Linking Network and Neuron Level Correlations via Renormalized Field Theory

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Introduction

It is hypothesized that cortical networks operate close to a critical point. But which one? Popular candidates are a transition to chaos and an avalanche like criticality. Both have been studied separately, but we want to pave the way to a proper comparison in one model.

Model and DMFT breakdown

- focus on SCS model

$$\dot{x}_i + x_i = \sum_{j=1}^{N} J_{ij}\phi(x_j) + \xi_i,$$
$$J_{ij} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\frac{\bar{g}}{N}, \frac{g^2}{N})$$
$$\langle \xi_i(t)\xi_j(s)\rangle = D\,\delta_{ij}\,\delta(t-s)$$

- g^2 controls transition to chaos
  - already well studied
- $\bar{g}$ for avalanche like criticality
  - out focus here

Mean-field approaches on fields like

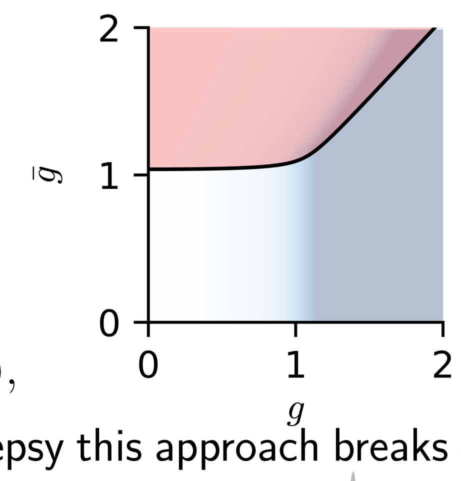
$$R(t) := \frac{\bar{g}}{N}\sum_{j=1}^{N}\phi_j(t),$$
$$Q(s,t) := \frac{g^2}{N}\sum_{j=1}^{N}\phi_j(s)\phi_j(t),$$

are common, giving a way to obtain an effective low-dimensional set of equations for the collective behavior. This means approximating

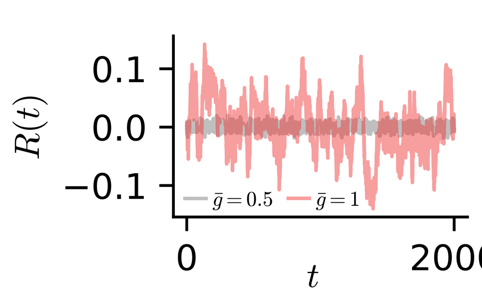
$$R^{\text{MF}}(t) = \bar{g}\mu_\phi(t) = \bar{g}\langle\phi(t)\rangle,$$
$$Q^{\text{MF}}(t,s) = g^2 C_{\phi\phi}(s,t)\rangle = g^2\langle\phi^2(s,t)\rangle,$$

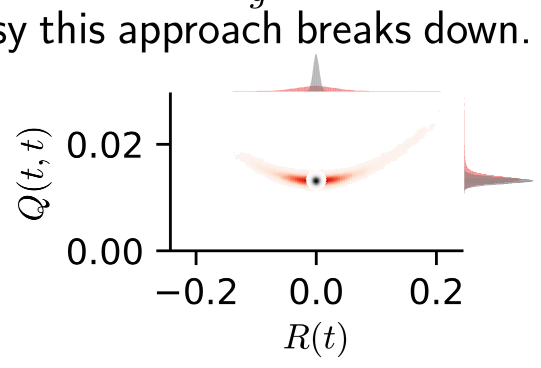
yielding a stochastic differential equation for a single Neuron

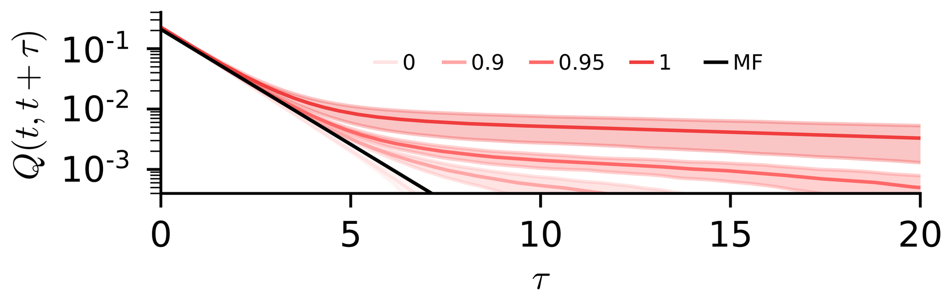
$$\dot{x} + x = \xi + \eta,$$
$$\langle\!\langle \eta(t)\rangle\!\rangle = \bar{g}\mu_\phi(t),$$
$$\langle\!\langle \eta(s)\eta(t)\rangle\!\rangle = g^2 C_{\phi\phi}(s,t),$$



near the transition to epilepsy this approach breaks down.







Thus renormalized theory...

Methods

- employ the MSRDJ formalism to get field theoretic description
- Insert sources for the first two cumulants of $y = (R, \tilde{R}, Q, \tilde{Q})$

$$Z[j,k] = \int_y e^{N\mathcal{W}[y] + j^{\mathrm{T}}y + \frac{1}{2}y^{\mathrm{T}}k^{\mathrm{T}}y},$$

- perform a 2nd order Legendre transform
  - create an ensemble described by cumulants
  - sources as observables

$$\Gamma[\beta_1,\beta_2] = \text{extr}_{\hat{j},k} - \ln\int_y e^{N\mathcal{W}[y] + \hat{j}^{\mathrm{T}}(y-\beta_1) + \frac{1}{2}k^{\mathrm{T}}[(y-\beta_1)^2 - \beta_2]},$$

- approximate the resulting effective action to 1-loop $(1/N)$ order.

$$\Gamma_{\text{1-loop}}[\beta_1,\beta_2] = -N\,\mathcal{W}[\beta_1] - \frac{1}{2}N\,\mathcal{W}^{(2)}[\beta_1]^{\mathrm{T}}\beta_2 + \frac{1}{2}\ln\det(\beta_2),$$

- solve such that the real sources a realised
- thus create system of self consistent equations for the cumulants

$$\beta_{11}(\omega) = \frac{1+\omega^2}{(1-\bar{g}\langle\phi'\rangle)^2 + \omega^2}\frac{\bar{g}^2}{N}\langle\!\langle \phi,\phi\rangle\!\rangle_*(\omega).$$

- here we can see that time constants diverge if $\bar{g}\langle\phi'\rangle = 1$

Finally we get:

$$\begin{aligned} Q^*(s,t) =& g^2\langle\!\langle \phi^2(s,t)\rangle\!\rangle_* \\ &+ \frac{1}{2}g^2\int_{u,v}\langle\!\langle \phi^2(s,t),\tilde{x}(u),\tilde{x}(v)\rangle\!\rangle_*\beta_{11}(u,v) \\ &+ g^2\bar{g}\int_{u,v}\langle\!\langle \phi^2(s,t),\tilde{x}(u),\phi(v)\rangle\!\rangle_*\beta_{12}(u,v) \\ &- \frac{g^4}{2}\int_{u_{1,2},v_{1,2}}\langle\!\langle \phi^2(s,t),\tilde{x}^2(u_1,u_2),\phi^2(v_1,v_2)\rangle\!\rangle_* \\ &\beta_{34}(u_1,u_2,v_1,v_2) \end{aligned}$$

Interpretation:

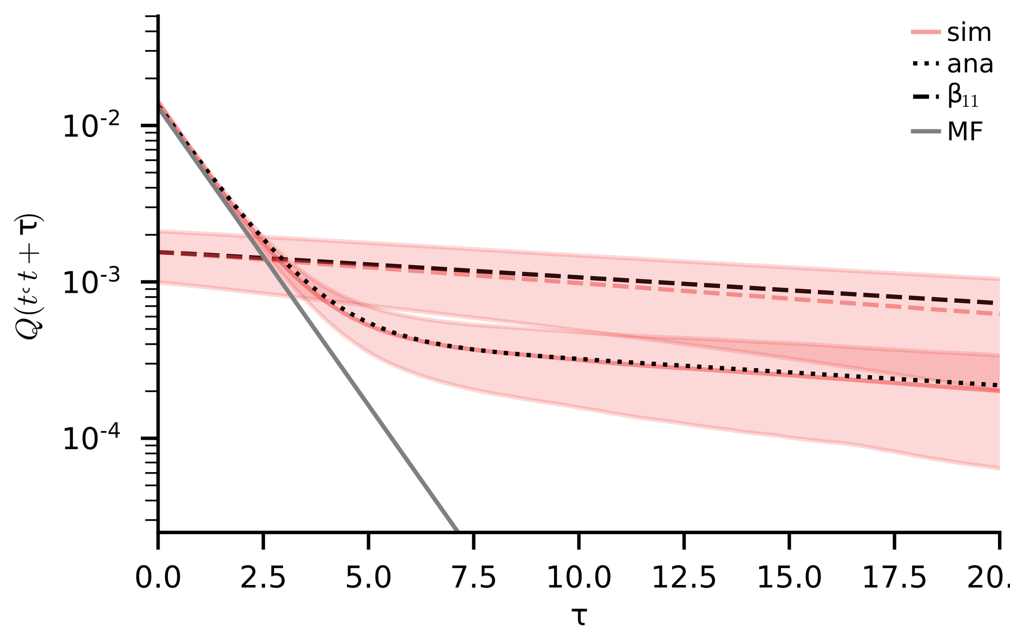
- mean-field neuron interacting with a bath
- first contribution equivalent to mean-field
- second contribution is the effect of fluctuations of R on variance of mean-field neurons input
- third term the effect of fluctuations neuronal activity echoed in the bath on the variance
- fourth term also echoes but due to fluctuations of ϕ^2

interesting Appendix things

- critical coupling strength in DMFT for $\phi(x) = \text{erf}(\sqrt{\pi}x/2)$
- $\langle \prod_i^n \tilde{x}(s_i)\prod_j^m \phi(x(t_j))\rangle$ in terns of response functions and expectation values of derivatives any ϕ which is zero at the origin
  - $\langle\langle \phi\tilde{x}\tilde{x}\rangle\rangle \propto \langle\phi''\rangle$
- Legendre transformation in cumulants instead of moments
- completely analytical results for linear network

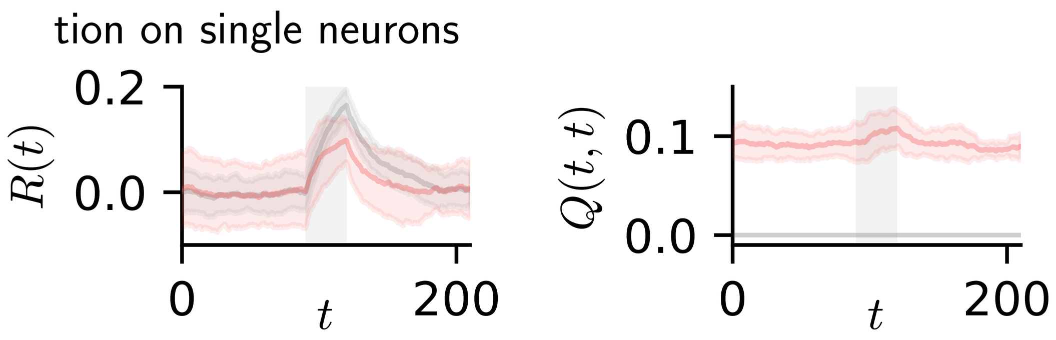
Results

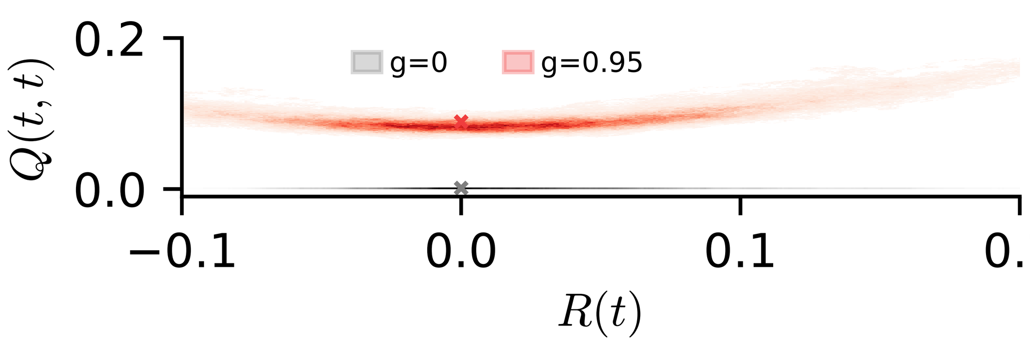
- Here we set $\phi(x) = \text{erf}(\sqrt{\pi}x/2)$
- consider only the corrections due to $\beta_{11}(t-s) = \langle\langle R(t)\,R(s)\rangle\rangle$



our theory

- describes the autocorrelation very well
- clearly shows the influence of population autocorrelation on single neurons





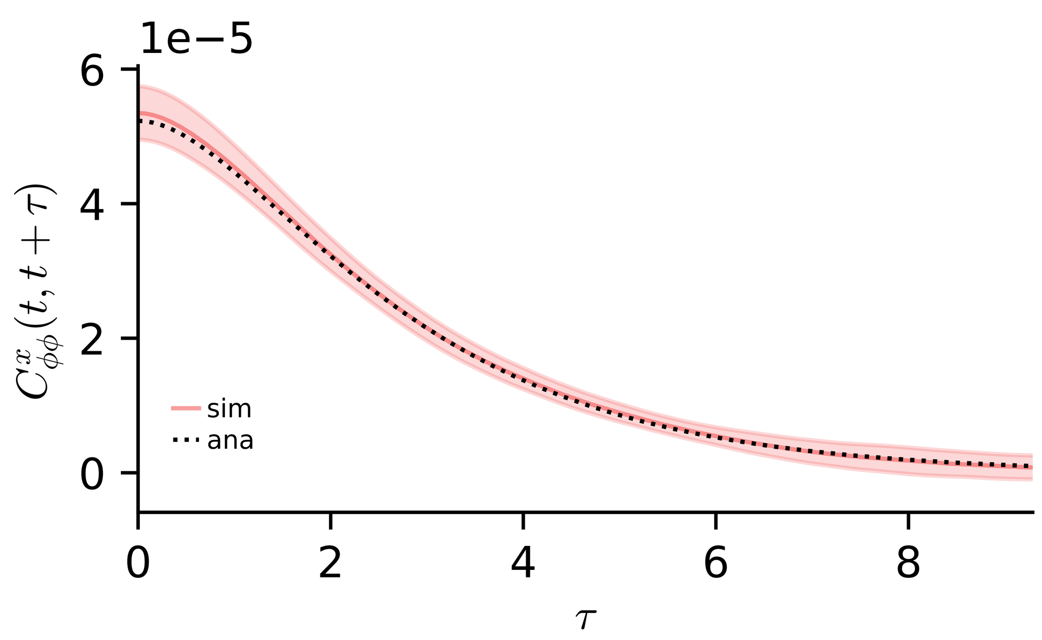
under common input:

- network with $g = 0$ behaves like capacitor
- for $g > 0$ increase in Q counteracts rise in R

$$\mu_\phi = \phi\Big(\frac{\mu_x}{\sqrt{1+\frac{\pi}{2}\sigma_x^2}}\Big)$$

Access to Q and β_{11} allows us to also calculate the cross correlations:

$$C^x_{\phi\phi}(t-s) := \frac{1}{N^2}\sum_{i\neq j}\phi_i(s)\phi_j(t) = \frac{\beta_{11}(s,t)}{\bar{g}^2} - \frac{Q(s,t)}{Ng^2}$$



References

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