Published for SISSA by Springer

Received: January 6, 2023
Accepted: February 6, 2023
Published: February 15, 2023

Local spatial densities for composite spin-3/2 systems

H. Alharazin,[a] B.-D. Sun,[b,c,d] E. Epelbaum,[a] J. Gegelia[a,e] and U.-G. Meißner[d,f,e]

[a] *Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

[b] *Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China*

[c] *Guangdong-Hong Kong Joint Laboratory of Quantum Matter, Southern Nuclear Science Computing Center, South China Normal University, Guangzhou 510006, China*

[d] *Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany*

[e] *Tbilisi State University, 0186 Tbilisi, Georgia*

[f] *Institute for Advanced Simulation, Institut für Kernphysik and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany*

E-mail: herz.alharazin@outlook.de, sun@hiskp.uni-bonn.de, evgeny.epelbaum@rub.de, Jambul.Gegelia@rub.de, meissner@hiskp.uni-bonn.de

Abstract: The definition of local spatial densities by using sharply localized one-particle states is applied to spin-3/2 systems. Matrix elements of the electromagnetic current and the energy-momentum tensor are considered and integral expressions of associated spatial distributions in terms of form factors are derived.

Keywords: Properties of Hadrons, Effective Field Theories of QCD

ArXiv ePrint: 2212.11505

https://doi.org/10.1007/JHEP02(2023)163

JHEP02(2023)163

Contents

1 Introduction

In close analogy with the electric charge density of hadrons [1–3] it has been suggested to interpret the Fourier transforms of the gravitational form factors in the Breit frame as local densities of physical quantities characterizing various composite systems [4–6]. The identification of the spatial densities with the Fourier transforms of the electromagnetic and gravitational form factors in the Breit frame, especially for systems with intrinsic sizes comparable to their Compton wavelengths, has been questioned in refs. [7–13]. This issue has raised much interest recently [14–28]. On the one hand, the formalism of Wigner phase space distributions is utilized in ref. [14] and in subsequent publications. On the other hand, two-dimensional densities in the transverse plane obtained in the formalism of light front dynamics by integrating over the x^- coordinate have been considered as the only possible true internal densities [13, 27]. Note that two-dimensional densities have been also considered earlier in ref. [14]. A definition of spatial densities of local operators using sharply localized wave packet states, applicable to systems with arbitrary Compton wavelengths, has been suggested in ref. [21], see also ref. [29] for a related earlier study. Specifying the one-particle state of a spin-0 system by a spherically symmetric wave packet localized in space and taking the size of the packet much smaller than all internal characteristic scales of the considered system, spatial charge distributions have been defined in the zero average momentum frame (ZAMF). The new definition has been also generalized to moving Lorentz frames. Recently this definition has been also applied to electromagnetic densities of spin-1/2 systems [20], and to gravitational densities of spin-0 and spin-1/2 systems, defined via the matrix elements of the energy-momentum tensor (EMT) [30].

In the current work we consider spatial densities corresponding to the electromagnetic current and the EMT for spin-3/2 systems. We obtain the corresponding expressions in terms of form factors and discuss their physical interpretation.

Our work is organized as follows. In section 2 we define spatial densities corresponding to the electromagnetic current for spin-3/2 systems. Section 3 deals with the one-particle matrix elements of the EMT. In section 4 we discuss the large-distance behavior of various distributions of the delta resonance, which is the most studied composite spin-3/2 system, in chiral EFT. We summarize our results in section 5. The appendix contains the lengthy expressions of various quantities.

2 Electromagnetic densities in the zero average momentum frame

We choose the four-momentum eigenstates $|p, s\rangle$ characterizing our spin-3/2 system to be normalized as

$$\langle p_f, s'|p_i, s\rangle = 2E(2\pi)^3\delta_{s's}\delta^{(3)}(\mathbf{p}_f - \mathbf{p}_i)\,, \tag{2.1}$$

where (p_i, s) and (p_f, s') are the momentum and polarization of the initial and final state, respectively. Further, $p = (E, \mathbf{p})$ with $E = \sqrt{m^2 + \mathbf{p}^2}$, where m is the particle's mass.

To define the spatial densities via the matrix elements of local operators we use normalizable Heisenberg-picture states written in terms of wave packets as follows:

$$|\Phi, \mathbf{X}, s\rangle = \int \frac{d^3p}{\sqrt{2E(2\pi)^3}}\, \phi(s, \mathbf{p})\, e^{-i\mathbf{p}\cdot\mathbf{X}}|p, s\rangle, \tag{2.2}$$

where the parameters $\mathbf{X}$ are interpreted as the coordinates of the center of the charge or mass distribution, corresponding to the operator under consideration, and the profile function satisfies the normalization condition

$$\int d^3p\, |\phi(s, \mathbf{p})|^2 = 1\,. \tag{2.3}$$

To *define* the density distributions of the system we use spherically symmetric wave packets and profile functions of which $\phi(s, \mathbf{p}) = \phi(\mathbf{p}) = \phi(|\mathbf{p}|)$ are also spin-independent. The average of the three-momentum of the system vanishes in states corresponding to such packets, thus they describe the system in the ZAMF. For our calculations it is convenient to define dimensionless profile functions

$$\phi(\mathbf{p}) = R^{3/2}\, \tilde{\phi}(R\mathbf{p})\,, \tag{2.4}$$

where R specifies the size of the wave packet. Small values of R correspond to sharp localization of the packet.

The matrix elements of the electromagnetic current operator between momentum eigenstates of a spin-3/2 system can be parameterized in terms of four form factors, see ref. [31] for a review. We use here the notation of ref. [32]:

$$\begin{aligned}\langle p_f, s'|J_\mu|p_i, s\rangle = -\bar{u}^\beta(p_f, s')\bigg[&\frac{P_\mu}{m}\left(g_{\alpha\beta}F^V_{1,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2}\,F^V_{1,1}(q^2)\right)\\ &+ \frac{i}{2m}\sigma_{\mu\rho}q^\rho\left(g_{\alpha\beta}F^V_{2,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2}\,F^V_{2,1}(q^2)\right)\bigg]u^\alpha(p_i, s)\,,\end{aligned} \tag{2.5}$$

where $P = (p_i + p_f)/2$, $q = p_f - p_i$. In terms of these variables, the energies are given as $E = (m^2 + \mathbf{P}^2 - \mathbf{P}\cdot\mathbf{q} + \mathbf{q}^2/4)^{1/2}$ and $E' = (m^2 + \mathbf{P}^2 + \mathbf{P}\cdot\mathbf{q} + \mathbf{q}^2/4)^{1/2}$. The spinors of the spin-3/2 states are defined as follows:

$$
\begin{aligned}
u^\mu(p,s) &= \sum_{\lambda,\sigma} \langle 1\lambda, \frac{1}{2}\sigma|\frac{3}{2}s\rangle e^\mu(p,\lambda) u(p,\sigma)\,,\\
e^\mu(p,\lambda) &= \left(\frac{\hat{\mathbf{e}}_\lambda\cdot\mathbf{p}}{m}, \hat{\mathbf{e}}_\lambda + \frac{\mathbf{p}(\hat{\mathbf{e}}_\lambda\cdot\mathbf{p})}{m(p_0+m)}\right)\,,\\
u(p,\sigma) &= \sqrt{p_0+m}\left(\chi_\sigma, \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{p_0+m}\,\chi_\sigma\right)^T\,,
\end{aligned}
\tag{2.6}
$$

where $\langle 1\lambda, \frac{1}{2}\sigma|\frac{3}{2}s\rangle$ are the pertinent Clebsh-Gordon coefficients and

$$
\hat{\mathbf{e}}_+ = -\frac{1}{\sqrt{2}}(1,i,0)\,, \qquad \hat{\mathbf{e}}_0 = (0,0,1)\,, \qquad \hat{\mathbf{e}}_- = \frac{1}{\sqrt{2}}(1,-i,0)\,. \tag{2.7}
$$

The Dirac spinors are normalized as $\bar{u}(p,s')u(p,s) = 2m\,\delta_{s's}$. The matrix element of the electromagnetic current operator in localized states is given by

$$
\begin{aligned}
j^\mu_\phi(s',s,\mathbf{r}) &\equiv \langle\Phi,\mathbf{X},s'|\hat{J}^\mu(\mathbf{x},0)|\Phi,\mathbf{X},s\rangle \\
&= -\int\frac{d^3P\,d^3q}{(2\pi)^3\sqrt{4EE'}}\,\bar{u}^\beta\left(P+\frac{q}{2},\sigma'\right)\left[\frac{P_\mu}{m}\left(g_{\alpha\beta}F^V_{1,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2}F^V_{1,1}(q^2)\right)\right.\\
&\quad\left. + \frac{i}{2m}\sigma_{\mu\rho}q^\rho\left(g_{\alpha\beta}F^V_{2,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2}F^V_{2,1}(q^2)\right)\right]u^\alpha\left(P-\frac{q}{2},\sigma\right)\phi\left(\mathbf{P}-\frac{\mathbf{q}}{2}\right)\phi^\star\left(\mathbf{P}+\frac{\mathbf{q}}{2}\right)e^{-i\mathbf{q}\cdot\mathbf{r}},
\end{aligned}
\tag{2.8}
$$

where $\mathbf{r} = \mathbf{x} - \mathbf{X}$. By applying the method of dimensional counting of ref. [33], the leading contributions in eq. (2.8) for $R \to 0$ can be obtained without specifying the form of the form-factors and the profile function $\phi(|\mathbf{p}|)$. Provided that the form factors $F^V_{1,0}(q^2)$, $F^V_{1,1}(q^2)$, $F^V_{2,0}(q^2)$ and $F^V_{2,1}(q^2)$ decay for large q^2 as $1/q^2$, $1/q^4$, $1/q^3$, $1/q^5$ (or faster), respectively, the only non-vanishing contribution for $R \to 0$ is generated from the region of integration where $\mathbf{P}$ is large. It can be obtained by substituting $\mathbf{P} = \mathbf{Q}/R$, expanding the resulting integrand in eq. (2.8) in powers of R around $R = 0$ and keeping the leading order term for each component separately. Introducing $\hat{n} = \mathbf{Q}/|\mathbf{Q}|$ and using the spherical symmetry of the wave packet, the integration over $|\mathbf{Q}|$ can be carried out without specifying the radial profile function. The result of the integration is given below in terms of irreducible tensors and the multipole operators which are defined as follows. The n-th rank irreducible tensors in coordinate and momentum spaces, respectively, are given by $(r \neq 0)$

$$
Y^{i_1 i_2\cdots i_n}_n(\Omega_r) = \frac{(-1)^n}{(2n-1)!!}r^{n+1}\partial^{i_1}\partial^{i_2}\cdots\partial^{i_n}\frac{1}{r}, \quad Y^{i_1 i_2\cdots i_n}_n(\Omega_p) = \frac{(-1)^n}{(2n-1)!!}p^{n+1}\partial^{i_1}\partial^{i_2}\cdots\partial^{i_n}\frac{1}{p}. \tag{2.9}
$$

The quadrupole- and octupole-operators, $\hat{Q}^{ij}$ and $\hat{O}^{ijk}$, for a spin-3/2 system are given in terms of the spin operator $\hat{S}^i$ via

$$\begin{aligned}\hat{Q}^{ij} &= \frac{1}{2}\left(\hat{S}^i\hat{S}^j + \hat{S}^j\hat{S}^i - \frac{2}{3}S(S+1)\delta^{ij}\right), \\ \hat{O}^{ijk} &= \frac{1}{6}\Bigg(\hat{S}^i\hat{S}^j\hat{S}^k + \hat{S}^j\hat{S}^i\hat{S}^k + \hat{S}^k\hat{S}^j\hat{S}^i + \hat{S}^j\hat{S}^k\hat{S}^i + \hat{S}^i\hat{S}^k\hat{S}^j + \hat{S}^k\hat{S}^i\hat{S}^j \\ &\qquad -\frac{6S(S+1)-2}{5}\left(\delta^{ij}\hat{S}^k + \delta^{ik}\hat{S}^j + \delta^{kj}\hat{S}^i\right)\Bigg), \end{aligned} \tag{2.10}$$

with $i,j,k = 1,2,3$.

The results for the matrix elements after integration over $|\mathbf{Q}|$ in eq. (2.8) have the form:

$$j^0_\phi(s',s,\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \frac{1}{4\pi}\int d^2\hat{n}\left\{ \mathcal{Z}_0(-q_\perp^2)\,\delta_{s's} + \left[\mathcal{Z}_1(-q_\perp^2)\,\hat{n}^k\hat{n}^l + \mathcal{Z}_2(-q_\perp^2)\,\frac{q_\perp^k q_\perp^l}{m^2}\right]\hat{Q}^{kl}_{s's}\right\} \tag{2.11}$$

$$= \rho_0^C(r)\,\delta_{s's} + \rho_2^C(r)\,Y_2^{kl}(\Omega_r)\,\hat{Q}^{kl}_{s's}, \tag{2.12}$$

$$\begin{aligned} j^i_\phi(s',s,\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \frac{i}{4\pi}\int d^2\hat{n}\Bigg\{&\left[\mathcal{A}_0(-q_\perp^2)\,\hat{n}^i\hat{n}^l\epsilon^{kln} + \mathcal{A}_1(-q_\perp^2)\left(\delta^{kl} - \hat{n}^k\hat{n}^l\right)\epsilon^{iln}\right]\frac{q_\perp^n}{m}\hat{S}^k_{s's} \\ &+\Bigg[\left(\mathcal{A}_2(-q_\perp^2)\,\hat{n}^t\hat{n}^z + \mathcal{A}_3(-q_\perp^2)\,\frac{q_\perp^t q_\perp^z}{m^2}\right)\hat{n}^i\hat{n}^l\epsilon^{kln} \\ &+\left(\mathcal{A}_4(-q_\perp^2)\hat{n}^t\hat{n}^z + \mathcal{A}_5(-q_\perp^2)\,\frac{q_\perp^t q_\perp^z}{m^2}\right)\epsilon^{iln}\left(\delta^{kl} - \hat{n}^k\hat{n}^l\right)\Bigg]\frac{q_\perp^n}{m}\hat{O}^{ktz}_{s's}\Bigg\} \end{aligned} \tag{2.13}$$

$$= i\epsilon^{ikn}\hat{S}^k_{s's}Y_1^n\frac{1}{m}\frac{d}{dr}\rho_1^M(r) + i\epsilon^{ikn}\hat{O}^{ktz}_{s's}Y_3^{ntz}\frac{r^3}{m^3}\left(\frac{1}{r}\frac{d}{dr}\right)^3\rho_3^M(r), \tag{2.14}$$

where $q_\perp^2 = \mathbf{q}^2 - (\mathbf{q}\cdot\hat{n})^2$ and the coefficient functions $\mathcal{Z}_i$ and $\mathcal{A}_i$ are given in the appendix. The quantities $\rho_0^C(r)$ and $\rho_2^C(r)$ are the monopole and quadrupole charge densities, respectively, whereas $\rho_1^M(r)$ and $\rho_3^M(r)$ are the dipole and octupole scalar magnetization densities, respectively. These quantities are given by

$$\begin{aligned}\rho_0^C(r) = \frac{1}{4\pi}\int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\Bigg\{&F^V_{1,0}(-q_\perp^2) + \frac{q_\perp^2}{6m^2}\left[-2F^V_{1,0}(-q_\perp^2) + F^V_{1,1}(-q_\perp^2) + F^V_{2,0}(-q_\perp^2)\right] \\ &+\frac{q_\perp^4}{24m^4}\left[-2F^V_{1,1}(-q_\perp^2) + F^V_{2,1}(-q_\perp^2)\right]\Bigg\}, \end{aligned} \tag{2.15}$$

$$\begin{aligned}\rho_2^C(r) = -\frac{r}{4\pi}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\,\frac{q_\perp^2}{12m^2q^2}\Bigg\{&4F^V_{1,0}(-q_\perp^2) + F^V_{1,1}(-q_\perp^2) - 2F^V_{2,0}(-q_\perp^2) \\ -\frac{3q_\perp^2}{q^2}\left[F^V_{1,0}(-q_\perp^2) + F^V_{1,1}(-q_\perp^2)\right] - \frac{q_\perp^2}{2m^2}&\left[-2F^V_{1,1}(-q_\perp^2) + F^V_{2,1}(-q_\perp^2)\right] - \frac{3q_\perp^4}{4m^2q^2}F^V_{1,1}(-q_\perp^2)\Bigg\}, \end{aligned} \tag{2.16}$$

$$\begin{aligned}\rho_1^M(r) = \frac{1}{4\pi}\int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\,\frac{q_\perp^2}{2q^2}\Bigg\{&F^V_{1,0}(-q_\perp^2) - \frac{1}{3}F^V_{2,0}(-q_\perp^2) \\ &+\frac{q_\perp^2}{30m^2}\left[-2F^V_{1,0}(-q_\perp^2) + 7F^V_{1,1}(-q_\perp^2) + 2F^V_{2,0}(-q_\perp^2) - 2F^V_{2,1}(-q_\perp^2)\right] \\ &+\frac{q_\perp^4}{60m^4}\left[F^V_{2,1}(-q_\perp^2) - F^V_{1,1}(-q_\perp^2)\right]\Bigg\}, \end{aligned} \tag{2.17}$$

$$\begin{aligned}\rho_3^M(r) = \frac{1}{4\pi}\int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\,\frac{q_\perp^4}{48q^4}\Bigg\{&-4F^V_{1,0}(-q_\perp^2) - F^V_{1,1}(-q_\perp^2) + 4F^V_{2,0}(-q_\perp^2) + F^V_{2,1}(-q_\perp^2) \\ +\frac{5q_\perp^2}{q^2}\left[F^V_{1,0}(-q_\perp^2) + F^V_{1,1}(-q_\perp^2) - F^V_{2,0}(-q_\perp^2) - F^V_{2,1}(-q_\perp^2)\right] &+ \frac{q_\perp^2}{m^2}\left[F^V_{2,1}(-q_\perp^2) - F^V_{1,1}(-q_\perp^2)\right] \\ &+\frac{5q_\perp^4}{4m^2q^2}\left[F^V_{1,1}(-q_\perp^2) - F^V_{2,1}(-q_\perp^2)\right]\Bigg\}. \end{aligned} \tag{2.18}$$

The standard expressions of the densities in terms of the form factors in the Breit frame, $F_{i,j}(q^2) = F_{i,j}(-\mathbf{q}^2)$, which we will refer to as "naive", are obtained by first approximating the integrand in eq. (2.8) by the two leading terms in the $1/m-$expansion[1] and subsequently localizing the wave packet by taking the limit $R \mapsto 0$. The resulting expressions have the form:

$$j^0_{\rm naive}(s',s,\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \left\{ \left[F^V_{1,0}\left(-\mathbf{q}^2\right) + \frac{\mathbf{q}^2}{6m^2} F^V_{1,1}\left(-\mathbf{q}^2\right) \right] \delta_{s's} - F^V_{1,1}\left(-\mathbf{q}^2\right) \frac{q^k q^l}{6m^2} \hat{Q}^{kl}_{s's} \right\}, \tag{2.19}$$

$$j^i_{\rm naive}(s',s,\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} i\epsilon^{ikn} \frac{q^n}{3m} \left\{ \left[F^V_{2,0}\left(-\mathbf{q}^2\right) + \frac{\mathbf{q}^2}{5m^2} F^V_{2,1}\left(-\mathbf{q}^2\right) \right] \hat{S}^k_{s's} - F^V_{2,1}\left(-\mathbf{q}^2\right) \frac{q^l q^z}{2m^2} \hat{O}^{klz}_{s's} \right\}.$$

3 Gravitational densities in the zero average momentum frame

One is often interested in matrix elements of the quark and gluon contributions to the EMT. As these are not separately conserved, we parameterize the matrix element of a symmetric EMT for spin-3/2 states in terms of ten form factors as follows [32, 34]:

$$\begin{aligned}
\langle p_f, s' | T_{\mu\nu}\left(\vec{x}\right) | p_i, s\rangle = -\bar{u}^\beta(p_f,s') \Bigg[& \frac{P_\mu P_\nu}{m} \left(g_{\alpha\beta} F_{1,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2} F_{1,1}(q^2) \right) \\
& + \frac{q_\mu q_\nu - \eta_{\mu\nu} q^2}{4m} \left(g_{\alpha\beta} F_{2,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2} F_{2,1}(q^2) \right) \\
& + m g^{\mu\nu} \left(g_{\alpha'\alpha} F_{3,0}(t) - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2} F_{3,1}(t) \right) \\
& + \frac{i}{2} \frac{\left(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho}\right) q^\rho}{m} \left(g_{\alpha\beta} F_{4,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2} F_{4,1}(q^2) \right) \\
& - \frac{1}{m} \left(g_{\nu\beta} q_\mu q_\alpha + g_{\mu\beta} q_\nu q_\alpha + g_{\nu\alpha} q_\mu q_\beta + g_{\mu\alpha} q_\nu q_\beta - 2 g_{\mu\nu} q_\alpha q_\beta \right. \\
& \left. - g_{\mu\beta} g_{\nu\alpha} q^2 - g_{\nu\beta} g_{\mu\alpha} q^2 \right) F_{5,0}(q^2) \\
& + m (g^\mu_{\alpha'} g^\nu_\alpha + g^\nu_{\alpha'} g^\mu_\alpha) F_{6,0}(t) \Bigg] u^\alpha(p_i,s) e^{-i\mathbf{q}\cdot\mathbf{r}} ,
\end{aligned} \tag{3.1}$$

where in case of a conserved EMT the form factors $F_{3,0}(t)$, $F_{3,1}(t)$ and $F_{6,0}(t)$ vanish. The matrix element of the EMT in localized states is written as

$$\begin{aligned}
t^{\mu\nu}_\phi(\mathbf{r}) \equiv\ & \langle \Phi, \mathbf{X}, s' | \hat{T}^{\mu\nu}(\mathbf{x},0) | \Phi, \mathbf{X}, s\rangle \\
= & -\int \frac{d^3P\, d^3q}{(2\pi)^3 \sqrt{4EE'}} \bar{u}^\beta\left(P + \frac{q}{2}, \sigma'\right) \Bigg[\frac{P_\mu P_\nu}{m} \left(g_{\alpha\beta} F_{1,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2} F_{1,1}(q^2) \right) \\
& + \frac{q_\mu q_\nu - \eta_{\mu\nu} q^2}{4m} \left(g_{\alpha\beta} F_{2,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2} F_{2,1}(q^2) \right) + \frac{i}{2} \frac{(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho}) q^\rho}{m} \left(g_{\alpha\beta} F_{4,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2} F_{4,1}(q^2) \right) \\
& - \frac{1}{m} (g_{\nu\beta} q_\mu q_\alpha + g_{\mu\beta} q_\nu q_\alpha + g_{\nu\alpha} q_\mu q_\beta + g_{\mu\alpha} q_\nu q_\beta - 2 g_{\mu\nu} q_\alpha q_\beta \\
& - g_{\mu\beta} g_{\nu\alpha} q^2 - g_{\nu\beta} g_{\mu\alpha} q^2) F_{5,0}(q^2) \Bigg] u^\alpha\left(P - \frac{q}{2}, \sigma\right) \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) \phi^\star\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}} .
\end{aligned} \tag{3.2}$$

[1] Factors of m introduced in eqs. (2.5) and (3.1) for dimensional reasons in the parametrization of the matrix elements in terms of form factors are not counted when expanding in $1/m$ neither here nor below in the case of EMT matrix elements.

JHEP02(2023)163

The matrix elements of the EMT in the localized states with $R \to 0$ can be obtained analogously to the electromagnetic case. As we will see below, the leading order contributions to $t^{00}_\phi(\mathbf{r})$ and $t^{0i}_\phi(\mathbf{r})$ are of the order of $1/R$, and the $t^{ij}_\phi(\mathbf{r})$ terms need to be treated differently from the others, when expanding in R. The reason for that is that, the components of $t^{ij}_\phi(\mathbf{r})$, unlike $t^{00}_\phi(\mathbf{r})$ and $t^{0i}_\phi(\mathbf{r})$, which contain only information about the energy and spin densities, respectively, encode information about the internal pressure and shear forces as well as about the motion of the system [26, 30]. That is, $t^{ij}_\phi(\mathbf{r})$ needs to be decomposed to a component $t^{ij}_{\phi,0}(\mathbf{r})$ that describes the motion of the system as whole, and a component that encodes information about pressure and shear forces $t^{ij}_{\phi,2}(\mathbf{r})$. Therefore, after expanding in R, we keep the leading order contribution of each of these terms. The resulting expressions have the form:

$$t^{00}_\phi(s',s,\mathbf{r}) = N_{\phi,R}\int \frac{d^3q}{(2\pi)^3}\,e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\left\{\,\mathcal{E}_0(q_\perp^2)\;\delta_{s's}+\left[\mathcal{E}_1(q_\perp^2)\;\hat{n}^k\hat{n}^l+\mathcal{E}_2(q_\perp^2)\;\frac{q_\perp^k q_\perp^l}{m^2}\right]\hat{Q}^{kl}_{s's}\right\}, \tag{3.3a}$$

$$\begin{aligned} t^{0i}_\phi(s',s,\mathbf{r}) = {}& i\,N_{\phi,R}\int \frac{d^3q}{(2\pi)^3}\,e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\,\left\{\;\left[\mathcal{C}_0(q_\perp^2)\,\epsilon^{kln}\hat{n}^l\hat{n}^i+\mathcal{C}_1(q_\perp^2)\,\epsilon^{iln}\left(\delta^{kl}-\hat{n}^k\hat{n}^l\right)\right]\frac{q_\perp^n}{m}\,\hat{S}^k_{s's}\right.\\ &+\left[\left(\mathcal{C}_2(q_\perp^2)\hat{n}^t\hat{n}^z+\mathcal{C}_3(q_\perp^2)\frac{q_\perp^t q_\perp^z}{m^2}\right)\,\epsilon^{kln}\hat{n}^l\hat{n}^i\right.\\ &\left.\left.+\left(\mathcal{C}_4(q_\perp^2)\hat{n}^t\hat{n}^z+\mathcal{C}_5(q_\perp^2)s\frac{q_\perp^t q_\perp^z}{m^2}\right)\epsilon^{iln}\left(\delta^{kl}-\hat{n}^k\hat{n}^l\right)\right]\frac{q_\perp^n}{m}\,\hat{O}^{ktz}_{s's}\right\}, \end{aligned} \tag{3.3b}$$

$$t^{ij}_\phi(s',s,\mathbf{r}) = t^{ij}_{\phi,0}(s',s,\mathbf{r})+t^{ij}_{\phi,2}(s',s,\mathbf{r}), \tag{3.3c}$$

where

$$t^{ij}_{\phi,0}(s',s,\mathbf{r}) = N_{\phi,R}\int\frac{d^3q}{(2\pi)^3}\;e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\hat{n}^i\hat{n}^j\;\left\{\;\mathcal{E}_0(q_\perp^2)\;\delta_{s's}+\left[\mathcal{E}_1(q_\perp^2)\;\hat{n}^k\hat{n}^l+\mathcal{E}_2(q_\perp^2)\;\frac{q_\perp^k q_\perp^l}{m^2}\right]\hat{Q}^{kl}_{s's}\right\} \tag{3.4a}$$

$$\begin{aligned} t^{ij}_{\phi,2}(s',s,\mathbf{r}) = {}& N_{\phi,R,2}\int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\\ &\times\left\{\frac{1}{2m^2}\left(q^iq^j-q_\perp^2\delta^{ij}\right)\left[\mathcal{W}_0(q_\perp^2)\delta_{s's}+\left[\mathcal{W}_1(q_\perp^2)\hat{n}^k\hat{n}^l+\mathcal{W}_2(q_\perp^2)\frac{q_\perp^k q_\perp^l}{m^2}\right]\hat{Q}^{kl}_{s's}\right]\right.\\ &\left.-2\delta^{ij}\left[\mathcal{U}_0(q_\perp^2)\delta_{s's}+\left[\mathcal{U}_1(q_\perp^2)\hat{n}^k\hat{n}^l+\mathcal{U}_2(q_\perp^2)\frac{q_\perp^k q_\perp^l}{m^2}\right]\hat{Q}^{kl}_{s's}\right]\right\}, \end{aligned} \tag{3.4b}$$

and the coefficient functions $\mathcal{E}_i, \mathcal{C}_i, \mathcal{W}_i$ and $\mathcal{U}_i$ are given in the appendix and further,

$$\begin{aligned} N_{\phi,R} &= \frac{1}{R}\int_0^\infty dQ\,Q^3|\tilde{\phi}(|\mathbf{Q}|)|^2\,,\\ N_{\phi,R,2} &= \frac{m^2R}{2}\int_0^\infty dQ\,Q|\tilde{\phi}(|\mathbf{Q}|)|^2\,. \end{aligned} \tag{3.5}$$

Below, we specify the multipole expansion of $t^{\mu\nu}_\phi(s',s,\mathbf{r})$. For the 00th component the result reads

$$t^{00}_\phi(s',s,\mathbf{r}) = N_{\phi,R}\left\{\rho^E_0\,(r)\,\delta_{s's}+\rho^E_2\,(r)\,Y^{kl}_2\,(\Omega_r)\,\hat{Q}^{kl}_{s's}\right\}, \tag{3.6}$$

JHEP02(2023)163

where the monopole and quadrupole energy densities are identified as ρ_0^E and ρ_2^E, respectively. Their expressions have the form:

$$\begin{aligned}\rho_0^E(r) =& \int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\Big\{F_{1,0}(-q_\perp^2)-\frac{2}{3}F_{6,0}(-q_\perp^2)\\&+\frac{q_\perp^2}{3m^2}\Big[-F_{1,0}(-q_\perp^2)+\frac{1}{2}F_{1,1}(-q_\perp^2)+F_{4,0}(-q_\perp^2)+2F_{5,0}(-q_\perp^2)\Big]-\frac{q_\perp^4}{12m^4}\big[F_{1,1}(-q_\perp^2)-F_{4,1}(-q_\perp^2)\big]\Big\},\end{aligned} \tag{3.7}$$

$$\begin{aligned}\rho_2^E(r) =& -r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\frac{1}{q^2}\Big\{\frac{2}{3}F_{6,0}(-q_\perp^2)-\frac{q_\perp^2}{q^2}F_{6,0}(-q_\perp^2)\\&+\frac{q_\perp^2}{12m^2}\big[4F_{1,0}(-q_\perp^2)+F_{1,1}(-q_\perp^2)-4F_{4,0}(-q_\perp^2)-8F_{5,0}(-q_\perp^2)\big]+\frac{q_\perp^4}{12m^4}\big[F_{1,1}(-q_\perp^2)-F_{4,1}(-q_\perp^2)\big]\\&+\frac{q_\perp^4}{4m^2q^2}\big[4F_{5,0}(-q_\perp^2)-F_{1,0}(-q_\perp^2)-F_{1,1}(-q_\perp^2)\big]-\frac{q_\perp^6}{16m^4q^2}F_{1,1}(-q_\perp^2)\Big\}.\end{aligned} \tag{3.8}$$

For the $0i$th components we have

$$t_\phi^{0i}(s',s,\mathbf{r}) = N_{\phi,R}\Big[\epsilon^{ikn}\hat{S}^k_{s's}Y_1^n\frac{1}{r}\rho_1^J(r) + \epsilon^{ikn}\hat{O}^{ktz}_{s's}Y_3^{ntz}\frac{1}{r}\rho_3^J(r)\Big], \tag{3.9}$$

where

$$\begin{aligned}\rho_1^J(r) =& \frac{r}{m}\frac{d}{dr}\int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\frac{q_\perp^2}{2q^2}\Big\{F_{1,0}(-q_\perp^2)-\frac{2}{3}F_{4,0}(-q_\perp^2)-\frac{4}{15}F_{6,0}(-q_\perp^2)\\&+\frac{q_\perp^2}{15m^2}\Big[-F_{1,0}(-q_\perp^2)+\frac{7}{2}F_{1,1}(-q_\perp^2)+2F_{4,0}(-q_\perp^2)-2F_{4,1}(-q_\perp^2)+4F_{5,0}(-q_\perp^2)\Big]\\&+\frac{q_\perp^4}{60m^4}\Big[-F_{1,1}(-q_\perp^2)+F_{4,0}(-q_\perp^2)+F_{4,1}(-q_\perp^2)\Big]\Big\},\end{aligned} \tag{3.10}$$

$$\begin{aligned}\rho_3^J(r) =& \frac{r^4}{m}\left(\frac{1}{r}\frac{d}{dr}\right)^3\int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\frac{q_\perp^2}{12q^4}\Big\{-4F^a_{6,0}(-q_\perp^2)+5\frac{q_\perp^2}{q^2}F^a_{6,0}(-q_\perp^2)\\&-\frac{q_\perp^2}{4m^2}\Big[4F^a_{1,0}(-q_\perp^2)+F^a_{1,1}(-q_\perp^2)-8F^a_{4,0}(-q_\perp^2)-2F^a_{4,1}(-q_\perp^2)-16F^a_{5,0}(-q_\perp^2)\Big]\\&+\frac{5q_\perp^4}{4m^2q^2}\Big[F^a_{1,0}(-q_\perp^2)+F^a_{1,1}(-q_\perp^2)-2F^a_{4,0}(-q_\perp^2)-2F^a_{4,1}(-q_\perp^2)-4F^a_{5,0}(-q_\perp^2)\Big]\\&-\frac{q_\perp^4}{4m^4}\Big[F^a_{1,1}(-q_\perp^2)-2F^a_{4,1}(-q_\perp^2)\Big]+\frac{5q_\perp^6}{16m^4q^2}\Big[F^a_{1,1}(-q_\perp^2)-2F^a_{4,1}(-q_\perp^2)\Big]\Big\}.\end{aligned} \tag{3.11}$$

The spin density is given by

$$\begin{aligned}J^i(\boldsymbol{r},s',s) = \epsilon^{ijk}r^jt_\phi^{0k}(s',s,\mathbf{r}) = N_{\phi,R}\Big\{&\left(\frac{2}{3}\delta^{il}Y_0-Y_2^{il}\right)\rho_1^J(r)\ \hat{S}^l_{s's}\\&+\Big[-Y_4^{iltz}+\frac{2}{35}\left(8\delta^{il}Y_2^{tz}+\delta^{it}Y_2^{lz}+\delta^{iz}Y_2^{lt}\right)\Big]\rho_3^J(r)\ \hat{O}^{ltz}_{s's}\Big\}.\end{aligned} \tag{3.12}$$

We identify the monopole angular momentum density ($J^i_{\rm mono}$) as the term with the structure $Y_0\hat{S}^i_{s's}$, i.e.

$$J^i_{\rm mono}(\mathbf{r},s',s) = \frac{2}{3}N_{\phi,R}\,\hat{S}^i_{s's}\rho_1^J(r)\,, \tag{3.13}$$

JHEP02(2023)163

and, following ref. [34], the averaged angular momentum density is given by

$$\rho_J(r) \equiv \frac{S}{\mathrm{Tr}\left[\hat{S}^2\right]} \sum_{s',s,i} S^i_{ss'} J^i_{\mathrm{mono}}(\vec{r},s',s) = N_{\phi,R}\, \rho^J_1(r) \ , \tag{3.14}$$

where $S = 3/2$ is the spin of the system.

Finally, for the ijth components we obtain

$$\begin{aligned} t^{ij}_{\phi,0}(s',s,\mathbf{r}) =\ & N_{\phi,R}\Bigg\{ \left[a_1(r)\,\delta^{ij} - \left(\frac{\delta^{ij}}{3}\partial^2 + Y^{ij}_2 r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\right) a_2(r)\right]\delta_{s's} + \hat{Q}^{ij}_{s's} a_3(r) \\ & + \hat{Q}^{kl}_{s's}\partial_k\partial_l \left(\frac{\delta^{ij}}{3}\partial^2 + Y^{ij}_2 r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\right) a_4(r) - \delta^{ij}\hat{Q}^{kl}_{s's}\partial_k\partial_l a_5(r) \\ & - \left[\frac{2}{3}\hat{Q}^{ij}_{s's}\partial^2 + \left(\hat{Q}^{iv}_{s's}Y^{jv}_2 + \hat{Q}^{jv}_{s's}Y^{iv}_2\right) r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\right] a_6(r) \Bigg\}, \end{aligned} \tag{3.15}$$

$$\begin{aligned} t^{ij}_{\phi,2}(s',s,\mathbf{r}) =\ & N_{\phi,R,2}\Bigg\{ -\frac{1}{2m^2}\left(\frac{\delta^{ij}}{3}\partial^2 + Y^{ij}_2 r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\right) w_0(r)\ \delta_{s's} \\ & + \frac{1}{2m^2}\hat{Q}^{kl}_{s's}\partial_k\partial_l \left[\left(\frac{\delta^{ij}}{3}\partial^2 + Y^{ij}_2 r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\right) w_1(r)\right] \\ & + \delta^{ij}\left[v_0(r)\ \delta_{s's} - \hat{Q}^{kl}_{s's}\partial_k\partial_l\, v_1(r)\right] \Bigg\}, \end{aligned} \tag{3.16}$$

where the coefficient functions $a(r)$, $w(r)$ and $v(r)$ are given in the appendix.

To obtain the pressure and shear force densities we consider a conserved EMT and take the part of $\tilde{t}^{ij}_{\phi,2}(s',s,\mathbf{r})$ linear in R (where the tilde means only conserved EMTs are considered), which we parametrize as follows [35]:

$$\begin{aligned} \tilde{t}^{ij}_{\phi,2}(s',s,\mathbf{r}) =\ & N_{\phi,R,2}\Bigg\{ p_0(r)\delta^{ij}\delta_{s's} + s_0(r)Y^{ij}_2\delta_{s's} + p_2(r)\hat{Q}^{ij}_{s's} + 2s_2(r)\left[\hat{Q}^{ik}_{s's}Y^{kj}_2 + \hat{Q}^{jk}_{s's}Y^{ki}_2 - \delta^{ij}\hat{Q}^{kl}_{s's}Y^{kl}_2\right] \\ & - \frac{1}{m^2}\hat{Q}^{kl}_{s's}\partial_k\partial_l\left[p_3(r)\delta^{ij} + s_3(r)Y^{ij}_2\right] \Bigg\}, \end{aligned} \tag{3.17}$$

where $p_0(r)$ and $s_0(r)$ are the pressure and shear force densities also appearing in the spherically symmetric hadrons, respectively, $p_2(r)$, $p_3(r)$ correspond to the quadrupole pressure densities, and $s_2(r)$, $s_3(r)$ are the quadrupole shear force densities.[2] Comparing eqs. (3.17) and (3.16) we obtain for the pressure and shear forces the following results:

$$\begin{aligned} p_0(r) &= \tilde{v}_0(r) - \frac{1}{6m^2}\partial^2 w_0(r)\,, & s_0(r) &= -\frac{1}{2m^2} r\frac{d}{dr}\frac{1}{r}\frac{d}{dr} w_0(r)\,, \\ p_2(r) &= 0, & s_2(r) &= 0, \\ p_3(r) &= m^2\tilde{v}_1(r) - \frac{1}{6}\partial^2 w_1(r)\,, & s_3(r) &= -\frac{1}{2} r\frac{d}{dr}\frac{1}{r}\frac{d}{dr} w_1(r)\,, \end{aligned} \tag{3.18}$$

[2] Another equivalent parametrization is given in ref. [36], where the normal and tangential forces can be defined in a compact way. However, it has been shown that the parametrization of ref. [35] has advantages in studying the mechanical structure, whenever performing an Abel transformation is involved [25].

where the coefficient functions $w(r)$ and $\tilde{v}(r)$ are given as

$$\begin{aligned}
w_0(r) &= \int \frac{d^2\hat{n}d^3q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \left[F_{2,0}(-q_\perp^2) + \frac{q_\perp^2}{6m^2}\left[-2F_{2,0}(-q_\perp^2)+F_{2,1}(-q_\perp^2)\right] - \frac{q_\perp^4}{12m^4}F_{2,1}(-q_\perp^2)\right], \qquad (3.19)\\
\tilde{v}_0(r) &= \int \frac{d^2\hat{n}d^3q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \left(-\frac{q_\perp^2}{2m^2}\right)\left[F_{2,0}(-q_\perp^2) + \frac{q_\perp^2}{6m^2}\left[-2F_{2,0}(-q_\perp^2)+F_{2,1}(-q_\perp^2)\right] - \frac{q_\perp^4}{12m^4}F_{2,1}(-q_\perp^2)\right],\\
w_1(r) &= \int \frac{d^2\hat{n}d^3q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \frac{q_\perp^2}{2m^2q^2}\left[\left(\frac{2}{3}-\frac{q_\perp^2}{2q^2}\right)F_{2,0}(-q_\perp^2)+\frac{1}{2}\left(\frac{1}{3}+\frac{q_\perp^2}{3m^2}-\frac{q_\perp^2}{q^2}-\frac{q_\perp^4}{4m^2q^2}\right)F_{2,1}(-q_\perp^2)\right],\\
\tilde{v}_1(r) &= \int \frac{d^2\hat{n}d^3q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \left(-\frac{q_\perp^4}{4m^4q^2}\right)\left[\left(\frac{2}{3}-\frac{q_\perp^2}{2q^2}\right)F_{2,0}(-q_\perp^2)+\frac{1}{2}\left(\frac{1}{3}+\frac{q_\perp^2}{3m^2}-\frac{q_\perp^2}{q^2}-\frac{q_\perp^4}{4m^2q^2}\right)F_{2,1}(-q_\perp^2)\right].
\end{aligned}$$

It is clear from eqs. (3.18) and (3.19) that the pressure and shear forces are expressed in terms of $F_{2,0}$ and $F_{2,1}$ only.[3]

Below we obtain the differential equation for the pressure and shear forces that follows from the conservation of the EMT. In that case, we have to make our matrix element in eq. (3.1) time-dependent, i.e. we substitute $e^{-i\mathbf{q}\cdot\mathbf{r}} \mapsto e^{iq_0 t - i\mathbf{q}\cdot\mathbf{r}}$. The conservation of the EMT leads to

$$\partial_\mu t_\phi^{\mu\nu}(s',s,\mathbf{r},t)|_{t=0} = \partial_0 t_\phi^{0\nu}(s',s,\mathbf{r},t)|_{t=0} + \partial_i t_\phi^{i\nu}(s',s,\mathbf{r},t)|_{t=0} = 0. \qquad (3.20)$$

Notice that ∂_0 and $|_{t=0}$ do not commute, i.e. to obtain $\partial_0 t_\phi^{0\nu}(s',s,\mathbf{r},t)|_{t=0}$ we first take the derivative of $t_\phi^{0\nu}(s',s,r)$, then put $t=0$ and after that perform the expansion around $R=0$. Since we are interested in the pressure and shear forces, we consider the case with $\nu=j$ and keep all contributions linear in R. We obtain the following differential equation, adhering to the notation of ref. [35],

$$p_n'(r) + \frac{2}{3}s_n'(r) + \frac{2}{r}s_n(r) = h_n'(r), \quad \text{with } n=0,2,3, \qquad (3.21)$$

where

$$\begin{aligned}
h_0(r) &= \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int d^2\hat{n} \frac{(\mathbf{q}\cdot\hat{n})^2}{2m^2} \mathcal{W}_0\left(q_\perp^2\right), \quad h_2(r)=0, \qquad (3.22)\\
h_3(r) &= \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int d^2\hat{n} \frac{(\mathbf{q}\cdot\hat{n})^2}{2q^2} \left[\mathcal{W}_1\left(q_\perp^2\right)\left(1-\frac{3q_\perp^2}{2q^2}\right)+\mathcal{W}_2\left(q_\perp^2\right)\frac{q_\perp^2}{2m^2}\left(\frac{3q_\perp^2}{q^2}-1\right)\right],
\end{aligned}$$

with $r=|\mathbf{r}|$, and the coefficient functions $\mathcal{W}_{0,1,2}(q_\perp^2)$ are given in the appendix. In contrast, there are no $h_n(r)$ terms in eq. (3.21) in the case of the Breit-frame because of the absence of the temporal dependence in $t^{0\nu}$ component (due to $q^0=0$).

The pressure densities $p_n(r)$ comply with the von Laue stability condition

$$\int d^3r\, p_n(r) = 0, \quad \text{with } n=0,2,3, \qquad (3.23)$$

as long as $\lim_{q_\perp^2\to 0}\left(q_\perp^2\right)^\delta F_{2,0}\left(-q_\perp^2\right)=0$ and $\lim_{q_\perp^2\to 0}\left(q_\perp^2\right)^\delta F_{2,1}\left(-q_\perp^2\right)=0$, for $\delta>0$.

[3] Notice that the feature $p_2=s_2=0$ is not identical with the result of the large N_c limit for baryons in the chiral soliton model as obtained in ref. [36], where another parameterization of the pressure and shear forces is used. This can be easily checked by converting the pressure and shear forces in eq. (3.18) to the notations used in ref. [36].

The dimensionless constants (generalized D-terms) are defined by

$$\mathcal{D}_n = -\frac{4}{15}\, m^2 \int \mathrm{d}^3r\, r^2 s_n(r) = m^2 \int d^3r\, r^2 \left[p_n(r) - h_n(r)\right], \quad \text{with} \quad n = 0, 2, 3. \tag{3.24}$$

Note that the above definition differs from that of the Breit-frame case [36] by the $h_n(r)$ terms.

The spherical components of the internal forces (dF_r, dF_θ and dF_φ) acting on the radial area element ($d\boldsymbol{S} = dS_r\hat{\boldsymbol{e}}_r + dS_\theta\hat{\boldsymbol{e}}_\theta + dS_\varphi\hat{\boldsymbol{e}}_\varphi$) are expressed as follows

$$\frac{dF_r}{dS_r} = N_{\phi,R,2}\left[\left(p_0(r) + \frac{2}{3}s_0(r)\right)\delta_{s's} + \left(p_2(r) + \frac{2}{3}s_2(r)\right)\hat{Q}^{rr}_{s's}\right. \tag{3.25}$$

$$\left. -\frac{1}{m^2}\hat{Q}^{rr}_{s's}\left(r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\left(p_3(r) + \frac{2}{3}s_3(r)\right) + s_3(r)\frac{2}{r^2}\right)\right], \tag{3.26}$$

$$\frac{dF_\theta}{dS_r} = N_{\phi,R,2}\left[\left(p_2(r) + \frac{2}{3}s_2(r)\right)\hat{Q}^{\theta r}_{s's} - \frac{2}{m^2}\hat{Q}^{\theta r}_{s's}\frac{d}{dr}\frac{s_3(r)}{r}\right], \tag{3.27}$$

$$\frac{dF_\varphi}{dS_r} = N_{\phi,R,2}\left[\left(p_2(r) + \frac{2}{3}s_2(r)\right)\hat{Q}^{\varphi r}_{s's} - \frac{2}{m^2}\hat{Q}^{\varphi r}_{s's}\frac{d}{dr}\frac{s_3(r)}{r}\right]. \tag{3.28}$$

Notice that for an unpolarized spin-3/2 hadron, the normal force acting on the radial area element (dF_r/dS_r) is solely due to $p_0(r) + \frac{2}{3}s_0(r)$.

As defined in ref. [6], the mechanical radius is given by

$$\langle r_n^2\rangle_{\rm mech} = \frac{\int d^3r\, r^2\left[p_n(r) + \frac{2}{3}s_n(r)\right]}{\int d^3r\, \left[p_n(r) + \frac{2}{3}s_n(r)\right]}. \tag{3.29}$$

Notice that eqs. (3.18), (3.19) and (3.29) lead to expressions for the radii which differ from those of the Breit frame.

Analogously to the case of the electromagnetic current, the static approximation is obtained by first expanding the integrand in $1/m$ and after that taking the limit of a sharply localized wave packet. The resulting naive densities read:

$$\begin{aligned}
t^{00}_{\rm naive}(s',s,\mathbf{r}) &= \int \frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\left\{\left[mF_{1,0}\left(-\mathbf{q}^2\right)+\frac{\mathbf{q}^2}{6m}F_{1,1}\left(-\mathbf{q}^2\right)\right]\delta_{s's}-\frac{1}{6m}F_{1,1}\left(-\mathbf{q}^2\right)q^kq^n\hat{Q}^{kn}_{s's}\right\},\\
t^{0i}_{\rm naive}(s',s,\mathbf{r}) &= -i\int \frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\epsilon^{ilk}q^l\left\{\frac{1}{3}\left[F_{4,0}\left(-\mathbf{q}^2\right)+\frac{\mathbf{q}^2}{5m^2}F_{4,1}\left(-\mathbf{q}^2\right)\right]\hat{S}^k_{s's}-\frac{F_{4,1}\left(-\mathbf{q}^2\right)}{6m^2}q^nq^t\hat{O}^{knt}_{s's}\right\},\\
t^{ij}_{\rm naive}(s',s,\mathbf{r}) &= \int \frac{d^3n}{R^2}|\tilde{\phi}(|\mathbf{n}|)|^2n^in^j\int \frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\left\{\left[\frac{1}{m}F_{1,0}\left(-\mathbf{q}^2\right)+\frac{\mathbf{q}^2}{6m^3}F_{1,1}\left(-\mathbf{q}^2\right)\right]\delta_{s's}\right.\\
&\quad\left.-\frac{1}{6m^3}F_{1,1}\left(-\mathbf{q}^2\right)q^kq^n\hat{Q}^{kn}_{s's}\right\}\\
&\quad+\int \frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\left\{\left(q^iq^j-\mathbf{q}^2\delta^{ij}\right)\left[\left(\frac{1}{4m}F_{2,0}\left(-\mathbf{q}^2\right)+\frac{\mathbf{q}^2}{24m^3}F_{2,1}\left(-\mathbf{q}^2\right)\right)\delta_{s's}\right.\right.\\
&\quad\left.-\frac{1}{24m^3}F_{2,1}\left(-\mathbf{q}^2\right)q^kq^n\hat{Q}^{kn}_{s's}\right]-\frac{4}{3m}F_{5,0}\left[\left(q^iq^j-\mathbf{q}^2\delta^{ij}\right)\delta_{s's}\right.\\
&\quad\left.+\frac{1}{2}\left(q^2\hat{Q}^{ij}_{s's}+\delta^{ij}q^kq^n\hat{Q}^{kn}_{s's}-q^kq^i\hat{Q}^{kj}_{s's}-q^kq^j\hat{Q}^{ki}_{s's}\right)\right]+\frac{F_{3,1}\left(-\mathbf{q}^2\right)}{6m}\delta^{ij}q^kq^n\hat{Q}^{kn}_{s's}\\
&\quad\left.-\left(mF_{3,0}\left(-\mathbf{q}^2\right)+\frac{\mathbf{q}^2}{6m}F_{3,1}\left(-\mathbf{q}^2\right)+\frac{2m}{3}F_{6,0}\left(-\mathbf{q}^2\right)\right)\delta^{ij}\delta_{s's}+\frac{2m}{3}F_{6,0}\left(-\mathbf{q}^2\right)\hat{Q}^{ij}_{s's}\right\}.
\end{aligned} \tag{3.30}$$

JHEP02(2023)163

The above results for $t^{00}_{\text{naive}}(s', s, \mathbf{r})$ and $t^{0i}_{\text{naive}}(s', s, \mathbf{r})$ agree with the corresponding Breit-frame expressions of ref. [34] modulo terms of higher-orders in the $1/m$ expansion, contained in the Breit-frame expressions. On the other hand, for the ijth component, ref. [34] only quotes the result corresponding to the second term of $t^{ij}_{\text{naive}}(s', s, \mathbf{r})$, contained in the last four lines of eq. (3.30), which is to be interpreted as characterizing the internal forces of the considered system [26, 30]. Again, the two expressions agree modulo terms of higher-orders in the $1/m$ expansion, contained in the Breit-frame expression of ref. [34].

4 Large distance behavior of the energy, spin, pressure and shear forces distributions of the delta resonance in chiral EFT

In ref. [37] the GFFs of the delta resonance have been calculated up to third chiral order. Using these results and restricting ourselves to the region of distances $1/\Lambda_{\text{strong}} \ll r \ll 1/M_\pi$ we obtained approximate asymptotic behavior of the spatial densities from the singularities of GFFs at $t = 0$ in the chiral limit. The third order one-loop calculations of ref. [37] led to the following expressions for corresponding leading non-analytical parts of the GFFs:

$$
\begin{aligned}
F_{1,0}(t) &= -\frac{25 g_1^2}{9216 F^2 m_\Delta}\, t\sqrt{-t}\,, \\
F_{1,1}(t) &= -\frac{5 g_1^2 m_\Delta}{1536 F^2}\, \sqrt{-t}\,, \\
F_{2,0}(t) &= \frac{5 g_1^2 m_\Delta}{768 F^2}\, \sqrt{-t}\,, \\
F_{2,1}(t) &= \frac{5 g_1^2 m_\Delta^3}{384 F^2}\, \frac{\sqrt{-t}}{t}\,, \\
F_{4,0}(t) &= -\frac{5 g_1^2}{1728 \pi^2 F^2}\, t \ln(-t/m_N^2)\,, \\
F_{4,1}(t) &= 0\,, \\
F_{5,0}(t) &= -\frac{5 g_1^2 m_\Delta}{9216 F^2}\, \sqrt{-t}\,,
\end{aligned} \tag{4.1}
$$

where $t = q^2$. Using the above results we obtain the following long-range behavior for the densities derived in the previous section

$$
\rho_0^E(r) = \frac{25 g_1^2}{1536 F^2 m_\Delta}\frac{1}{r^6} - \frac{10 g_1^2}{81 \pi^2 F^2 m_\Delta^2}\frac{1}{r^7} + \mathcal{O}\left(\frac{1}{r^8}\right), \tag{4.2}
$$

$$
\rho_2^E(r) = \frac{35 g_1^2}{6144 F^2 m_\Delta}\frac{1}{r^6} + \frac{35 g_1^2}{162 \pi^2 F^2 m_\Delta^2}\frac{1}{r^7} + \mathcal{O}\left(\frac{1}{r^8}\right), \tag{4.3}
$$

$$
\rho_1^J(r) = \frac{5 g_1^2}{162 \pi^2 F^2 m_\Delta}\frac{1}{r^5} - \frac{125 g_1^2}{3072 F^2 m_\Delta^2}\frac{1}{r^6} + \mathcal{O}\left(\frac{1}{r^7}\right), \tag{4.4}
$$

$$
\rho_3^J(r) = -\frac{625 g_1^2}{24576 F^2 m_\Delta^2}\frac{1}{r^6} + \frac{5 g_1^2}{54 \pi^2 F^2 m_\Delta^3}\frac{1}{r^7} + \mathcal{O}\left(\frac{1}{r^8}\right), \tag{4.5}
$$

$$
p_0(r) = -\frac{25 g_1^2}{2304 F^2 m_\Delta}\frac{1}{r^6} - \frac{75 g_1^2}{1024 F^2 m_\Delta^3}\frac{1}{r^8} + \mathcal{O}\left(\frac{1}{r^9}\right), \tag{4.6}
$$

$$s_0(r) = \frac{5g_1^2}{96F^2 m_\Delta}\frac{1}{r^6} + \frac{15g_1^2}{64F^2 m_\Delta^3}\frac{1}{r^8} + \mathcal{O}\left(\frac{1}{r^8}\right), \tag{4.7}$$

$$p_3(r) = \frac{85g_1^2 m_\Delta}{221184F^2}\frac{1}{r^4} - \frac{155g_1^2}{196608F^2 m_\Delta}\frac{1}{r^6} + \mathcal{O}\left(\frac{1}{r^8}\right), \tag{4.8}$$

$$s_3(r) = -\frac{25g_1^2 m_\Delta}{9216F^2}\frac{1}{r^4} + \frac{15g_1^2}{4096F^2 m_\Delta}\frac{1}{r^6} + \mathcal{O}\left(\frac{1}{r^8}\right). \tag{4.9}$$

Notice that while the delta resonances are unstable particles, our expressions satisfy the general stability conditions of ref. [6], i.e. $\rho_0^E(r) > 0$ and $\frac{2}{3}s_0(r) + p_0(r) > 0$. This result is in agreement with the observation of ref. [6] that the general stability conditions are necessary but not sufficient for a system to be stable.

5 Summary and conclusions

In this work we applied the novel definition of local spatial densities using sharply localized wave packets [21] to spin-3/2 systems. Matrix elements of the electromagnetic current and the energy-momentum tensor in the ZAMF were considered and integral representations of associated spatial densities in terms of form factors were derived. Following ref. [11], the corresponding expressions in the Breit-frame were obtained by first expanding the integrands in inverse powers of the mass of the system and then taking the limit of sharply localized wave packets. This corresponds to considering packet sizes that are much larger than the Compton wavelength of the system. To apply the new definition as well as the Breit-frame formulas one needs to take the packet sizes much smaller than any length scales characterizing internal structure of the system. This makes clear that the Breit-frame spatial densities cannot be used for systems whose Compton wavelengths and the radii have comparable sizes [11]. However, the novel definition used here does not impose any lower bound on the size of the wave packet and therefore can be applied to any systems.

Considering the spatial components of the matrix elements of the EMT we obtained the expressions of the pressure and the shear forces inside the spin 3/2-systems. We also obtained a differential equation satisfied by these quantities due to the conservation of the EMT.

The formalism can be extended to the $\Delta \to N$ transition form factors, however, this is not straightforward and requires a separate investigation. Work along such lines is under way.

Acknowledgments

We thank J. Panteleeva for numerous discussions and for checking some formulas. This work was supported in part by BMBF (Grant No. 05P21PCFP1), by DFG and NSFC through funds provided to the Sino-German CRC 110 "Symmetries and the Emergence of Structure in QCD" (NSFC Grant No. 11621131001, DFG Project-ID 196253076 — TRR 110), by CAS through a President's International Fellowship Initiative (PIFI) (Grant No. 2018DM0034), by the VolkswagenStiftung (Grant No. 93562), by the EU Horizon 2020 research and innovation program (STRONG-2020, grant agreement No. 824093), and by the Heisenberg-Landau Program 2021, by Guangdong Provincial funding with Grant No. 2019QN01X172, the National Natural Science Foundation of China with Grant No. 12035007 and No. 11947228, Guangdong Major Project of Basic and Applied Basic Research No. 2020B0301030008, and the Department of Science and Technology of Guangdong Province with Grant No. 2022A0505030010.

A Coefficient functions

Coefficient functions for $j_\phi^0(s',s,\mathbf{r})$ and $j_\phi^i(s',s,\mathbf{r})$:

$$\begin{aligned}
\mathcal{Z}_0(-q_\perp^2) &= F_{1,0}^V(-q_\perp^2)+\frac{q_\perp^2}{6m^2}\left[-2F_{1,0}^V(-q_\perp^2)+F_{1,1}^V(-q_\perp^2)+F_{2,0}^V(-q_\perp^2)\right] \\
&\quad +\frac{q_\perp^4}{24m^4}\left[-2F_{1,1}^V(-q_\perp^2)+F_{2,1}^V(-q_\perp^2)\right], \qquad \text{(A.1)}
\end{aligned}$$

$$\mathcal{Z}_1(-q_\perp^2) = \frac{q_\perp^2}{6m^2}\left[3F_{1,0}^V(-q_\perp^2)-2F_{2,0}^V(-q_\perp^2)\right]+\frac{q_\perp^4}{24m^4}\left[3F_{1,1}^V(-q_\perp^2)-2F_{2,1}^V(-q_\perp^2)\right], \qquad \text{(A.2)}$$

$$\mathcal{Z}_2(-q_\perp^2) = -\frac{1}{6}\left[-2F_{1,0}^V(-q_\perp^2)+F_{1,1}^V(-q_\perp^2)+2F_{2,0}^V(-q_\perp^2)\right]+\frac{q_\perp^2}{12m^2}\left[F_{1,1}^V(-q_\perp^2)-F_{2,1}^V(-q_\perp^2)\right], \qquad \text{(A.3)}$$

$$\mathcal{A}_0(-q_\perp^2) = F_{1,0}^V(-q_\perp^2)+\frac{q_\perp^2}{30m^2}\left[-2F_{1,0}^V(-q_\perp^2)+7F_{1,1}^V(-q_\perp^2)\right]-\frac{q_\perp^4}{60m^4}F_{1,1}^V(-q_\perp^2), \qquad \text{(A.4)}$$

$$\mathcal{A}_1(-q_\perp^2) = \frac{1}{3}F_{2,0}^V(-q_\perp^2)+\frac{q_\perp^2}{15m^2}\left[-F_{2,0}^V(-q_\perp^2)+F_{2,1}^V(-q_\perp^2)\right]-\frac{q_\perp^4}{60m^4}F_{2,1}^V(-q_\perp^2), \qquad \text{(A.5)}$$

$$\mathcal{A}_2(-q_\perp^2) = \frac{q_\perp^2}{6m^2}F_{1,0}^V(-q_\perp^2)+\frac{q_\perp^4}{24m^4}F_{1,1}^V(-q_\perp^2), \qquad \text{(A.6)}$$

$$\mathcal{A}_3(-q_\perp^2) = -\frac{1}{6}F_{1,1}^V(-q_\perp^2), \qquad \text{(A.7)}$$

$$\mathcal{A}_4(-q_\perp^2) = \frac{q_\perp^2}{6m^2}F_{2,0}^V(-q_\perp^2)+\frac{q_\perp^4}{24m^4}F_{2,1}^V(-q_\perp^2), \qquad \text{(A.8)}$$

$$\mathcal{A}_5(-q_\perp^2) = -\frac{1}{6}F_{2,1}^V(-q_\perp^2). \qquad \text{(A.9)}$$

Coefficient functions for $t_\phi^{00}(s',s,\mathbf{r})$ and $t_{\phi,0}^{ij}(s',s,\mathbf{r})$:

$$\begin{aligned}
\mathcal{E}_0\left(q_\perp^2\right) &= F_{1,0}(-q_\perp^2)-\frac{2}{3}F_{6,0}(-q_\perp^2)+\frac{q_\perp^2}{3m^2}\left[-F_{1,0}(-q_\perp^2)+\frac{1}{2}F_{1,1}(-q_\perp^2)+F_{4,0}(-q_\perp^2)+2F_{5,0}(-q_\perp^2)\right] \\
&\quad +\frac{q_\perp^4}{12m^4}\left[-F_{1,1}(-q_\perp^2)+F_{4,1}(-q_\perp^2)\right], \qquad \text{(A.10)}
\end{aligned}$$

$$\begin{aligned}
\mathcal{E}_1\left(q_\perp^2\right) &= \frac{2}{3}F_{6,0}(-q_\perp^2)+\frac{q_\perp^2}{m^2}\left[\frac{1}{2}F_{1,0}(-q_\perp^2)-\frac{2}{3}F_{4,0}(-q_\perp^2)-\frac{2}{3}F_{5,0}(-q_\perp^2)\right] \\
&\quad +\frac{q_\perp^4}{m^4}\left[\frac{1}{8}F_{1,1}(-q_\perp^2)-\frac{1}{6}F_{4,1}(-q_\perp^2)\right], \qquad \text{(A.11)}
\end{aligned}$$

$$\mathcal{E}_2\left(q_\perp^2\right) = \frac{1}{3}\left[F_{1,0}(-q_\perp^2)-\frac{1}{2}F_{1,1}(-q_\perp^2)-2F_{4,0}(-q_\perp^2)\right]+\frac{q_\perp^2}{12m^2}\left[F_{1,1}(-q_\perp^2)-2F_{4,1}(-q_\perp^2)\right], \qquad \text{(A.12)}$$

$$\begin{aligned}
a_1(r) &= \int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\,\frac{q_\perp^2}{2q^2}\Bigg\{F_{1,0}(-q_\perp^2)-\frac{2}{3}F_{6,0}(-q_\perp^2) \qquad \text{(A.13)} \\
&\quad -\frac{q_\perp^2}{3m^2}\left[F_{1,0}(-q_\perp^2)-\frac{1}{2}F_{1,1}(-q_\perp^2)-F_{4,0}(-q_\perp^2)-2F_{5,0}(-q_\perp^2)\right]-\frac{q_\perp^4}{12m^4}\left[F_{1,1}(-q_\perp^2)-F_{4,1}(-q_\perp^2)\right]\Bigg\},
\end{aligned}$$

$$\begin{aligned}
a_2(r) &= \int\frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\,\frac{1}{q^2}\left(1-\frac{3q_\perp^2}{2q^2}\right)\Bigg\{F_{1,0}(-q_\perp^2)-\frac{2}{3}F_{6,0}(-q_\perp^2) \qquad \text{(A.14)} \\
&\quad -\frac{q_\perp^2}{3m^2}\left[F_{1,0}(-q_\perp^2)-\frac{1}{2}F_{1,1}(-q_\perp^2)-F_{4,0}(-q_\perp^2)-2F_{5,0}(-q_\perp^2)\right]-\frac{q_\perp^4}{12m^4}\left[F_{1,1}(-q_\perp^2)-F_{4,1}(-q_\perp^2)\right]\Bigg\},
\end{aligned}$$

JHEP02(2023)163

$$a_3(r) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int d^2\hat{n}\, \frac{q_\perp^4}{4q^4} \Big\{ \frac{2}{3} F_{6,0}\left(-q_\perp^2\right) + \frac{q^2}{6m^2}\left[2F_{1,0}\left(-q_\perp^2\right) - F_{1,1}\left(-q_\perp^2\right) - 4F_{4,0}\left(-q_\perp^2\right)\right]$$

$$+\frac{q_\perp^2}{6m^2}\left[F_{1,0}\left(-q_\perp^2\right) + F_{1,1}\left(-q_\perp^2\right) - 4F_{5,0}\left(-q_\perp^2\right)\right] + \frac{q^2 q_\perp^2}{12m^4}\left[F_{1,1}\left(-q_\perp^2\right) - 2F_{4,1}\left(-q_\perp^2\right)\right]$$

$$+\frac{q_\perp^4}{24m^4} F_{1,1}\left(-q_\perp^2\right) \Big\}, \tag{A.15}$$

$$a_4(r) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int d^2\hat{n}\, \frac{1}{3q^4} \Big\{ 2F_{6,0}\left(-q_\perp^2\right) - \frac{10q_\perp^2}{q^2} F_{6,0}\left(-q_\perp^2\right) + \frac{35q_\perp^4}{4q^4} F_{6,0}\left(-q_\perp^2\right)$$

$$+\frac{q_\perp^2}{4m^2}\left[4F_{1,0}\left(-q_\perp^2\right) + F_{1,1}\left(-q_\perp^2\right) - 4F_{4,0}\left(-q_\perp^2\right) - 8F_{5,0}\left(-q_\perp^2\right)\right]$$

$$-\frac{5q_\perp^4}{8m^2q^2}\left[5F_{1,0}\left(-q_\perp^2\right) + \frac{7}{2}F_{1,1}\left(-q_\perp^2\right) - 2F_{4,0}\left(-q_\perp^2\right) - 16F_{5,0}\left(-q_\perp^2\right)\right]$$

$$+\frac{q_\perp^4}{4m^4}\left[F_{1,1}\left(-q_\perp^2\right) - F_{4,1}\left(-q_\perp^2\right)\right] + \frac{35q_\perp^6}{16m^2q^4}\left[F_{1,0}\left(-q_\perp^2\right) + F_{1,1}\left(-q_\perp^2\right) - 4F_{5,0}\left(-q_\perp^2\right)\right]$$

$$+\frac{5q_\perp^6}{32m^4q^2}\left[-5F_{1,1}\left(-q_\perp^2\right) + 2F_{4,1}\left(-q_\perp^2\right)\right] + \frac{35q_\perp^8}{64m^4q^4} F_{1,1}\left(-q_\perp^2\right) \Big\}, \tag{A.16}$$

$$a_5(r) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int d^2\hat{n}\, \frac{q_\perp^2}{12q^4} \Big\{ 4F_{6,0}\left(-q_\perp^2\right) - \frac{5q_\perp^2}{q^2} F_{6,0}\left(-q_\perp^2\right)$$

$$+\frac{q_\perp^2}{2m^2}\left[5F_{1,0}\left(-q_\perp^2\right) + \frac{1}{2}F_{1,1}\left(-q_\perp^2\right) - 6F_{4,0}\left(-q_\perp^2\right) - 8F_{5,0}\left(-q_\perp^2\right)\right]$$

$$-\frac{5q_\perp^4}{4m^2q^2}\left[F_{1,0}\left(-q_\perp^2\right) + F_{1,1}\left(-q_\perp^2\right) - 4F_{5,0}\left(-q_\perp^2\right)\right] + \frac{q_\perp^4}{8m^4}\left[5F_{1,1}\left(-q_\perp^2\right) - 6F_{4,1}\left(-q_\perp^2\right)\right]$$

$$-\frac{5q_\perp^6}{16m^4q^2} F_{1,1}\left(-q_\perp^2\right) \Big\}, \tag{A.17}$$

$$a_6(r) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int d^2\hat{n}\, \frac{q_\perp^2}{3q^4} \Big\{ 2F_{6,0}\left(-q_\perp^2\right) - \frac{5q_\perp^2}{2q^2} F_{6,0}\left(-q_\perp^2\right)$$

$$+\frac{q_\perp^2}{8m^2}\left[2F_{1,0}\left(-q_\perp^2\right) + 5F_{1,1}\left(-q_\perp^2\right) + 4F_{4,0}\left(-q_\perp^2\right) - 16F_{5,0}\left(-q_\perp^2\right)\right]$$

$$-\frac{5q_\perp^4}{8m^2q^2}\left[F_{1,0}\left(-q_\perp^2\right) + F_{1,1}\left(-q_\perp^2\right) - 4F_{5,0}\left(-q_\perp^2\right)\right] + \frac{q_\perp^4}{16m^4}\left[F_{1,1}\left(-q_\perp^2\right) + 2F_{4,1}\left(-q_\perp^2\right)\right]$$

$$-\frac{5q_\perp^6}{32m^4q^2} F_{1,1}\left(-q_\perp^2\right) \Big\}. \tag{A.18}$$

Coefficient functions for $t^{ij}_{\phi,2}(s', s, \mathbf{r})$:

$$\mathcal{W}_0\left(q_\perp^2\right) = F_{2,0}(-q_\perp^2) + \frac{q_\perp^2}{6m^2}\left[-2F_{2,0}(-q_\perp^2) + F_{2,1}(-q_\perp^2)\right] - \frac{q_\perp^4}{12m^4} F_{2,1}(-q_\perp^2), \tag{A.19}$$

$$\mathcal{W}_1\left(q_\perp^2\right) = \frac{q_\perp^2}{2m^2} F_{2,0}(-q_\perp^2) + \frac{q_\perp^4}{8m^4} F_{2,1}(-q_\perp^2), \tag{A.20}$$

$$\mathcal{W}_2\left(q_\perp^2\right) = \frac{1}{6}\left[2F_{2,0}(-q_\perp^2) - F_{2,1}(-q_\perp^2)\right] + \frac{q_\perp^2}{12m^2} F_{2,1}(-q_\perp^2), \tag{A.21}$$

$$\mathcal{U}_0\left(q_\perp^2\right) = F_{3,0}(-q_\perp^2) + \frac{q_\perp^2}{6m^2}\left[-2F_{3,0}(-q_\perp^2) + F_{3,1}(-q_\perp^2)\right] - \frac{q_\perp^4}{12m^4} F_{3,1}(-q_\perp^2), \tag{A.22}$$

$$\mathcal{U}_1\left(q_\perp^2\right) = \frac{q_\perp^2}{2m^2} F_{3,0}(-q_\perp^2) + \frac{q_\perp^4}{8m^4} F_{3,1}(-q_\perp^2), \tag{A.23}$$

$$\mathcal{U}_2\left(q_\perp^2\right) = \frac{1}{6}\left[2F_{3,0}(-q_\perp^2) - F_{3,1}(-q_\perp^2)\right] + \frac{q_\perp^2}{12m^2} F_{3,1}(-q_\perp^2), \tag{A.24}$$

$$v_0(r) = \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \int d^2\hat{n} \Big\{ -2F_{3,0}(-q_\perp^2) + \frac{q_\perp^2}{3m^2}\left[-\frac{3}{2}F_{2,0}(-q_\perp^2) + 2F_{3,0}(-q_\perp^2) - F_{3,1}(-q_\perp^2)\right]$$

$$+\frac{q_\perp^4}{6m^4}\left[F_{2,0}(-q_\perp^2) - \frac{1}{2}F_{2,1}(-q_\perp^2) + F_{3,1}(-q_\perp^2)\right] + \frac{q_\perp^6}{24m^6} F_{2,1}(-q_\perp^2) \Big\}, \tag{A.25}$$

$$v_1(r) = \int \frac{d^2\hat{n} d^3 q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \frac{q_\perp^2}{2m^2 q^2} \left\{ \left(\frac{2}{3} - \frac{q_\perp^2}{2q^2}\right)\left(-\frac{q_\perp^2}{2m^2} F_{2,0}(-q_\perp^2) - 2F_{3,0}(-q_\perp^2)\right), \right.$$
$$\left. + \frac{1}{2}\left(\frac{1}{3} + \frac{q_\perp^2}{3m^2} - \frac{q_\perp^2}{q^2} - \frac{q_\perp^4}{4m^2 q^2}\right)\left(-\frac{q_\perp^2}{2m^2} F_{2,1}(-q_\perp^2) - 2F_{3,1}(-q_\perp^2)\right)\right\}. \tag{A.26}$$

Coefficient functions for $t_\phi^{0i}(s', s, \mathbf{r})$:

$$\begin{aligned} \mathcal{C}_0\left(q_\perp^2\right) = & F_{1,0}(-q_\perp^2) - \frac{1}{3} F_{4,0}(-q_\perp^2) - \frac{4}{15} F_{6,0}(-q_\perp^2) \\ & + \frac{q_\perp^2}{15m^2}\left[-F_{1,0}(-q_\perp^2) + \frac{7}{2} F_{1,1}(-q_\perp^2) + F_{4,0}(-q_\perp^2) - F_{4,1}(-q_\perp^2) + 4F_{5,0}(-q_\perp^2)\right] \\ & + \frac{q_\perp^4}{60m^4}\left[-F_{1,1}(-q_\perp^2) + F_{4,0}(-q_\perp^2)\right], \end{aligned} \tag{A.27}$$

$$\mathcal{C}_1\left(q_\perp^2\right) = \frac{1}{3} F_{4,0}(-q_\perp^2) + \frac{q_\perp^2}{15m^2}\left[-F_{4,0}(-q_\perp^2) + F_{4,1}(-q_\perp^2)\right] - \frac{q_\perp^4}{60m^4} F_{4,1}(-q_\perp^2), \tag{A.28}$$

$$\begin{aligned} \mathcal{C}_2\left(q_\perp^2\right) = & \frac{2}{3} F_{6,0}(-q_\perp^2) + \frac{q_\perp^2}{6m^2}\left[F_{1,0}(-q_\perp^2) - F_{4,0}(-q_\perp^2) - 4F_{5,0}(-q_\perp^2)\right] \\ & + \frac{q_\perp^4}{24m^4}\left[F_{1,1}(-q_\perp^2) - F_{4,1}(-q_\perp^2)\right], \end{aligned} \tag{A.29}$$

$$\mathcal{C}_3\left(q_\perp^2\right) = \frac{1}{6}\left[-F_{1,1}(-q_\perp^2) + F_{4,1}(-q_\perp^2)\right], \tag{A.30}$$

$$\mathcal{C}_4\left(q_\perp^2\right) = \frac{q_\perp^2}{6m^2} F_{4,0}(-q_\perp^2) + \frac{q_\perp^4}{24m^4} F_{4,1}(-q_\perp^2), \tag{A.31}$$

$$\mathcal{C}_5\left(q_\perp^2\right) = -\frac{1}{6} F_{4,1}(-q_\perp^2). \tag{A.32}$$

References

[1] R. Hofstadter, F. Bumiller and M.R. Yearian, *Electromagnetic structure of the proton and neutron*, *Rev. Mod. Phys.* **30** (1958) 482.

[2] F.J. Ernst, R.G. Sachs and K.C. Wali, *Electromagnetic form factors of the nucleon*, *Phys. Rev.* **119** (1960) 1105 [INSPIRE].

[3] R.G. Sachs, *High-energy behavior of nucleon electromagnetic form factors*, *Phys. Rev.* **126** (1962) 2256 [INSPIRE].

[4] M.V. Polyakov and A.G. Shuvaev, *On 'dual' parametrizations of generalized parton distributions*, Tech. Rep. RUB-TP2-12-02 (2002) [hep-ph/0207153] [INSPIRE].

[5] M.V. Polyakov, *Generalized parton distributions and strong forces inside nucleons and nuclei*, *Phys. Lett. B* **555** (2003) 57 [hep-ph/0210165] [INSPIRE].

[6] M.V. Polyakov and P. Schweitzer, *Forces inside hadrons: pressure, surface tension, mechanical radius, and all that*, *Int. J. Mod. Phys. A* **33** (2018) 1830025 [arXiv:1805.06596] [INSPIRE].

[7] M. Burkardt, *Impact parameter dependent parton distributions and off forward parton distributions for* $\zeta \to 0$, *Phys. Rev. D* **62** (2000) 071503 [*Erratum ibid.* **66** (2002) 119903] [hep-ph/0005108] [INSPIRE].

[8] G.A. Miller, *Charge density of the neutron*, *Phys. Rev. Lett.* **99** (2007) 112001 [arXiv:0705.2409] [INSPIRE].

[9] G.A. Miller, *Singular charge density at the center of the pion?*, *Phys. Rev. C* **79** (2009) 055204 [arXiv:0901.1117] [INSPIRE].

[10] G.A. Miller, *Transverse charge densities*, *Ann. Rev. Nucl. Part. Sci.* **60** (2010) 1 [arXiv:1002.0355] [INSPIRE].

[11] R.L. Jaffe, *Ambiguities in the definition of local spatial densities in light hadrons*, *Phys. Rev. D* **103** (2021) 016017 [arXiv:2010.15887] [INSPIRE].

[12] G.A. Miller, *Defining the proton radius: a unified treatment*, *Phys. Rev. C* **99** (2019) 035202 [arXiv:1812.02714] [INSPIRE].

[13] A. Freese and G.A. Miller, *Forces within hadrons on the light front*, *Phys. Rev. D* **103** (2021) 094023 [arXiv:2102.01683] [INSPIRE].

[14] C. Lorcé, H. Moutarde and A.P. Trawiński, *Revisiting the mechanical properties of the nucleon*, *Eur. Phys. J. C* **79** (2019) 89 [arXiv:1810.09837] [INSPIRE].

[15] C. Lorcé, *Charge distributions of moving nucleons*, *Phys. Rev. Lett.* **125** (2020) 232002 [arXiv:2007.05318] [INSPIRE].

[16] C. Lorcé, P. Schweitzer and K. Tezgin, *2D energy-momentum tensor distributions of nucleon in a large-N_c quark model from ultrarelativistic to nonrelativistic limit*, *Phys. Rev. D* **106** (2022) 014012 [arXiv:2202.01192] [INSPIRE].

[17] Y. Chen and C. Lorcé, *Pion and nucleon relativistic electromagnetic four-current distributions*, *Phys. Rev. D* **106** (2022) 116024 [arXiv:2210.02908] [INSPIRE].

[18] Y. Guo, X. Ji and K. Shiells, *Novel twist-three transverse-spin sum rule for the proton and related generalized parton distributions*, *Nucl. Phys. B* **969** (2021) 115440 [arXiv:2101.05243] [INSPIRE].

[19] J.Y. Panteleeva and M.V. Polyakov, *Forces inside the nucleon on the light front from 3D Breit frame force distributions: Abel tomography case*, *Phys. Rev. D* **104** (2021) 014008 [arXiv:2102.10902] [INSPIRE].

[20] J.Y. Panteleeva, E. Epelbaum, J. Gegelia and U.-G. Meißner, *Definition of electromagnetic local spatial densities for composite spin-1/2 systems*, *Phys. Rev. D* **106** (2022) 056019 [arXiv:2205.15061] [INSPIRE].

[21] E. Epelbaum, J. Gegelia, N. Lange, U.G. Meißner and M.V. Polyakov, *Definition of local spatial densities in hadrons*, *Phys. Rev. Lett.* **129** (2022) 012001 [arXiv:2201.02565] [INSPIRE].

[22] J.-Y. Kim and H.-C. Kim, *Transverse charge distributions of the nucleon and their Abel images*, *Phys. Rev. D* **104** (2021) 074003 [arXiv:2106.10986] [INSPIRE].

[23] J.-Y. Kim and H.-C. Kim, *Energy-momentum tensor of the nucleon on the light front: Abel tomography case*, *Phys. Rev. D* **104** (2021) 074019 [arXiv:2105.10279] [INSPIRE].

[24] J.-Y. Kim, *Electromagnetic multipole structure of a spin-one particle: Abel tomography case*, *Phys. Rev. D* **106** (2022) 014022 [arXiv:2204.08248] [INSPIRE].

JHEP02(2023)163

[25] J.-Y. Kim, B.-D. Sun, D. Fu and H.-C. Kim, *Mechanical structure of a spin-1 particle*, Tech. Rep. INHA-NTG-06/2022 (2022) [arXiv:2208.01240] [INSPIRE].

[26] A. Freese and G.A. Miller, *Unified formalism for electromagnetic and gravitational probes: densities*, *Phys. Rev. D* **105** (2022) 014003 [arXiv:2108.03301] [INSPIRE].

[27] A. Freese and G.A. Miller, *On the true meaning of spatial densities of hadrons*, Tech. Rep. NT@UW-22-17 (2022) [arXiv:2210.03807] [INSPIRE].

[28] C.E. Carlson, *Comments and extensions of a recent suggestion for a relativistic charge density definition*, arXiv:2208.00826 [INSPIRE].

[29] G.N. Fleming, *Charge distributions from relativistic form factors*, in *Physical reality and mathematical description*, Springer, Dordrecht, The Netherlands (1974), p. 357.

[30] J.Y. Panteleeva, E. Epelbaum, J. Gegelia and U.-G. Meißner, *Definition of gravitational local spatial densities for spin-0 and spin-1/2 systems*, arXiv:2211.09596 [INSPIRE].

[31] V. Pascalutsa, M. Vanderhaeghen and S.N. Yang, *Electromagnetic excitation of the $\Delta(1232)$-resonance*, *Phys. Rept.* **437** (2007) 125 [hep-ph/0609004] [INSPIRE].

[32] S. Cotogno, C. Lorcé, P. Lowdon and M. Morales, *Covariant multipole expansion of local currents for massive states of any spin*, *Phys. Rev. D* **101** (2020) 056016 [arXiv:1912.08749] [INSPIRE].

[33] J. Gegelia, G.S. Japaridze and K.S. Turashvili, *Calculation of loop integrals by dimensional counting*, *Theor. Math. Phys.* **101** (1994) 1313 [INSPIRE].

[34] J.-Y. Kim and B.-D. Sun, *Gravitational form factors of a baryon with spin-3/2*, *Eur. Phys. J. C* **81** (2021) 85 [arXiv:2011.00292] [INSPIRE].

[35] M.V. Polyakov and B.-D. Sun, *Gravitational form factors of a spin one particle*, *Phys. Rev. D* **100** (2019) 036003 [arXiv:1903.02738] [INSPIRE].

[36] J.Y. Panteleeva and M.V. Polyakov, *Quadrupole pressure and shear forces inside baryons in the large N_c limit*, *Phys. Lett. B* **809** (2020) 135707 [arXiv:2004.02912] [INSPIRE].

[37] H. Alharazin, E. Epelbaum, J. Gegelia, U.-G. Meißner and B.D. Sun, *Gravitational form factors of the delta resonance in chiral EFT*, *Eur. Phys. J. C* **82** (2022) 907 [arXiv:2209.01233] [INSPIRE].

JHEP02(2023)163