# Nonlinear dimensionality reduction with normalizing flows for analysis of electrophysiological recordings

Peter Bouss[1,2], Sandra Nestler[1,2], Kirsten Fischer[1,2], Claudia Merger[1,3], Alexandre René[1,3,4], Moritz Helias[1,3]

[1] Institute of Neuroscience and Medicine (INM-6) and Institute for Advanced Simulation (IAS-6) and JARA-Institute Brain Structure-Function Relationships (INM-10), Forschungszentrum Jülich, Jülich, Germany
[2] RWTH Aachen University, Germany
[3] Department of Physics, Faculty 1, RWTH Aachen University, Aachen, Germany
[4] Department of Physics, University of Ottawa, Ottawa, Canada

JÜLICH Forschungszentrum

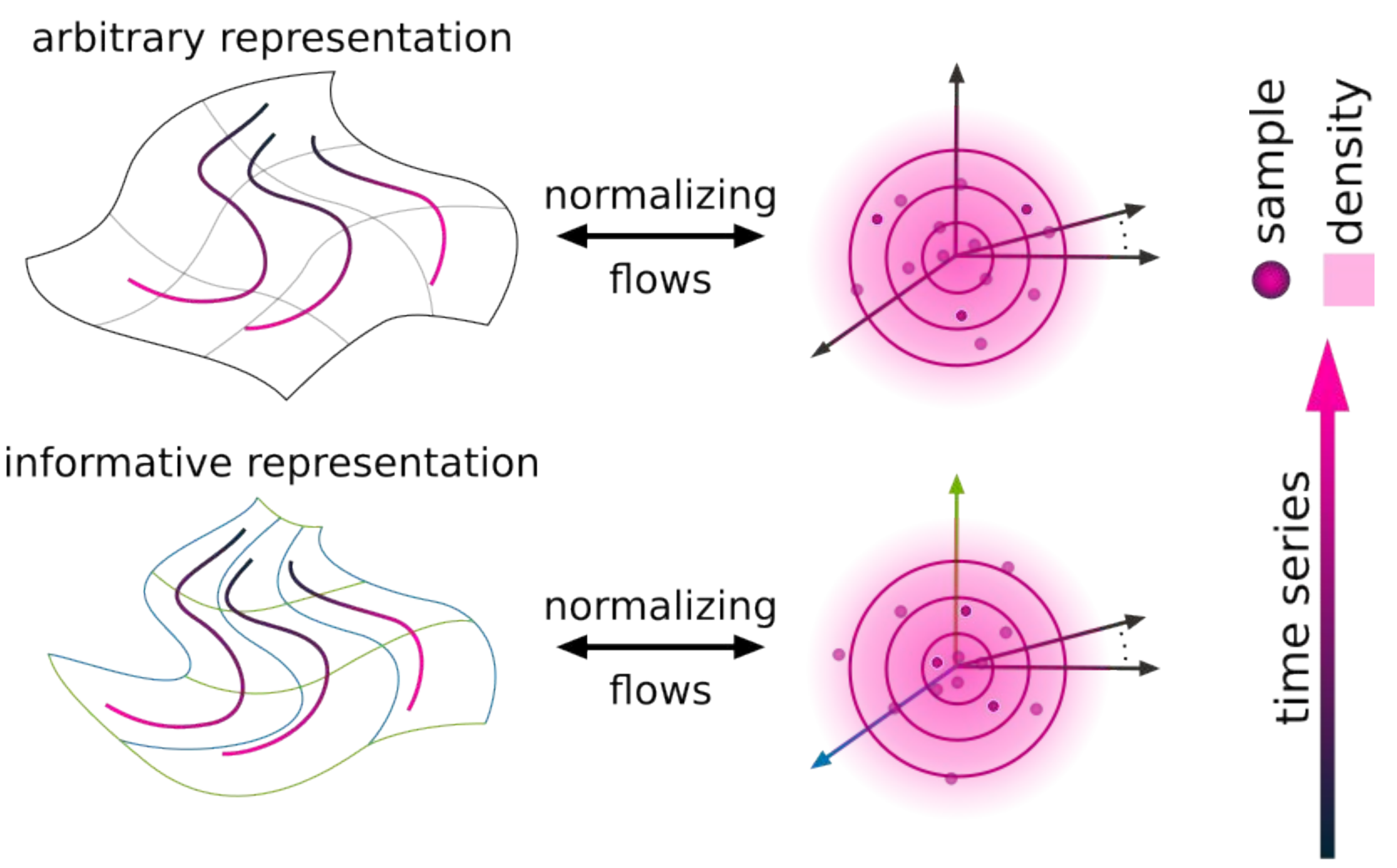
RTG 2416 MultiSenses Scales

RWTH AACHEN UNIVERSITY

## Summary

- We demonstrate nonlinear dimensionality reduction using invertible neural networks (INNs).
- This is achieved by enforcing reconstruction with few (meaningful) dimensions.
- The method shows comparable results to PCA on close to Gaussian data.
- The network models require much fewer parameters on larger datasets than linear models.

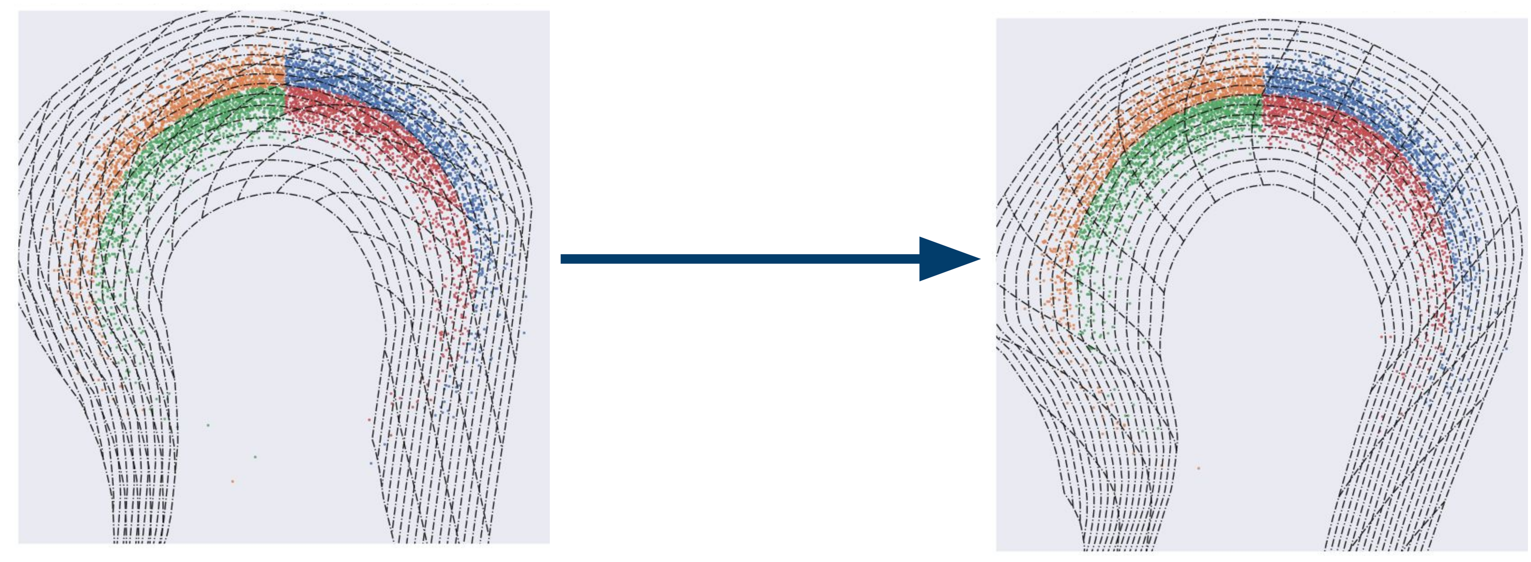
## Normalizing flows




- the standard normalizing flow is based on the log-likelihood loss [1,2]

$$\mathcal{L}^{\mathrm{ll}}(\theta) = -\left(\sum_{x\in X} \log\left(p_z(f_\theta(x))\right) + \log\left(\left|\det\left(\frac{\partial f_\theta(x)}{\partial x^T}\right)\right|\right)\right)$$

## Enforce meaningful latent dimensions



## Reconstruct data with few dimensions

- In the latent space, data features are entangled and use all dimensions.
- To reduce dimensionality, we identify data-relative dimensions in the latent space by projecting into subspaces of the latent space.

$$\tilde{z}_i(x) = \sum_{k=1}^{i} f_\theta^k(x)\hat{e}_k \qquad \tilde{x}_i(x) = f_\theta^{-1}(\tilde{z}_i(x))$$

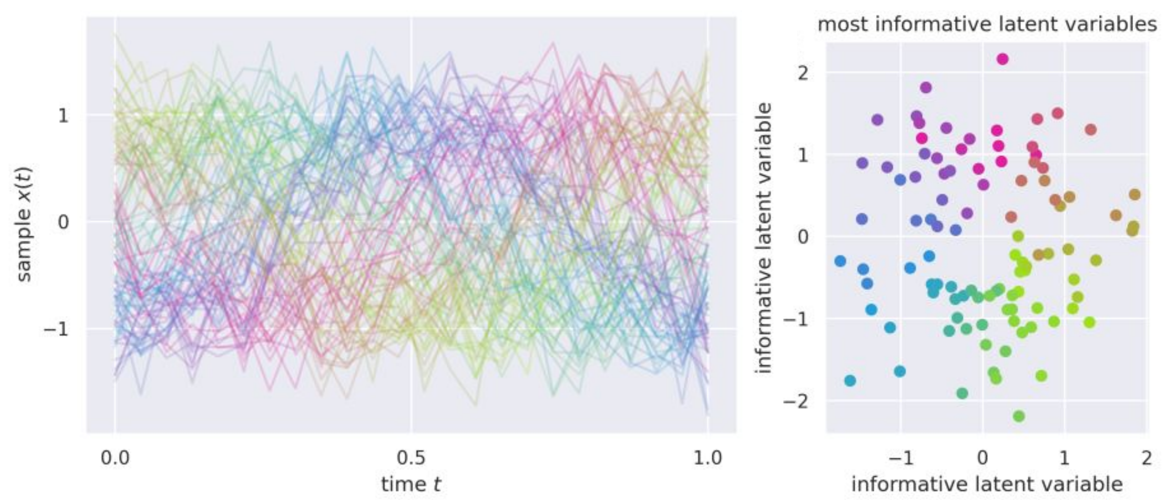
- the explained variance of these projections yields the extra loss

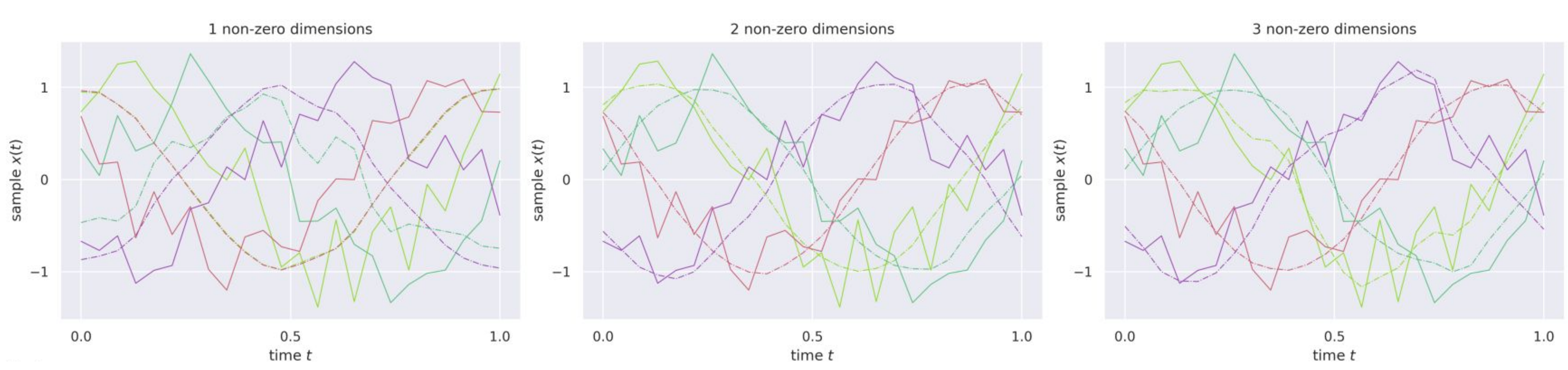
$$R_i = \left\langle |\tilde{x}_i(x) - x|^2 \right\rangle_x \qquad \mathcal{L}^{\mathrm{dim}}(\theta) = \gamma_{\mathrm{dim}} \frac{\sum_{i=1}^{N} R_i}{N}$$

## Identified latent dimensions enable effective reconstruction and sampling

### Oscillations

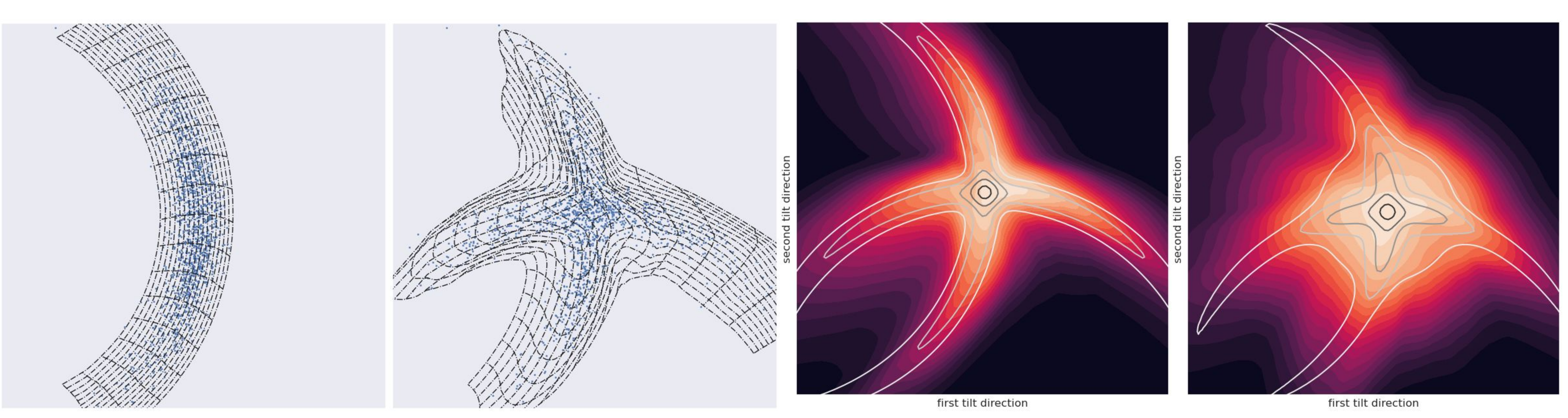
- Oscillations with phases drawn from bimodal Gaussian
- Added white noise
- Two latents reconstruct noise-free sines



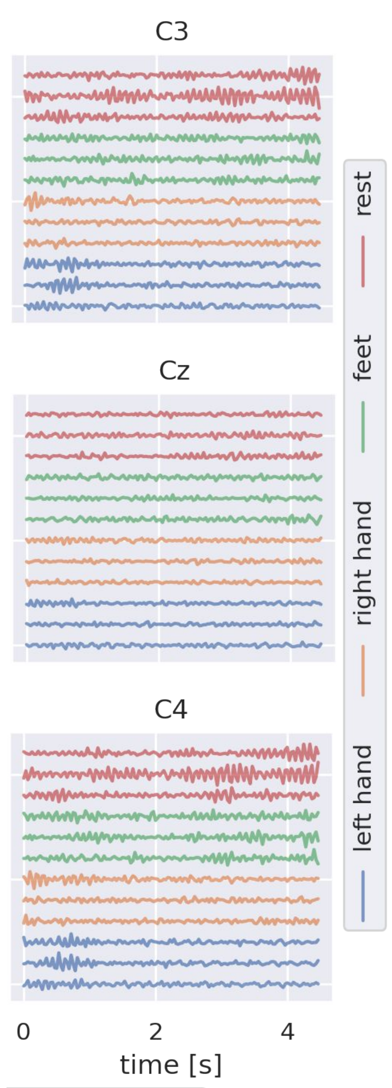

### Mixture distributions

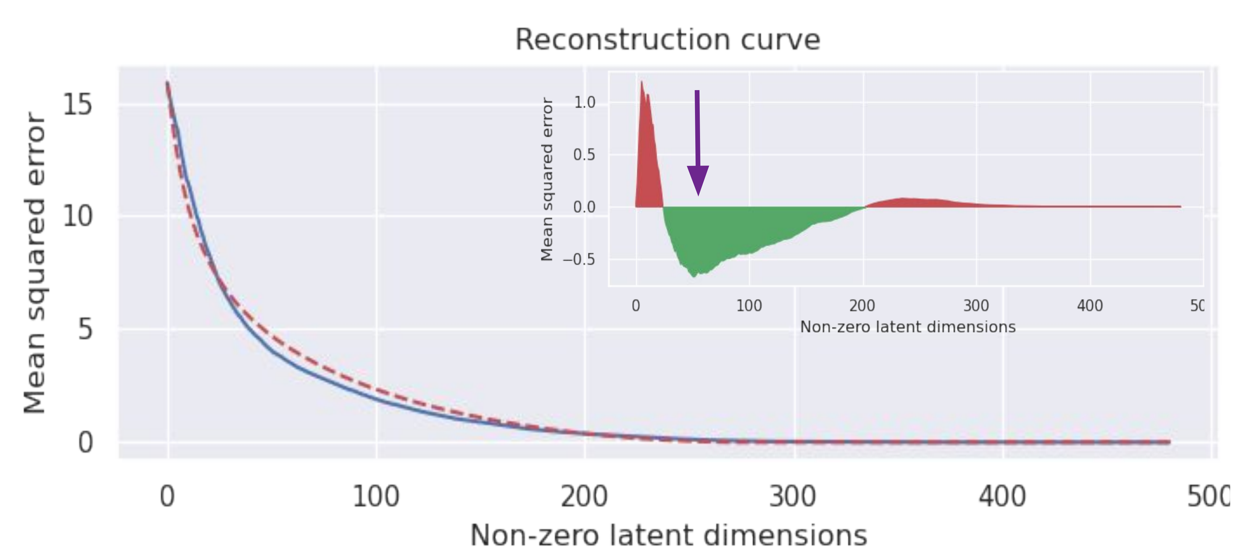
- N-dimensional curved manifolds [4] $p(x) \sim \exp(-\beta(\frac{1}{2\sigma}(\|x-x_0\|^2 - c^2)^2 + j^T(x-x_0)))$
- Learned density similar to ground truth, but smeared out

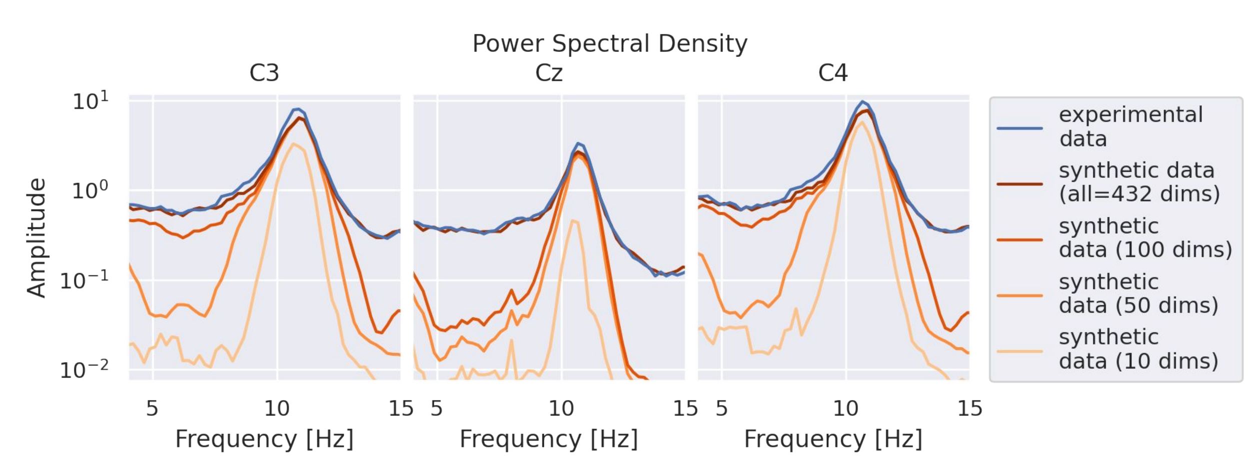


### EEG recordings

- High Gamma dataset [3], subsampled to 32 Hz, 3 selected channels
- Detect curved manifold underlying data
- Better reconstruction than PCA for 20 to 200 dimensions
- Few dominant dimensions (<=10) contain prominent peak at 11 Hz.
- Dimensionality < 30% sufficient to reproduce spectra with high fidelity.




## References

[1] Dinh et al. (2016). Density estimation using real NVP. arXiv:1605.08803.
[2] Kingma & Dhariwal (2018). Glow: Generative flow with invertible 1x1 convolutions. Advances in neural information processing systems.
[3] Schirrmeister et al. (2017). Deep learning with convolutional neural networks for EEG decoding and visualization. Human brain mapping.
[4] Nestler et al. (2023). Neuronal architecture extracts statistical temporal patterns. arXiv:2301.10203.