

# Dimensionality reduction with normalizing flows

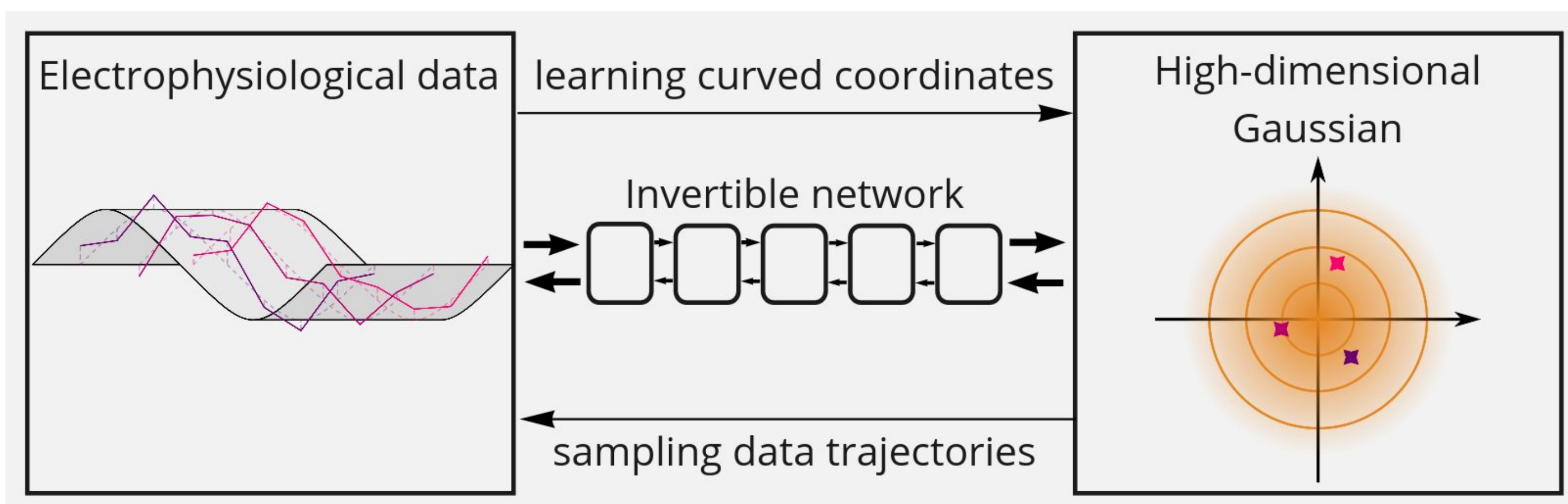
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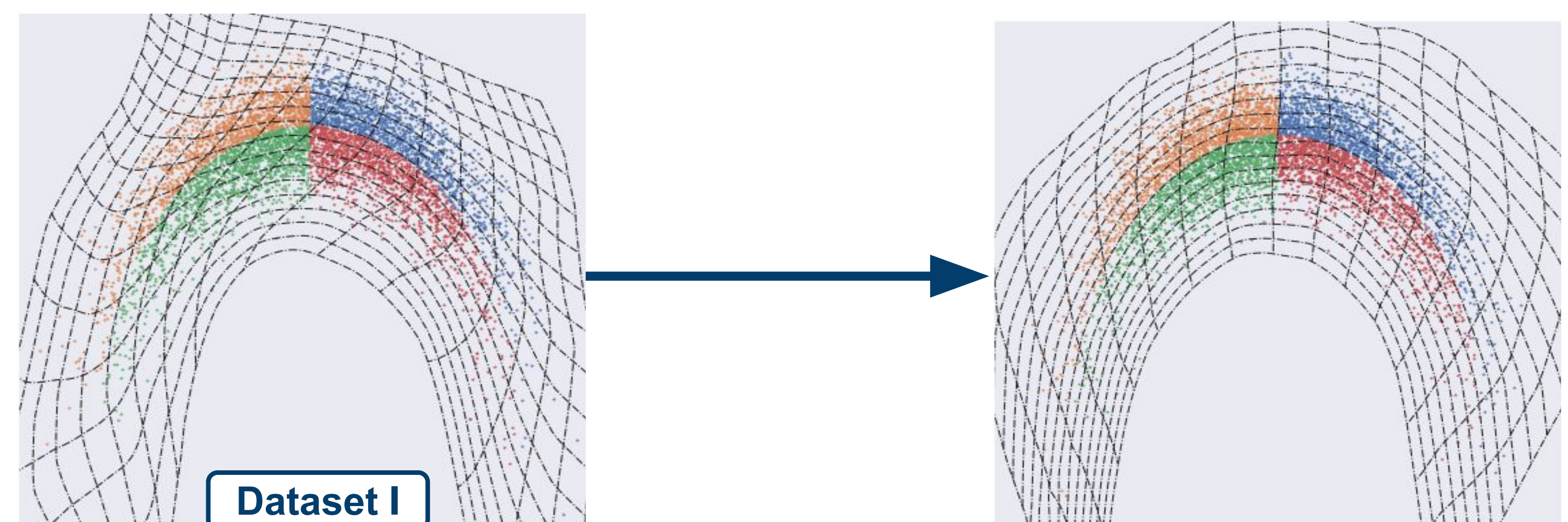
## Summary

- We demonstrate nonlinear dimensionality reduction using invertible neural networks (INNs).
- This is achieved by enforcing reconstruction with few (meaningful) dimensions.
- The method shows comparable results to PCA on close to Gaussian data.
- The network models require much fewer parameters on larger datasets than linear models.

## Normalizing flows

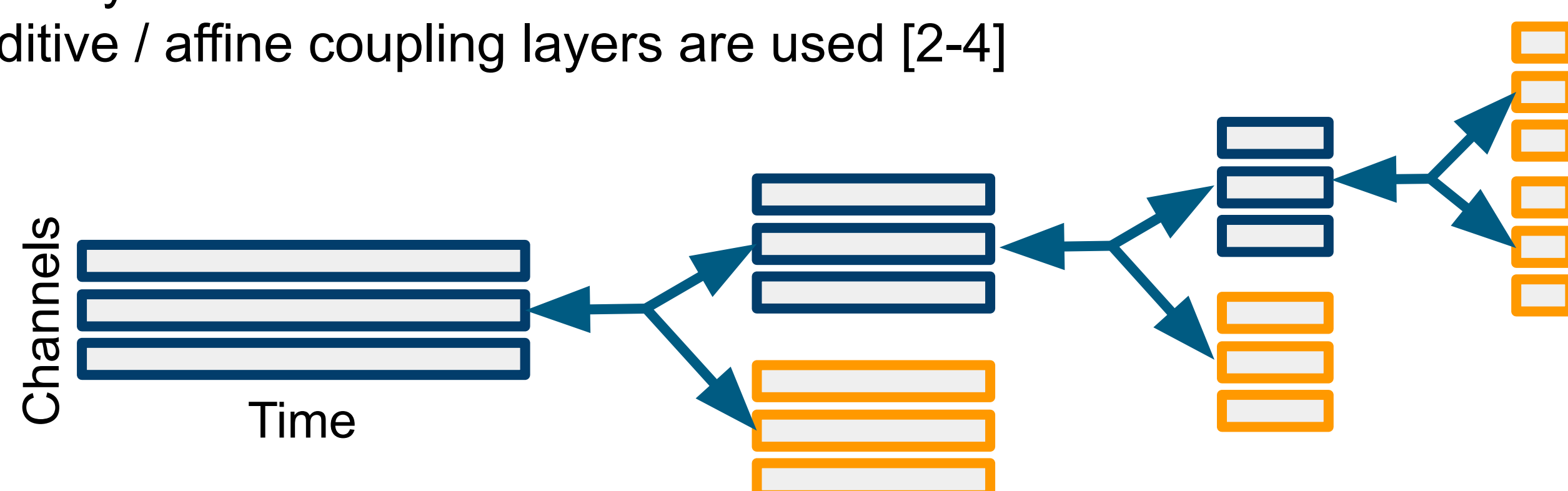


## Enforce meaningful latent dimensions



## Multiscale Architecture

- At every scale: Invertible convolutions - additive / affine coupling layers are used [2-4]



## Reconstruct data with few dimensions

- In the latent space, data features are entangled and use all dimensions.
- To reduce dimensionality, we identify data-relative dimensions in the latent space.

$$\tilde{x}_i(x) = f^{-1} \left( \sum_{k=1}^i f_k(x) \hat{e}_k \right)$$

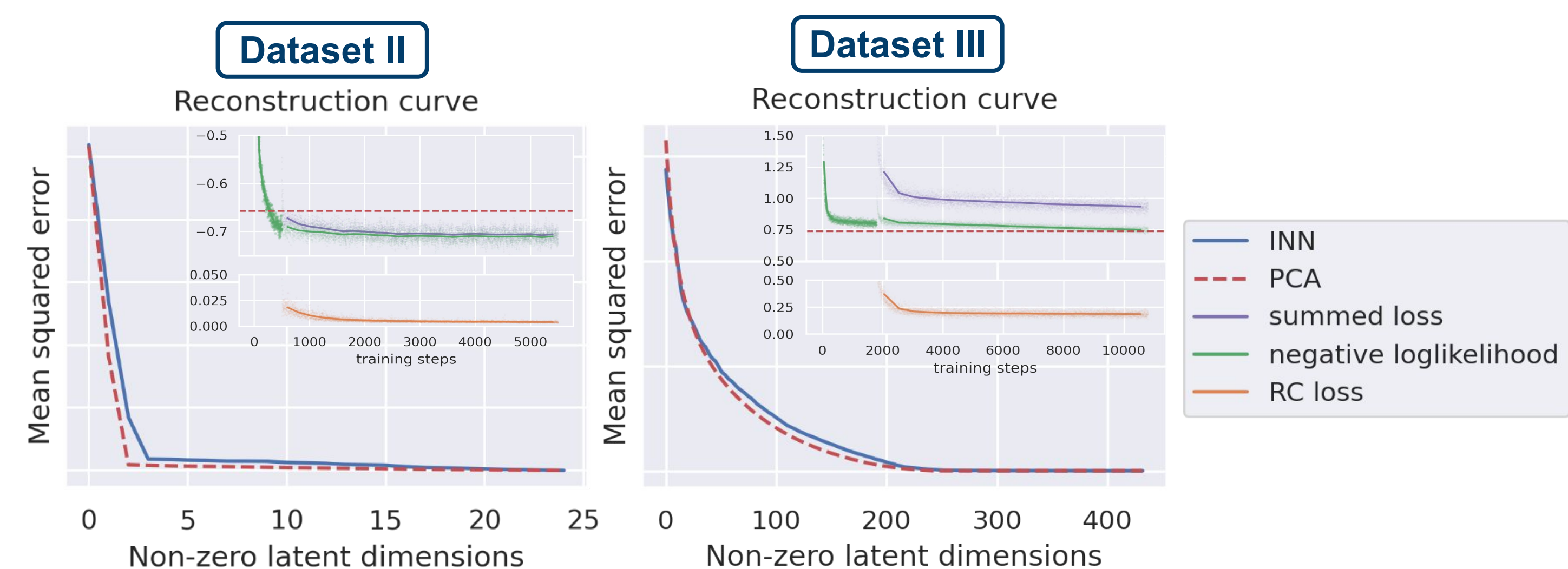
- Reconstructed data with  $i$  dimensions
- Mean squared error for  $i$  dimensions
- Loss term added to standard loss enforcing low-dimensional reconstruction

$$\tilde{x}_0(x) = f^{-1}(0)$$

$$R_i = \langle |\tilde{x}_i(x) - x|^2 \rangle_x$$

$$\mathcal{L}_{\text{dim}} = \gamma_{\text{dim}} \cdot \langle R_i \rangle_i$$

- For both data sets, INN and PCA have very similar performance → data is close to multivariate Gaussian
- Controlled data is clearly reconstructed by a few dimensions



## Data sets

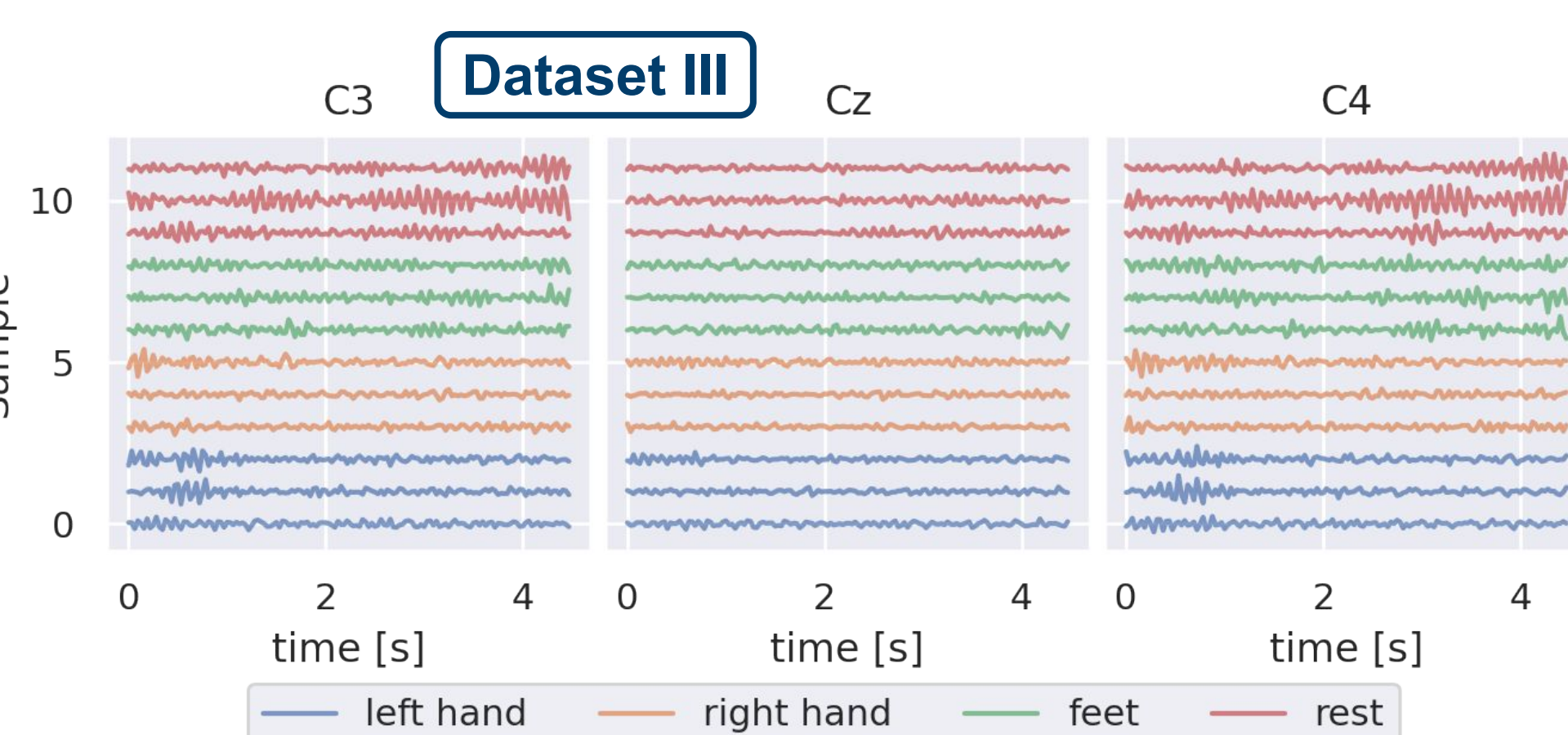
### I: Gaussian on Circle

- Phases and radii drawn from Gaussian distributions



### II: Oscillations

- Oscillations with phases drawn from bimodal Gaussian
- Added white noise

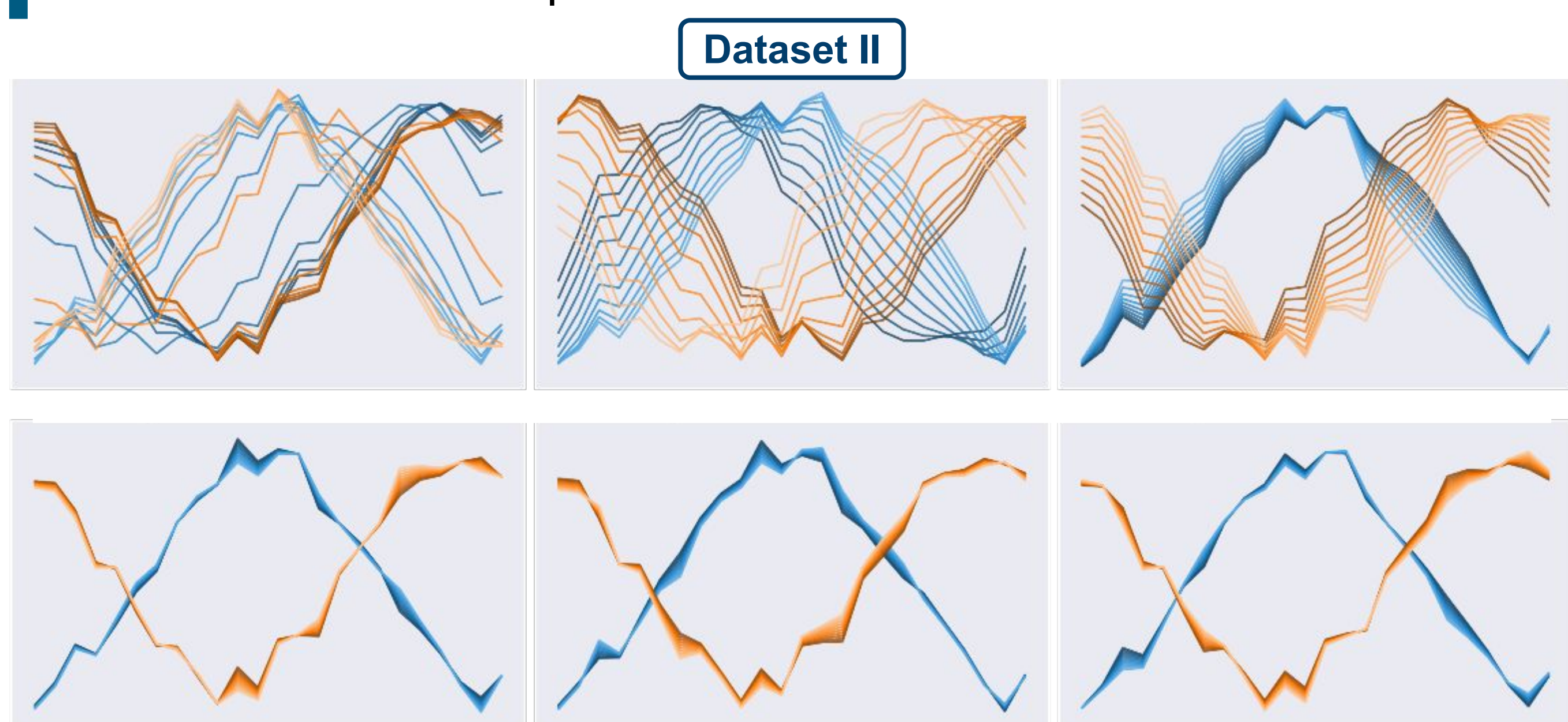


### III: EEG recordings

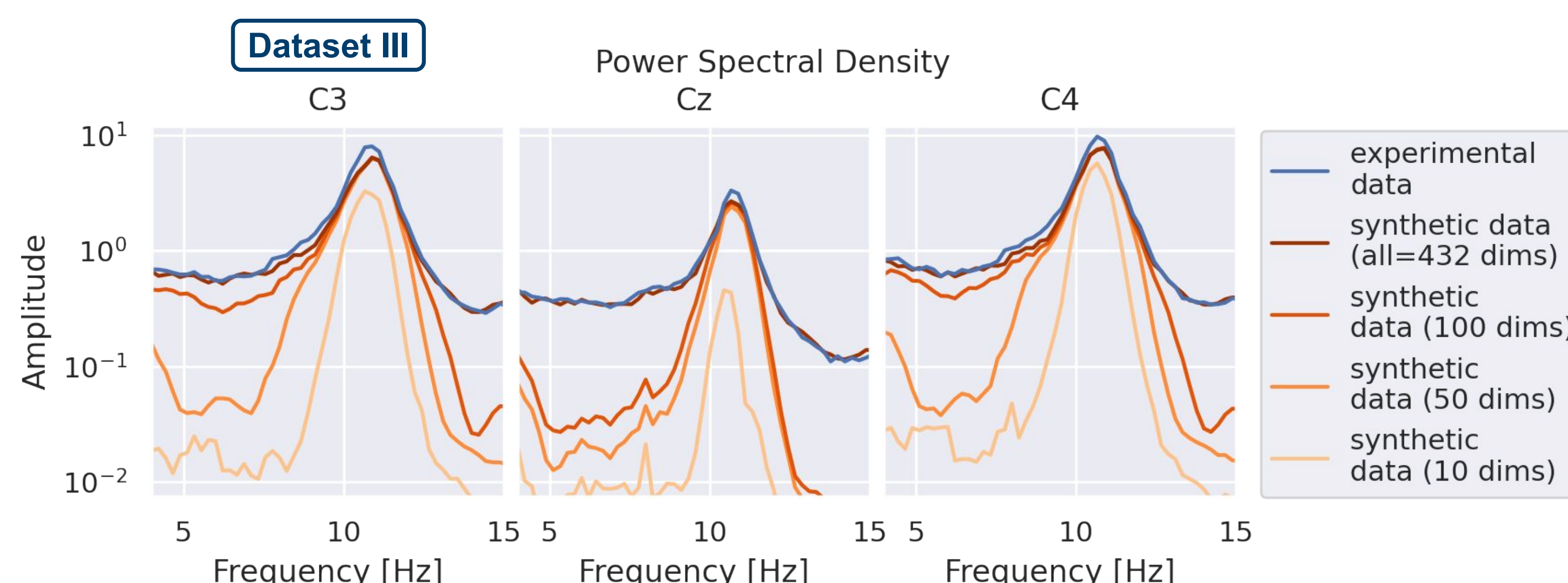
- High Gamma dataset from Tonio Ball's lab [1]
- Selected three channels subsampled to 32 Hz

## Identified latent dimensions enable effective reconstruction and sampling

- In controlled setting, only three latent dimensions modulate main information.
- Further dimensions provide the noise contribution.



- Few dominant dimensions ( $\leq 10$ ) contain prominent peak at 11 Hz.
- Dimensionality of  $< 30\%$  sufficient to reproduce spectra with high fidelity.



## References

- <sup>1</sup> Schirrneister et al. (2017). Deep learning with convolutional neural networks for EEG decoding and visualization. Human brain mapping.  
<sup>2</sup> Dinh et al. (2014). Nice: Non-linear independent components estimation. arXiv.

- <sup>3</sup> Dinh et al. (2016). Density estimation using real nvp. arXiv preprint arXiv.  
<sup>4</sup> Kingma & Dhariwal (2018). Glow: Generative flow with invertible 1x1 convolutions. Advances in neural information processing systems.