Investigating various possibilities to solve the Hubbard model using the kinetic energy part for quantum annealing

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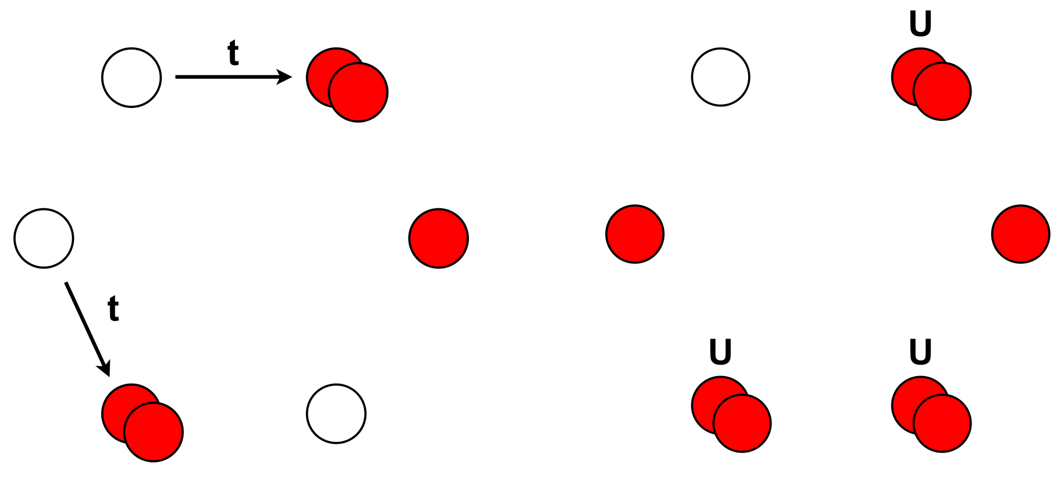
Goal

To find the ground-state of the Hubbard Hamiltonian for various fillings and total Spin-Z

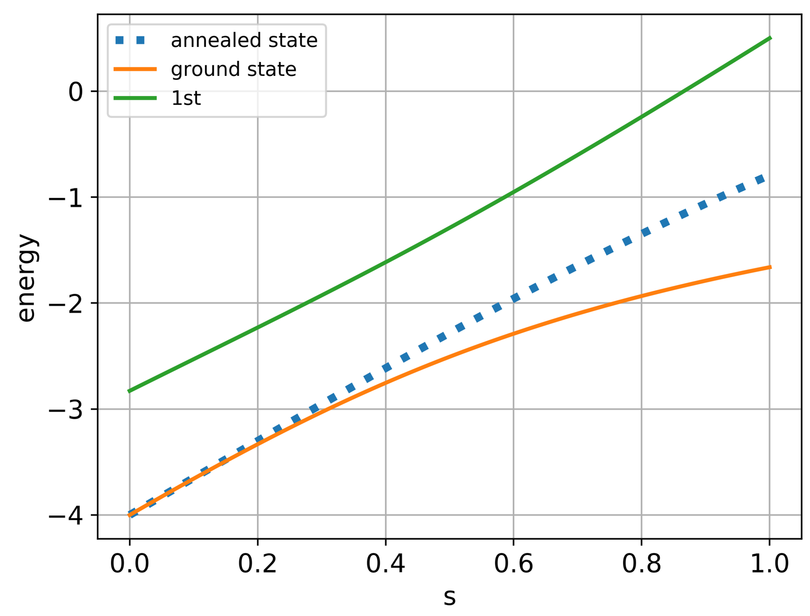
$$H_H = -t \sum_{\langle i,j\rangle,\sigma}^{L} (c^\dagger_{i\sigma} c_{j\sigma} + c^\dagger_{j\sigma} c_{i\sigma}) + U \sum_{i}^{L} n_{i\downarrow} n_{i\uparrow}$$



How?

Use quantum annealing with a driving Hamiltonian (H_D)

$$H(s) = (1-s)H_D + sH_H$$

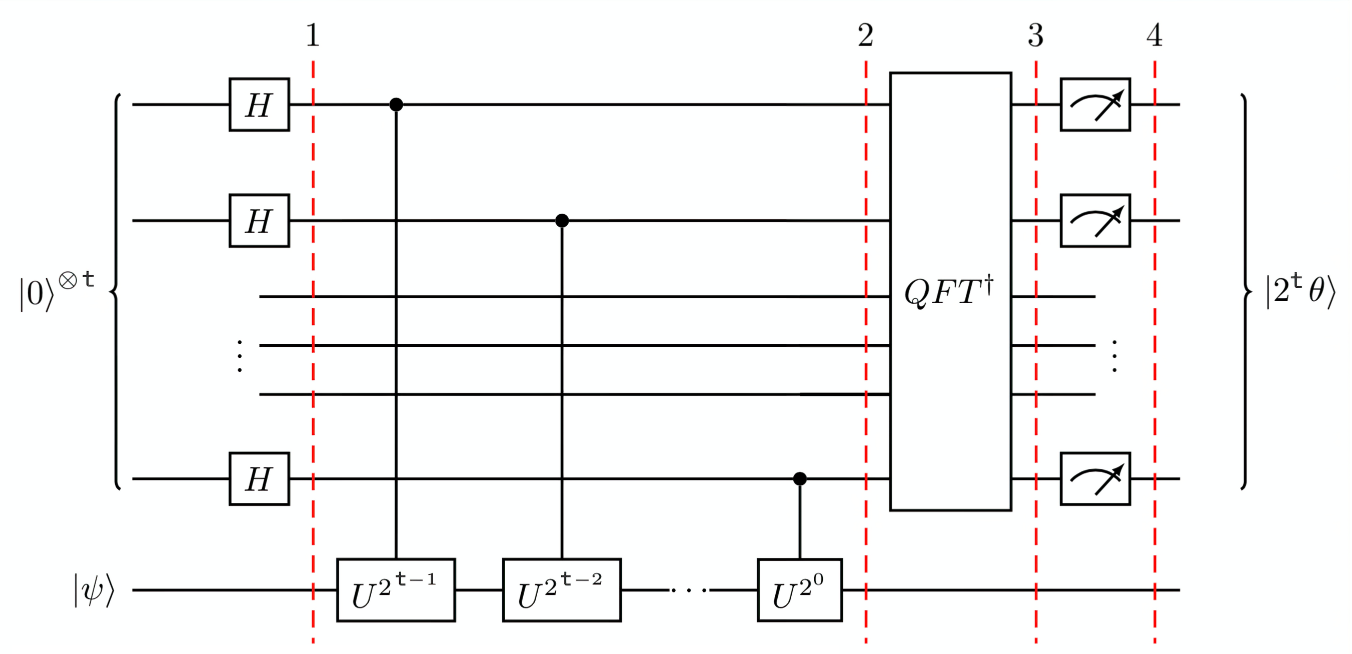


- With a good choice of the driving Hamiltonian, we expect to find a spectrum of the evolving Hamiltonian such that there are gaps between the ground state and the excited states.
- One of the possible choices for this is the kinetic energy part of the Hubbard model.

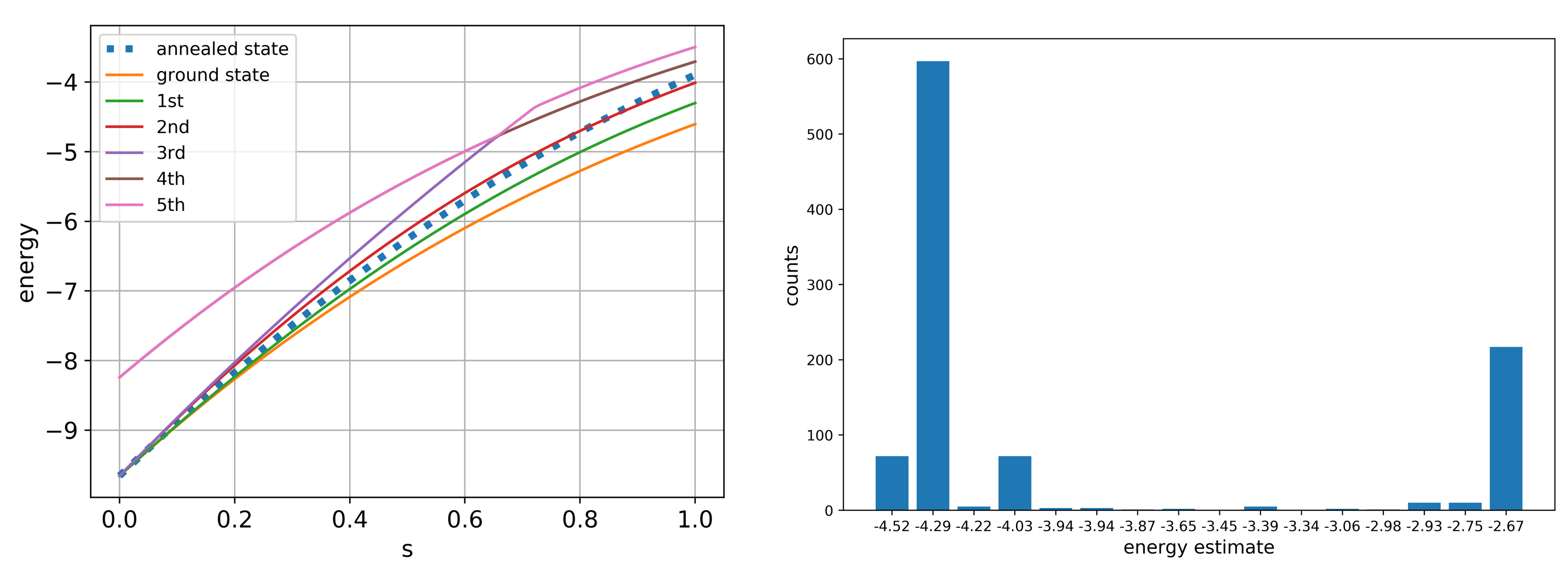
$$H_D = -t \sum_{\langle i,j\rangle,\sigma}^{L} (c^\dagger_{i\sigma} c_{j\sigma} + c^\dagger_{j\sigma} c_{i\sigma})$$

Quantum Phase Estimation

For some cases, the ground state of the driving Hamiltonian is degenerate. QPE routine can be used after annealing to sample out the ground state for such cases.



The figure shows the quantum circuit for phase estimation



The figure on the left shows the spectrum for system with L = 8, $\sum n_\uparrow = 4$, $\sum n_\downarrow = 4$
The figure on the right shows QPE sampling for the same with 4 ancilla qubits

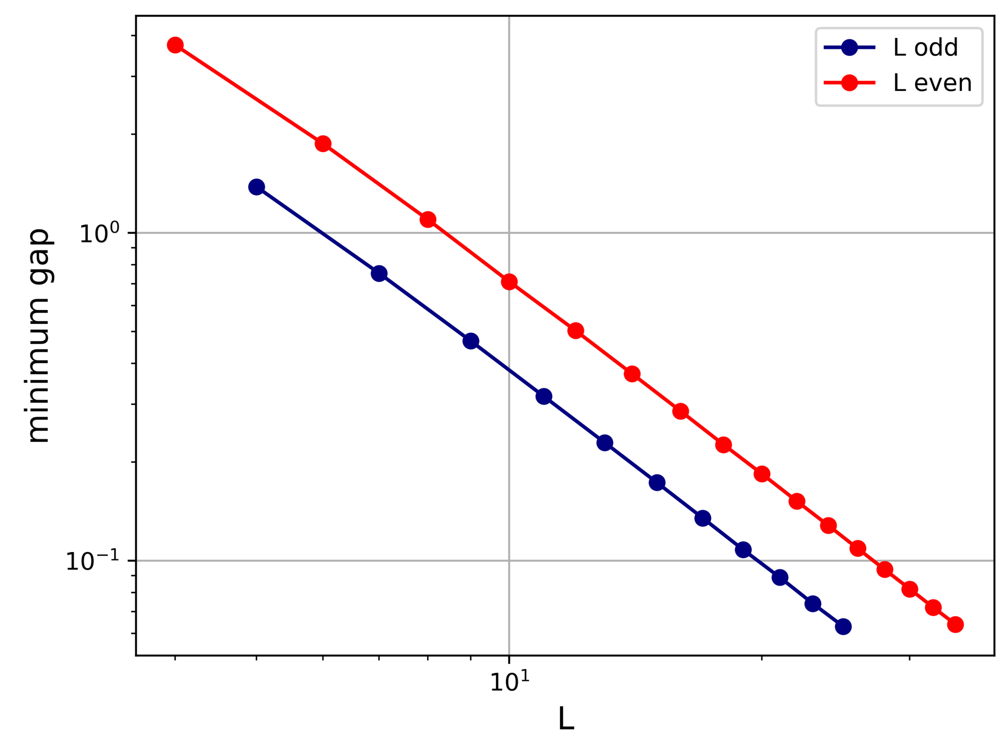
Minimum gap

Due to translational invariance, only states that have the same total momentum can get excited. The momentum operator can be diagonalized simultaneously with the Hamiltonian thereby enabling the calculation of minimum gap for certain cases.

$$\Pi = \phi\left(\sum_{m=1}^{L-1} \frac{1}{2} + \frac{S^m}{e^{-i\phi m} - 1}\right)$$

where S is the operator that generates a shift on the lattice by 1 site

The figure shows how minimum gap with the relevant state scales with system size for half-filled systems with $\sum n_\downarrow = 1$



Observations

- The kinetic energy part of the Hubbard model can be used as a driving Hamiltonian to calculate the ground state.
- For cases with degeneracy, quantum phase estimation would be required after quantum annealing to sample the ground state.
- The scaling of minimum gap with system size for half-filled systems with $\sum n_\downarrow = 1$ is a polynomial decrease which would mean that such cases can be solved in polynomial time.

Further plans

- To study minimum gap scaling for different values of $\sum n_\downarrow$.
- Further, we plan to investigate such scaling for 2 dimensional systems as well.
- Finally, we want to investigate the effect of environmental temperature on the quantum annealing process. We plan to do this by coupling the annealing Hamiltonian to some bath and then study how the system evolves.

$$H = H(s) + H_B + H_I$$

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