

# Derivation of spontaneously broken gauge symmetry from the consistency of effective field theory I: Massive vector bosons coupled to a scalar field

D. Djukanovic,<sup>1</sup> J. Gegelia,<sup>2,3</sup> and Ulf-G. Meißner<sup>4,2</sup>

<sup>1</sup>*Helmholtz Institute Mainz, University of Mainz, D-55099 Mainz, Germany*

<sup>2</sup>*Institute for Advanced Simulation,  
Institut für Kernphysik and Jülich Center for Hadron Physics,  
Forschungszentrum Jülich, D-52425 Jülich, Germany*

<sup>3</sup>*Tbilisi State University, 0186 Tbilisi, Georgia*

<sup>4</sup>*Helmholtz Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics,  
Universität Bonn, D-53115 Bonn, Germany*

(Dated: 17 March, 2018)

## Abstract

We revisit the problem of deriving local gauge invariance with spontaneous symmetry breaking in the context of an effective field theory. Previous derivations were based on the condition of tree-order unitarity. However, the modern point of view considers the Standard Model as the leading order approximation to an effective field theory. As tree-order unitarity is in any case violated by higher-order terms in an effective field theory, it is instructive to investigate a formalism which can be also applied to analyze higher-order interactions. In the current work we consider an effective field theory of massive vector bosons interacting with a massive scalar field. We impose the conditions of generating the right number of constraints for systems with spin-one particles and perturbative renormalizability as well as the separation of scales at one-loop order. We find that the above conditions impose severe restrictions on the coupling constants of the interaction terms. Except for the strengths of the self-interactions of the scalar field, that can not be determined at this order from the analysis of three- and four-point functions, we recover the gauge-invariant Lagrangian with spontaneous symmetry breaking taken in the unitary gauge as the leading order approximation to an effective field theory. We also outline the additional work that is required to finish this program.

PACS numbers: 04.60.Ds, 11.10.Gh, 03.70.+k,

Keywords: Effective field theory; Quantization; Constraints; Renormalization

## I. INTRODUCTION

The standard model (SM) is widely accepted as the established consistent theory of the strong, electromagnetic and weak interactions [1]. Invariance under Lorentz and local gauge  $SU(3)_C \times SU(2)_L \times U(1)$  transformations is taken as the underlying symmetry of the SM. Despite the tremendous success of the SM its structure leaves some unanswered questions. In particular, the electromagnetic and gravitational forces are long-ranged and therefore if they are indeed mediated by massless photons and gravitons, then the corresponding local Lorentz-invariant quantum field theories must be gauge theories [1]. On the other hand, as the weak interaction is mediated by massive particles, one might wonder why it should be described by a gauge theory with the spontaneous symmetry breaking? A gauge-invariant theory with the spontaneous symmetry breaking has been derived by demanding tree-order unitarity of the S-matrix in Refs. [2–5]. This result could be considered as a (more or less) satisfactory answer to the above raised question, however, the modern point of view considers the SM as an effective field theory (EFT) [1] which inevitably violates the tree-order unitarity condition at sufficiently high energies. This motivates us to revisit the problem.

In the current work we address the issue of deriving the most general theory of massive vector bosons by demanding self-consistency in the sense of an EFT. The Lagrangian of an EFT consists of an infinite number of terms, however, the contributions of non-renormalizable interactions in physical quantities are suppressed for energies much lower than some large scale. Renormalizability in the sense of a fundamental theory is replaced by the renormalizability in the sense of an EFT, i.e. that all divergences can be absorbed by renormalizing an infinite number of parameters of the effective Lagrangian. Notice that the condition of perturbative renormalizability in the sense of EFT is not equivalent to the condition of tree-order unitarity. While the tree-order unitarity implies renormalizability in the traditional sense, perturbative renormalizability in the sense of EFT is a much weaker condition and it does not imply tree-order unitarity. On the other hand for an EFT to be “effective” it is crucial that the scales are separated, i.e. the contributions of higher order operators in physical quantities are suppressed by powers of some large scale. This condition is much more restrictive than just renormalizability in the sense of EFT. Renormalizability alone can be achieved without introducing scalars, i.e. considering a theory of massive vector bosons and fermions [6]. However, in such a theory divergences generated from the leading order Lagrangian are removed by renormalizing the parameters of higher order interactions. This leaves the scales of the renormalized couplings of the higher order terms much too low to explain the tremendous success of the SM. Therefore, in what follows we analyze the constraint structure and the conditions of perturbative renormalizability and scale separation for the most general Lorentz-invariant effective Lagrangian of massive vector bosons interacting with a scalar field. The performed analysis is similar to that of Refs. [7–9] but here we do not assume parity conservation.

The most general Lorentz-invariant effective Lagrangian contains an infinite number of interaction terms. It is assumed that all coupling constants of “non-renormalizable” interactions, i.e. terms with couplings of negative mass-dimensions, are suppressed by powers of some large scale. Massive vector bosons are spin-one particles and therefore they are described by Lagrangians with constraints. To have a system with the right number of degrees of freedom, the coupling constants of the Lagrangian have to satisfy some non-trivial relations. Additional consistency conditions are imposed on the couplings by demanding perturbative renormalizability in the sense of EFT and the separation of scales. Restrict-

tions on the couplings appear because while all loop diagrams can be made finite in any quantum field theory if we include an infinite number of counter terms in the Lagrangian, it is by no means guaranteed that these counter terms are consistent with constraints of the theory of spin-one particles and that the scale separation is not violated.

The paper is organized as follows: In section II we specify the assumptions and conditions imposed on the effective Lagrangian. In section III we give the effective Lagrangian and carry out the analysis of the constraints. The conditions of perturbative renormalizability and scale separation are obtained in section IV. We summarize and discuss the obtained results in section V.

## II. STARTING ASSUMPTIONS AND REQUIRED CONSTRAINTS

The aim of the current work is to construct the most general consistent Lorentz-invariant EFT Lagrangian of three<sup>1</sup> interacting massive vector bosons and a scalar. The free massive vector bosons are described by the Proca Lagrangian which incorporates the second class constraints such that the right number of independent dynamical degrees of freedom are left (three coordinates for each particle). To have a consistent theory of interacting massive vector bosons, the pertinent interaction terms have to be consistent with the second class constraints. This generates some non-trivial relations between the coupling constants of the interaction terms of the most general Lorentz-invariant Lagrangian of a scalar and vector bosons. The next condition we impose is the renormalizability in the sense of an EFT, i.e. that all divergences can be absorbed by renormalizing an infinite number of parameters of the effective Lagrangian. As (some of) the couplings of the effective Lagrangian are already related due to the second class constraints, the condition of perturbative renormalizability cannot be satisfied unless the coupling constants satisfy further restricting conditions. Further constraints on our EFT Lagrangian are imposed by the following considerations: in an EFT, in which the divergences generated by the leading order Lagrangian are removed by renormalizing the parameters of higher order interactions, the scale of renormalized couplings of higher order interaction terms is set by the mass of the vector bosons. The SM is considered to be the leading order approximation to an EFT. One expects that in this EFT the contributions of higher order operators in physical quantities are suppressed by powers of some large scale. The value of this large scale is determined by new physics, that is physics beyond the SM. This leads us to the next condition imposed on our EFT - the separation of scales. That is, we demand that the divergences of the loop diagrams contributing to *physical* scattering amplitudes generated by the leading order Lagrangian should be removable by renormalizing the parameters of the leading order Lagrangian. This condition is much more restrictive than just renormalizability in the sense of EFT. It is actually equivalent to demanding renormalizability of the leading order EFT Lagrangian in the traditional sense, however, not for off-shell Green functions but for the on-shell S-matrix. Notice here that perturbative renormalizability in the sense of EFT in general does not lead to tree-order unitarity. Indeed, renormalizability alone can be achieved without introducing scalars, i.e. considering an EFT of massive vector bosons (and fermions) [6]. However, as is well known, massive Yang-Mills theory is not renormalizable in the traditional sense and it violates tree-order unitarity condition.

---

<sup>1</sup> The number of massive vector bosons is taken as an input here.

To illustrate the problem with the scale separation when divergences are removed by renormalizing the couplings of higher-order operators let us consider an EFT specified by the following Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{M^2}{2} W_\mu^a W^{a\mu} + \mathcal{L}_{\text{ho}}, \quad (1)$$

where  $W_\mu^a$  is the triplet of  $SU(2)$  vector bosons,  $F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon^{abc}W_\mu^b W_\nu^c$  is the corresponding field strength tensor and  $\mathcal{L}_{\text{ho}}$  contains all possible local terms with coupling constants of inverse mass dimensions which are invariant under local  $SU(2)$  gauge transformations. The quantum field theory specified by the Lagrangian of Eq. (1) is perturbatively renormalizable in the sense of EFT, i.e. all divergences can be absorbed in the redefinition of fields and an infinite number of coupling constants [10]. It is well known that massive Yang-Mills theory is perturbatively non-renormalizable. Therefore to get rid of the divergences of loop diagrams generated by interaction terms with dimensionless coupling constants, contained in the first term in Eq. (1), one needs to renormalize the couplings of  $\mathcal{L}_{\text{ho}}$ , i.e. couplings with inverse mass dimensions. This has the consequence that even if these couplings are suppressed by some scale much larger than the mass of the vector boson -  $M$  for some fixed renormalization condition, slight changes of the renormalization scale will lead to renormalized couplings suppressed only by powers of  $M$  divided by some power of the dimensionless coupling  $g$ . To be more specific let us consider an example of the vector boson self-energy. Calculating divergent parts of two one-loop diagrams generated by interactions with dimensionless coupling  $g$  we obtain

$$\Sigma_{\text{div}}^{ab,\mu\nu}(p) = \frac{g^2 \delta_{ab}}{96\pi^2 M^4 (n-4)} [84M^4 - 14M^2 p^2 - p^4] (p^\mu p^\nu - p^2 g^{\mu\nu}), \quad (2)$$

where we have used dimensional regularization (see, e.g., Ref. [11]) with  $n$  spacetime dimensions. The divergence, corresponding to the first term in the square brackets is absorbed in the renormalization of the vector field. The second term in the square brackets is removed by renormalizing the coupling constant of the following term (contained in  $\mathcal{L}_{\text{ho}}$ )<sup>2</sup>:

$$\mathcal{L}_{HD} = \frac{g_{HD}}{4} D_\mu^{ab} F_{\nu\lambda}^b D^{ac,\mu} F^{c\nu\lambda}, \quad (3)$$

where  $D_\mu^{ab} = \delta^{ab} \partial_\mu - g\epsilon^{abc}W_\mu^c$ . Renormalization of the coupling  $g_{HD}$  by absorbing the divergence corresponding to the second term in Eq. (2) leads to the following renormalized coupling:

$$g_{HD}(\mu) = g_{HD}(\mu_0) - \frac{7g^2 \ln \frac{\mu}{\mu_0}}{96\pi^2 M^2}. \quad (4)$$

Even if the renormalized coupling  $g_{HD}(\mu_0)$ , corresponding to  $\mu = \mu_0$  is suppressed by some large scale  $\Lambda \gg M$ , for  $\mu \sim e \times \mu_0$  the renormalized coupling will become  $g_{HD}(\mu) \sim \frac{g^2}{16\pi^2} \frac{1}{M^2}$ . Analogously, for all other couplings with inverse mass dimensions the scale of the renormalized couplings is set by  $M^2$ .

---

<sup>2</sup> We will not consider the third term of the square brackets here.

### III. CONSTRAINT ANALYSIS OF AN EFT LAGRANGIAN OF MASSIVE VECTOR BOSONS AND A SCALAR

We start with the most general Lorentz-invariant effective Lagrangian of a scalar and three massive vector boson fields respecting electromagnetic charge conservation (even though we do not consider the explicit coupling to the  $U(1)$  gauge field in the following). Two charged vector particles are represented by vector fields  $V_\mu^\pm = (V_\mu^1 \mp iV_\mu^2)/\sqrt{2}$ , the third component,  $V_\mu^3$ , and the scalar field  $\Phi$  are charge-neutral. The effective Lagrangian contains an infinite number of interaction terms and hence depends on an infinite number of parameters. We assume that coupling constants with negative mass dimensions are independent from those of positive and zero mass dimensions. Below we analyse the Lagrangian containing only interaction terms with coupling constants of non-negative dimensions (as explained in detail below). Thus, the effective Lagrangian under consideration can be written as follows:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} V_{\mu\nu}^a V^{a\mu\nu} + \frac{M_a^2}{2} V_\mu^a V^{a\mu} - g_V^{abc} V_\mu^a V_\nu^b \partial^\mu V^{c\nu} \\ & - g_A^{abc} \epsilon^{\mu\nu\alpha\beta} V_\mu^a V_\nu^b \partial_\alpha V_\beta^c - h^{abcd} V_\mu^a V_\nu^b V^{c\mu} V^{d\nu} \\ & + \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{m^2}{2} \Phi^2 - a \Phi - \frac{b}{3!} \Phi^3 - \frac{\lambda}{4!} \Phi^4 \\ & - g_{vss} \partial_\mu V^{3\mu} \Phi^2 - g_{vvs}^{ab} V^{a\mu} V_\mu^b \Phi - g_{vvss}^{ab} V^{a\mu} V_\mu^b \Phi^2, \end{aligned} \quad (5)$$

where  $V_{\mu\nu}^a = \partial_\mu V_\nu^a - \partial_\nu V_\mu^a$ ,  $M_a$  are vector boson masses ( $M_1 = M_2 = M$ ),  $m$  is the mass of the scalar and the summation over all repeated indices runs from 1 to 3. The non-interacting part of the vector fields, i.e. the first two terms in Eq. (5), are given by the well-known Proca Lagrangian. This guarantees that free massive vector bosons have the right number of dynamical degrees of freedom corresponding to spin-one particles. Note that since the vector bosons are not related to some gauge symmetry, the three- and four-boson couplings have independent coupling constants. We did not include the mixed term  $\partial_\mu V^{3\mu} \Phi$  as it can be eliminated by a suitable field redefinition. Due to the electromagnetic charge conservation not all coupling constants of the above Lagrangian are independent from each other. The interaction terms of the scalar field with two vector fields can be written in terms of four real parameters

$$g_{1,s} = g_{vvs}^{11} = g_{vvs}^{22}, \quad g_{2,s} = g_{vvs}^{33}, \quad g_{1,ss} = g_{vvss}^{11} = g_{vvss}^{22}, \quad g_{2,ss} = g_{vvss}^{33}, \quad (6)$$

and all other  $g_{vvs}^{ab}$  and  $g_{vvss}^{ab}$  couplings do not contribute in the effective Lagrangian. The coefficient of the linear term  $a$  vanishes at tree order and further corrections can be fixed by demanding that the vacuum expectation value (vev) of the scalar field vanishes.

The three-boson interaction term of the Lagrangian depends on ten real parameters,

$$\begin{aligned} g_V^{333} &= g_1, & g_V^{113} &= g_2, & g_V^{123} &= -g_3, & g_V^{213} &= g_3, \\ g_V^{223} &= g_2, & g_V^{311} &= g_4, & g_V^{321} &= -g_5, & g_V^{312} &= g_5, \\ g_V^{322} &= g_4, & g_V^{131} &= g_6, & g_V^{231} &= -g_7, & g_V^{132} &= g_7, & g_V^{232} &= g_6, \\ g_A^{213} &= -g_A^{123} = g_{A1}, & g_A^{311} &= g_A^{322} = -g_A^{131} = -g_A^{232} = g_{A2}, \\ g_A^{312} &= -g_A^{321} = -g_A^{132} = g_A^{231} = g_{A3}. \end{aligned} \quad (7)$$

Electromagnetic charge conservation relates the coupling constants of the four-boson interaction  $h^{abcd}$  to each other as follows

$$h^{1111} = h^{2222} = \frac{d_1 + d_2}{4}, \quad h^{1112} = -h^{1121} - h^{1211} - h^{2111},$$

$$\begin{aligned}
h^{1122} &= d_2 - h^{2112} - h^{1221} - h^{2211}, & h^{1212} &= \frac{1}{2} (d_1 - d_2 - 2 h^{2121}), \\
h^{1323} &= -h^{2313} - h^{3132} - h^{3231}, & h^{2122} &= -h^{1222} - h^{2212} - h^{2221}, \\
h^{2323} &= \frac{1}{2} (d_4 - 2 h^{3232}), & h^{3113} &= \frac{1}{2} [d_3 - 2 (h^{1133} + h^{1331} + h^{3311})], \\
h^{3223} &= \frac{1}{2} [d_3 - 2 (h^{2233} + h^{2332} + h^{3322})], \\
h^{3123} &= -h^{1233} - h^{1332} - h^{2133} - h^{2331} - h^{3213} - h^{3312} - h^{3321}, \\
h^{3131} &= \frac{1}{2} (d_4 - 2 h^{1313}), & h^{3333} &= d_5,
\end{aligned} \tag{8}$$

and the effective Lagrangian of Eq. (5) depends only on  $d_1, \dots, d_5$ .

Details of the canonical formalism followed below can be found in Ref. [12]. Our analysis is closely related to that of Ref. [6], which considered an EFT without the scalar field, and it is similar to the one of Ref. [7], with the difference that in Ref. [7], parity conservation has been taken as an input.

The canonical momenta corresponding to  $\Phi$ ,  $V_0^a$  and  $V_i^a$  are defined as

$$p = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = \dot{\Phi}, \tag{9}$$

$$\pi_0^a = \frac{\partial \mathcal{L}}{\partial \dot{V}_0^a} = -g_V^{bca} V_0^b V_0^c - g_{vss} \delta_{a3} \Phi^2, \tag{10}$$

$$\pi_i^a = \frac{\partial \mathcal{L}}{\partial \dot{V}_i^a} = V_{0i}^a + g_V^{bca} V_0^b V_i^c + g_A^{bca} \epsilon^{ijk0} V_j^b V_k^c. \tag{11}$$

Eq. (10) leads to the primary constraints

$$\phi_1^a = \pi_0^a + g_V^{bca} V_0^b V_0^c + g_{vss} \delta_{a3} \Phi^2. \tag{12}$$

On the other hand, from Eqs. (9) and (11) we solve

$$\begin{aligned}
\dot{V}_i^a &= \pi_i^a + \partial_i V_0^a - g_V^{bca} V_0^b V_i^c - g_A^{bca} \epsilon^{ijk0} V_j^b V_k^c, \\
\dot{\Phi} &= p.
\end{aligned} \tag{13}$$

For the total Hamiltonian [12] we have:

$$H_1 = \int d^3 \mathbf{x} (\phi_1^a z^a + \mathcal{H}) \tag{14}$$

with

$$\begin{aligned}
\mathcal{H} &= \frac{\pi_i^a \pi_i^a}{2} + \pi_i^a \partial_i V_0^a + \frac{1}{4} V_{ij}^a V_{ij}^a - \frac{M_a^2}{2} V_\mu^a V^{a\mu} - g_V^{abc} V_0^a V_i^b \pi_i^c - g_A^{abc} \epsilon^{ijk0} V_j^a V_k^b \pi_i^c \\
&- g_V^{abc} V_0^a V_i^b \partial_i V_0^c - g_V^{abc} V_i^a V_0^b \partial_i V_0^c + g_V^{abc} V_i^a V_j^b \partial_i V_j^c + \frac{1}{2} g_V^{abc} g_V^{a'b'c} V_0^a V_i^b V_0^{a'} V_i^{b'} \\
&+ \frac{1}{2} g_A^{bca} g_A^{b'c'a} \epsilon^{ijk0} \epsilon^{ij'k'0} V_j^b V_{j'}^{b'} V_k^c V_{k'}^{c'} + g_A^{abc} g_V^{b'c'c} \epsilon^{ijk0} V_j^a V_k^b V_0^{b'} V_i^{c'} \\
&+ g_A^{abc} \epsilon^{ijk0} V_0^a V_j^b \partial_i V_k^c - g_A^{abc} \epsilon^{ijk0} V_j^a V_0^b \partial_i V_k^c + h^{abcd} V_\mu^a V_\nu^b V^{\mu\nu} \\
&+ \frac{p^2}{2} + \frac{\partial_i \Phi^2}{2} + \frac{m^2}{2} \Phi^2 + a \Phi + \frac{b}{3!} \Phi^3 + \frac{\lambda}{4!} \Phi^4 \\
&+ g_{vss} \partial_i V^{3i} \Phi^2 + g_{vvs}^{ab} V^{a\mu} V_\mu^b \Phi + g_{vvs}^{ab} V^{a\mu} V_\mu^b \Phi^2,
\end{aligned} \tag{15}$$

and the  $z^a$  are arbitrary functions which must be determined.

The primary constraints  $\phi_1^a$  have to be conserved in time, i.e. their Poisson brackets with the Hamiltonian must vanish for each  $a = 1, 2, 3$ . Calculating the Poisson brackets we obtain

$$\begin{aligned} \{\phi_1^a, H_1\} = & (g_V^{bca} + g_V^{cba} - g_V^{acb} - g_V^{cab}) V_0^c z^b + \partial_i \pi_i^a + g_V^{abc} V_i^b \pi_i^c + (g_V^{abc} + g_V^{bac}) V_i^b \partial_i V_0^c \\ & - g_V^{bca} \partial_i (V_0^b V_i^c) - g_V^{cba} \partial_i (V_0^b V_i^c) + M_a^2 V_0^a - g_V^{abc} g_V^{a'b'c} V_i^b V_0^{a'} V_i^{b'} \\ & - g_A^{a'bc} g_V^{ac'c} \epsilon^{ijk0} V_j^{a'} V_k^b V_i^{c'} - g_A^{abc} \epsilon^{ijk0} V_j^b \partial_i V_k^c + g_A^{bac} \epsilon^{ijk0} V_j^b \partial_i V_k^c \\ & - (h^{abcd} + h^{badc} + h^{cbad} + h^{dcba}) V_\mu^b V_0^c V^{d\mu} \\ & + 2 g_{vss} \delta_{a3} p \Phi - 2 g_{vvs}^{ab} V_0^b \Phi - 2 g_{vvs}^{ab} V_0^b \Phi^2 \equiv A^{ab} z^b + \chi^a. \end{aligned} \quad (16)$$

The  $3 \times 3$  matrix  $A$  is given by

$$A = \begin{pmatrix} 0 & -2\gamma_1 V_0^3 & \gamma_2 V_0^1 - \gamma_1 V_0^2 \\ 2\gamma_1 V_0^3 & 0 & \gamma_1 V_0^1 + \gamma_2 V_0^2 \\ -(\gamma_2 V_0^1 - \gamma_1 V_0^2) & -(\gamma_1 V_0^1 + \gamma_2 V_0^2) & 0 \end{pmatrix}, \quad (17)$$

where  $\gamma_1 = g_5 + g_7$  and  $\gamma_2 = g_4 + g_6 - 2g_2$ . The determinant of  $A$  vanishes and therefore the system of equations

$$A^{ab} z^b = -\chi^a \quad (18)$$

can be satisfied only if the right-hand side satisfies the secondary constraint

$$\phi_2 = \chi^1 (\gamma_1 V_0^1 + \gamma_2 V_0^2) + \chi^2 (\gamma_1 V_0^2 - \gamma_2 V_0^1) - \chi^3 2\gamma_1 V_0^3 = 0. \quad (19)$$

If at least one of  $\gamma_1$  or  $\gamma_2$  is non-zero then for non-vanishing  $V_0^1$  and/or  $V_0^2$  we obtain from Eq. (18) that

$$z^1 = \frac{\chi_3 + \gamma_1 z^2 V_0^1 + \gamma_2 z^2 V_0^2}{\gamma_1 V_0^2 - \gamma_2 V_0^1}, \quad z^3 = \frac{\chi_1 + 2\gamma_1 z^2 V_0^3}{\gamma_2 V_0^1 - \gamma_1 V_0^2} \quad (20)$$

and  $z^2$  can be solved from time conservation of the constraint  $\phi_2$ ,  $\{\phi_2, H_1\} = 0$ . However, in this case we obtain four constraints of the second class instead of six for our system of three massive vector fields, i.e. we would have two extra degrees of freedom. Therefore, for a self-consistent theory we must require

$$\gamma_1 = \gamma_2 = 0 \Rightarrow g_7 = -g_5, \quad 2g_2 = g_4 + g_6. \quad (21)$$

Thus we are left with secondary constraints:

$$\begin{aligned} \{\phi_1^a, H_1\} = & \partial_i \pi_i^a + g_V^{abc} V_i^b \pi_i^c + (g_V^{abc} + g_V^{bac}) V_i^b \partial_i V_0^c - g_V^{bca} \partial_i (V_0^b V_i^c) - g_V^{cba} \partial_i (V_0^b V_i^c) + M_a^2 V_0^a \\ & - g_V^{abc} g_V^{a'b'c} V_i^b V_0^{a'} V_i^{b'} - g_A^{a'bc} g_V^{ac'c} \epsilon^{ijk0} V_j^{a'} V_k^b V_i^{c'} - g_A^{abc} \epsilon^{ijk0} V_j^b \partial_i V_k^c + g_A^{bac} \epsilon^{ijk0} V_j^b \partial_i V_k^c \\ & - (h^{abcd} + h^{badc} + h^{cbad} + h^{dcba}) V_\mu^b V_0^c V^{d\mu} \\ & + 2 g_{vss} \delta_{a3} p \Phi - 2 g_{vvs}^{ab} V_0^b \Phi - 2 g_{vvs}^{ab} V_0^b \Phi^2 \equiv \phi_2^a, \quad a = 1, 2, 3. \end{aligned} \quad (22)$$

If no more constraints appear then our Lagrangian describes a system with the right number of constraints for three massive vector bosons interacting with a scalar particle. If this is the case, then the  $z^a$  have to be solvable from the condition of the constraints  $\phi_2^a$  being conserved in time.

From the condition of conservation of  $\phi_2^a$  in time we obtain

$$\{\phi_2^a, H_1\} = \mathcal{M}^{ab} z^b + Y^a = 0, \quad a = 1, 2, 3, \quad (23)$$

where

$$\begin{aligned} \mathcal{M}^{ab} = & M_a^2 \delta^{ab} - (g_V^{bca} + g_V^{cba}) \partial_i V_i^c - [g_V^{ace} g_V^{bde} - (h^{acbd} + h^{cadb} + h^{bcad} + h^{dbca})] V_i^c V_i^d \\ & - (h^{abcd} + h^{badc} + h^{cbad} + h^{dcba} + h^{adcb} + h^{dabc} + h^{cdab} + h^{bcda} + h^{acbd} + h^{cadb} \\ & + h^{bcad} + h^{dbca}) V_0^c V_0^d - 4 g_{vss}^2 \delta_{a3} \delta_{b3} \Phi^2 - 2 g_{vvs}^{ab} \Phi - 2 g_{vvs}^{ab} \Phi^2, \end{aligned} \quad (24)$$

and the particular form of  $Y^a$  is not important for our purposes. To obtain a self-consistent field theory we demand that  $\det \mathcal{M}$  does not vanish. For small fluctuations about the vacuum this is indeed the case and we proceed by quantizing these small fluctuations and deriving further constraints on the couplings by investigating the conditions of perturbative renormalizability and scale separation.

Thus, we obtained that consistency with the constraints implies relationships between the couplings of Eq. (21). As shown in the next section, these relations applied in combination with the conditions of perturbative renormalizability and scale separation lead to very non-trivial restrictions on the interaction terms of the leading order EFT Lagrangian.

#### IV. PERTURBATIVE RENORMALIZABILITY

Below we analyze one-loop order diagrams using dimensional regularization. To that end we use the standard procedure of quantizing systems with second class constraints [12]. After lengthy calculations we obtain the following generating functional

$$Z[J^{a\mu}, I] = \int \mathcal{D}V \mathcal{D}\Phi \mathcal{D}_c \mathcal{D}\bar{c} \mathcal{D}\lambda e^{i \int d^4x (\mathcal{L} + \mathcal{L}_{\text{aux}}(\bar{c}, c, \lambda, \Phi, V) + J^{a\mu} V_\mu^a + I\Phi)}, \quad (25)$$

where  $\lambda^a$ ,  $c^a$  and  $\bar{c}^a$  are auxiliary fields. In analogous expressions in standard gauge theories the integration over the  $\lambda^a$  variables is usually carried out explicitly and one is left with “ghost” fields  $c^a$ ,  $\bar{c}^a$  and the original fields.<sup>3</sup> The particular form of  $\mathcal{L}_{\text{aux}}(\bar{c}, c, \lambda, \Phi, V)$  is unimportant as it generates vanishing contributions to Feynman diagrams if dimensional regularization is applied. This is because due to the absence of field-independent terms with two derivatives in Eq. (24) the  $\lambda^a$ ,  $c^a$  and  $\bar{c}^a$  fields do not have kinetic parts. In the calculations of the loop diagrams below we used the programs FeynCalc [13, 14] and Form [15] independently. The divergent parts of the one-loop integrals have been checked with the expressions obtained in Ref. [16].

We impose the on-mass-shell renormalization condition, i.e. require that all divergences in physical quantities should be removable by redefining the parameters of the effective Lagrangian.

We start by calculating the one-loop contribution to the scattering amplitude  $V^3 V^3 \rightarrow V^3 V^3$ , shown in Fig. 1. The coefficient of the divergence is a polynomial of the Mandelstam

<sup>3</sup> Let us remind the reader that a standard gauge theory also becomes a system with second class constraints after gauge fixing. Notice further that BRST quantization used in gauge theories is not applicable in case of the EFT considered here, as local gauge invariance is not taken as an input.



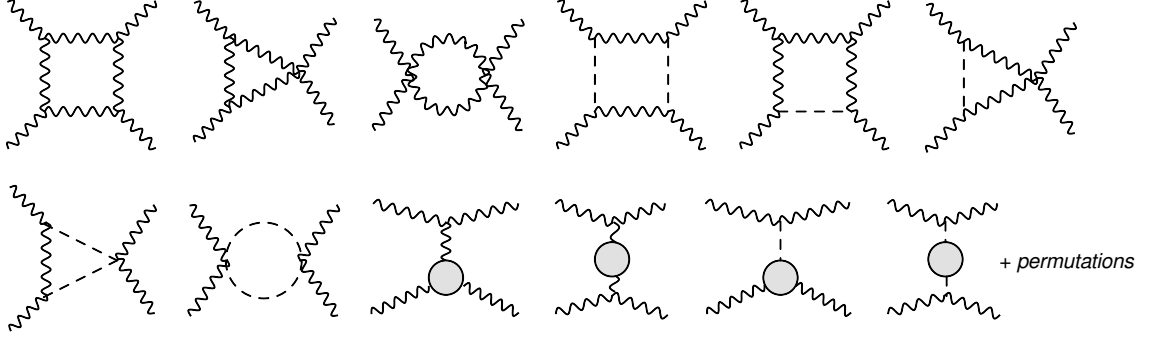


FIG. 1: One-loop contributions to the four-vector vertex function. The dashed and the wiggly lines correspond to the scalar and the vector-boson, respectively. Blobs indicate the corresponding one-loop two- and three-point vertex functions. In last four diagrams only the one-particle-irreducible parts are taken into account.

variables  $(s, t, u)$  divided by powers of the vector boson masses. If these divergences are removed by renormalizing the coupling constants of the higher order operators, i.e. four-vector interaction terms with derivatives, then the scale of these couplings will be set by the masses of vector bosons. This would mean that the contributions of higher order operators in physical quantities would not be suppressed by powers of a large scale (but rather by the vector boson mass divided by some power of a dimensionless coupling constant). Therefore to have a self-consistent perturbative EFT with clear scale-separation divergences generated by interactions with dimensionless (or positive mass dimension-) couplings should either cancel each other or be removable by renormalizing this set of leading order couplings. Notice here that non-pole parts of one-particle reducible diagrams in Fig. 1 have to be taken into account together with one-particle irreducible diagrams. The dressed vertices and self-energies of the scalar and vector particles contributing in one-particle reducible diagrams are given by respective diagrams in Figs. 2, 3 and 4.<sup>4</sup> We do not give very lengthy expressions of the divergent parts of the loop diagrams but rather write their sum in the form of a polynomial in terms of the Mandelstam variables:

$$V^{\mu\nu\lambda\sigma} = \sum_{i,j=0}^4 u^i s^j C_{ij}^{\mu\nu\lambda\sigma}, \quad (26)$$

where the coefficients  $C_{ij}^{\mu\nu\lambda\sigma}$  depend on the momenta, masses and coupling constants. Consider first the term proportional to  $u^4$  in the coefficient of  $g^{\mu\nu}g^{\lambda\sigma}$  in Eq. (26),

$$\frac{i\pi^2}{120} \left( \frac{g_1^4}{M_3^8} + \frac{2g_2^4}{M^8} \right) u^4 g^{\lambda\sigma} g^{\mu\nu}, \quad (27)$$

where  $\mu, \nu, \lambda$  and  $\sigma$  are Lorentz indices corresponding to the external vector lines. Demanding that the factor in the round brackets vanishes leads to

$$g_1^4 M_3^8 + 2g_2^4 M^8 = 0, \Rightarrow g_1 = 0, \quad g_2 = 0. \quad (28)$$

<sup>4</sup> In all figures only those diagrams are shown which contribute to the divergent parts of the corresponding scattering amplitudes.

Notice further that if the conditions of Eq. (28) are not satisfied, then the divergent piece of Eq. (27) has to be cancelled by some higher-order term which generates a tree-order contribution in the vertex function:

$$ic u^4 g^{\lambda\sigma} g^{\mu\nu}, \quad (29)$$

where  $c$  is a coupling constant of mass dimension minus eight. As discussed in section II, the scale of the renormalized coupling  $c_R$  will be set by  $M$  and/or  $M_3$ . This will clearly upset the scale separation according to which higher order terms have to be suppressed by some large scale, large in comparison to the  $M$  and  $M_3$  scales.

The next condition is obtained by demanding that the term proportional to  $u^2$  also vanishes. This leads to the following condition on certain couplings:

$$\begin{aligned} & \left[ \frac{1}{4} \left( -20 (g_{A1} + g_{A3})^2 - 20g_{A2}^2 - 5g_3^2 - 10g_5g_3 - g_4^2 - g_5^2 \right) + d_3 \right]^2 \\ & + \frac{1}{16} \left[ 40 \left( 44g_{A2}^2 + 96g_3^2 + 11g_4^2 + 11(g_3 + g_5)^2 \right) (g_{A1} + g_{A3})^2 + \frac{32M^4}{M_3^4} d_5^2 \right. \\ & \left. + 880 (g_{A1} + g_{A3})^4 + 55 \left( 4g_{A2}^2 + g_4^2 + (g_3 + g_5)^2 \right)^2 \right] = 0. \end{aligned} \quad (30)$$

Eq. (30) leads to

$$d_5 = 0, \quad g_4 = 0, \quad g_5 = -g_3, \quad g_{A2} = 0, \quad g_{A3} = -g_{A1}, \quad d_3 = -g_3^2. \quad (31)$$

Further, demanding the vanishing of the term proportional to  $s^2$ , we obtain

$$d_4 = g_3^2. \quad (32)$$

Taking into account Eqs. (28), (31) and (32) the full expression of the divergent part of the amplitude  $V^3V^3 \rightarrow V^3V^3$  becomes proportional to

$$\left[ 8M^8 \left( 2M_3^2 g_{2,ss} + g_{2,s}^2 \right)^2 + M_3^4 \left( g_3^2 M_3^4 - 4M^2 g_{1,s} g_{2,s} \right)^2 \right] \left( g^{\lambda\sigma} g^{\mu\nu} + g^{\lambda\nu} g^{\mu\sigma} + g^{\lambda\mu} g^{\nu\sigma} \right). \quad (33)$$

For  $d_5 = g_1 = 0$ , the one-particle-irreducible tree-order contribution to  $V^3V^3 \rightarrow V^3V^3$  amplitude vanishes and therefore we have to demand that the expression in Eq. (33) also vanishes. Doing so we obtain:

$$g_{2,ss} = -\frac{g_3^4 M_3^6}{32M^4 g_{1,s}^2}, \quad g_{2,s} = \frac{g_3^2 M_3^4}{4M^2 g_{1,s}}. \quad (34)$$

Next, as there is no tree order one-particle irreducible contribution in the amplitude  $V^1V^1 \rightarrow V^1V^1$ , we have to demand that the divergent part of the corresponding one-loop contribution vanishes (diagrams shown in Fig.1). By demanding that the terms proportional to  $s^2$  and  $st$  vanish, we obtain the following conditions:

$$\begin{aligned} & (d_1 + d_2) \left( d_2 + \frac{g_3^2}{2} \right) = 0, \\ & \left( d_2 + \frac{g_3^2}{2} \right)^2 + (d_1 + d_2)^2 + \frac{1}{4} g_3^4 \left( 1 - \frac{M^4}{M_3^4} \right) = 0. \end{aligned} \quad (35)$$

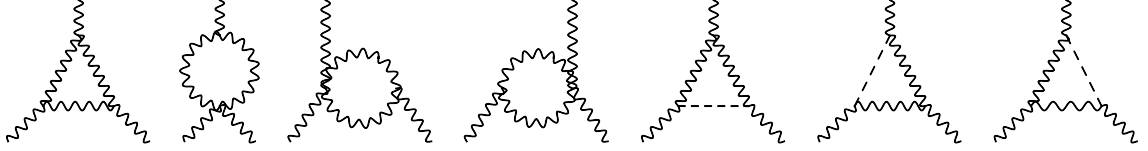


FIG. 2: One-loop contributions to the three-vector vertex function. The dashed and wiggly lines correspond to the scalar and the vector-boson, respectively.

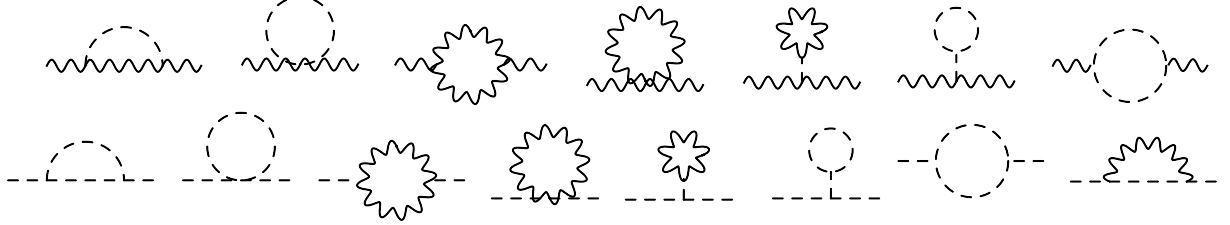


FIG. 3: One-loop contributions to the vector-boson and the scalar self-energies. The first and second lines represent the vector boson and scalar self-energies. The dashed and the wiggly lines correspond to the scalar and the vector-boson, respectively.

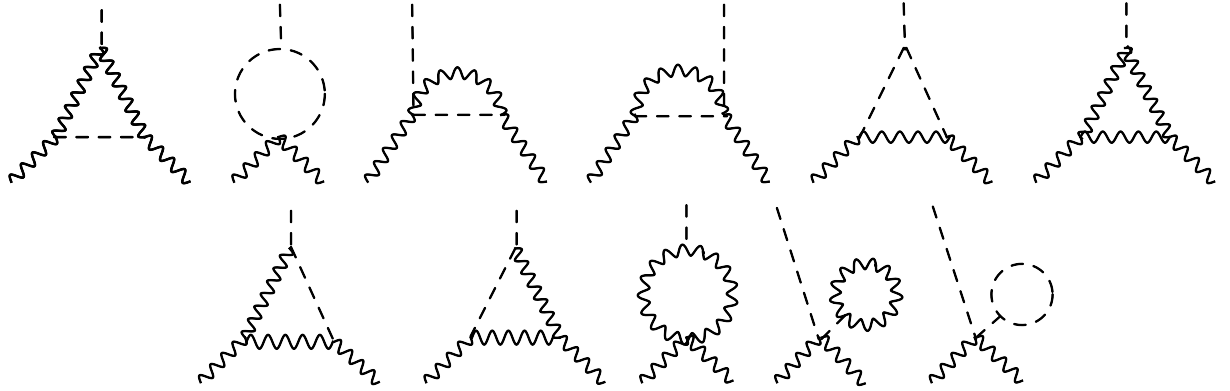


FIG. 4: One-loop contributions to the scalar-vector-vector vertex function. The dashed and the wiggly lines correspond to the scalar and the vector-boson, respectively.

Considering the amplitude of the scalar boson decaying into two vectors and requiring that the divergences of corresponding diagrams, shown in Fig. 4, do not contribute in the renormalization of the couplings of the higher-order operators, i.e. that they do not violate the scale separation, we find that the following condition has to be satisfied:

$$g_{1,s} \left( (d_1 + d_2) + d_2 + \frac{g_3^2}{2} \right) (m^2 - 10M^2) = 0. \quad (36)$$

The coupling  $g_{1,s}$  cannot be vanishing due to the condition of Eq. (34) and therefore from Eqs. (35) and (36) we obtain

$$d_1 = -d_2 = \frac{g_3^2}{2}, \quad M_3 = M. \quad (37)$$

Using all conditions imposed on couplings so far and analyzing the vertex function  $V_1 V_2 V_3$  and demanding that the divergent part of the sum of loop diagrams, shown in Fig. 2, has the same Lorentz structure as the tree one, we obtain

$$g_{1,s} = g_{2,s}. \quad (38)$$

Eqs. (34) and (38) lead to

$$g_{1,s} = g_{2,s} = \pm \frac{g_3 M}{2}. \quad (39)$$

Going back to the  $V_1 V_1 \rightarrow V_1 V_1$  amplitude and taking into account Eqs. (37) and (39), the condition of the vanishing of its divergent part reduces to

$$(8g_{1,ss} + g_3^2)^2 = 0, \quad (40)$$

from which we obtain

$$g_{1,ss} = -\frac{g_3^2}{8}. \quad (41)$$

Next, we have calculated the divergent parts of one-loop diagrams contributing to the  $\Phi V_3 \rightarrow \Phi V_3$  scattering amplitude. As the coupling constant of the  $V_\mu^3 V^{3\mu} \Phi^2$  interaction term is given by  $g_{2,ss} = -g_3^2/8$ , i.e. in terms of the coupling of the three-vector and four-vector interaction terms, the divergent pieces of the corresponding amplitudes have to be correlated. In a self-consistent theory the renormalized value for the coupling  $g_3$  should be independent from the process that was used to fix it. After a lengthy one-loop calculation we found that this consistency condition requires that the coupling  $g_{vss}$  has to vanish.

We checked in explicit calculations that all one-loop divergences appearing in processes with three and four particles are absorbed in a redefinition of the coupling constants and the masses and no further conditions on the couplings are obtained.

To summarize, all obtained relations among couplings and masses can be written as

$$\begin{aligned} M_1 &= M_2 = M_3 = M, & g_V^{abc} &= -g_3 \epsilon^{abc}, & g_A^{abc} &= g_{A1} \epsilon^{abc}, \\ h^{abcd} &= \frac{1}{4} g_V^{abe} g_V^{cde}, & g_{vss} &= 0, & g_{1,s} &= g_{2,s} = \frac{g_3 M}{2}, & g_{1,ss} &= g_{2,ss} = -\frac{g_3^2}{8}. \end{aligned} \quad (42)$$

The sign of the couplings  $g_{1,s}$  and  $g_{2,s}$  can be changed to the opposite by redefining the scalar field. We have chosen the positive sign displayed above.

For the couplings in Eq. (42) the effective Lagrangian can be written in a compact form, denoting  $g_3 = g$ ,

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \frac{1}{2} V_\mu^a V^{a\mu} \left( M - \frac{g}{2} \Phi \right)^2 - g_{A1} \epsilon^{abc} \epsilon^{\mu\nu\alpha\beta} V_\mu^a V_\nu^b \partial_\alpha V_\beta^c, \\ &+ \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{m^2}{2} \Phi^2 - a \Phi - \frac{b}{3!} \Phi^3 - \frac{\lambda}{4!} \Phi^4, \end{aligned} \quad (43)$$

where

$$G_{\mu\nu}^a = V_{\mu\nu}^a - g \epsilon^{abc} V_\mu^b V_\nu^c. \quad (44)$$

This Lagrangian coincides with the SU(2) locally gauge invariant Lagrangian of scalars and vector bosons with spontaneous symmetry breaking in the unitary gauge except for the self-interaction terms of the scalars. The coefficient of the term linear in  $\Phi$  is fixed by the

condition that the vacuum expectation value (i.e. one-point function) of  $\Phi$ -field vanishes. Note in particular the gauge-type form of the vector boson field strength. We checked by explicit calculations that no further constraints on couplings are generated by the condition of perturbative renormalizability of the three- and four-point functions of scalar and vector bosons. This leaves the two scalar self-interaction couplings unfixed. Notice here that to fix the couplings of the scalar self-interaction terms tree-order unitarity conditions of five-point functions have been analysed in Refs. [3, 4]. “Squaring” the tree order diagrams to obtain the loop contributions to the scattering amplitudes we expect that the investigation of the one-loop diagrams contributing in five and six-point functions and/or two-loop order analysis of four-point functions will fix the couplings of three and four scalar self-interactions such that the Lagrangian with spontaneous symmetry breaking taken in unitary gauge results as an unique self-consistent EFT of a massive scalar and massive vector bosons. Notice further that the most general leading order Lagrangian of Eq. (43) does not include the well known “chiral electroweak Lagrangian” (see e.g. Refs. [17, 18] and references therein). This is because to remove the one-loop divergences generated by the leading order chiral effective Lagrangian one needs to renormalize couplings of the next-to-leading order Lagrangian, i.e. the condition of separation of scales is not satisfied by this EFT.

## V. SUMMARY AND DISCUSSIONS

In the current work we revisited the problem of the uniqueness of a theory with spontaneously broken gauge symmetry as a consistent framework for describing the electroweak interactions. Following the modern point of view of the Standard Model being the leading order approximation of an effective field theory we analyzed the most general Lorentz-invariant leading order effective Lagrangian of massive vector bosons interacting with a massive scalar field. Here, under leading order we mean interaction terms with couplings of non-negative mass dimensions.

Massive spin-one particles are described by theories with constraints. The interaction terms of the effective Lagrangian have to be consistent with the constraints so that the theory describes the dynamics of the right number of degrees of freedom. Using the standard canonical formalism, we analyzed the constraint structure of our effective Lagrangian and obtained consistency conditions which must be satisfied by the various coupling constants. Further conditions are obtained by requiring perturbative renormalizability. In particular, using dimensional regularization we calculated the divergent parts of one-loop Feynman diagrams contributing to various physical quantities and analyzed the conditions of renormalizability.

By applying dimensional regularization we can keep track of only logarithmic divergences. However, this is sufficient as we are looking for necessary conditions of perturbative renormalizability. We imposed the condition that all logarithmic divergences generated by the interaction terms of the leading order effective Lagrangian should be removable from physical quantities in such a way that the perturbative contributions of higher-order operators remain suppressed by large scales. In combination with the relations between the coupling constants obtained from the constraint analysis these conditions impose severe restrictions on the coupling constants such that we end up with the Lagrangian of spontaneously broken gauge symmetry in unitary gauge except that the coupling constants of the self-interactions of the scalar field remain unfixed. These are not pinned down by the analysis of the UV divergences of all three- and four-point functions at one-loop order. We expect that the con-

dition of perturbative renormalizability for three- and four-point functions at two-loop order or/and one-loop order amplitudes with more external legs will fix these two free couplings such that the Lagrangian with spontaneously broken  $SU(2)$  gauge symmetry taken in unitary gauge appears as an unique leading-order Lagrangian of a self-consistent EFT of a massive scalar interacting with massive vector bosons. As it is well known, the S-matrix generated by such a Lagrangian is ultraviolet finite being identical to the one of the renormalizable gauge [19]. Extending our analysis to pin down the couplings of self-interactions of the scalar fields together with the inclusion of the electromagnetic interaction (analogously to Ref. [8]) and fermions is relegated to forthcoming publications.

## Acknowledgments

We thank Jürg Gasser for useful remarks on the manuscript. This work was supported in part by the DFG and NSFC through funds provided to the Sino-German CRC 110 “Symmetries and the Emergence of Structure in QCD” (NSFC Grant No. 11621131001, DFG Grant No. TRR110), by the VolkswagenStiftung (Grant No. 93562), by the CAS President’s International Fellowship Initiative (PIFI) (Grant No. 2018DM0034) and by the Georgian Shota Rustaveli National Science Foundation (Grant No. FR17-354).

- 
- [1] S. Weinberg, *The Quantum Theory Of Fields. Vol. 1: Foundations* (Cambridge University Press, Cambridge, England, 1995).
  - [2] C. H. Llewellyn Smith, Phys. Lett. B **46**, 233 (1973).
  - [3] J. M. Cornwall, D. N. Levin and G. Tiktopoulos, Phys. Rev. Lett. **30**, 1268 (1973) [Erratum-ibid. **31**, 572 (1973)].
  - [4] J. M. Cornwall, D. N. Levin and G. Tiktopoulos, Phys. Rev. D **10**, 1145 (1974) [Erratum-ibid. D **11**, 972 (1975)].
  - [5] S. D. Joglekar, Annals Phys. **83**, 427 (1974).
  - [6] J. Gegelia, arXiv:1207.0156 [hep-ph].
  - [7] D. Djukanovic, J. Gegelia and S. Scherer, Int. J. Mod. Phys. A **25**, 3603 (2010).
  - [8] D. Djukanovic, M. R. Schindler, J. Gegelia, S. Scherer, Phys. Rev. Lett. **95**, 012001 (2005).
  - [9] D. Djukanovic, M. R. Schindler, J. Gegelia, G. Japaridze and S. Scherer, Phys. Rev. Lett. **93**, 122002 (2004)
  - [10] J. Gegelia and G. Japaridze, Mod. Phys. Lett. A **27**, 1250128 (2012).
  - [11] J. C. Collins, *Renormalization* (Cambridge University Press, Cambridge, UK, 1984).
  - [12] D. M. Gitman and I. V. Tyutin, *Canonical Quantization of Fields with Constraints* (Nauka, Moscow, 1986; extended English version: Springer, Berlin, 1990).
  - [13] R. Mertig, M. Bohm and A. Denner, Comput. Phys. Commun. **64**, 345 (1991).
  - [14] V. Shtabovenko, R. Mertig and F. Orellana, Comput. Phys. Commun. **207** (2016) 432
  - [15] J. Kuipers, T. Ueda, J. A. M. Vermaseren and J. Vollinga, Comput. Phys. Commun. **184**, 1453 (2013)
  - [16] A. Denner and S. Dittmaier, Nucl. Phys. B **734**, 62 (2006)
  - [17] G. Buchalla, O. Cat and C. Krause, Nucl. Phys. B **880**, 552 (2014) Erratum: [Nucl. Phys. B **913**, 475 (2016)]
  - [18] G. Buchalla, O. Cata, A. Celis, M. Knecht and C. Krause, Nucl. Phys. B **928**, 93 (2018).

[19] G. 't Hooft, Nucl. Phys. B **33**, 173 (1971).