Rapid Communications

Few-body universality in the deuteron-α system

Jin Lei,[1,*] L. Hlophe,[2,†] Ch. Elster,[1] A. Nogga,[3] F. M. Nunes,[2] and D. R. Phillips[1,4,5]

[1]*Institute of Nuclear and Particle Physics, and Department of Physics and Astronomy, Ohio University, Athens, Ohio 45701, USA*

[2]*National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*

[3]*IAS-4, IKP-3, JHCP, and JARA-HPC, Forschungszentrum Jülich, D-52428 Jülich, Germany*

[4]*Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany*

[5]*ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany*

(Received 17 September 2018; published 27 November 2018)

We treat ^{6}Li as an effective three-body (n-p-α) system and compute the d-α S-wave scattering length and three-body separation energy of ^{6}Li for a wide variety of nucleon-nucleon and α-nucleon potentials which have the same (or nearly the same) phase shifts. The Coulomb interaction in the p-α subsystem is omitted. The results of all calculations lie on a one-parameter curve in the plane defined by the d-α S-wave scattering length and the amount by which ^{6}Li is bound with respect to the n-p-α threshold. We argue that these aspects of the n-p-α system can be understood using few-body universality and that ^{6}Li can thus usefully be thought of as a two-nucleon halo nucleus.

DOI: 10.1103/PhysRevC.98.051001

Introduction. Few-body universality is a powerful tool to analyze the low-energy properties of quantum mechanical systems that are weakly bound [1,2]. Applications of few-body universality range from atomic and molecular physics, e.g., atomic species near a Feshbach resonance [3] or dimers and trimers of ^{4}He atoms [4], to nuclear physics, e.g., few-nucleon systems [5] and halo nuclei [6], to hadronic physics, e.g., the $X(3872)$ and other "exotic" mesons near two-meson thresholds. All these systems have in common that their two-body separation energy is small enough that the wave function of the effective low-energy degrees of freedom (e.g., atoms, nucleons, D and $\bar{D}$ mesons) has much of its support in a region outside the interaction potential, i.e., in the tunneling regime. The properties of the two-body systems are then, to a first approximation, independent of details of the potential, and are correlated solely with the separation energy. The qualitative picture of two-body universality laid out in this paragraph can be systematically organized in terms of an effective field theory (EFT) expansion in powers of R, the range of the two-body potential, times $\gamma = \sqrt{2\mu E_2}$, the binding momentum of the two-body bound state (with E_2 the two-body separation energy and μ the two-body reduced mass) since γ determines the exponential fall-off of the two-body wave function outside the potential.

Many of these systems also exhibit three-body bound states. However, the three-body separation energy E_3 is not solely determined by the two-body separation energy, although it does depend on it. At leading order (LO) in the γR expansion, one three-body observable must be used to fix a "three-body parameter." All other properties of the three-body system are then determined by the three-body observable chosen (e.g., the separation energy E_3) and E_2 [7–9]. It is important to note—especially in the context of our calculation presented below—that the three-body parameter need not arise from "intrinsic" three-body forces. It may, instead, in part or in whole, reflect off-shell properties of two-body forces that are not observable in the two-body system, and first have experimental consequences in the three-body system [10,11]. If E_2 is small compared to E_3 and $\sqrt{2\nu E_3}R$ (with ν the 2+1 reduced mass) is also small, then there is the possibility to observe a sequence of three-body bound states, which are related to one another by a scaling transformation, as predicted by Efimov [12,13]. But even in systems where the conditions for the emergence of bound excited Efimov states are not met, universality still connects disparate three-body systems to one another and provides insights that aid in organizing their phenomenology [14,15].

For example, one important consequence of universality in the three-body system is that E_3 is correlated with the scattering length of the third particle and the two-body bound state. This correlation persists to much smaller 2+1 scattering lengths a_{21} than does the correlation obtained by considering the three-body system to be weakly bound with respect to the 2+1 threshold, $E_3 = \frac{1}{2\nu a_{21}^2} + E_2$. In the three-nucleon system, the E_3-a_{21} correlation—which in this case is with the scattering length in the total-spin-1/2 channel, where the three-body bound state, the triton, resides—was first demonstrated by Phillips [16] and is known as "the Phillips line." This Phillips line still emerges for nucleon-nucleon (NN) potentials that are fitted much more accurately to data than were those originally examined by Phillips [17]. Efimov [18] demonstrated that such a correlation is a consequence of the

*jinl@ohio.edu
†hlophe@nscl.msu.edu

shallow binding of the two-body system, and it has been computed at LO and next-to-leading order (NLO) in the EFT that encodes universality in the three-nucleon system [9,19].

In this paper we show that a similar universal correlation occurs between the three-body separation energy of ^{6}Li and the d-α S-wave scattering length $a_{d\alpha}$. We do this by modeling the d-α system as an effective three-body problem, in which the neutron, proton, and α particle are viewed as basic degrees of freedom that interact via pairwise forces. This is justified because the first excited state of the α particle is $\approx$20 MeV above its ground state and the α particle is compact with respect to ^{6}Li. Our ansatz follows a large body of work treating ^{6}Li as a three-body problem; see e.g., [20–23].

We note that there is also a study of the implications of universality for ^{6}Li as a six-body system. In Ref. [24] Stetcu *et al.* constructed an EFT for the no-core shell model and determined the leading-order NN and three-nucleon forces in the EFT by demanding that the experimental binding energies of the deuteron, triton, and α particle are exactly reproduced. Their six-body calculation then had ^{6}Li unbound with respect to the d-α threshold; $a_{d\alpha}$ could not be computed. In contrast, our three-body model of ^{6}Li avoids the need to compute the emergent low-energy scales in ^{5}He and ^{6}Li *ab initio* from NN and three-nucleon forces. Instead, it takes those scales as input and elucidates their consequences for the low-energy dynamics of the d-α system.

For the purpose of this work we ignore the Coulomb effects between the α particle and the proton. The α interacts with the nucleons predominantly in P waves, while the neutron and proton interaction is mainly in the S wave. The resulting three-body system thus has different dynamics to the three-nucleon case described above, since it contains two P-wave attractive interactions and only one S-wave one.

Framework. We take the neutron-proton (np) force in the $^3S_1$-$^3D_1$ channel, and the αN force in the $P_{3/2}$, $P_{1/2}$, and $S_{1/2}$ partial waves. The three-body separation energy of ^{6}Li is obtained by solving bound-state Faddeev equations with separable representations of these forces as outlined in Ref. [25]. (The "three-body separation energy" of ^{6}Li is the amount by which it is bound compared to the n-p-α threshold, and thus is equal to its d-α separation energy plus the n-p separation energy of deuteron.) The work of Ref. [25] showed that in this system the solution of the Faddeev equations with separable forces is numerically indistinguishable from the solution with nonseparable forces provided the separable basis is appropriately chosen.

For d-α scattering, we solve the momentum space Faddeev-AGS equations [26],

$$U_{ij}(E) = \bar{\delta}_{ij} G_0^{-1}(E) + \sum_{k=1}^{3} \bar{\delta}_{ik} t_k(E) G_0(E) U_{kj}(E), \qquad (1)$$

with $\bar{\delta}_{ij} = 1 - \delta_{ij}$, and $G_0(E) = (E + i0 - H_0)^{-1}$ the free resolvent at the available energy E. The free three-particle Hamiltonian is denoted by H_0, while $t_k = v_k + v_k G_0(E) t_k$ is the two-body transition matrix. Here the index k stands for the channel corresponding to the configuration where the particle k is the spectator and the remaining two form the pair (ij). Since here we are working with three distinguishable particles, cyclic permutations of (ijk) leads to the three required transition operators in Eq. (1). Since we are interested only in very low-energy scattering, we do not have to treat breakup singularities, and the numerical solution of Eq. (1) is straightforward. As in the bound-state calculation [25] we employ the separable representation of the interactions in the two-body subsystems, which was shown to lead to numerically the same observables as a solution with nonseparable forces for the continuum [27]. In addition, we employ the same model space in the scattering calculation as is used to calculate the three-body separation energy of ^{6}Li; this is sufficient when studying the low-energy parameters in the d-α channel with $J^\pi = 1^+$ and total isospin $T = 0$.

In order to investigate if there is a correlation between the three-body separation energy of ^{6}Li and the d-α S-wave scattering length, one needs to solve for these quantities using different sets of potentials which describe the low-energy behavior in the subsystems with the same quality, i.e., potentials that are phase-shift equivalent. In the case of the np interaction this is relatively easy to achieve, since all modern NN interactions are fitted to describe the deuteron binding energy, the np low-energy parameters (scattering length and effective range), and phase shifts in the energy range we are considering. The situation is quite different in the case of effective αN interactions. There have been several efforts to construct effective αN interactions of varying degrees of sophistication (e.g., [28–31]). However, the condition of phase-shift equivalence was imposed rather loosely compared to the NN subsystem. Thus we need to consider a different approach to construct phase-shift equivalent αN potentials. Following the suggestion of Refs. [32,33] we employ a unitary transformation (UT) of the αN Hamiltonian $H_{2b} = h_0 + v$ with h_0 being the two-body kinetic energy operator and v the effective two-body interaction. Following [32,33] we define a transformed Hamiltonian

$$\tilde{H}_{2b} = U H_{2b} U^\dagger = h_0 + \tilde{v}, \qquad (2)$$

where $\tilde{v}$ is the transformed potential keeping the phase shifts unchanged. The operator for the UT is defined as

$$U = 1 - 2|h\rangle\langle h|. \qquad (3)$$

Following Ref. [34] we choose for $|h\rangle$

$$\langle r Y_l^m | h \rangle = N r^l \mathrm{e}^{-cr}(1 - br), \qquad (4)$$

where N is evaluated through the normalization condition $\langle h|h\rangle = 1$ for each partial wave. In our calculations, we only consider the UT on P waves. We include the factor of r^l in accord with Ref. [34], and pick $b = 1$ fm^{-1} for simplicity. We vary the parameter c, thereby changing the range of the transformation. If the starting potential v is separable and of rank 1, the transformed potential $\tilde{v}$ will be of rank 3 [32]. In the case of an arbitrary local or nonlocal v, the transformed potential will have to be numerically calculated, leading to a nonlocal potential $\tilde{v}$.

Results. To study a possible correlation between the three-body separation energy of ^{6}Li and the corresponding S-wave scattering length in the d-α channel, we start by using very simple, rank-1 separable interactions in the two-body subsystems. The form factors of the separable interactions are

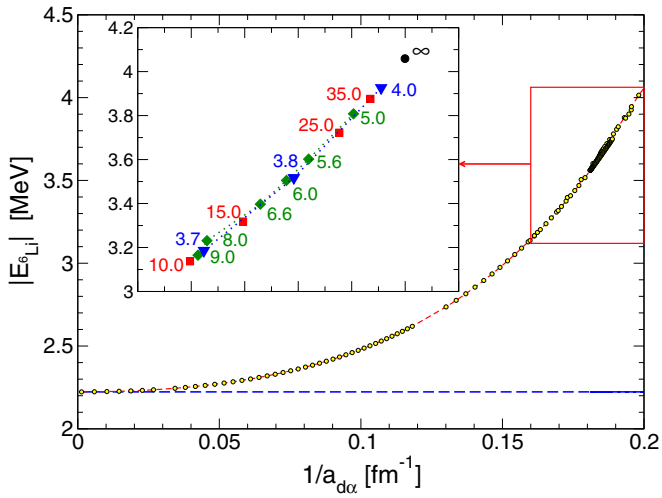

FIG. 1. Absolute value of the three-body separation energy of ^{6}Li as function of the inverse of the d-α S-wave scattering length $a_{d\alpha}$ for phase-shift equivalent interactions obtained by unitarily transforming the interactions in the $n\alpha$ $P_{3/2}$ and $P_{1/2}$ channels. The insert magnifies the marked rectangle and indicates the value c of the exponent in the transformation of Eq. (4). The dashed horizontal line indicates the deuteron breakup threshold.

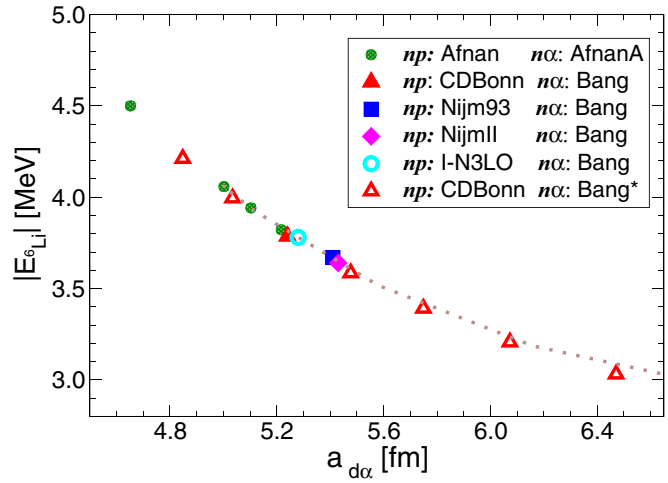

FIG. 2. Absolute value of the three body separation energy of ^{6}Li as a function of d-α S-wave scattering length $a_{d\alpha}$ calculated with a variety of interactions, as described in the text and indicated by the legend. The faint dotted line picks up points from Fig. 1 and is intended to guide the eye.

of Yukawa type, and the parameters are fitted to reproduce the deuteron binding energy and np low-energy scattering parameters in the case of the np interaction, and the αN S- and P-wave phase shifts up to 10 MeV in the case of the αN interaction. Specifically, for the αN interaction we employ model A from Ref. [20] and for the np interaction we choose the parameters from that work that lead to a deuteron D-state probability of 4%.

We then apply the UT of Eqs. (3) and (4) to the P waves of the αN interaction and reduce the parameter c in Eq. (4), starting from a value $c = 35$ fm^{-1} until we reach values at which ^{6}Li is no longer bound. The result of these calculations is summarized in Fig. 1, which shows the dependence of the three-body separation energy of ^{6}Li as a function of the inverse S-wave scattering length $a_{d\alpha}$. (Almost exactly the same correlation of inverse scattering length and three-body separation energy is obtained if the UT is only employed in the $P_{3/2}$ channel, and a very similar result is obtained if only the $P_{1/2}$ αN partial wave is unitarily transformed. Including the $S_{1/2}$ αN partial wave in the UT does not change the results either.) The insert magnifies the regime when c varies from 35 to 4 fm^{-1}, and also shows the calculation using the unmodified αN interaction as a solid circle (labeled by ∞). First, a decrease in c from 35 to 10 fm^{-1} leads to a decrease in the ^{6}Li separation energy together with an increase in the scattering length, forming a line along which the loci of separation energy versus inverse scattering length sit (red solid squares). When c is further decreased, this trend reverses, with the loci now following the previous line, but in the opposite direction—as indicated by the green diamonds in the inset of Fig. 1. This phenomenon of directional reversal on the correlation line has also been observed in Ref. [33], where the UT was applied to NN potentials in the three-nucleon problem. Once the value of c drops below 4 fm^{-1}, the separation energy decreases uniformly as a function of the inverse scattering length until the deuteron breakup threshold is reached at $c = 3.9$ fm^{-1}. At this point ^{6}Li becomes unbound, and $1/a_{d\alpha} \to 0$. Figure 1 shows that all calculations determine a single parametric curve.

The large variation of the parameter c in the UT of the αN interaction in the P wave may appear somewhat artificial. Thus as the next step we consider "realistic" interactions in the two-body subsystem. For the αN interaction we choose the Bang interaction [30], where we set the strength parameter of the central Woods-Saxon term to -44 MeV as in Ref. [25], while for the np interaction we employ the CD-Bonn potential [35]. This αN interaction generates a Pauli-forbidden S-wave αN bound state, which we remove from the two-body spectrum using the methods described in Ref. [25]. Omitting the Coulomb interaction we then obtain a ^{6}Li three-body separation energy of 3.78 MeV and a scattering length of 5.29 fm, indicated in Fig. 2 as a solid red upward triangle. As a guide to the eye a subset of the points from Fig. 1 is also displayed in Fig. 2 as a faint dotted line; we see that this calculation based on "realistic" interactions falls almost on top of the line determined previously by the rank-1 separable interactions. This indicates that off-shell and high-momentum details of the two-body forces do not influence the low-energy behavior of the d-α system—except to the extent that a particular force's high-momentum behavior determines the particular point on the correlation line at which it resides. To check if this is indeed the case, we employ a series of np interactions which have quite different off-shell and high-momentum behavior but are all fitted to the deuteron binding energy and the $^3S_1$-$^3D_1$ phase shift with high precision. The calculation based on the Nijmegen-93 potential [36] is indicated by the blue solid square, the Nijmegen-II potential [36] by the magenta solid diamond, and the Idaho-N4LO potential [37] by the open

cyan circle. Though the realistic NN interactions are located very close to each other in Fig. 2, they all fall on the line established by the previous calculations shown in Fig. 1. In addition to the modern NN interactions we also include as filled green circles the rank-1 np interaction from Ref. [20] in which the deuteron D-state probability is varied for the np interaction.

In order to further explore this behavior for more sophisticated potentials we also modify the strength of the Woods-Saxon potential in the central part of the Bang αN interaction from -42 to -45 MeV; this preserves the general characteristics of the αN system, i.e., leaves it unbound, but causes the agreement with the αN phase shifts to deteriorate and the $P_{3/2}$ resonance position to move. Keeping the np interaction fixed while making this change yields results for the three-body system that are represented in Fig. 2 by the red open upward triangles. They are consistent with the line established earlier. This is a non-phase-equivalent variation of the Bang interaction, so it is somewhat surprising that the $E_{^6\mathrm{Li}}$-$a_{d\alpha}$ curve is unaffected. In contrast, changing the strength of the NN interaction, so altering the deuteron binding energy, yields a $E_{^6\mathrm{Li}}$-$a_{d\alpha}$ curve whose linear portion has a different slope (not shown). The correlation seems to be more sensitive to the on-shell NN input than it is to the on-shell αN input.

Interpretation and implications. We compute the universal correlation between $a_{d\alpha}$ and $E_{^6\mathrm{Li}}$ by evaluating both quantities using several np potentials that have different off-shell and high-momentum behavior, but almost the same np phase shifts, together with a continuous family of αN potentials that have different off-shell and high-momentum behavior but exactly the same αN phase shifts. Arbitrary combinations of these two-body potentials yield results for the three-body observables that lie on a single curve in the $a_{d\alpha}$- $E_{^6\mathrm{Li}}$ plane.

The $a_{d\alpha}$-$E_{^6\mathrm{Li}}$ correlation displayed here is certainly related to the well-known Phillips line of the neutron-deuteron system: it is not surprising that NN interactions with different off-shell behavior produce points along a curve in the $a_{d\alpha}$-$E_{^6\mathrm{Li}}$ plane. The novel feature of the n-p-α system is that varying the off-shell properties of the P-wave nucleon-α potential also produces points on the same curve. This kind of correlation is typical of weakly bound systems and is a consequence of few-body universality. It is in accord with analyses of ^{6}He that show universal correlations are expected for weakly bound, three-body systems where the same angular-momentum structure of two-body potentials occurs as in ^{6}Li [38,39].

The existence of an $a_{d\alpha}$-$E_{^6\mathrm{Li}}$ correlation thus suggests that ^{6}Li can be thought of as a "deuteron halo". Indeed, the experimental d-α separation energy of ^{6}Li (1.47 MeV) [40] is comparable to the deuteron binding energy ($B_d = 2.22$ MeV), and certainly much smaller than the energy associated with α-particle excitation. Recent work on infra-red extrapolations of the ^{6}Li binding energy in *ab initio* no-core shell model calculations using sophisticated NN and three-nucleon forces also show a typical momentum that is much smaller than that of the α particle, supporting its identification as a halo nucleus [41].

The portion of the curve at very large $a_{d\alpha}$, i.e., very small deuteron separation energy, is well described by an effective-range expansion in the $d\alpha$ system. However, such a two-body description is only valid when $|E_{^6\mathrm{Li}}| - B_d < B_d$, i.e., the deuteron separation energy of ^{6}Li is significantly less than the deuteron binding energy. When ^{6}Li is more bound the $a_{d\alpha}$-$E_{^6\mathrm{Li}}$ correlation is linear, with a slope that depends on low-energy NN observables. In this domain changes of the NN interaction that alter the NN phase shifts and the deuteron binding energy yield a different relation between $a_{d\alpha}$ and $E_{^6\mathrm{Li}}$. We conclude that, at least for realistic ^{6}Li binding, the connection between $a_{d\alpha}$ and $E_{^6\mathrm{Li}}$ is a consequence of universality in the three-body n-p-α system, and cannot be understood using a low-order effective-range expansion for the d-α system.

As is well known from three-nucleon systems [9], such a strict correlation suggests that one three-body force can absorb the dependence on the unitary transformation at leading order in the γR expansion. We caution that here we have only examined the existence of such a correlation in the α-n-p channel with total angular momentum $J = 1$, positive parity, and total isospin $T = 0$. But following the example of the three-nucleon case, we anticipate that other low-energy d-α observables—not just $a_{d\alpha}$—are correlated with the three-body separation energy. If that is the case then d-α scattering should be accurately predicted starting from α-nucleon and np interactions as long as the three-body separation energy is reproduced.

In Ref. [42] Ryberg *et al.* performed an EFT calculation of the αnn system and argued that, for the ^{6}He channel where $J = 0$ and $T = 1$, there were at least two three-body force structures if both the $P_{3/2}$ and $P_{1/2}$ channels were included in the αN interaction. In contrast, we found that the $a_{d\alpha}$-$E_{^6\mathrm{Li}}$ correlation is very similar regardless of whether only $P_{3/2}$, only $P_{1/2}$, or both αN channels are unitarily transformed. Thus we have no indication that a second three-body force structure contributes to low-energy αd observables at leading order in the γR expansion, even if both P-wave αN channels are included nonperturbatively in the three-body calculation. The extent to which other observables are correlated with the ^{6}Li binding energy is an interesting topic for future work, as is the identification of the leading three-body force in all of the ^{6}Li three-body channels [43].

As mentioned before, we have not included the Coulomb repulsion between the α particle and the proton in this analysis. It seems reasonable to expect that the halo nature of the ^{6}Li system unveiled in this study will still be present once Coulomb effects are included (cf. Ref. [44] for a study of this issue in a two-body model). In Ref. [25] a subset of the authors computed the amount by which that force reduces the three-body separation energy of ^{6}Li, but those results were only for the ^{6}Li bound state. Once we have the ability to include the Coulomb force when solving the scattering Faddeev-AGS equations with separable interactions, it will be worthwhile to revisit the calculations present here and assess the impact of the repulsive αp electrostatic interaction on the universal correlations in the ^{6}Li system.

Acknowledgments. We thank Hans-Werner Hammer for useful discussions and for comments on the manuscript. This work was performed in part under the auspices of the National Science Foundation under Contract No. NSF-PHY-1520972

with Ohio University and No. NSF-PHY-1520929 with Michigan State University, of the U.S. Department of Energy under Contract No. DE-FG02-93ER40756 with Ohio University, and of DFG and NSFC through funds provided to the Sino-German CRC 110 "Symmetries and the Emergence of Structure in QCD" (NSFC Grant No. 11621131001, DFG Grant No. TRR110), and by the ExtreMe Matter Institute EMMI at the GSI Helmholtzzentrum für Schwerionenphysik, Darmstadt, Germany. This research used resources of the National Energy Research Scientific Computing Center (NERSC), a U.S. Department of Energy Office of Science User Facility operated under Contract No. DE-AC02-05CH11231.

[1] E. Braaten and H. W. Hammer, Phys. Rep. **428**, 259 (2006).
[2] S. König, H. W. Grießhammer, H. W. Hammer, and U. van Kolck, Phys. Rev. Lett. **118**, 202501 (2017).
[3] F. Ferlaino, S. Knoop, M. Berninger, W. Harm, J. P. D'Incao, H. C. Nagerl, and R. Grimm, Phys. Rev. Lett. **102**, 140401 (2009).
[4] M. Kunitski *et al.*, Science **348**, 551 (2015).
[5] P. F. Bedaque and U. van Kolck, Ann. Rev. Nucl. Part. Sci. **52**, 339 (2002).
[6] H. W. Hammer, C. Ji, and D. R. Phillips, J. Phys. G **44**, 103002 (2017).
[7] P. F. Bedaque, H. W. Hammer, and U. van Kolck, Nucl. Phys. A **646**, 444 (1999).
[8] P. F. Bedaque, H. W. Hammer, and U. van Kolck, Phys. Rev. Lett. **82**, 463 (1999).
[9] P. F. Bedaque, H. W. Hammer, and U. van Kolck, Nucl. Phys. A **676**, 357 (2000).
[10] P. Naidon, S. Endo, and M. Ueda, Phys. Rev. Lett. **112**, 105301 (2014).
[11] W. N. Polyzou and W. Glöckle, Few-Body Syst. **9**, 97 (1990).
[12] V. Efimov, Phys. Lett. B **33**, 563 (1970).
[13] V. Efimov, Yad. Fiz. **29**, 1058 (1979) [Sov. J. Nucl. Phys. **29**, 546 (1979)].
[14] H.-W. Hammer and L. Platter, Ann. Rev. Nucl. Part. Sci. **60**, 207 (2010).
[15] P. Naidon and S. Endo, Rept. Prog. Phys. **80**, 056001 (2017).
[16] A. C. Phillips, Nucl. Phys. A **107**, 209 (1968).
[17] H. Witala, A. Nogga, H. Kamada, W. Glöckle, J. Golak, and R. Skibiński, Phys. Rev. C **68**, 034002 (2003).
[18] V. Efimov and E. Tkachenko, Few-Body Syst. **4**, 71 (1988).
[19] H. W. Hammer and T. Mehen, Phys. Lett. B **516**, 353 (2001).
[20] A. Eskandarian and I. R. Afnan, Phys. Rev. C **46**, 2344 (1992).
[21] D. R. Lehman, Phys. Rev. C **25**, 3146 (1982).
[22] D. R. Lehman, M. Rai, and A. Ghovanlou, Phys. Rev. C **17**, 744 (1978).
[23] N. W. Schellingerhout, L. P. Kok, S. A. Coon, and R. M. Adam, Phys. Rev. C **48**, 2714 (1993); **52**, 439(E) (1995).
[24] I. Stetcu, B. R. Barrett, and U. van Kolck, Phys. Lett. B **653**, 358 (2007).
[25] L. Hlophe, J. Lei, C. Elster, A. Nogga, and F. M. Nunes, Phys. Rev. C **96**, 064003 (2017).
[26] E. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. B **2**, 167 (1967).
[27] T. Cornelius, W. Glöckle, J. Haidenbauer, Y. Koike, W. Plessas, and H. Witala, Phys. Rev. C **41**, 2538 (1990).
[28] H. Kanada, T. Kaneko, S. Nagata, and M. Nomoto, Prog. Theor. Phys. **61**, 1327 (1979).
[29] E. Garrido, D. Fedorov, and A. Jensen, Nucl. Phys. A **617**, 153 (1997).
[30] J. Bang, J. J. Benayoun, C. Gignoux, and I. J. Thompson, Nucl. Phys. A **405**, 126 (1983).
[31] J. Bang and C. Gignoux, Nucl. Phys. A **313**, 119 (1979).
[32] I. R. Afnan and F. J. D. Serduke, Phys. Lett. B **44**, 143 (1973).
[33] A. W. Thomas and I. R. Afnan, Phys. Lett. B **55**, 425 (1975).
[34] M. I. Haftel and F. Tabakin, Phys. Rev. C **3**, 921 (1971).
[35] R. Machleidt, Phys. Rev. C **63**, 024001 (2001).
[36] V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, Phys. Rev. C **49**, 2950 (1994).
[37] D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001 (2003).
[38] J. Rotureau and U. van Kolck, Few Body Syst. **54**, 725 (2013).
[39] C. Ji, C. Elster, and D. R. Phillips, Phys. Rev. C **90**, 044004 (2014).
[40] F. Ajzenberg-Selove, Nucl. Phys. A **490**, 1 (1988).
[41] C. Forssén, B. D. Carlsson, H. T. Johansson, D. Sääf, A. Bansal, G. Hagen, and T. Papenbrock, Phys. Rev. C **97**, 034328 (2018).
[42] E. Ryberg, C. Forssén, and L. Platter, Few-Body Syst. **58**, 143 (2017).
[43] H. W. Griesshammer, Nucl. Phys. A **760**, 110 (2005).
[44] H. W. Hammer and R. Higa, Eur. Phys. J. A **37**, 193 (2008).