

Final state interactions in the decays $J/\psi \rightarrow VPP$

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Abstract

We investigate the interplay between crossed channel final state interactions and the constraints from two-particle unitarity for the reactions $J/\psi \rightarrow V\pi\pi$ and $VK\bar{K}$, where V is either ω or ϕ . Using a model where the parameters are largely constrained by other sources, we find that, although small, crossed channel final state interaction can influence the amplitudes considerably, in special areas of phase space. These results cast doubt on the inapplicability of unitarity constraints on production amplitudes as recently claimed in the literature.

1 Introduction

In recent years, much interest and effort have been put into extracting information about two-body interactions from their Final State Interaction (FSI) in production reactions. To study the final state interaction among multiple final particles rigorously, one in principle needs to solve complicated many-body equations, where not all of the necessary input is known. However, if one of the pair-wise interactions in the final state is much stronger than all the others, one may use a simplified method, first introduced by Watson [1]. The theorem may be derived from the unitarity relation for two-particle interactions from a production reaction

$$\text{Im}(A_i) = -\sigma_k T_{ik}^* A_k, \quad (1)$$

where A_k is the full production amplitude into channel k , σ_k is the two-body phase space factor for channel k and T_{ik}^* is the complex conjugate of the scattering amplitude connecting channels i and

k . Here summation over k is assumed. This relation is also called Extended Unitarity (EU) [2] and has been used in many analyses. We will also use this phrase in what follows. The case studied by Watson was the single channel case. Since $\text{Im}(A)$ is a real number, it automatically follows then that the phase of the production amplitude equals to that of elastic scattering — this is known as the Watson theorem. Then the final state interaction is well described by the Jost–function for the scattering amplitude [3], which for large scattering lengths and small momenta agrees with the scattering amplitude. However, in many applications, as the J/ψ decays we will study in this work, the mentioned conditions are not met in the whole kinematic regime. For example above the $\bar{K}K$ threshold coupled channels become relevant. Since Eq. (1) contains a summation, there is no connection anymore between the phase of the scattering amplitudes and that of the production amplitudes. In addition, there might be effects from crossed channels. Those were most recently discussed in Ref. [4]. The effects of coupled channels on the mentioned J/ψ decays were first investigated in Ref. [5]. The idea was refined in Ref. [6] and later improved in Ref. [7] — especially here the data of Refs. [8, 9, 10] was included. Crossed channel effects were studied in Refs. [11, 12], however, no detailed comparison to the most recent data was performed.

Recently, Bugg [13] argued that the EU is in conflict with experimental data when applied to the case with multiple resonances being present in one partial wave, such as the scalar meson channel. Based on the incorrect assumption¹ that Watsons theory was originally derived without overlapping resonances, the author argues that the extension of this theory to describe the scalar meson production, in which $f_0(600)$ (the so-called σ meson) and $f_0(980)$ are present, is not reliable. The central statement of Ref. [13] is that, since EU predicts the same phase of production amplitude and scattering amplitude (and he assumes this to be the case even above the opening of inelastic channels), analyticity would further require that the relative magnitude of the two resonances should also be the same for both processes. He then demonstrated that this is in conflict with the PWA results of the s -wave contribution to the $\pi\pi$ invariant mass spectrum in the $J/\psi \rightarrow \omega\pi\pi$, where there is no signal of a deep dip around the $K\bar{K}$ threshold as it would follow from the reasoning presented above. In this work we investigate, if this observation indeed shows that Eq. (1) is wrong. Before addressing this issue, one should notice that the naive EU as given in Eq. (1) only tells us the relation between the production amplitude which only includes the information of FSI of two final particles and the corresponding elastic scattering amplitude of the two particles. This means

¹The reasoning by Watson is given for a single continuum channel. No statement is made on the number of resonances allowed.

that the FSI between these two particles and other hadrons in the final state, which will be called crossed channel FSI in the following, has been neglected. Sometimes, such an approximation can be employed. For instance, for the dipion transition between charmonia $\psi' \rightarrow J/\psi \pi \pi$, if there is no resonance which couples strongly to $J/\psi \pi$ or $\psi' \pi$. In addition, the interaction between J/ψ and ψ' and the pions is OZI suppressed and therefore small [14]. However, the situation is different for the J/ψ decays into three light flavored mesons. For the decay $J/\psi \rightarrow \omega \pi \pi$, there is a well-established axial vector meson, the $b_1(1235)$, which couples strongly to both the $\omega \pi$ and $J/\psi \pi$ [15] (certainly the coupling strength of $b_1 \omega \pi$ is much larger than that of $J/\psi b_1 \pi$ since the latter is OZI suppressed).

It should also be stressed that a linear relation like Eq. (1) is putting a lot weaker constraints on the amplitudes as a non-linear relation like the corresponding one for elastic scattering does. In particular, the amplitude gets fixed only up to a polynomial. Thus, it is not correct that the requirement of unitarity and analyticity can determine the relative strengths from different resonances as well. On the contrary, based on Eq. (1) there is still the freedom to adjust the relative coupling between different resonances.

The main aim of this work is to demonstrate that the conflict between the experimental data analysis and the result that follows from EU can be largely removed by the crossed channel effects. Here we mainly concentrate on the production of one vector meson and two pseudoscalar mesons in J/ψ decays where the two pseudoscalars are in relative s -wave. We follow the ideas and formalisms in Refs. [6, 7, 11]. In Ref. [7], the authors constructed the production amplitude from two pieces. The first part is the coupling of the J/ψ to a vector meson and a scalar source, and the second part is the coupling of the scalar source to two pseudoscalar mesons. The latter is described by scalar form factors. Final state interaction is incorporated through a chiral unitary description with coupled channels, in which the scalar mesons (σ and $f_0(980)$) are produced dynamically from the s -wave interactions between the Goldstone bosons. In this way, it is possible to investigate the OZI violating effect and to extract the low energy constants (LECs) of chiral perturbation theory (ChPT), if scalar form factors are calculated up to next-to-leading order (NLO). In Ref. [7] a deep dip appears right above the $K\bar{K}$ threshold in the s -wave $\pi\pi$ invariant mass spectrum of the $J/\psi \rightarrow \omega \pi \pi$ as discussed above. However, this is apparently in conflict with the PWA results by the BES Collaboration [8]. Bugg considered this as a signal that EU is wrong. However, it should be noted that in Ref. [7], the OZI violation parameter is assumed to be real. This means that the contribution of crossed channels was neglected. In fact, in Ref. [11] the authors have investigated

the contribution from those mechanisms. In their work, the crossed channel FSI are modelled by some three-meson loop diagrams. It is shown that the contribution of those mechanisms is not negligible. It is clear that the crossed channel FSI can distort the prediction of the naive EU. After considering those diagrams, it is possible to largely remove the conflict between the prediction of EU and the BES PWA results as will be shown in this paper.

The formalism of our analysis is presented in section 2. In section 3, we give the results of our analysis and the parameters from fitting to the data. The conclusion and some discussions are given in section 4.

2 Formalism

The experimental data of the J/ψ decaying into a vector meson and two pseudoscalar mesons are collected in a series of papers [8, 9, 10, 16, 17]. The latest data are published by the BES Collaboration in Refs. [8, 9, 10] where also a partial wave analysis (PWA) is provided. In this way the s -wave contribution of the final $\pi\pi$ and $K\bar{K}$ pairs was isolated from the other partial waves. It is convenient for us to use these PWA results because we can concentrate on the scalar meson channel and avoid the complication from the contribution of other resonances. Hence, our analysis is based on the PWA results [8, 9]. However, it should be stressed that $\pi\pi$ s -wave in the region of interest is only a very small part of the full signal which emerges from interferences of this partial waves with others. As a result, the extracted s -wave strength should depend on its assumed phase motion. We expect that the amplitudes of our analysis will show a different phase motion compared to those used in the analysis so far. In addition, in the BES analysis some possible tree level resonance contributions were neglected, such as the subthreshold contributions from the ρ (for $\omega\pi\pi$) and the K^* (for $\phi K\bar{K}$), and the contribution from K_1 mesons (for $\phi K\bar{K}$) as shown in Fig. 1. It has been stressed in Ref. [18] that the contribution from the ρ is important and may have significant influence on PWA result. This uncertainty also enters our work because our analysis is based on BES PWA result. Because of these reasons we do not aim at a high quality description of the data, but only at pointing at possible deficits of previous analyses, especially of Ref. [13]. Eventually the full data set should be reanalyzed based on improved amplitudes.

Usually in PWA only some tree level amplitudes are constructed, and loop contributions are effectively absorbed into the coupling constants. This is the reason why in some phenomenological analyses the coupling constants are allowed to take complex values. But it should be noted that in Ref. [7] the OZI violation parameter and coupling constant are set to be real. This implicitly means

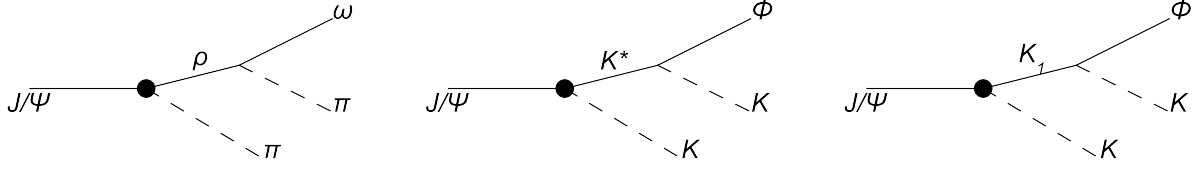


Figure 1: The tree level Feynman diagrams included in our work for the decays $J/\psi \rightarrow \omega\pi\pi$ and $\phi K\bar{K}$.

that the crossed-channel contributions are not included. However, the diagrams shown in Fig. 2 can contribute as stated in the Introduction, if the outgoing light meson pairs are in the s -wave. The formalism we will use to evaluate the mentioned mechanisms follows Refs. [7, 11]. Here we just illustrate the formalism briefly, and the detailed description can be found in Refs. [7, 11].

The Feynman diagram that only includes the FSI between the pseudoscalar mesons is given in Fig. 3. Using the definition and normalization of Ref. [7], the corresponding amplitudes can be written as

$$\mathcal{M}_\phi^{\pi\pi} = \sqrt{\frac{2}{3}} C_\phi \langle 0 | s\bar{s} + \lambda_\phi n\bar{n} | \pi\pi \rangle^{I=0}, \quad (2)$$

$$\mathcal{M}_\phi^{K\bar{K}} = \sqrt{\frac{1}{2}} C_\phi \langle 0 | s\bar{s} + \lambda_\phi n\bar{n} | K\bar{K} \rangle^{I=0}. \quad (3)$$

Here $\mathcal{M}_\phi^{\pi\pi}$ and $\mathcal{M}_\phi^{K\bar{K}}$ are the amplitudes for the processes $J/\psi \rightarrow \phi\pi\pi$ and $J/\psi \rightarrow \phi K\bar{K}$, respectively. λ_ϕ is the OZI violation parameter. The amplitudes for $J/\psi \rightarrow \omega\pi\pi$ and $J/\psi \rightarrow \omega K\bar{K}$ can be similarly written as

$$\mathcal{M}_\omega^{\pi\pi} = \sqrt{\frac{2}{3}} C_\omega \langle 0 | s\bar{s} + \lambda_\omega n\bar{n} | \pi\pi \rangle^{I=0}, \quad (4)$$

$$\mathcal{M}_\omega^{K\bar{K}} = \sqrt{\frac{1}{2}} C_\omega \langle 0 | s\bar{s} + \lambda_\omega n\bar{n} | K\bar{K} \rangle^{I=0}. \quad (5)$$

with

$$C_\omega = \lambda_\phi C_\phi, \quad (6)$$

$$\lambda_\omega = \frac{\lambda_\phi + \sqrt{2}}{\sqrt{2}\lambda_\phi}. \quad (7)$$

The scalar form factors are defined as

$$\sqrt{2}B_0\Gamma_1^n(s) = \langle 0 | \bar{n}n | \pi\pi \rangle^{I=0}, \quad (8)$$

$$\sqrt{2}B_0\Gamma_2^n(s) = \langle 0 | \bar{n}n | K\bar{K} \rangle^{I=0}. \quad (9)$$

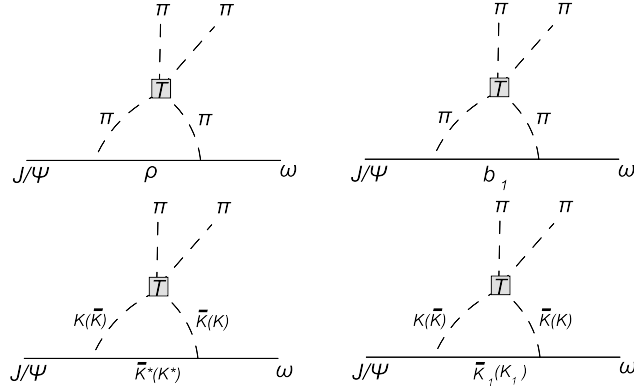


Figure 2: Three-meson loop diagrams for $J/\psi \rightarrow \omega \pi \pi$. (The corresponding diagrams for other channels can be obtained by changing ω to ϕ and $\pi\pi$ to $K\bar{K}$ in the last two diagrams (for $J/\psi \rightarrow \phi K\bar{K}$) or changing ω to ϕ in the last two diagrams (for $J/\psi \rightarrow \phi \pi \pi$) respectively.)

Here B_0 denotes the strength of the scalar quark condensate. The scalar form factors can be calculated within ChPT to a given order. However, because we are interested in the energy range up to 1.2 GeV, where the energy is too high for ChPT to be applied, the Chiral Unitary Approach [6, 19] is adopted. This method is described in detail in Ref. [19], and the expressions for the scalar form factors up to NLO can be found in Ref. [7].

As discussed in the Introduction, crossed channel effects are potentially important. We model them by some three-meson loop diagrams following Ref. [11]. Those diagrams are shown in Fig. 2. The effective Lagrangians for the corresponding vertices can be written as [11],

$$\begin{aligned}
\mathcal{L}_{VVP} &= \frac{G}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \langle \partial_\mu V_\nu \partial_\alpha V_\beta P \rangle \\
\mathcal{L}_{\psi VP} &= \frac{\bar{G}}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \psi_\nu \langle \partial_\alpha V_\beta P \rangle \\
\mathcal{L}_{A(B)VP} &= D \langle V_{\mu\nu} \{ B^{\mu\nu}, P \} \rangle - iF \langle V_{\mu\nu} [A^{\mu\nu}, P] \rangle \\
\mathcal{L}_{A(B)\psi P} &= \bar{D} \psi_{\mu\nu} \langle B^{\mu\nu} P \rangle
\end{aligned} \tag{10}$$

where $\langle \dots \rangle$ means SU(3) flavor trace and $V(P)$ are the SU(3) matrix representations of the vector (pseudoscalar) mesons as usual. Using the tensor field formalism to describe the spin-1 mesons [20], $A^{\mu\nu}$ and $B^{\mu\nu}$ are the SU(3) matrices for axial-vector mesons with $J^{PC} = 1^{++}$ and 1^{+-} , respectively, and $V^{\mu\nu}$ is the SU(3) matrix for vector mesons.

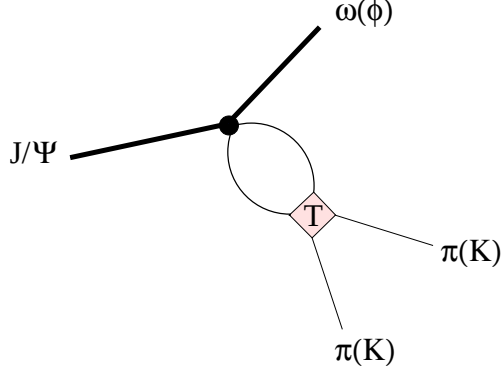


Figure 3: Feynman diagram of the J/ψ decays into one vector meson and two pseudoscalar mesons with the FSI of two pseudoscalar mesons.

The propagators of vector and axial vector mesons in tensor formalism are given by [20]

$$\begin{aligned}
 i\mathcal{D}_{\mu\nu\rho\sigma} &\equiv \langle 0|T\{W_{\mu\nu}W_{\rho\sigma}\}|0\rangle \\
 &= i\frac{M_W^{-2}}{M_W^2 - P^2 - i\epsilon} \left[g_{\mu\rho} g_{\nu\sigma} (M_W^2 - P^2) + g_{\mu\rho} P_\nu P_\sigma - g_{\mu\sigma} P_\nu P_\rho - (\mu \leftrightarrow \nu) \right], \quad (11)
 \end{aligned}$$

where M_W and P are the mass and momentum of the spin-1 meson, respectively.

In the standard way, it is easy to write down the amplitudes of those diagrams. The squares in the diagrams mean the full meson–meson scattering amplitudes involving the loop resummation through the BS equation [19]. The three–meson loop integrals are divergent. They are regularized using a cut-off as described in detail in Ref. [21].

Using the amplitudes described above, it is straightforward to calculate the invariant mass spectra and fit the free parameters in the amplitudes to the data².

3 Numerical Results

In our work, we will only consider the J/ψ decays into the $\omega\pi\pi$, $\phi\pi\pi$ and $\phi K\bar{K}$ channels. Although it will be straightforward to include the $\omega K\bar{K}$ channel as well, as pointed out in Ref. [7], we do not include this channel in the fitting procedure due to its large uncertainty. For illustration our results for that channel are shown below. The free parameters basically come from the diagrams shown in Fig. 3, which include the NLO LECs L_4 , L_5 , L_6 and L_8 appearing in the scalar form

²Note, the data as given in the publications are to be corrected for an invariant mass dependent acceptance. See Ref. [7] for details.

factors, the parameter C_ϕ and the OZI violation parameter λ_ϕ . Other parameters such as C_ω and λ_ω can be related to C_ϕ and λ_ϕ through Eqs. [6,7]. The cut-off in the two-meson loop is set to be 0.9 GeV as Ref. [7]. Most of the parameters appear in the three-meson loop diagrams such as the coupling constants, the mixing angle between K_1 mesons are fixed in Ref. [11] and some related papers [22]. In order to reduce the number of free parameters, we adopt those values and keep them fixed in our calculation. The uncertainties due to those coupling constants are included in the error band in our results shown in Figs. 5 and 6. We adopt $\Lambda = 1.0$ GeV for the cut-off parameter used in three-meson loop calculation. Note that the quality of the fit does not change considerably when the cut-off is varied around that value — we varied Λ in the range between 0.85 and 1.15 GeV; the resulting variation in the observables is smaller than the error bands shown, if the parameters are refit for each cut-off. Besides the magnitudes of the coupling constants, their relative signs are also important. Most of the relative signs have been fixed in Ref. [11]. But there is still a relative sign between the $J/\psi VP$ and $J/\psi AP$ couplings which cannot be determined. This leads to two sets of solutions in Ref. [11], and thus we keep it as an adjustable freedom in our work.

As mentioned above, we assume that the tree level contributions shown in Fig. 1 are effectively absorbed into the partial wave amplitudes of the BES analysis. So in our calculation, we include the contributions from ρ , K^* and K_1 exchange.

Another difference between our work and Ref. [7] is that we use equal weights for different channels and fit the s -wave contribution up to 1.2 GeV for every decay including the $\omega\pi\pi$ channel, while in Ref. [7] the fitting range stopped before the deep dip appearing in their description of the $\omega\pi\pi$ channel.

With the considerations described above, we totally have 6 free parameters and a relative sign between the coupling $J/\psi VP$ and $J/\psi AP$, which we fit to the invariant mass spectra using the MINUIT program [23]. The results for the best fit and a comparison with the BES data are shown in Figs. 5 and 6, where also the $\omega\bar{K}K$ channel is shown for illustration, although it was not included in the fit. The resulting parameters are given in Table 1. The uncertainties from the coupling constants and the free parameters are included in the error band.

4 Discussion and Conclusion

Before discussing the results in detail we illustrate the effect of the three-meson loop diagrams in some more detail. In Fig. 4 we show a comparison of magnitude and phase motion for the two-kaon

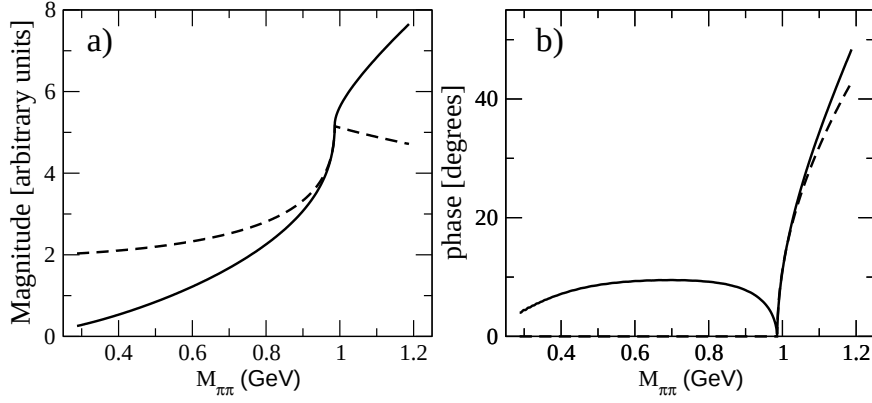


Figure 4: Comparison of magnitude and phase motion of the two-kaon loop (dashed line) and the three-meson loop with an additional K^* exchange (solid line).

Table 1: Parameter values obtained by fitting to BES results for the J/ψ decays into the $\omega\pi\pi$, $\phi\pi\pi$ and $\phi K\bar{K}$. All values of the renormalized LECs L_i^r are quoted at the scale $\mu = m_\rho$, and the relative sign between the $J/\psi VP$ and $J/\psi AP$ couplings is even ($\chi^2/d.o.f. \approx 2.1$). The results of Ref. [7] is also given for comparison.

	$L_4^r[10^{-3}]$	$L_5^r[10^{-3}]$	$L_6^r[10^{-3}]$	$L_8^r[10^{-3}]$	$C_\phi[\text{keV}^{-1/2}]$	λ_ϕ	$q_{max}[\text{GeV}]$
Our work	$0.76^{+0.02}_{-0.02}$	$0.54^{+0.05}_{-0.05}$	$-0.17^{+0.02}_{-0.02}$	$0.65^{+0.02}_{-0.02}$	$64.0^{+1.9}_{-1.9}$	$0.134^{+0.013}_{-0.013}$	0.9
Fit I of [7]	$0.84^{+0.06}_{-0.05}$	$0.45^{+0.08}_{-0.09}$	$0.03^{+0.16}_{-0.13}$	$0.33^{+0.14}_{-0.17}$	$42.1^{+5.0}_{-5.0}$	$0.132^{+0.018}_{-0.015}$	0.9 ± 0.025

loop (Fig. 3) and a corresponding three-meson loop with additional K^* exchange (third diagram of Fig. 2), where for illustration the scattering T -matrix is replaced by a constant. As one can see, the three-meson loop leads to both a non-trivial variation in magnitude and phase compared to what emerges from the kaon loop alone. Especially, there is already a phase motion present for $\pi\pi$ invariant masses below the two-kaon threshold. This is a result of the consideration of the width of the K^* by which the kinematically allowed $K\bar{K}\pi$ can contribute. Other three-meson loops show different patterns. Therefore crossed channel effects lead to contributions that cannot be included by just using complex coupling constants.

Figures 5 and 6 show that we describe the data quite well, although the description near the $K\bar{K}$ threshold in $\omega\pi\pi$ channel where the deep dip originally appears is not good enough, it has been improved a lot compared to Ref. [7]. The LECs we get are consistent with other works within

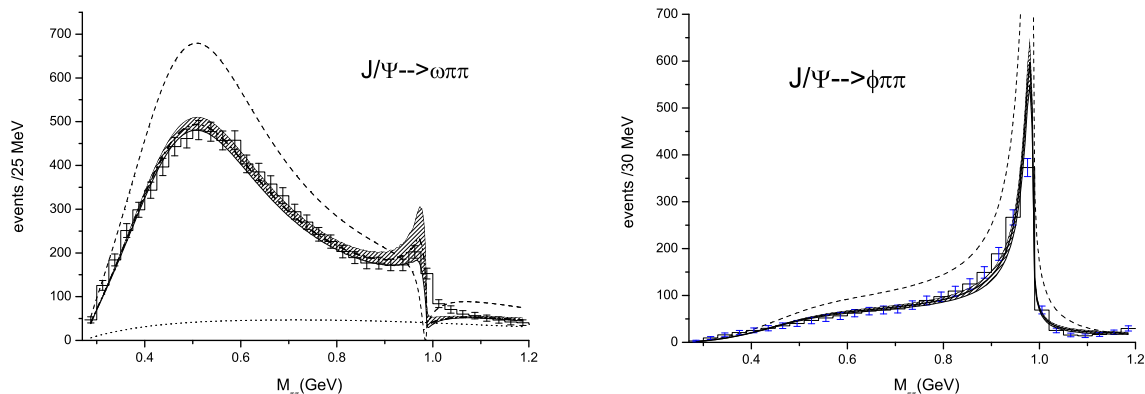


Figure 5: Solid line: full results for $J/\psi \rightarrow V\pi\pi$ corresponding to parameters shown in Table 1; dotted line: tree level contributions from ρ ; dashed line: results without the contributions from the three-meson loop diagrams and the tree level diagrams. The BES PWA results for the s -wave contribution are shown by the histograms.

uncertainties. However, the parameter C_ϕ which describes the coupling of J/ψ to vector mesons and a scalar source is about a factor of 1.5 larger than that in Ref. [7]. This is needed to cancel some contribution of new mechanisms we added, otherwise we cannot fit the data well.

In our work, we investigated the interplay between crossed channel effects and the prediction of EU. It seems from our calculation that the prediction of EU can be influenced by crossed channel FSI greatly in those regions of phase space where the amplitudes from the two-body interactions are small. So any conclusion about the conflict of EU with experimental data, which is based on those regions, should be questioned, because there might be other mechanisms besides those governed by EU that are relevant. Note, however, that overall the description from just keeping the two-body $\pi\pi$ and $\bar{K}K$ interaction is quite impressive. To separate the influence from crossed channel effects from the leading amplitudes is difficult and may be model dependent. In our work, we follow the ideas of Ref. [11] to calculate the crossed channel FSI. The calculation is model dependent and not complete in principle, but it offers some information about the interplay between EU prediction and crossed channel FSI. It seems that we cannot confirm a conflict of EU predictions and the experimental analysis.

We also addressed uncertainties in the PWA by the BES collaboration. In their analysis some possible tree level contributions were neglected, which contribute in principle and are not negligible

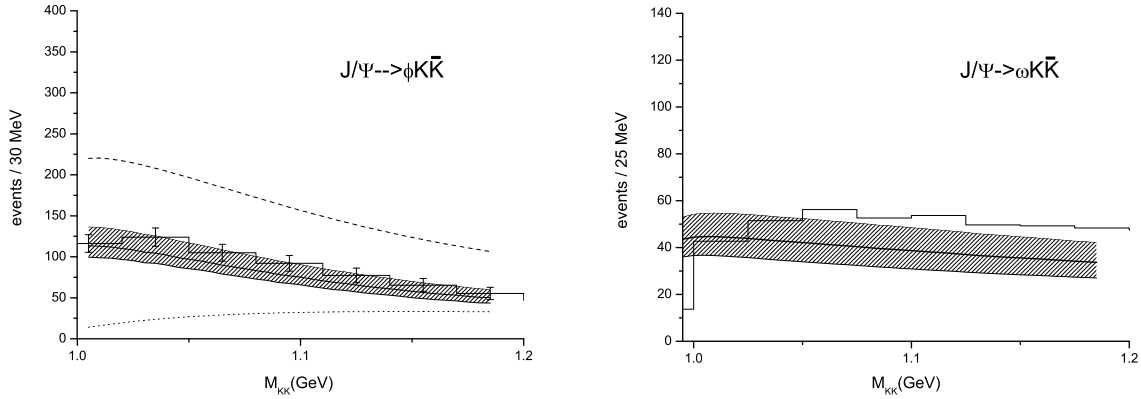


Figure 6: Solid line: full results for $J/\psi \rightarrow VK\bar{K}$ corresponding to parameters shown in Table 1; dotted line: tree level contributions K^* and K_1 mesons; dashed line: results without the contributions from the three-meson loop diagrams and the tree level diagrams. The BES PWA results for the s -wave contribution are shown by the histograms.

in our calculation (see dotted lines in Figs. 5 and 6). With these uncertainties in mind, we conclude that there is no conflict between EU and experimental data. Especially, there is no justification to replace Eq. (1) by something else that has no theoretical ground, as was done in Ref. [13].

Acknowledgments

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