Mapping NP-Complete Problems to Physics-Based QUBO Solvers: Quantitative Comparison and Understanding

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Introduction

NP-complete problems like 3-SAT can be mapped and solved by emerging physics-based hardware such as Ising systems, quantum annealers, or Hopfield neural networks. Such systems natively handle quadratic unconstrained binary optimization (QUBO) problems, while higher order and constrained problem classes can be transformed into these simpler QUBO formulations. However, there are often multiple possible mappings for such transformations, with substantial performance differences. Here, we compared several different mappings from 3-SAT to a QUBO solver and quantified the differences in resources required (additional auxiliary variables) and final time-to-solution. Notably, while the global minimum of the 3-SAT problem matches the global minimum of the QUBO problem, we find stark differences in other portions of the landscape in terms of gradient directions. We attempt to explain the observed differences between the mappings utilizing a simplified under-sampling metric and showed good predictive capability. Our chosen platform was a Hopfield neural network, with different annealing techniques and neuron update rules compared.

Conclusion

Quadratization comes with many challenges, which includes larger search space, more rugged landscape, and not so faithful to the original landscape. Yet, quadratization is needed when mapped to physic-based solver such as Ising machines or neuromorphic solvers such as Hopfield networks. We presented that not all mapping are equals notwithstanding they all share the same global minimum. Moreover, update rules in the solver heuristics may produce different solution qualities such as ability to find a solution and time-to-solution (TTS).

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Different Mapping

Mapping 3-SAT into a QUBO solver can be implemented with different methods, each differs in dimension, energy landscape and time-to-solution. In this work we studied three different mappings, two from the literature [1, 2] and the third we developed to reduce auxiliary variable resulted from [1]. Table. 1 presents the variation in dimensionality by these three mappings. Although, our mapping aimed to reduced dimensionality and thence reduce TTS, we discovered that dimensionality of the mapping cannot be a predictor for the solver performance. Some instances benefited from the huge jump of dimensions resulted by [2], while others favored [1] as a sweet point in number of dimensions.

Problem Size	[1]	[2]	Ours
20	111	273	[58-73]
50	268	654	[178-206]
75	400	975	[88-318]

Table 1: Original 3-SAT problem sizes and their corresponding sizes after using the described QUBO mapping

Results Heading 2

Many metrics were introduced in the literature tried to measure the hardness of a 3-SAT instance such as [6]. We developed an under-sampling landscape analysis for the mappings and predicted with 90% accuracy the relative performance.Our goal is to make it easier and faster to calculate the metric than solving the problem.

We studied three different annealing techniques, two from literature. First is the classical Hopfield Neural Network asynchronous update [3], second is the digital annealer (DA) by the Fujitsu team [4]. Additionally, we simulated our own proposed stochastic group update (batch update) , wherein neurons are randomly grouped at each step and perform a parallel update for the whole group. The performance of each technique was studied quantitatively with inferior results by DA.

Figure 1: Top: TTS between [1],[2]and ours on selected instances from SATLIB [5]. Bottom: Our metric expected relative performance for [1] and [2] with 90% accuracy.

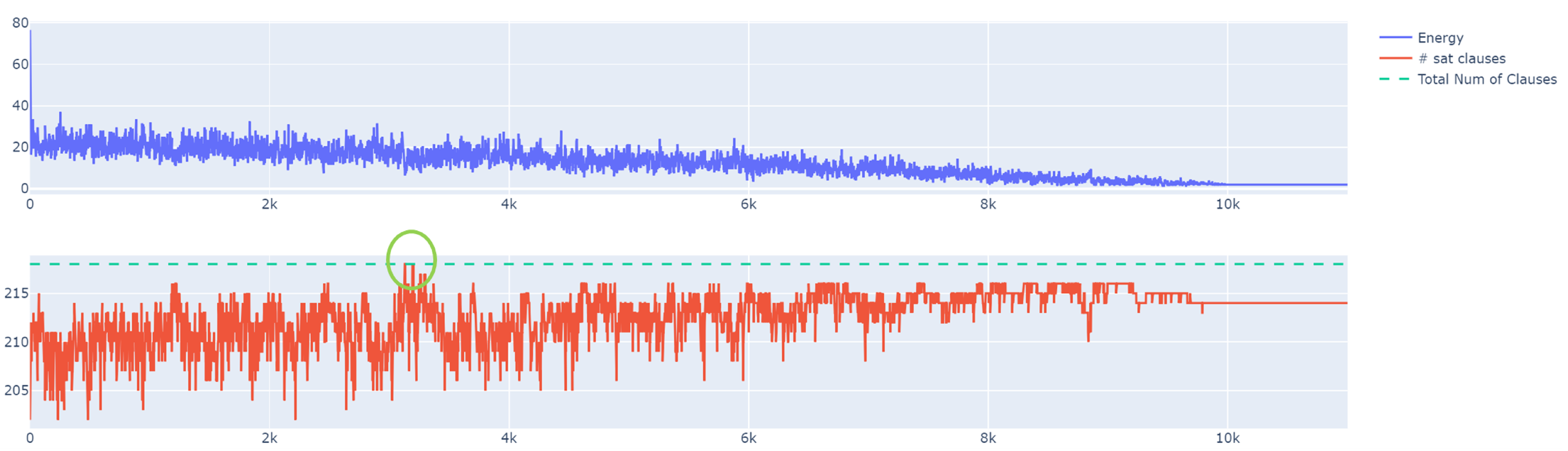

Figure 2: Top curve is the value of constructed objective function while bottom curve is number of satisfied clauses (original objective function) during a run of simulated annealing (x axis shows steps) of problem size 50 with 218 clauses. Green circle highlights when prob. was sated but not a global optimum in QUBO.

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