GUIDED QUANTUM WALK

A New Algorithm between Quantum Walk and Quantum Annealing

7. NOVEMBER 2023 I DENNIS WILLSCH

Guided quantum walk

Sebastian Schulz,[1,*] Dennis Willsch,[1] and Kristel Michielsen[1,2,3]

[1] *Jülich Supercomputing Centre, Institute for Advanced Simulation, Forschungszentrum Jülich, 52425 Jülich, Germany*
[2] *AIDAS, 52425 Jülich, Germany*
[3] *RWTH Aachen University, 52056 Aachen, Germany*

arXiv:2308.05418

WE COMPUTE WITH QUANTUM COMPUTERS

supported by Bundesministerium für Bildung und Forschung
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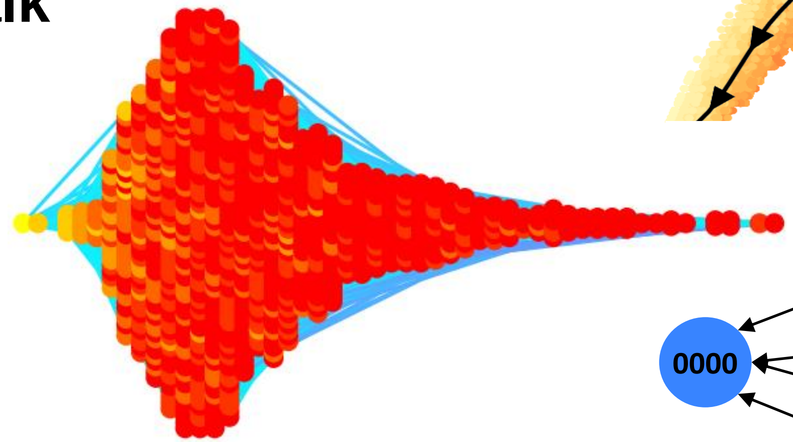
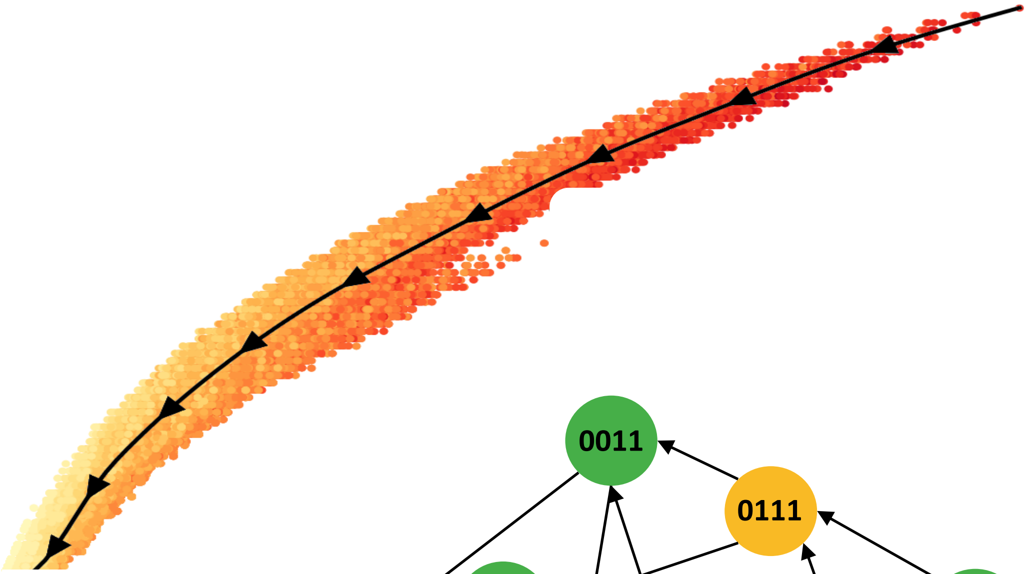
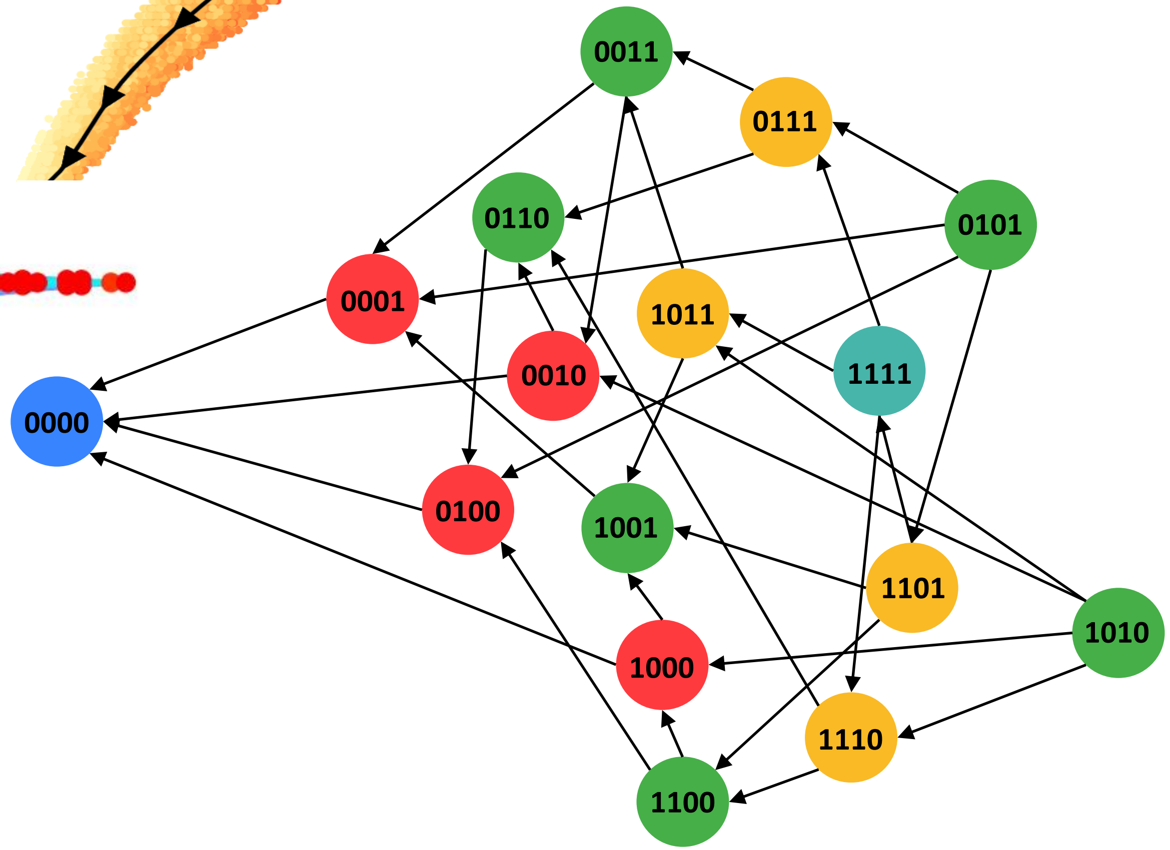
Contents

- **Local Amplitude Transfer (LAT) Theory**
- **Guided Quantum Walk**
  - Theoretical GQW
  - Practical GQW
- **Performance**
  - Scaling Analysis
  - Comparison to QA, QW, QAOA, AQA
  - Accounting for Classical Optimization Phase







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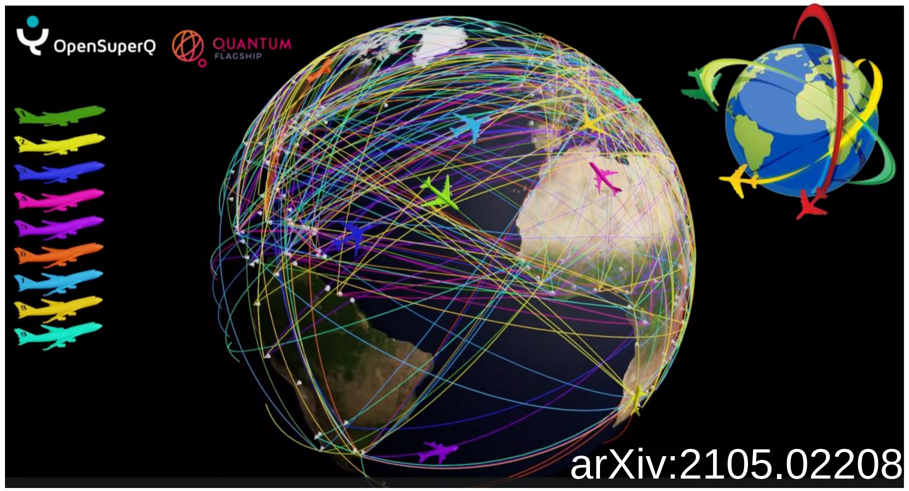
Local Amplitude Transfer (LAT) Theory

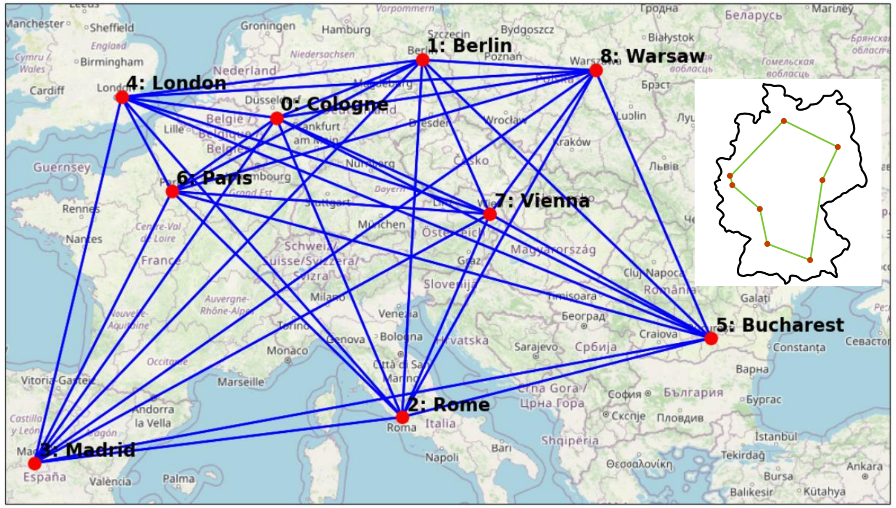
- ➢ Hamiltonian:

$$H_{GQW}(t) = \underbrace{\Gamma(t)}_{\text{Time-dependent hopping rate}} \cdot \underbrace{\left(-\sum_{i=1}^{N}\sigma_i^x\right)}_{\text{Driver Hamiltonian } H_D} + \underbrace{\left(-\sum_{i=1}^{N} h_i\sigma_i^z + \sum_{i\neq j} J_{ij}\sigma_i^z\sigma_j^z\right)}_{\text{Cost Hamiltonian } H_C}$$

- ➢ We consider three problems:



Exact Cover (EC)
from Tail Assignment Problems



Traveling Salesperson (TSP)

Garden Optimization (GO)

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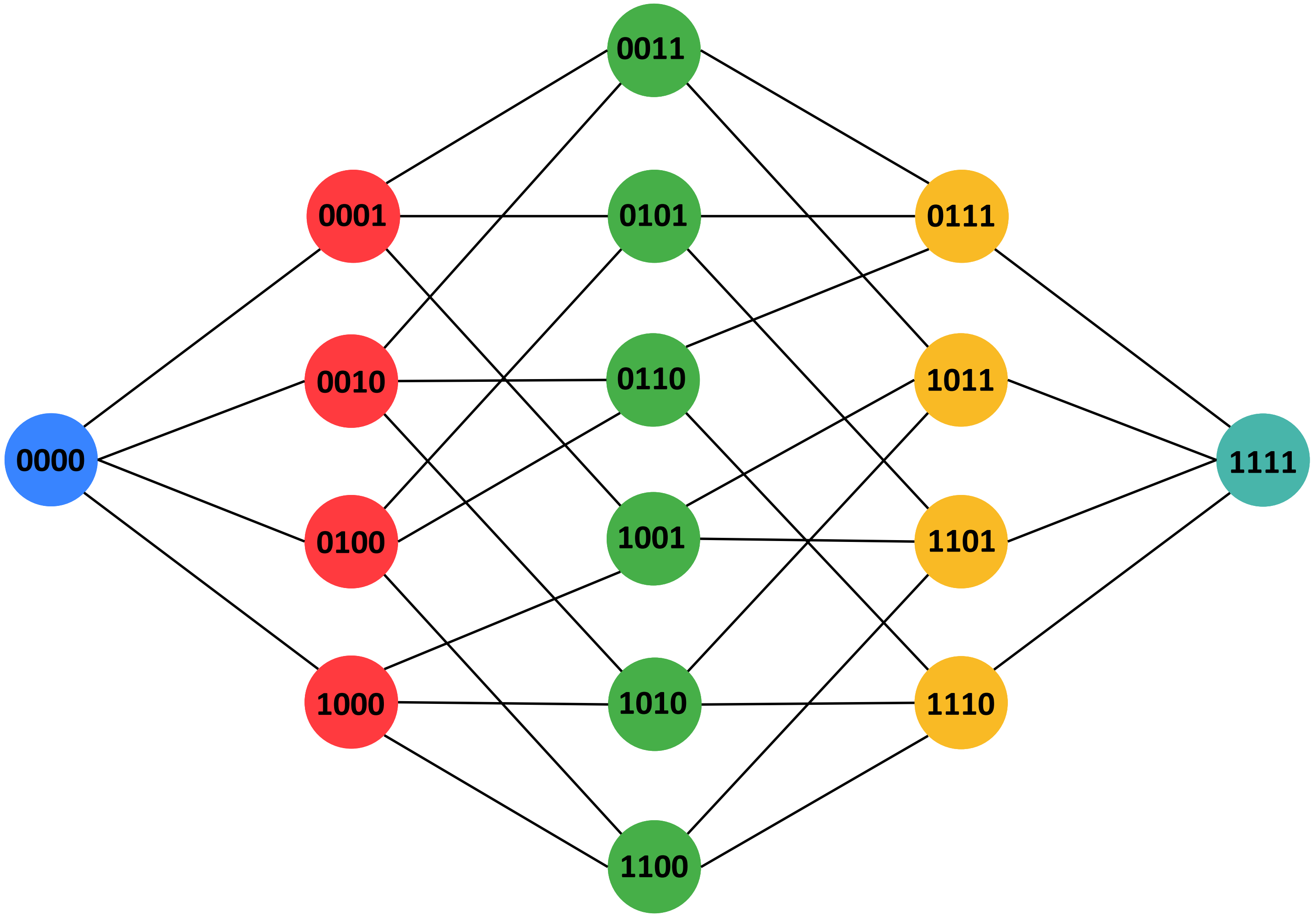
Local Amplitude Transfer (LAT) Theory

$$H_{GQW}(t) = \Gamma(t) \cdot \underbrace{\left(-\sum_{i=1}^{N} \sigma_i^x\right)}_{H_D} + \underbrace{\left(-\sum_{i=1}^{N} h_i \sigma_i^z + \sum_{i \neq j} J_{ij} \sigma_i^z \sigma_j^z\right)}_{H_C}$$

Generic Energy Spectrum of H_C

- States with Hamming distance 1 are coupled by H_D
- Order states by energy E



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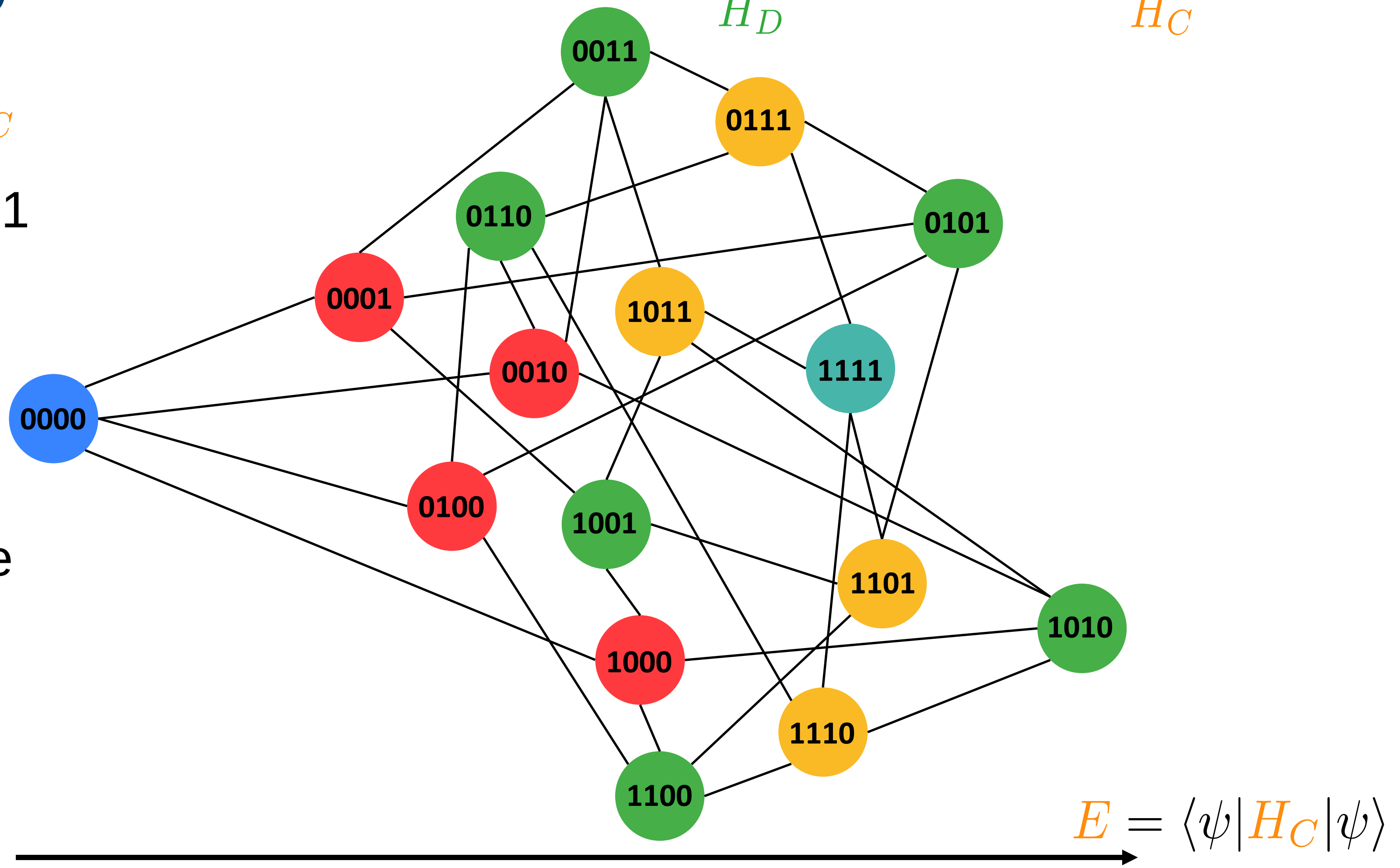
- **Goal:** Drive probability amplitude towards ground state



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Generic Energy Spectrum of H_C

- States with Hamming distance 1 are coupled by H_D
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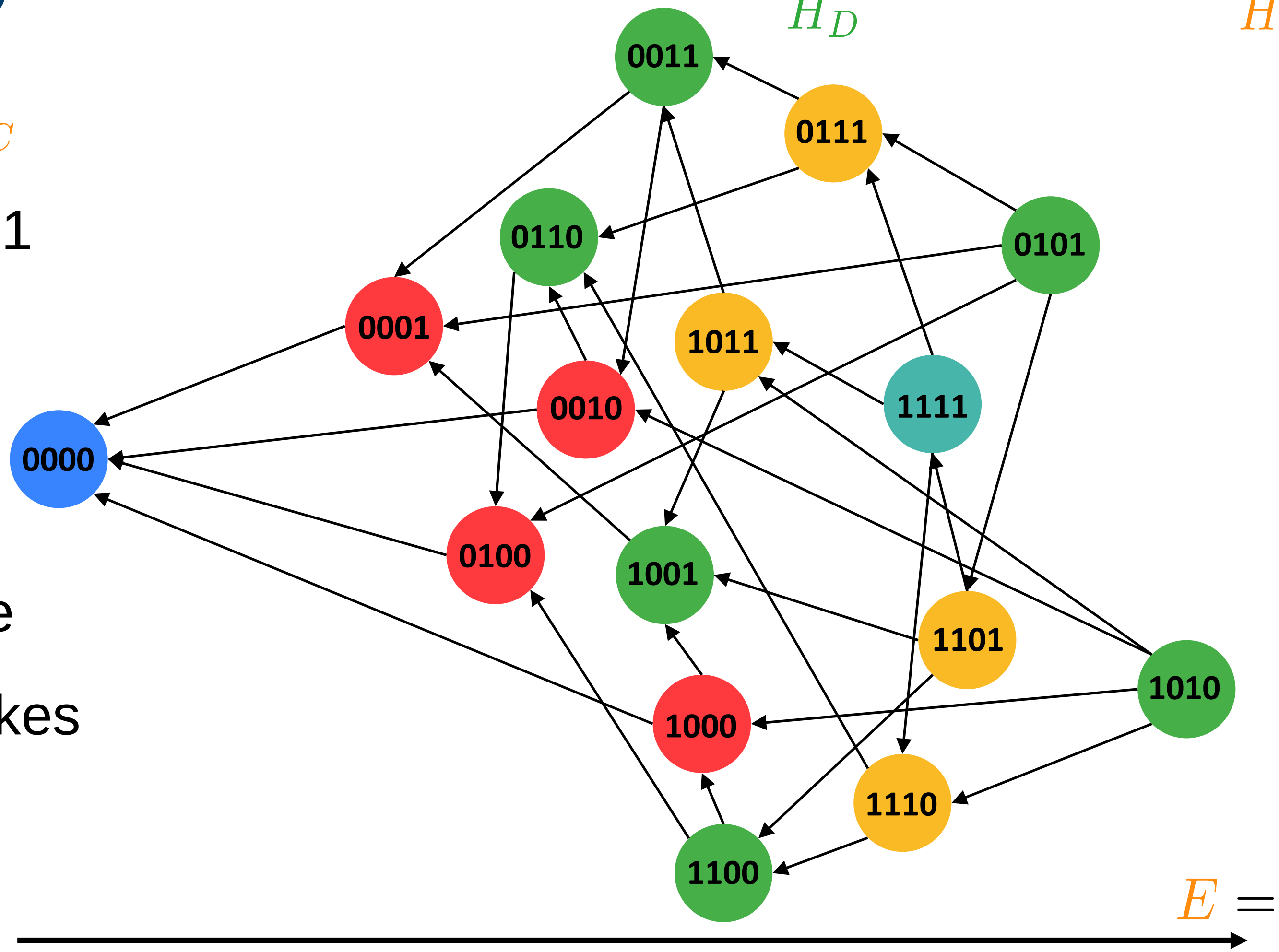
- **For large problems:** Graph takes generic "Onion" shape



$$E = \langle \psi | H_C | \psi \rangle$$

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Local Amplitude Transfer (LAT) Theory

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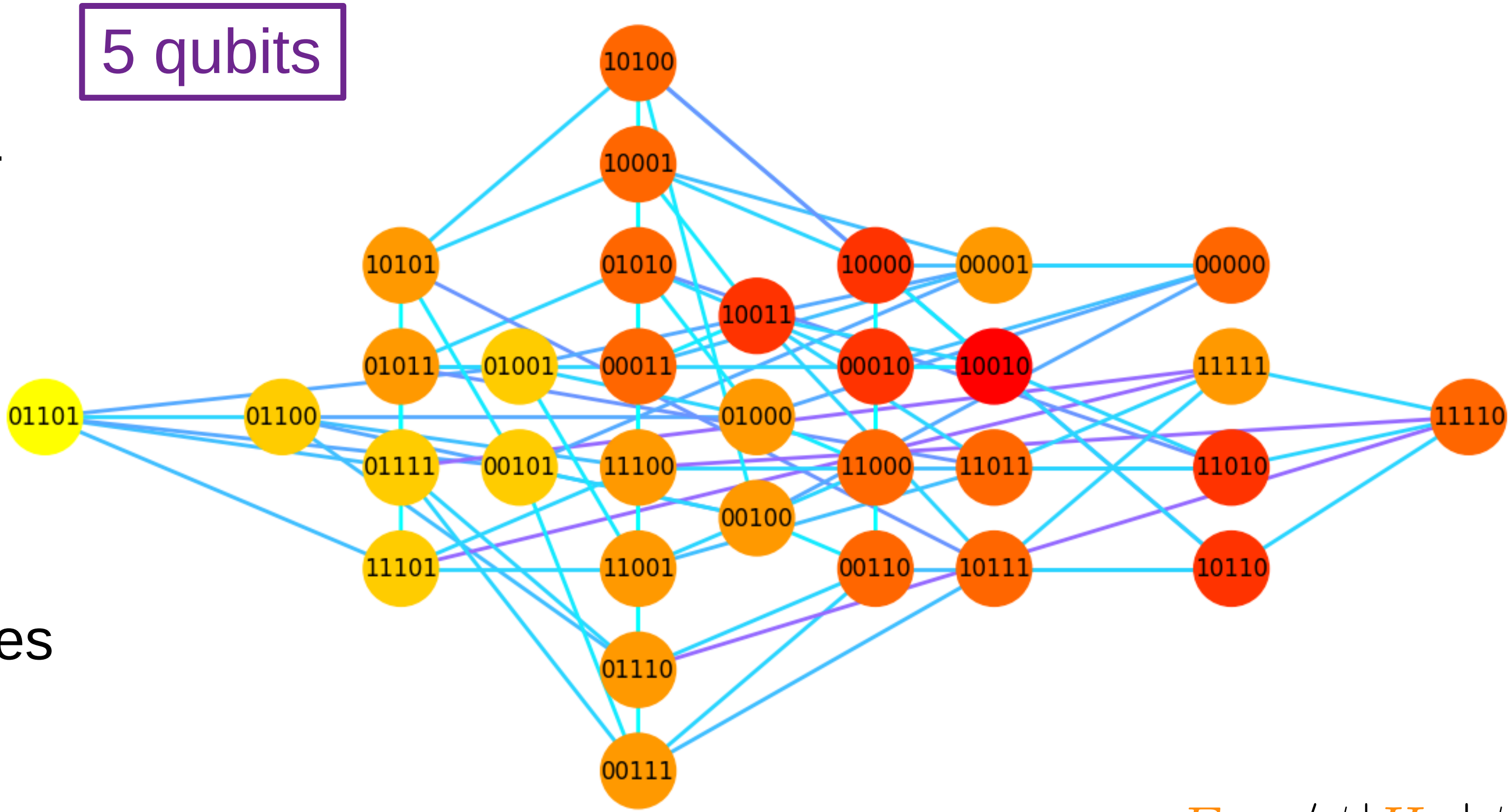

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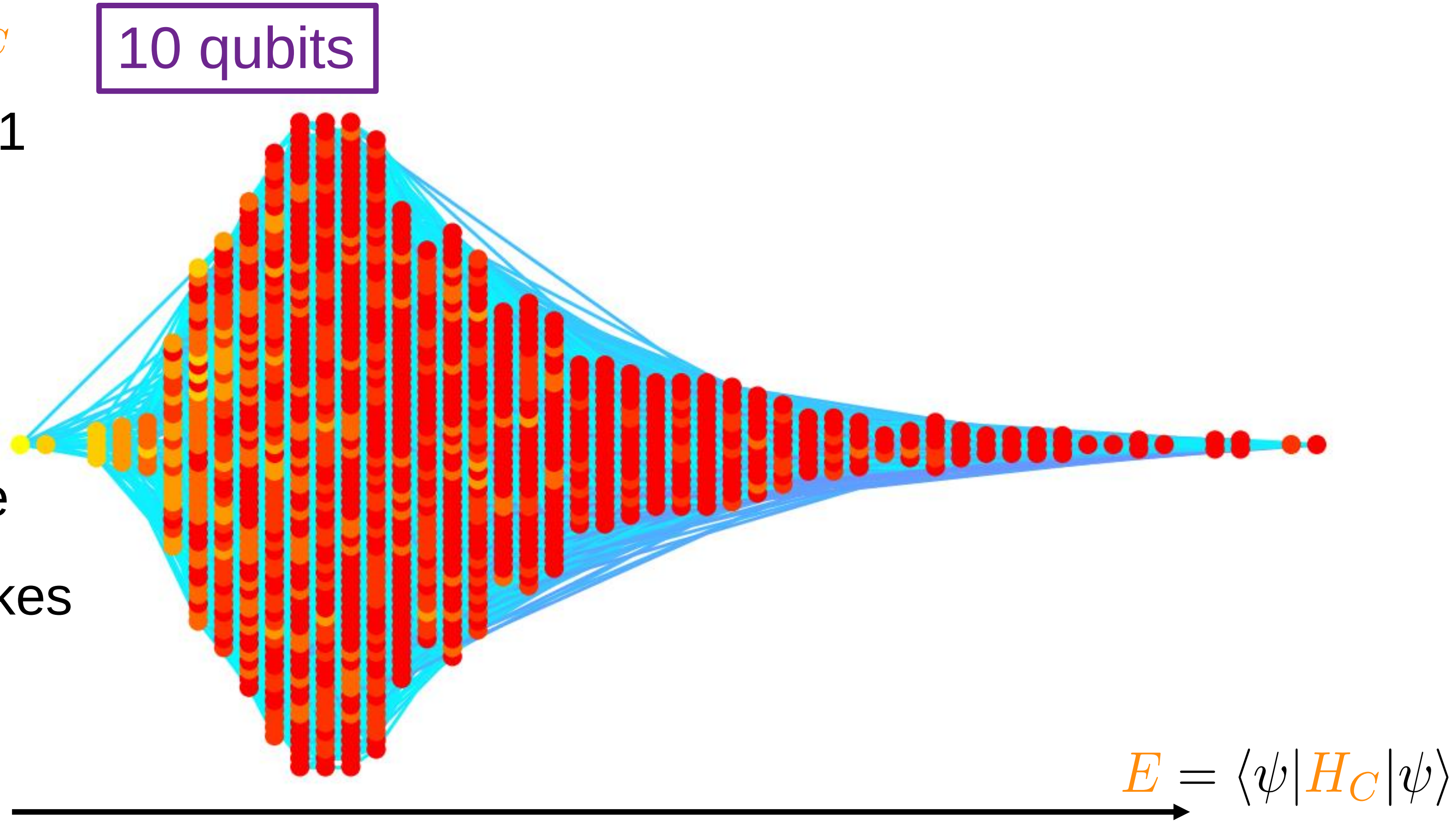

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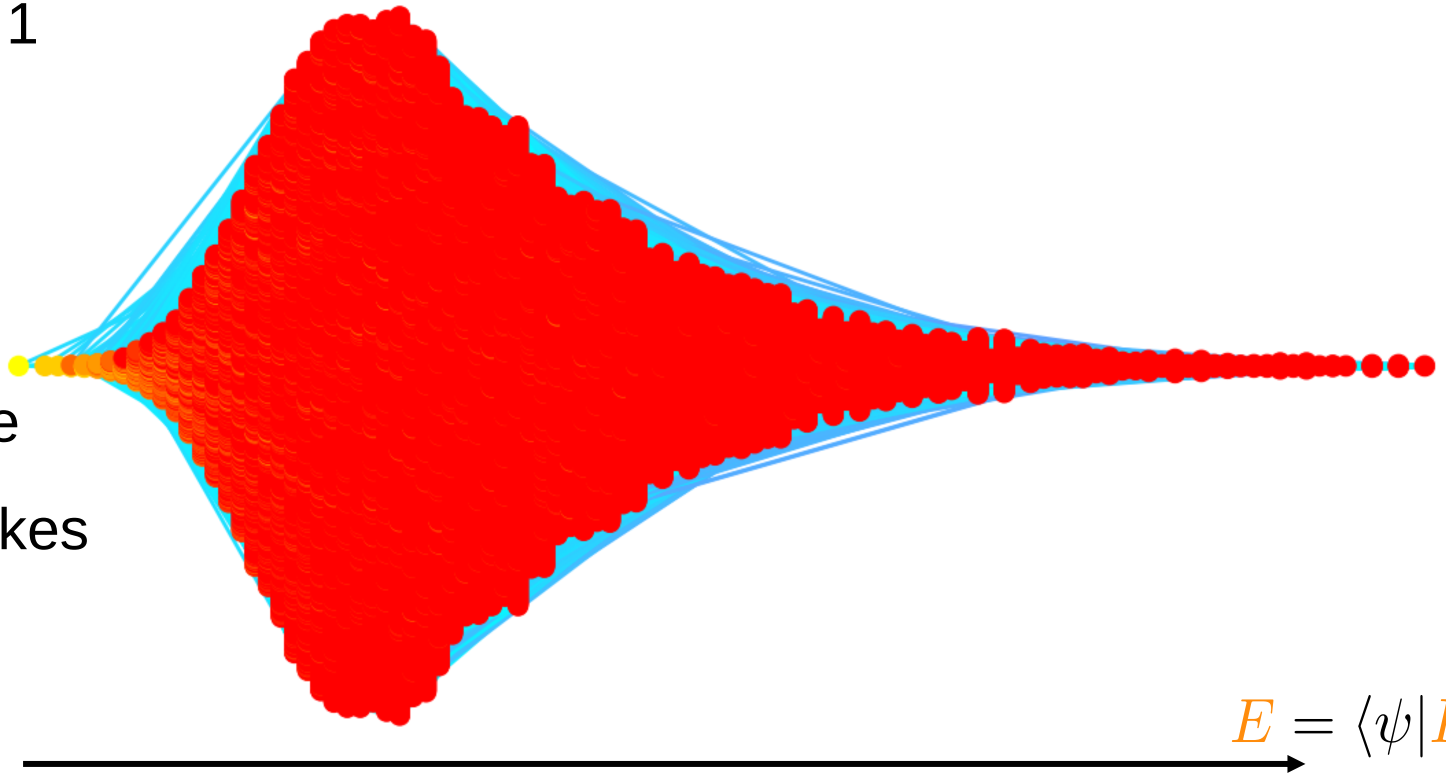
$\boxed{\text{15 qubits}}$



$E = \langle \psi | H_C | \psi \rangle$

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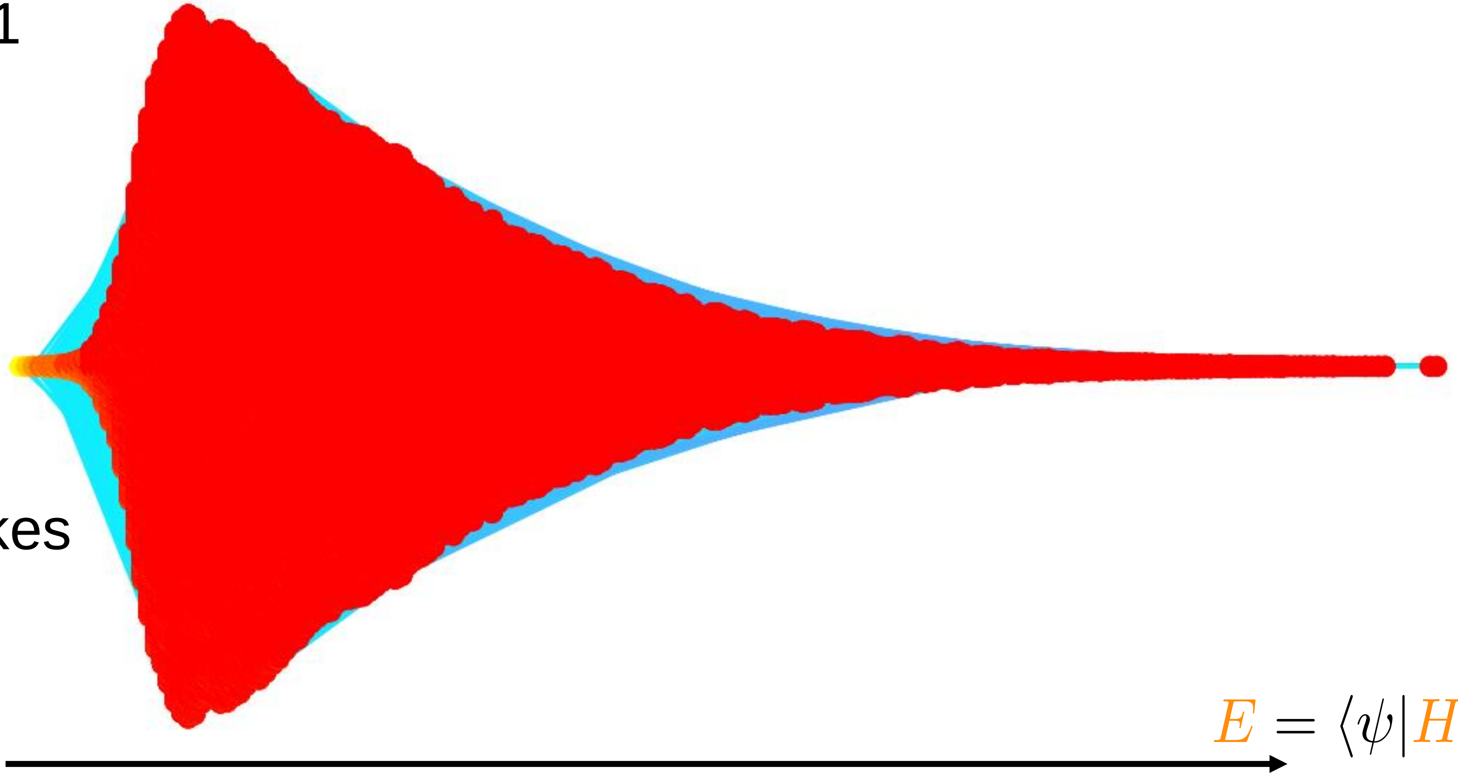
18 qubits



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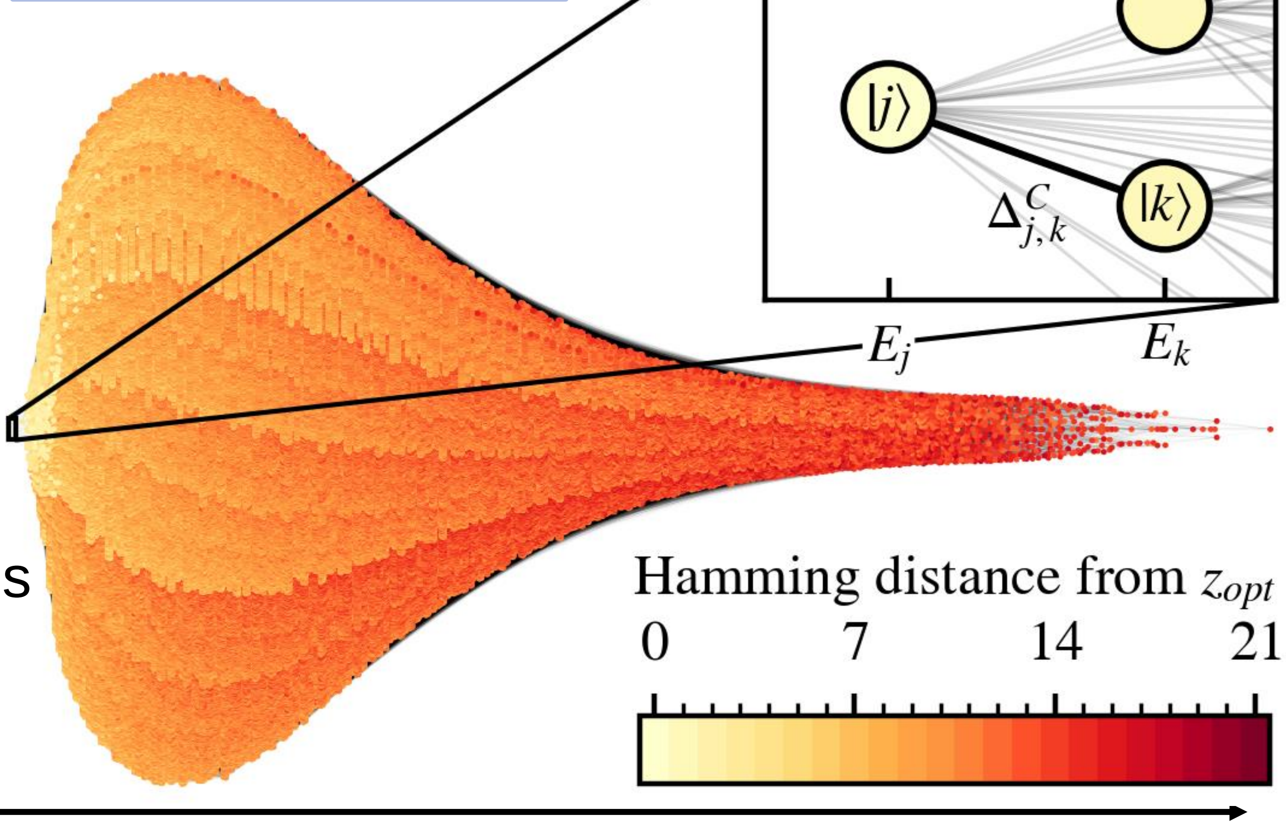
Hypercube Graph:



$$E_j = \langle j | H_C | j \rangle$$

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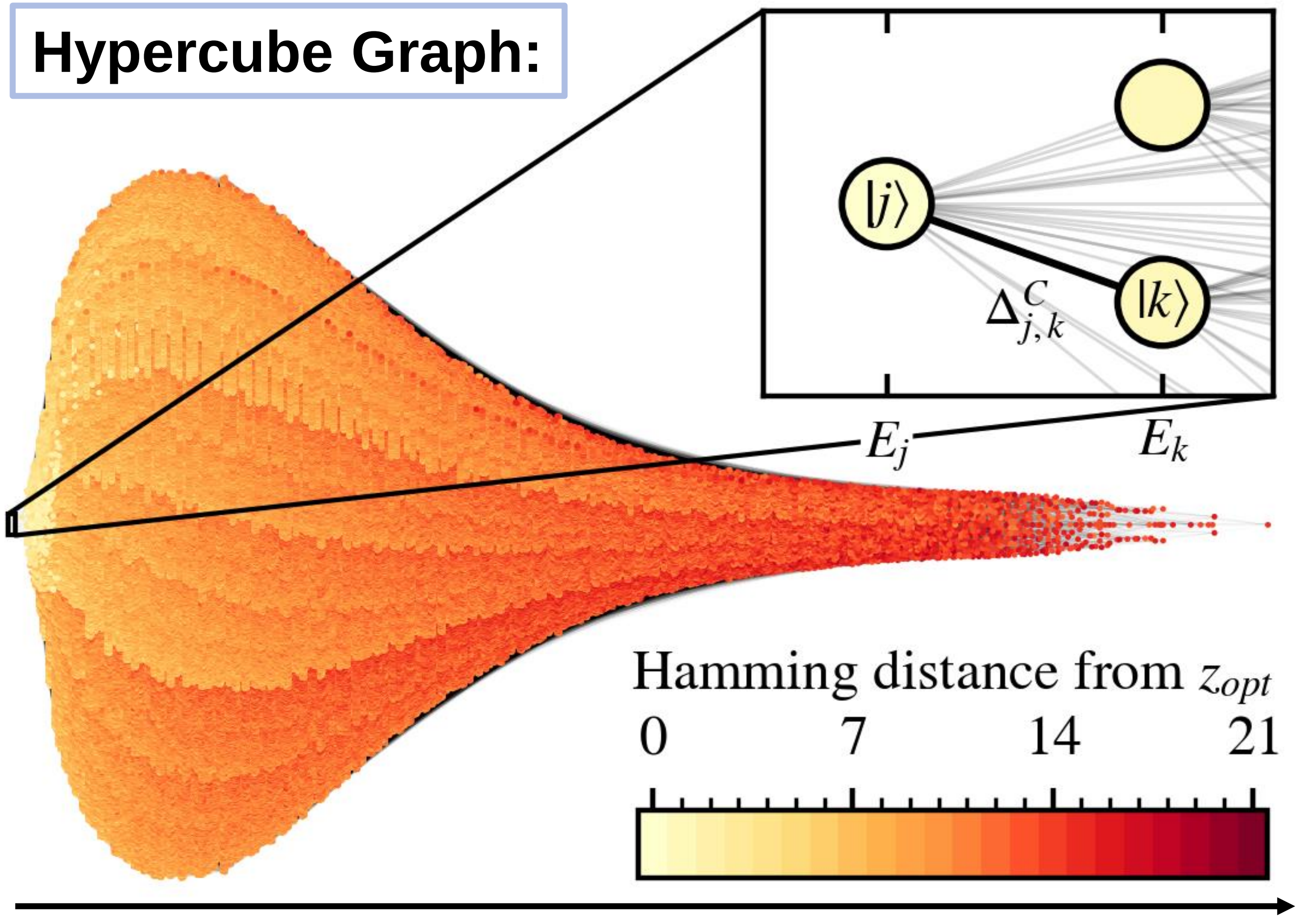

$E_j = \langle j | H_C | j \rangle$

- Look at **Local** Hamiltonian

$$H^{(j,k)} = \Gamma \cdot H_D^{(j,k)} + H_C^{(j,k)}$$

$$H_D^{(j,k)} = -\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad H_C^{(j,k)} = \begin{pmatrix} E_j & 0 \\ 0 & E_k \end{pmatrix}$$

- Drives **Amplitude Transfer** $|j\rangle \longleftrightarrow |k\rangle$

$$P_{\rightarrow|j\rangle}(t) = \frac{1}{2} + \frac{\Gamma \Delta^C_{j,k}}{2\Omega^2} \sin^2 \Omega t$$

  - **Local Energy Gaps** $\Delta^C_{j,k} = E_k - E_j$

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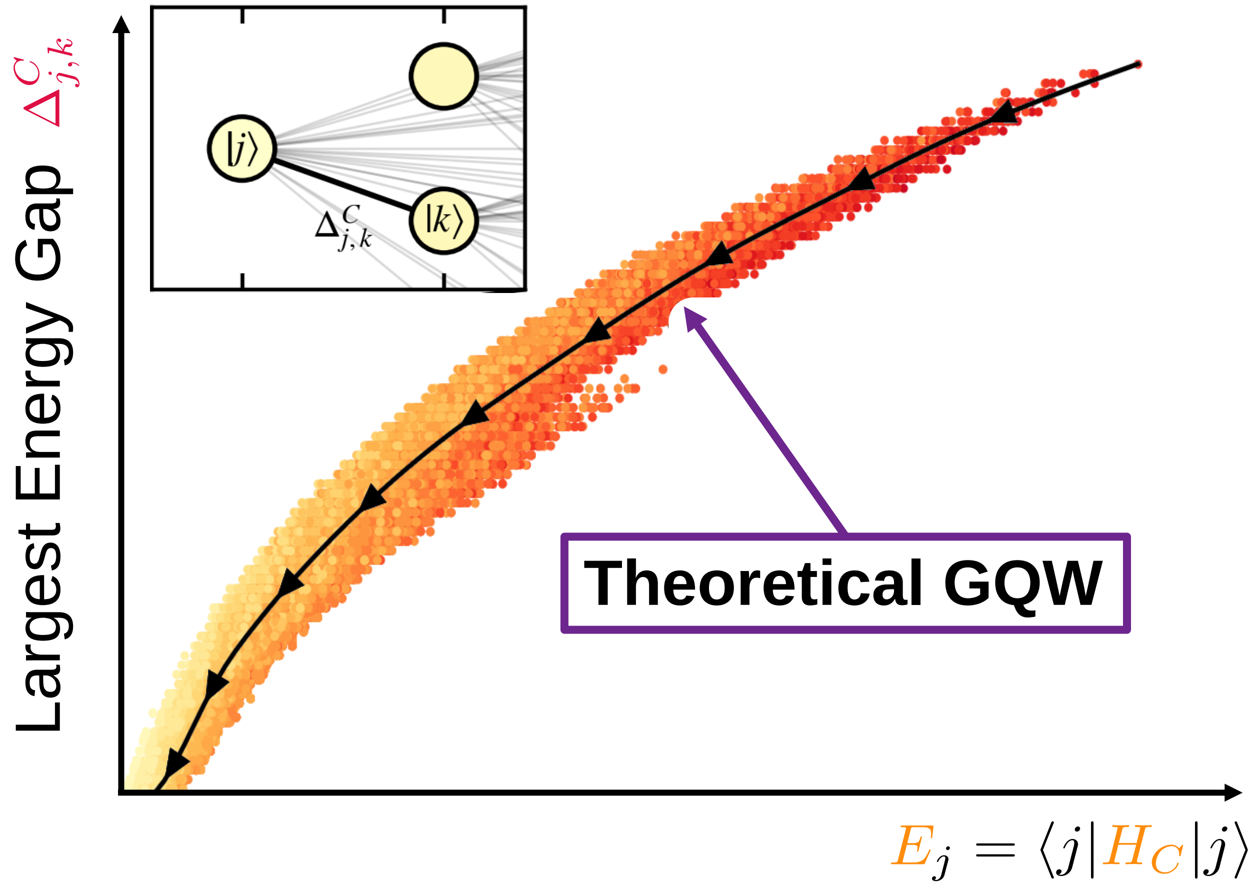

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  - **Local Energy Gaps** $\Delta^C_{j,k} = E_k - E_j$
  - **Local Rabi oscillations** $\Omega = \sqrt{\Gamma^2 + \left(\frac{\Delta^C_{j,k}}{2}\right)^2}$

- **LAT Theory**: Optimal Balance at

$$\Gamma \approx \Delta^C_{j,k} \,/\, 2$$

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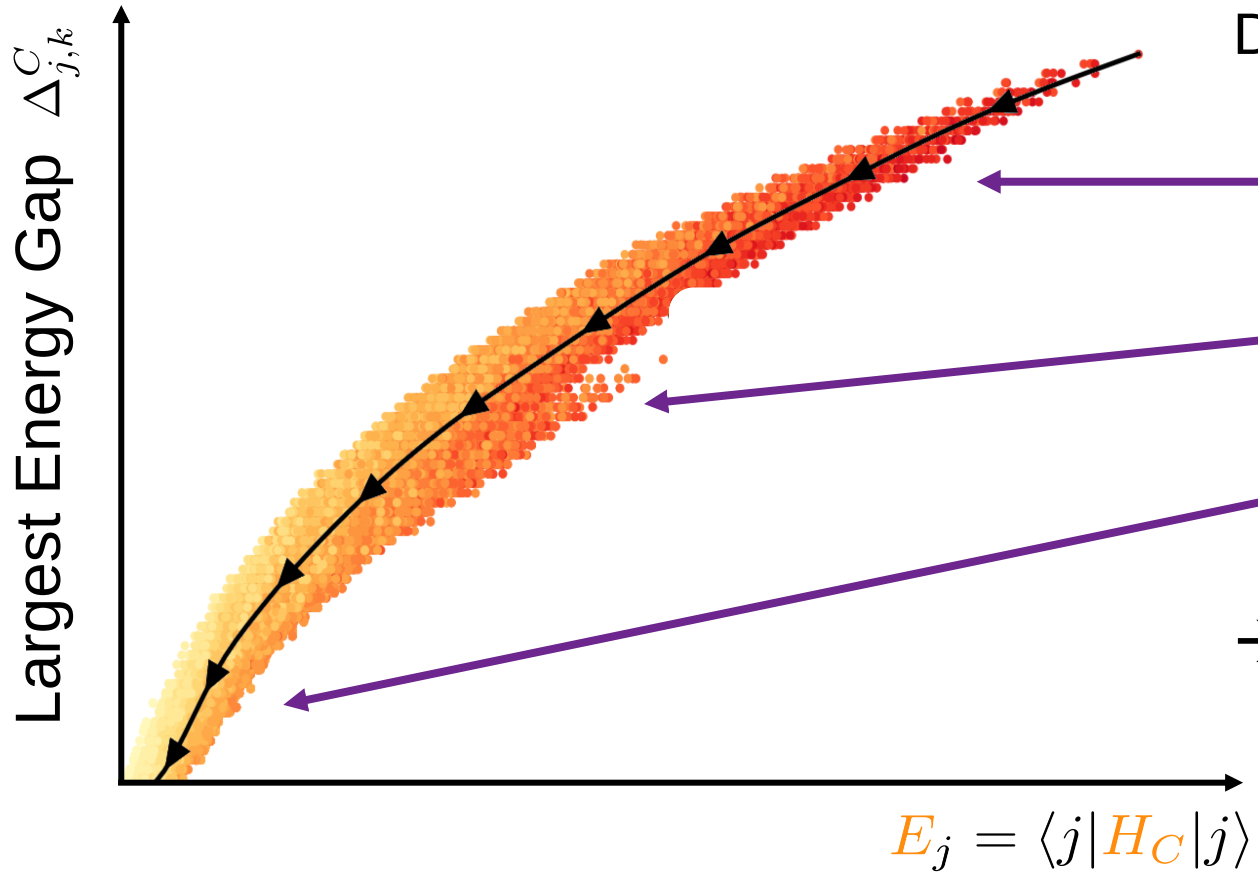
$$H_{GQW}(t) = \Gamma(t)\cdot\left(-\sum_{i=1}^{N}\sigma_i^x\right) + \left(-\sum_{i=1}^{N}h_i\sigma_i^z + \sum_{i\neq j}J_{ij}\sigma_i^z\sigma_j^z\right)$$

Theoretical GQW

LAT Theory: $\Gamma \approx \Delta^C_{j,k}\,/\,2$



Drive Amplitude Through Hypercube Graph:

Beginning $\Gamma(t) \approx \langle\Delta^C\rangle_{\text{large } E_j}\,/\,2$

Intermediate $\Gamma(t) \approx \langle\Delta^C\rangle_{\text{medium } E_j}\,/\,2$

End $\Gamma(t) \approx \langle\Delta^C\rangle_{\text{small } E_j}\,/\,2$

→ Hopping Rate “Schedule”

$$\Gamma(t) \approx \langle\Delta^C\rangle(E(t))\,/\,O(1)$$

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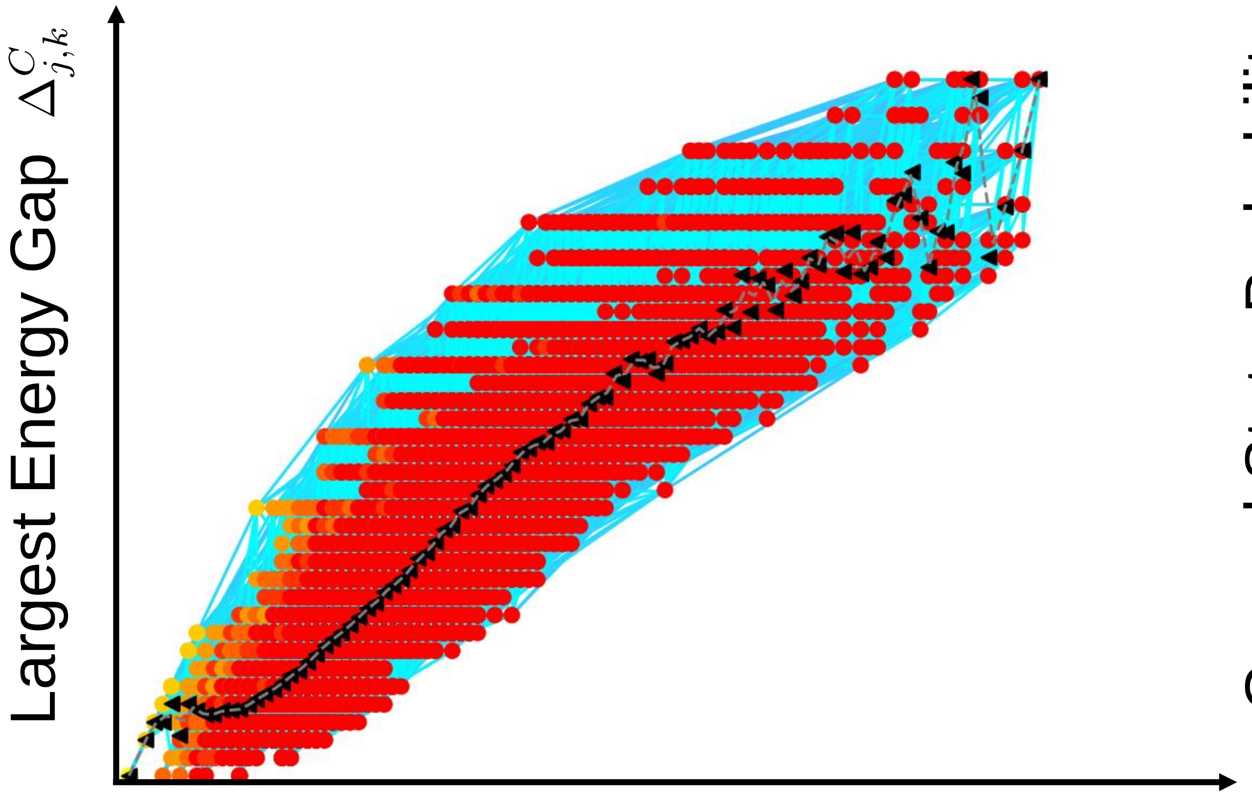

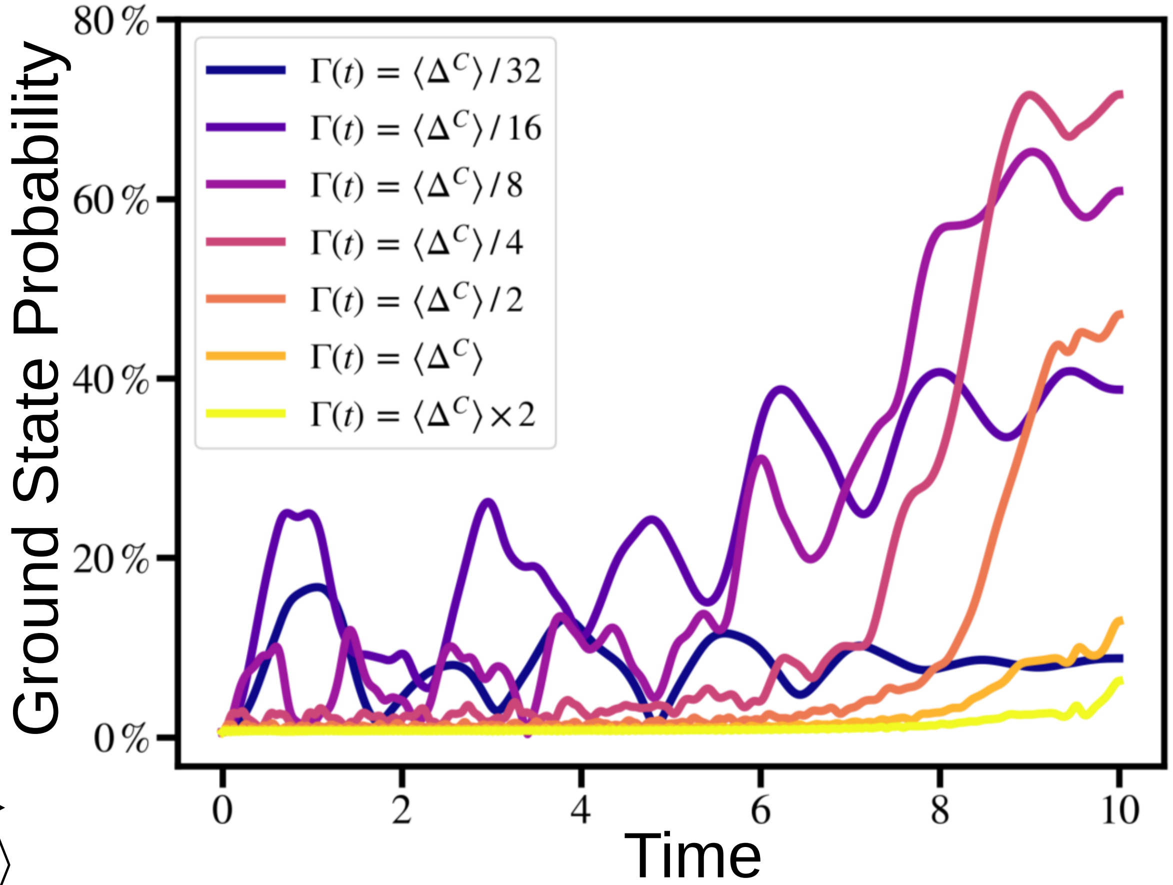

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$$H_{GQW}(t) = \Gamma(t) \cdot \left(-\sum_{i=1}^{N} \sigma_i^x \right) + \left(-\sum_{i=1}^{N} h_i \sigma_i^z + \sum_{i \neq j} J_{ij} \sigma_i^z \sigma_j^z \right)$$

Practical GQW

- **In practice:** $\langle \Delta^C \rangle (E(t))$ is not known
- **Observation:**
  - Generic energy spectrum
  - $\Gamma(t)$ takes generic shape

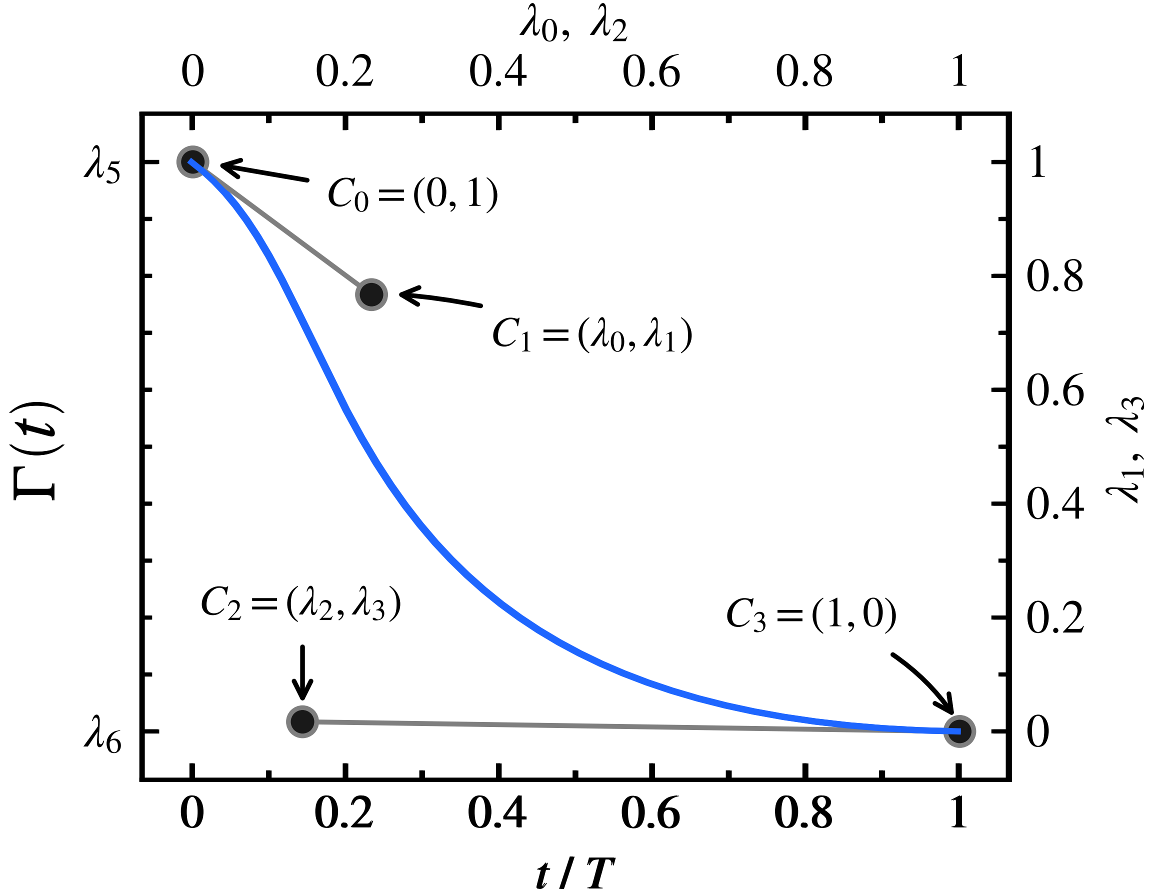
"Typical" Quantum Annealing Schedule!

  - Deviations can be modelled with

Bezier curves

  - Optimize only 6 hyperparameters!

LAT Theory: $\Gamma(t) \approx \langle \Delta^C \rangle (E(t)) \, / \, O(1)$



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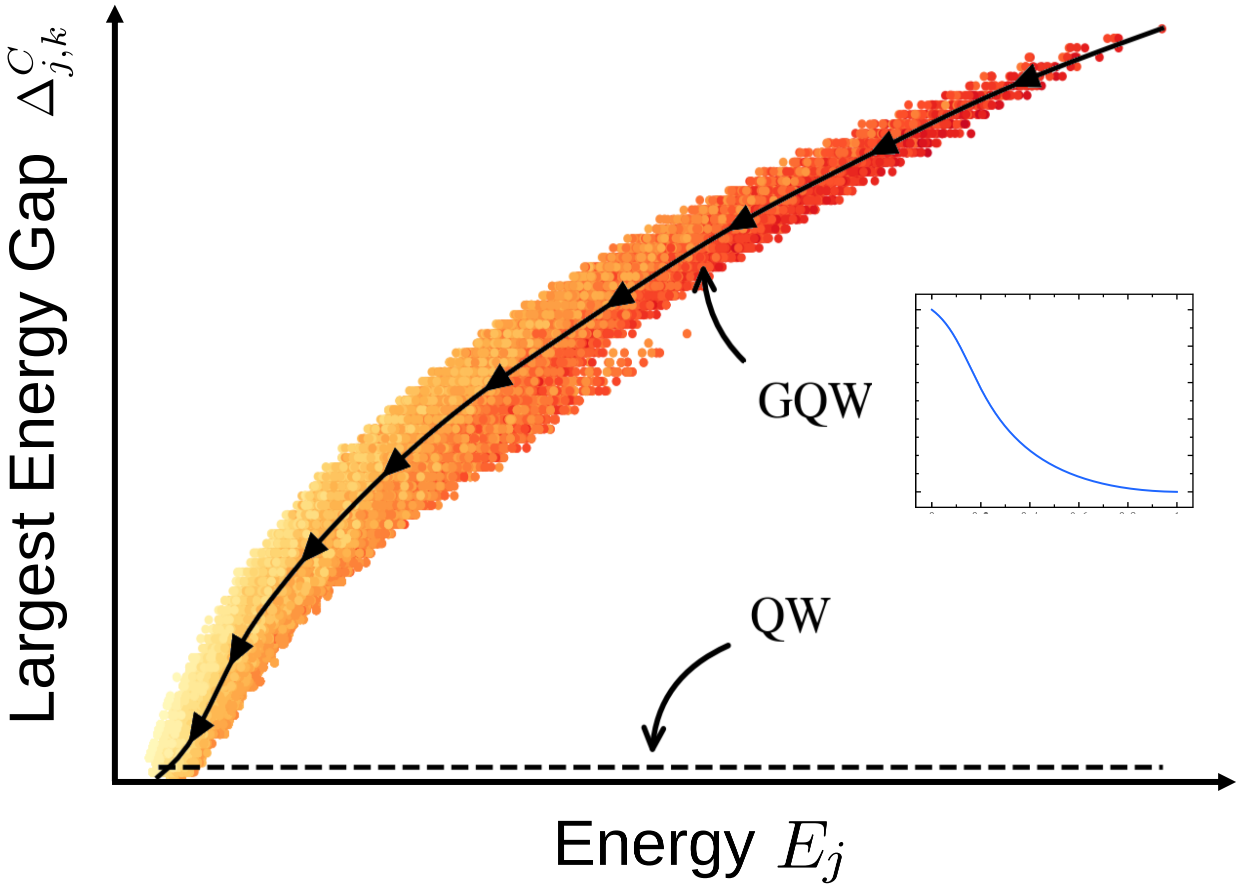
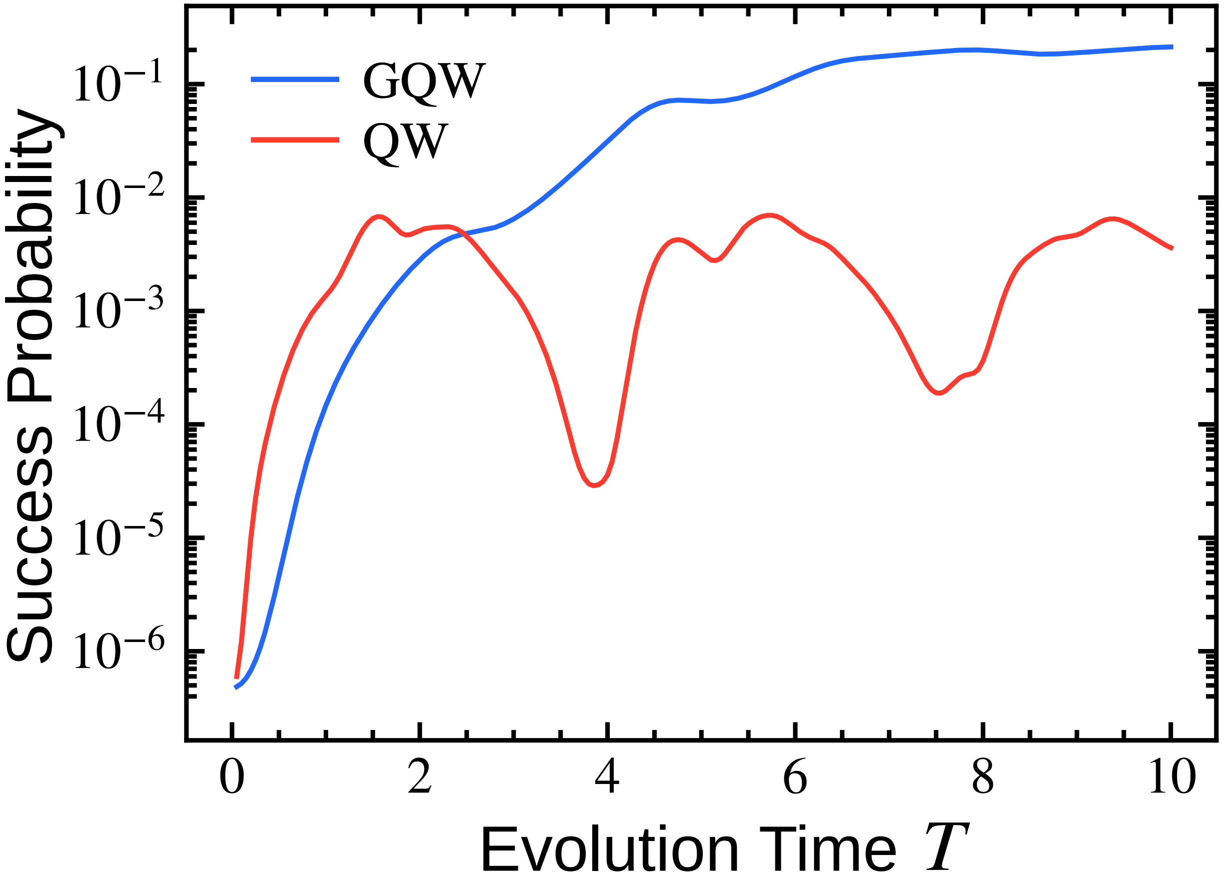
Performance vs. Quantum Walk





arXiv:2308.05418

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Performance vs. Quantum Annealing

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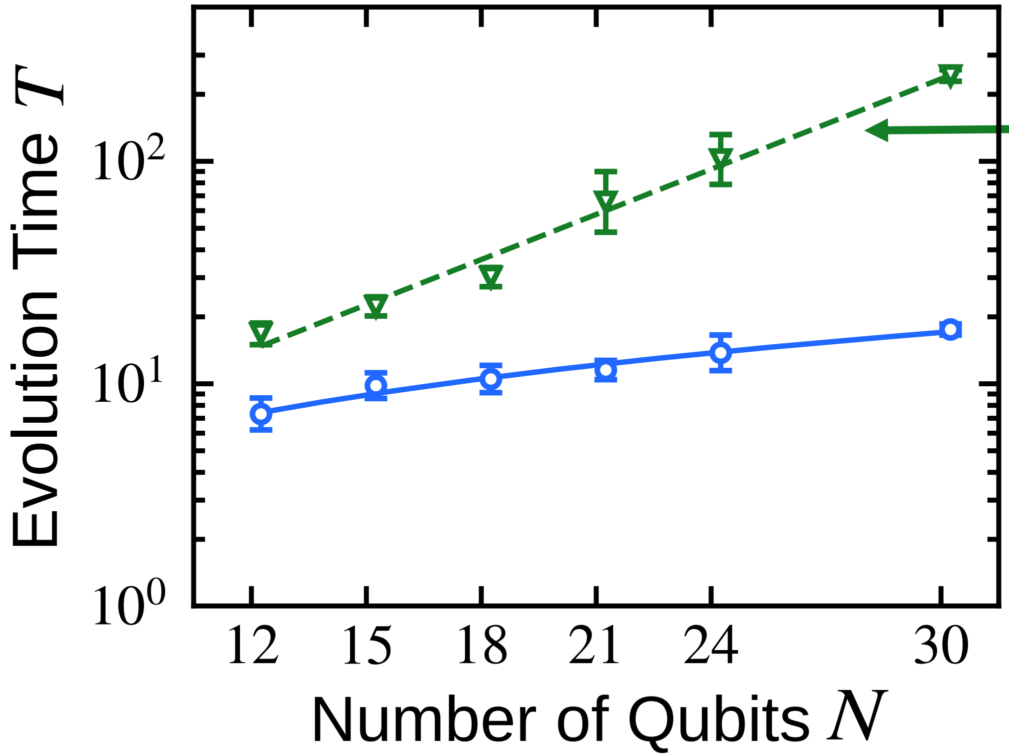
Time to achieve 90% success probability:



QA: $T \sim 2^{0.22\,N}$

GQW: $T \sim 0.54\,N$

How can the scaling be **linear**?

→ Need to properly account for the **cost** of the **classical optimization** phase!

GUIDED QUANTUM WALK

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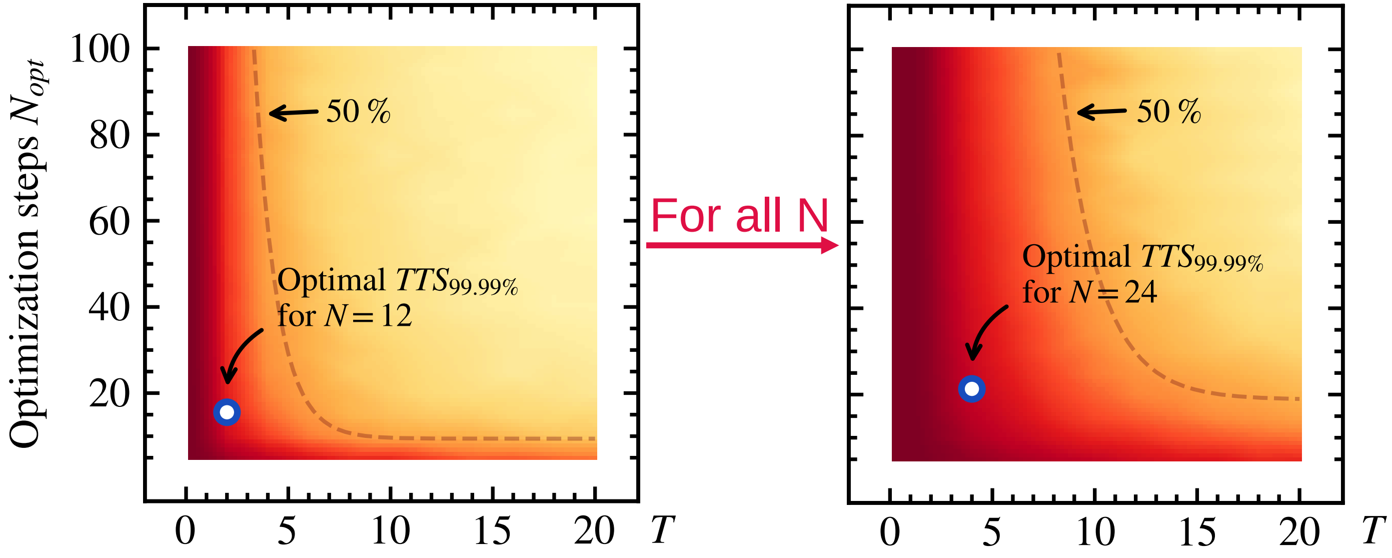
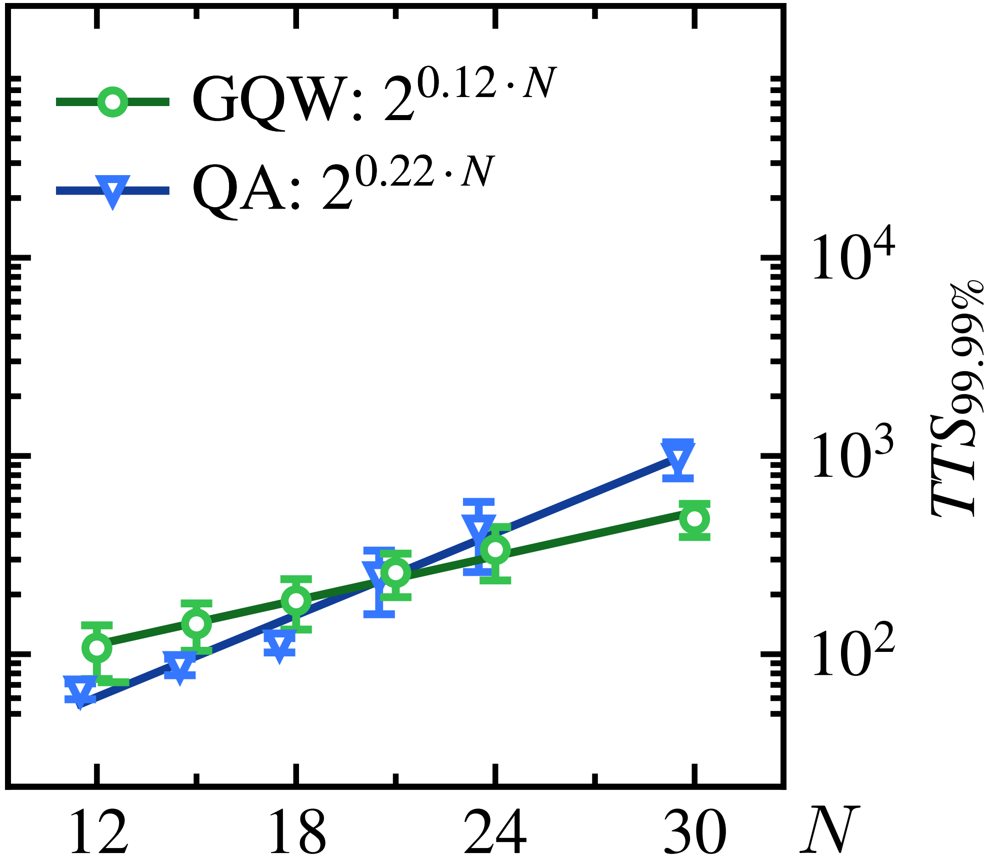
Accounting for Cost of Classical Optimizer

Time to achieve given success probability:

$$\mathrm{TTS}_{P_{\mathrm{target}}} = \frac{\ln(1 - P_{\mathrm{target}})}{\ln(1 - P_{\mathrm{groundstate}})} T + N_{\mathrm{opt}}\, T$$





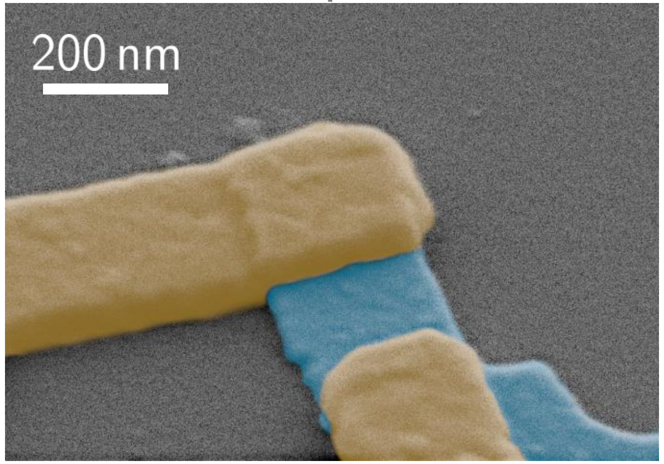
Proper accounting recovers **exponential scaling**

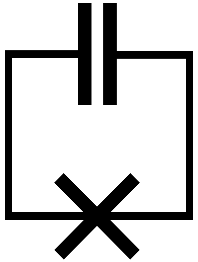
→ GQW scales favourably without cheating

INTERLUDE: JOSEPHSON HARMONICS

Using the Right Model to Describe Data

Standard Josephson Junction





$\Longleftrightarrow$

$$\boxed{H = 4E_c n^2 - \cancel{E_J \cos\varphi} - \sum_m E_{Jm} \cos m\varphi}$$

Paper: arXiv:2302.09192

Talk: https://youtu.be/dWu3lCwyXBM

Observation of Josephson Harmonics in Tunnel Junctions

Dennis Willsch,[1,*] Dennis Rieger,[2,3,*] Patrick Winkel,[2,3,4,5] Madita Willsch,[1,6] Christian Dickel,[7] Jonas Krause,[7] Yoichi Ando,[7] Raphaël Lescanne,[8,9] Zaki Leghtas,[8] Nicholas T. Bronn,[10] Pratiti Deb,[10] Olivia Lanes,[10] Zlatko K. Minev,[10] Benedikt Dennig,[2,3] Simon Geisert,[2] Simon Günzler,[2] Sören Ihssen,[2] Patrick Paluch,[2,3] Thomas Reisinger,[2] Roudy Hanna,[11,12] Jin Hee Bae,[11] Peter Schüffelgen,[11] Detlev Grützmacher,[11,12] Luiza Buimaga-Iarinca,[13] Cristian Morari,[13] Wolfgang Wernsdorfer,[2,3] David P. DiVincenzo,[14,12] Kristel Michielsen,[1,6,12] Gianluigi Catelani,[15,16] and Ioan M. Pop[2,3,†]

GUIDED QUANTUM WALK

Generalization of Schedules to Larger Problems

$$H_{GQW}(t) = \Gamma(t) \cdot \underbrace{\left(-\sum_{i=1}^{N} \sigma_i^x\right)}_{H_D} + \underbrace{\left(-\sum_{i=1}^{N} h_i \sigma_i^z + \sum_{i \neq j} J_{ij} \sigma_i^z \sigma_j^z\right)}_{H_C}$$

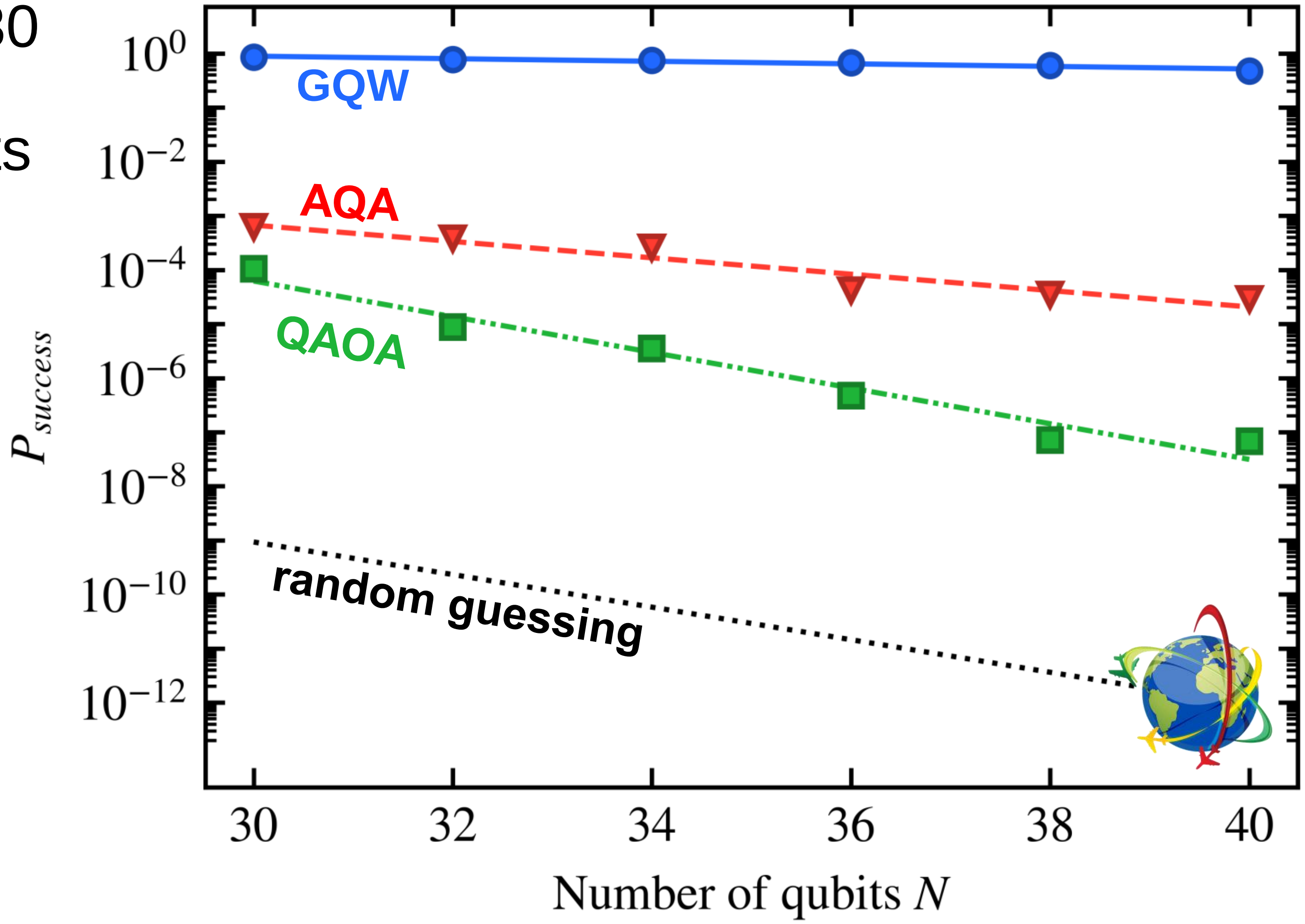
- Optimize GQW schedule $\Gamma(t)$ for N=30
- Apply same schedule until N=40 qubits
- Comparison vs.

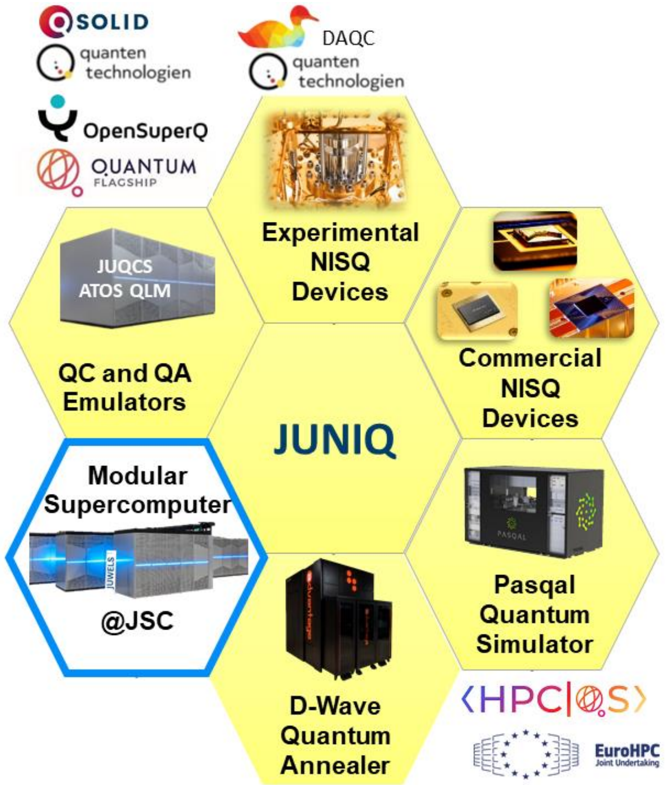
QAOA (2p = 12 parameters)

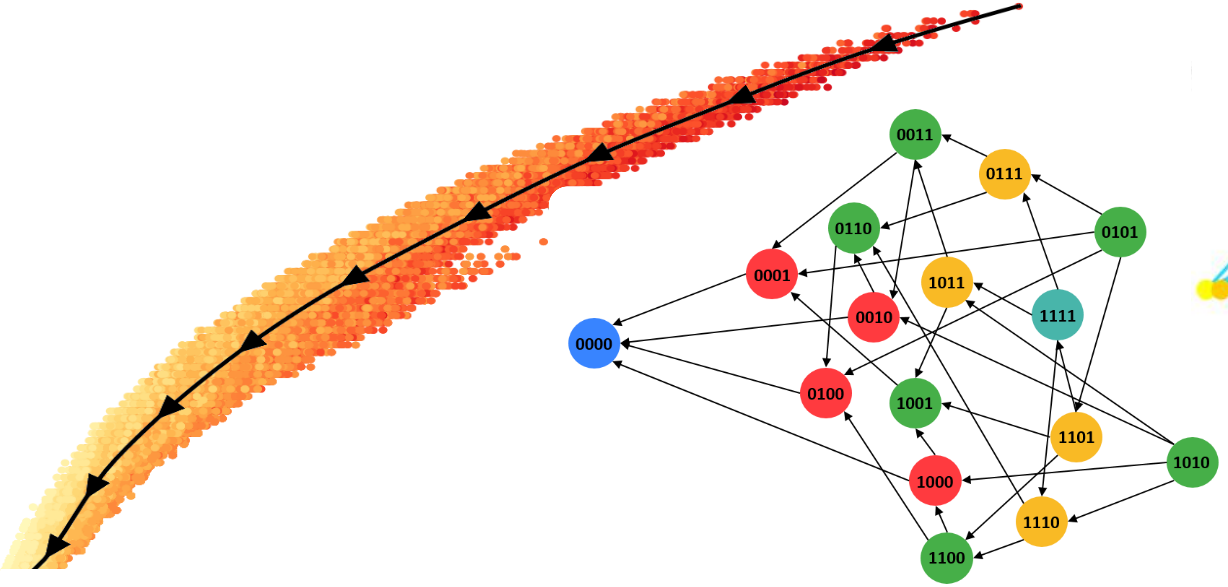
$$|\beta, \gamma\rangle = e^{-i\beta_p H_D} e^{-i\gamma_p H_C} \cdots e^{-i\beta_1 H_D} e^{-i\gamma_1 H_C} \, |+\rangle^{\otimes N}$$

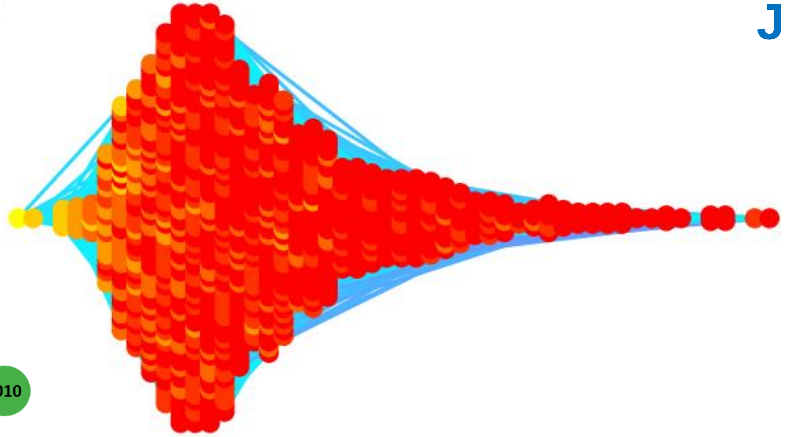
AQA (1 parameter) arXiv:2104.03293

$$\beta_k = -\tau(A(s_{k+1}) + A(s_k))/2, \qquad k = 1, ..., p-1,$$
$$\beta_p = -\tau A(s_p)/2,$$
$$\gamma_k = \tau B(s_k), \qquad k = 1, ..., p,$$









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