right] \\
&+ \left\{0,\ (E_{cm}'M-\mathbf{p}'^2)\left[\delta(\mathbf{p}_2'-\mathbf{p}_2)\mathbf{\Gamma}_1(\mathbf{p}',\mathbf{p}) - \delta(\mathbf{p}_1'-\mathbf{p}_1)\mathbf{\Gamma}_2(\mathbf{p}',\mathbf{p})\right]\frac{\theta(\Lambda-p')-\theta(\Lambda-p)}{p'^2-p^2}\right\} \\
&= \left[\delta(\mathbf{p}_2'-\mathbf{p}_2)\Gamma_1^\mu(\mathbf{p}_1',\mathbf{p}_1) + \delta(\mathbf{p}_1'-\mathbf{p}_1)\Gamma_2^\mu(\mathbf{p}_2',\mathbf{p}_2)\right]\theta(\Lambda-p') \\
&+ \left\{0,\ \left[\delta(\mathbf{p}_2'-\mathbf{p}_2)\mathbf{\Gamma}_1(\mathbf{p}',\mathbf{p}) - \delta(\mathbf{p}_1'-\mathbf{p}_1)\mathbf{\Gamma}_2(\mathbf{p}',\mathbf{p})\right]\frac{\theta(\Lambda-p')-\theta(\Lambda-p)}{p'^2-p^2}(E_{cm}'M-\mathbf{p}'^2)\right\} \\
&= \delta(\mathbf{p}_2'-\mathbf{p}_2)\left\{\Gamma_1^\mu(\mathbf{p}_1',\mathbf{p}_1)\theta(\Lambda-p') + [0,\ \mathbf{\Gamma}_1(\mathbf{p}',\mathbf{p})]\frac{\theta(\Lambda-p')-\theta(\Lambda-p)}{p'^2-p^2}(E_{cm}'M-\mathbf{p}^2)\right\} \\
&\quad + \ (1\leftrightarrow 2).
\end{aligned}
\tag{A.5}
$$

However, in order to satisfy time reversal invariance, it is better to use the symmetrized form $(G_0\Theta)^\mu = \frac{1}{2}\{[G_0\Theta]^\mu + [\Theta G_0]^\mu\}$:

$$
\begin{aligned}
\langle \mathbf{p}'|G_0^{-1}(G_0\Theta)^\mu G_0^{-1}|\mathbf{p}\rangle &= \frac{1}{2}\delta(\mathbf{p}_2'-\mathbf{p}_2)\left\{\Gamma_1^\mu(\mathbf{p}_1',\mathbf{p}_1)\left[\theta(\Lambda-p)+\theta(\Lambda-p')\right]\right. \\
&+ \left.\left[0,\ \mathbf{\Gamma}_1(\mathbf{p}',\mathbf{p})\frac{\theta(\Lambda-p')-\theta(\Lambda-p)}{p'^2-p^2}(E_{cm}'M-\mathbf{p}'^2+E_{cm}M-\mathbf{p}^2)\right]\right\} + (1\leftrightarrow 2)
\end{aligned}
\tag{A.6}
$$

which is the form for the IA vertex current specified in Eq. (9).

It is instructive to rewrite Eq. (A.4) in a different form for the case where the single nucleon vertex current is of the form given in Eq. (20). We first write Eq. (A.4) as

$$
\langle \mathbf{p}'|G_0^{-1}[G_0\Theta]^\mu G_0^{-1}|\mathbf{p}\rangle = \delta(\mathbf{p}_2'-\mathbf{p}_2)\,A^\mu + (1\leftrightarrow 2)
\tag{A.7}
$$

where

$$
A^\mu = \Gamma_1^\mu(\mathbf{p}_1',\mathbf{p}_1)\theta(\Lambda-p) + [0,\ \mathbf{\Gamma}_1(\mathbf{p}',\mathbf{p})]\frac{\theta(\Lambda-p')-\theta(\Lambda-p)}{p'^2-p^2}(E_{cm}M-\mathbf{p}^2).
\tag{A.8}
$$

The zeroth and spatial components of A^μ can then be simplified as follows:

$$
\begin{aligned}
A_0/(ie_1) &= \theta(\Lambda - p)\,, && \text{(A.9a)}\\
\mathbf{A}/(ie_1) &= \frac{1}{2M}(\mathbf{p}_1' + \mathbf{p}_1)\theta(\Lambda - p) + \frac{1}{2M}(\mathbf{p}' + \mathbf{p})\frac{\theta(\Lambda - p') - \theta(\Lambda - p)}{p'^2 - p^2}(E_{cm}M - \mathbf{p}^2),\\
&= \frac{1}{4M}(\mathbf{P}' + \mathbf{P})\theta(\Lambda - p) + \frac{1}{2M}(\mathbf{p}' + \mathbf{p})\frac{\theta(\Lambda - p') - \theta(\Lambda - p)}{p'^2 - p^2}E_{cm}M\\
&\quad + \frac{1}{2M}(\mathbf{p}' + \mathbf{p})\frac{\theta(\Lambda - p)\mathbf{p}'^2 - \theta(\Lambda - p')\mathbf{p}^2}{p'^2 - p^2}\\
&= \frac{1}{4M}(\mathbf{P}' + \mathbf{P})\theta(\Lambda - p) + \frac{1}{2M}(\mathbf{p}' + \mathbf{p})\frac{\theta(\Lambda - p') - \theta(\Lambda - p)}{\mathbf{p}'^2 - \mathbf{p}^2}(E_{cm}M + \frac{\mathbf{p}'^2}{2} + \frac{\mathbf{p}^2}{2})\\
&\quad + \frac{1}{4M}(\mathbf{p}' + \mathbf{p})[\theta(\Lambda - p) + \theta(\Lambda - p')]\,. && \text{(A.9b)}
\end{aligned}
$$

One can easily verify the WT identity for the operator in Eq. (A.4). Expressing Eq. (A.4) as in Eq. (A.7) we have

$$
\begin{aligned}
q_\mu A^\mu/(ie_1) &= (P' - P)_\mu \Bigg[\left(1, \frac{\mathbf{p}_1' + \mathbf{p}_1}{2M}\right)^\mu \theta(\Lambda - p')\\
&\qquad + \left(0, \frac{\mathbf{p}' + \mathbf{p}}{2M}\right)^\mu \frac{\theta(\Lambda - p) - \theta(\Lambda - p)}{\mathbf{p}'^2 - \mathbf{p}^2}(E_{cm}M - \mathbf{p}^2)\Bigg]\\
&= [(E' - E) - \mathbf{p}_1'^2/2M + \mathbf{p}_1^2/2M]\,\theta(\Lambda - p)\\
&\qquad - (\mathbf{p}'^2 - \mathbf{p}^2)\frac{\theta(\Lambda - p') - \theta(\Lambda - p)}{\mathbf{p}'^2 - \mathbf{p}^2}(E_{cm} - \mathbf{p}^2/M)\\
&= (E' - \mathbf{p}_1'^2/2M - \mathbf{p}_2'^2/2M)\theta(\Lambda - p) - \theta(\Lambda - p')(E_{cm} - \mathbf{p}^2/M)\\
&= G_0^{-1}(p')\theta(\Lambda - p) - \theta(\Lambda - p')G_0^{-1}(p)\\
&= G_0^{-1}(p')\,[G_0(p)\theta(\Lambda - p) - G_0(p')\theta(\Lambda - p')]\,G_0^{-1}(p)\,. && \text{(A.10)}
\end{aligned}
$$

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