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Once more on the Higgs decay into two photons

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Abstract

We comment on the recently reiterated claim that the contribution of the W-boson loop to the Higgs boson decay into two photons leads to different expressions in the R_ξ gauge and the unitary gauge. By applying a gauge-symmetry preserving regularization with higher-order covariant derivatives we reproduce once again the "classical" gauge-independent result.

The original calculations of the W-boson loop contribution to the Higgs boson decay into two photons [1–3] have been challenged in Refs. [4–6]. While the old result has been confirmed in a variety of calculations (see e.g. Refs. [7–9]), it has been shown in Refs. [24,25] how to get the correct result working in the unitarity gauge. In Ref. [10] it has been argued that the dispersion theory calculation confirms the discrepancy. However, the careful and detailed studies of Ref. [11] revealed that unregulated and unsubtracted results in the unitary gauge are incorrect in spite of being finite. Results consistent with Ref. [11] have been also obtained for $H \to Z + \gamma$ in Ref. [12].

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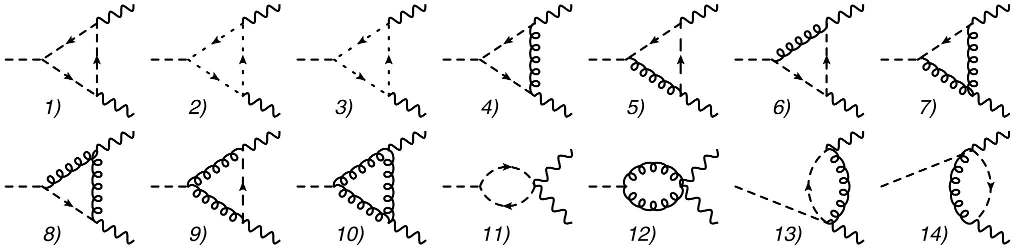

Fig. 1. One-loop diagrams of the W-boson loop contribution in the Higgs boson decay into two photons. Crossed diagrams are not shown. Curved, wiggled, dashed, dashed with arrows and dotted lines correspond to the photon, W-boson, Higgs scalar, Goldstone bosons and the Faddeev–Popov ghosts, respectively. Notice that diagrams 2) and 3) correspond to two different ghost lines (see Ref. [14] for details).

The issue of the gauge (in)dependence of the Higgs decay amplitude has been raised again in a recent publication [13] where it has been claimed that the results of the R_ξ gauge and the unitary gauge are explicitly verified to be different.

Using the Feynman rules (and notations) of Ref. [14] we obtained that in R_ξ gauge all ultraviolet divergences of one-loop diagrams appearing in the W-boson loop contribution to the Higgs boson decay into two photons (diagrams are shown in Fig. 1) cancel at the level of integrands except

$$\frac{e^3 M_Z\big(2(D-1)M_W^2+m_\phi^2\big)}{M_W\sqrt{M_Z^2-M_W^2}}\int\frac{d^Dq}{(2\pi)^D}\,\frac{4q^\mu q^\nu-q^2g^{\mu\nu}}{\left[q^2-M_W^2\right]^3}, \tag{1}$$

with M_Z, M_W and m_ϕ the masses of the Z-boson, the W-boson and the Higgs particle, respectively, and e is the conventional electromagnetic coupling constant. This integral is also finite, however, only after the loop integration has been carried out. Note that while we explicitly worked in D dimensions, one can also do the algebra in four space-time dimensions which amounts to setting $D=4$. This is exactly the same integral which has been identified as the source of the discrepancy between the unitary and R_ξ gauges [4–6]. The problem with the finite loop integral in Eq. (1) is that it is a difference of two logarithmically divergent integrals and cannot be calculated without regularization (see, e.g., Refs. [15,16]). While the divergent parts of these two integrals have the same coefficient for any Lorentz-invariant regularization, different regularizations generate different finite pieces and therefore the final result depends on the applied regularization scheme. Thus the problem actually is not with the unitary and R_ξ gauges leading to different results, but rather the result being dependent on the way we calculate the divergent integrals. If we deal with the expression of Eq. (1) the same way as done in Refs. [4–6] we get a result different from that of the dimensional regularization also in R_ξ gauge.

To verify once again that the dimensional regularization leads to a correct result and that the problem is caused by the incorrect treatment of the integral of Eq. (1), we applied a gauge symmetry preserving regularization with higher-order covariant derivatives [17,18] to the electroweak theory by adding the following regularizing terms to the Lagrangian (we use the notations and the parametrization of Ref. [14])

$$\begin{aligned}\mathcal{L}_{HD} &= g_{HD}\, D_\mu^{ab} F_{\nu\lambda}^b D^{ac,\mu} F^{c\nu\lambda}\,,\\ D_\mu^{ab} &= \delta^{ab}\partial_\mu - g f^{abc} W_\mu^c\,,\end{aligned} \tag{2}$$

where W^a_μ is the triplet of SU(2) vector bosons and $F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{abc} W^b_\mu W^c_\nu$ is the corresponding field strength tensor and the subscript HD refers to higher order derivatives. The addition of the term of Eq. (2) to the Lagrangian of the electroweak theory leads to modifications of the Feynman rules. Below we specify only those which are relevant for our calculation. The modified propagator of the W-boson has the form

$$\frac{1}{4g_{\rm HD}(k^2)^2 + k^2 + i\epsilon - M_W^2}\left[g^{\mu\nu} - \frac{k^\mu k^\nu\left(1-\alpha\left(1+4g_{\rm HD}k^2\right)\right)}{k^2+i\epsilon-\alpha M_W^2}\right], \tag{3}$$

with α the gauge parameter associated with the W-boson. There is an additional $A_\alpha(k)W^-_\beta(p)$ $W^+_\gamma(q)$ vertex with all momenta incoming,

$$\begin{aligned}
&-4eg_{\rm HD}\, g^{\alpha\beta}\left[p^\gamma(2k\cdot p + p\cdot q) - k^\gamma(2k\cdot p + k\cdot q)\right] \\
&+4eg_{\rm HD}\, g^{\beta\gamma}\left[q^\alpha(k\cdot q + 2p\cdot q) - p^\alpha(k\cdot p + 2p\cdot q)\right] \\
&+4eg_{\rm HD}\, g^{\alpha\gamma}\left[k^\beta(k\cdot p + 2k\cdot q) - q^\beta(2k\cdot q + p\cdot q)\right) \\
&+4eg_{\rm HD}\left[k^\beta\left(k^\gamma\left(p^\alpha - q^\alpha\right) - p^\alpha p^\gamma\right) + q^\beta\left(q^\alpha\left(k^\gamma - p^\gamma\right) + p^\alpha p^\gamma\right)\right],
\end{aligned} \tag{4}$$

and an additional $W^+_\alpha(p)W^-_\beta(q)A_\gamma(r)A_\delta(k)$ vertex with all momenta incoming,

$$\begin{aligned}
&4e^2 g_{\rm HD}\Big[g^{\alpha\gamma}g^{\beta\delta}(2k\cdot q + k\cdot r + p\cdot q + 2p\cdot r) + g^{\alpha\delta}g^{\beta\gamma}(2k\cdot p + k\cdot r + p\cdot q + 2q\cdot r) \\
&-g^{\alpha\beta}g^{\gamma\delta}(k\cdot p + k\cdot q + 4k\cdot r + 4p\cdot q + p\cdot r + q\cdot r)\Big] \\
&-4e^2 g_{\rm HD}\Big[-2g^{\gamma\delta}k^\alpha k^\beta + p^\delta g^{\beta\gamma}k^\alpha + p^\beta g^{\gamma\delta}k^\alpha + k^\gamma\left(g^{\alpha\delta}k^\beta + g^{\beta\delta}k^\alpha + p^\delta g^{\alpha\beta}\right. \\
&\left.-2p^\beta g^{\alpha\delta} + q^\delta g^{\alpha\beta} - 2q^\alpha g^{\beta\delta} - 2r^\delta g^{\alpha\beta} - r^\beta g^{\alpha\delta} - r^\alpha g^{\beta\delta}\right) + q^\delta g^{\alpha\gamma}k^\beta + q^\alpha g^{\gamma\delta}k^\beta \\
&-r^\delta g^{\alpha\gamma}k^\beta - r^\delta g^{\beta\gamma}k^\alpha + 2r^\beta g^{\gamma\delta}k^\alpha + 2r^\alpha g^{\gamma\delta}k^\beta + 2p^\delta q^\gamma g^{\alpha\beta} - p^\delta q^\alpha g^{\beta\gamma} - p^\beta q^\delta g^{\alpha\gamma} \\
&-p^\beta q^\gamma g^{\alpha\delta} - 2p^\beta q^\alpha g^{\gamma\delta} - 2p^\beta r^\delta g^{\alpha\gamma} + p^\beta r^\alpha g^{\gamma\delta} + p^\beta p^\delta g^{\alpha\gamma} \\
&+p^\gamma\left(-2p^\delta g^{\alpha\beta} + p^\beta g^{\alpha\delta} + 2q^\delta g^{\alpha\beta} - q^\alpha g^{\beta\delta} + r^\delta g^{\alpha\beta} + r^\alpha g^{\beta\delta}\right) + q^\gamma r^\delta g^{\alpha\beta} + q^\gamma r^\beta g^{\alpha\delta} \\
&-2q^\alpha r^\delta g^{\beta\gamma} + q^\alpha r^\beta g^{\gamma\delta} - 2q^\gamma q^\delta g^{\alpha\beta} + q^\alpha q^\delta g^{\beta\gamma} + q^\alpha q^\gamma g^{\beta\delta} + r^\beta r^\delta g^{\alpha\gamma} \\
&+r^\alpha r^\delta g^{\beta\gamma} - 2r^\alpha r^\beta g^{\gamma\delta}\Big].
\end{aligned} \tag{5}$$

We add these two vertices to the corresponding expressions of the Feynman rules specified in Ref. [14] so that the topologies and the number of Feynman diagrams remain the same. All other additional vertices generated by the term of Eq. (2) are not relevant for the current calculation. There are twenty six one-loop diagrams in the W-loop contribution to the Higgs boson decay into two photons, shown in Fig. 1. For $\alpha = 0$ all diagrams containing at least one W-boson propagator are finite for non-vanishing g_{HD}. Diagrams 1), 2), 3), their crossed partners and diagram 11) are regularized by subtracting the analogous loop diagrams with propagators with a heavy mass Λ, amounting to gauge symmetry preserving Pauli–Villars regularization [17]. For convenience in the calculations we take $g_{HD} = 1/(4\Lambda^2)$ so that the removed regulator limit is obtained by taking the limit $\Lambda \to \infty$ after performing the loop integration (and subtracting divergences — if there were any).

In the calculation of the loop diagrams we apply the method of dimensional counting of Ref. [19], that is similar to the "strategy of regions" of Refs. [20,21]. This method allows to represent each regulated loop diagram in four space-time dimensions as the sum of two expressions, both calculated by applying dimensional regularization. The first expression for each diagram is obtained by expanding the integrand of the one-loop integral in inverse powers of Λ and interchanging the integration and summation. For $\Lambda \to \infty$ these expressions exactly coincide to the standard Feynman diagrams obtained using the Feynman rules of Ref. [14] and applying dimensional regularization. The corresponding second part for each diagram is obtained by rescaling the integration variable $k \to q\Lambda$, expanding the resulting integrand in inverse powers of Λ and interchanging the integration and the summation.

Let us briefly demonstrate the method of dimensional counting for a simple massless one-loop integral regulated using a Pauli–Villars type regulator,

$$
\begin{aligned}
I &= \int \frac{d^4k}{(2\pi)^4} \frac{-\Lambda^2}{[k^2-\Lambda^2+i\epsilon]} \frac{1}{[k^2+i\epsilon][(k+p)^2+i\epsilon]} \Rightarrow I_1 + I_2\,, \\
I_1 &= \int \frac{d^Dk}{(2\pi)^D} \frac{1}{[k^2+i\epsilon][(k+p)^2+i\epsilon]} + \frac{1}{\Lambda^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k+p)^2+i\epsilon} + \mathcal{O}\left(\frac{1}{\Lambda^4}\right), \\
I_2 &= \Lambda^{D-4}\left(\int \frac{d^Dq}{(2\pi)^D} \frac{-1}{(q^4+i\epsilon)(q^2-1+i\epsilon)} + \frac{1}{\Lambda}\int \frac{d^Dq}{(2\pi)^D} \frac{2p\cdot q}{(q^6+i\epsilon)(q^2-1+i\epsilon)} \right. \\
&\quad \left. + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)\right).
\end{aligned}
$$

Calculating the dimensionally regulated integrals I_1 and I_2 and expanding at $D = 4$, one finds that the $1/(D-4)$ poles cancel and obtains

$$
I = -\frac{i}{16\pi^2}\left(-1 + \ln\frac{-p^2-i\epsilon}{\Lambda^2}\right) + \mathcal{O}\left(\frac{1}{\Lambda}\right). \tag{6}
$$

Let us return to our original problem. If the sum of the second parts of all twenty six diagrams is non-vanishing in the limit $\Lambda \to \infty$ for $D = 4$ that would mean that dimensional regularization and the symmetry preserving regularization with higher covariant derivatives give different results.

The sum of the regularized diagrams 1), 2), 3), their crossed partners and diagram 11) regularized by subtracting the analogous expressions with a heavy mass Λ has the form

$$
\begin{aligned}
&\frac{e^3 m_\phi^2 M_Z}{M_W\sqrt{M_Z^2-M_W^2}} \\
&\quad \times \int \frac{d^4k}{(2\pi)^4}\left\{\left(\frac{g^{\mu\nu}}{[k^2-\Lambda^2][(k+p_1+p_2)^2-\Lambda^2]} - \frac{g^{\mu\nu}}{[k^2][(k+p_1+p_2)^2]}\right)\right. \\
&\quad + 2k^\mu k^\nu\left[\frac{1}{[k^2]}\left(\frac{1}{[(k+p_1)^2]} + \frac{1}{[(k+p_2)^2]}\right)\frac{1}{[(k+p_1+p_2)^2]}\right. \\
&\quad \left. - \frac{1}{[k^2-\Lambda^2]}\left(\frac{1}{[(k+p_1)^2-\Lambda^2]} + \frac{1}{[(k+p_2)^2-\Lambda^2]}\right)\frac{1}{[(k+p_1+p_2)^2-\Lambda^2]}\right] \\
&\quad + k^\nu p_2^\mu\left[\frac{2}{[k^2][(k+p_2)^2][(k+p_1+p_2)^2]}\right. \\
&\quad \left. - \frac{2}{[k^2-\Lambda^2][(k+p_2)^2-\Lambda^2][(k+p_1+p_2)^2-\Lambda^2]}\right]
\end{aligned}
$$

$$+k^\mu p_1^\nu \left[\frac{2}{[k^2][(k+p_1)^2][(k+p_1+p_2)^2]} - \frac{2}{[k^2-\Lambda^2][(k+p_1)^2-\Lambda^2][(k+p_1+p_2)^2-\Lambda^2]} \right]\Bigg\}. \tag{7}$$

Rescaling $k \to q\Lambda$, expanding the integrand in inverse powers of Λ and interchanging the integration and summation we obtain in the limit $\Lambda \to \infty$:

$$\frac{e^3 m_\phi^2 M_Z \Lambda^{D-4} g^{\mu\nu}}{D M_W \sqrt{M_Z^2 - M_W^2}} \int \frac{d^D q}{(2\pi)^D} \frac{2(D-6)q^4 - 3(D-4)q^2 + D - 4}{q^4(q^2-1)^3}, \tag{8}$$

which is easily integrated to give exactly zero for $D = 4$.

Because of the complicated expressions below we only give the rescaled parts in the $\Lambda \to \infty$ limit for remaining diagrams.

The rescaled expressions of diagrams 4)–9) give vanishing integrands in the $\Lambda \to \infty$ limit. The rescaled part of diagram 10) plus its crossed partner in the limit $\Lambda \to \infty$ reduces to

$$-\frac{8(D-1)e^3 M_W M_Z \Lambda^{D-4}}{\sqrt{M_Z^2 - M_W^2}} \int \frac{d^D q}{(2\pi)^D} \frac{(1-2q^2)^2 q^\mu q^\nu}{q^6(q^2-1)^3}. \tag{9}$$

The analogous expression for diagram 12) reads

$$\frac{2e^3 M_W M_Z \Lambda^{D-4}}{\sqrt{M_Z^2 - M_W^2}} \int \frac{d^D q}{(2\pi)^D} \frac{q^2\big((2D-3)q^2 - D + 2\big)g^{\mu\nu} + \big((4D-3)q^2 - 1\big)q^\mu q^\nu}{q^6(q^2-1)^2}. \tag{10}$$

The rescaled part for diagrams 13), 14) and their crossed partners in the limit $\Lambda \to \infty$ sum up to

$$\frac{2e^3 M_W M_Z \Lambda^{D-4}}{\sqrt{M_Z^2 - M_W^2}} \int \frac{d^D q}{(2\pi)^D} \frac{q^2 g^{\mu\nu} - q^\mu q^\nu}{q^6(q^2-1)}. \tag{11}$$

It is easily verified that the sum of integrals in Eqs. (9)–(11) give exactly zero for $D = 4$.

Thus, the sum of all diagrams regulated by applying higher covariant derivatives in the $\Lambda \to \infty$ limit exactly coincides with the sum of the corresponding dimensionally regularized diagrams obtained using the standard Feynman rules of Ref. [14], taken at $D = 4$. Using FeynCalc [22,23] we checked that we indeed reproduce the old finite gauge-independent result.

Thus we confirm once again that the problem raised in Refs. [4–6] originates from the incorrect treatment of the cancelling divergent integrals. We also notice here that it is trivial to check by using FeynCalc [22,23] that vanishing results are generated if dimensional regularization is applied to the expressions of Eqs. (82) and (88) of Ref. [13] which are claimed in that work to be the source of the discrepancy between unitary and R_ξ gauges if these expressions are treated more carefully.

We hope that we could convince the reader that our study refutes the reiterated claims of Ref. [13] that the unitary and R_ξ gauges lead to different results for the W-boson loop contribution to the Higgs decay into two photons and puts this issue at rest, finally.

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