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## Parameter estimation and dynamic optimization of an industrial fed-batch reactor

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### Abstract

Modeling and optimization of fed-batch reactors with several multi-step reaction pathways is challenging due to the nonlinear dynamic system behavior and large number of kinetic parameters. We showcase the model-based optimization of an industrial (20 m<sup>3</sup>) fed-batch reactor by using our open-source dynamic optimization software DyOS. First, we build a detailed mechanistic model of the fed-batch reactor. Second, we conduct parameter estimation of the mechanistic model with 25 states and 44 fitting parameters using historic time-series industrial production data. Third, we perform dynamic multi-stage optimization and identify optimal feeding profiles for the operation, targeting improvements in economic profit over the established experience-based production routine. We demonstrate substantial economic improvement: The optimized production recipe can save up to 10% of raw material at the same yield of main product. Our findings underline the strong capabilities of model-based process optimization and its application to industrial challenges in process design and operation.

**Keywords:** Dynamic process optimization, Fed-batch reactor, Industrial application, Reaction optimization.

### 1. Introduction

The determination of feeding strategies is of paramount importance in the design and control of fed-batch reactors. Finding optimal feeding strategies is typically based on the dynamic optimization of a reactor model. However, modeling reaction processes with several multi-step reaction pathways can be challenging due to nonlinear system behavior that needs to be expressed by many equations and parameters, and a lack of mechanistic knowledge about the considered reaction, e.g., reaction kinetics and corresponding parameters. When experimental data of the reactor is available, kinetic parameters can be estimated using dynamic optimization. Subsequently, dynamic optimization can be employed for determining optimal feeding profiles with respect to a defined objective, e.g., reaction yield, product quality, and economic targets. Dynamic optimization of fed-batch reactors has been performed successfully for several applications including the production of (bio-)polymers (Zavala et al, 2005; Lopez et al., 2010, Jung et al. 2015), yeast (Hjersted & Henson, 2006) and drugs (Banga et al., 2005). We have also recently

optimized fed-batch reactors for microgel (Jung et al., 2019) and polymer (Faust et al. 2021) production. To this end, we have employed our dynamic optimization software DyOS (Caspari et al., 2019), which is an open-source framework for the optimization of large-scale differential algebraic equation (DAE) systems.

Herein, we use DyOS for the optimization of feeding strategies for a fed-batch reactor of industrial scale. Specifically, we conduct three steps with the overall goal to identify optimal feeding strategies: (i) mechanistic modeling of the fed-batch reactor with the reaction kinetics, (ii) estimation of kinetic reaction and heat transfer parameters, and (iii) dynamic multi-stage optimization of raw material feeding profiles. In the first step, we develop together with the plant operators a mechanistic reactor model in the Modelica modeling language (Modelica, 2022). The parameter estimation in the second step includes the collection of historic industrial production data of the currently established experienced-based feeding profiles that we then use to fit model parameters with DyOS. In the third step, we start from the established feeding profiles and apply DyOS to identify economically optimized feeding strategies. Our in-silico results suggest that the optimized feeding strategies can lead to substantial economic improvements and savings in raw materials.

Due to confidentiality reasons, we cannot provide detailed insights on the model, the corresponding data, and the literature sources we used to collect information about the considered reaction process. We thus focus on the methodological approach of using dynamic optimization for fitting model parameters and optimizing feeding profiles. In the following, we schematically describe the considered fed-batch reactor, explain the three steps (i – iii) in detail, and present the modeling and optimization results.

## 2. Fed-Batch Reactor Model Development

We consider an industrial stirred-tank fed-batch reactor producing one main product in a multi-step reaction. The reactor has a volume of 20 m<sup>3</sup>. A schematic illustration of the reactor is shown in Figure 1. The inputs to the reactor are composed of four raw materials (RM1, RM2, RM3, RM4), water (H<sub>2</sub>O), and a catalyst (CAT). All input components are fed in liquid form, either in pure species or in aqueous solutions. RM2, H<sub>2</sub>O, and CAT are filled into the reactor before the reactions start. When RM1 and RM3 are introduced to the reactor, several multi-step reactions involving multiple intermediate products (IPs) are initiated. RM1 and RM3 are dynamically fed during the reaction process and can be used to influence the reaction rates. The production goal is the main product (MP). Additionally, several undesired byproducts (BPs) occur in the reaction competing with the MP production. The reactor temperature during operation is controlled with cooling water (CW) that flows through the cooling jacket of the reactor and enters with ambient temperature. To stop the reaction process an inhibitor (RM4) is fed to the reactor. After feeding RM4, the species concentrations remain approximately constant and the reactor can be drained.

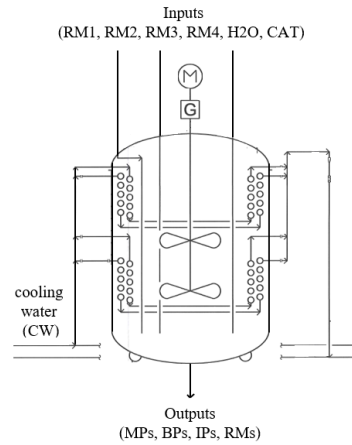


Figure 1. Schematic illustration of fed-batch reactor with input, output, and cooling water flows.

### 2.1. Mechanistic Modeling

We develop a dynamic model of the reactor and reaction system. The model includes 25 differential equations with 44 fitting parameters corresponding to the mass balance with the reaction kinetics and to the energy balance. The reaction kinetics involve 21 reactions for which reaction constants are calculated using the Arrhenius equation, i.e., two parameters in the form of the pre-exponential factor and the activation energy. Note that 3 side reactions have the same rate constant such that the reaction kinetics effectively have 38 fitting parameters in total. The energy balance accounts for the enthalpy of reactions, enthalpy flows, and cooling heat transfer rate affecting the temperature of the reaction mixture. Five reaction enthalpy parameters need to be estimated as several side reactions are modeled by the same enthalpy parameter. For the cooling, the heat transfer coefficient represents another fitting parameter, whereas we calculate the heat exchange surface and the cooling-water hold up based on the reactor geometry. For all other model parameters, e.g., as part of density and heat capacity correlations, we use standard values from the literature. In addition, we neglect other energy flows, e.g., heat loss over the reactor surface. Next to the differential equations, several algebraic equations, e.g., for pH, density, heat capacity, and yield calculation, are part of the model.

For modeling the dynamic input streams, we discretize the feed flows of RM1, RM3, RM4, and CW in one-minute intervals. Since RM2, H<sub>2</sub>O, and CAT are loaded to the reactor without any main reactions occurring, we use the respective total amount loaded to the reactor as initial values.

### 2.2. Implementation

We implement the reactor model in the modeling language Modelica (Modelica, 2022). Further, we apply our dynamic optimization framework DyOS (Caspari et al., 2019) with integrator NIXE (Hannemann et al., 2010) and NLP solver SNOPT (Gill et al., 2005). We use the Python interface of DyOS to implement and run the case study in Python. We use a Microsoft Windows Server with an Intel(R) Xeon(R) E5-2630 v2 processor running at 2.6 GHz and 128 GB RAM; simulating the model with given parameters took ca. 2 sec.

## 3. Parameter estimation

The model constitutes overall 44 fitting parameters that we initialize by data obtained from laboratory experiments and literature. However, as the simulation results when using these parametric values did not match the measured temperature profiles and final concentrations available from historical reactor data, we performed parameter estimation. To this end, we measured concentrations of some main components and the reactor temperature profile for one fed-batch run. In Figure 2, the feeding strategy for this run for (a) RM1, RM3, RM4, and (b) CW is illustrated. Subsequently, we used this experimental data for fitting the parameters of the model.

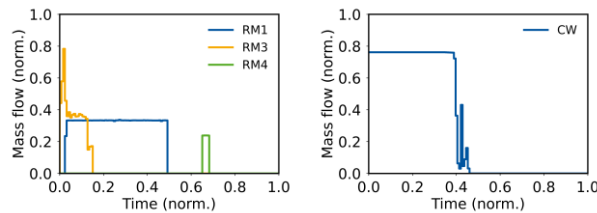


Figure 2. Feeding profiles of one fed-batch run for (a) RM1, RM3, RM4 and (b) CW.

In the parameter estimation process, we utilized DyOS to fit the 44 model parameters with the objective to minimize the mean squared error (MSE) of the model deviation to the measured time-series of the concentration profiles and the reactor temperature profile.

We solve the optimization problem using direct single shooting with optimality and feasibility tolerances of  $10^{-3}$ . We apply absolute and relative integrator tolerances of  $10^{-4}$ . Figure 3 shows the simulation results with the initial and fitted parameters in comparison to the experimental data for the temperature profile and the concentration profiles of the MP and RM2. In contrast to the initial model, the results from the fitted model match the temperature profile with slight deviations. The fitted model also predicts the concentration measurements of MP and RM2 with high accuracy. We stress that due to the large model size and large number of parameters, we did not perform a parameter identifiability study. Other parameter values might also yield a good fit but were not explored since we find a sufficient match of the experimental data using the fitted model.

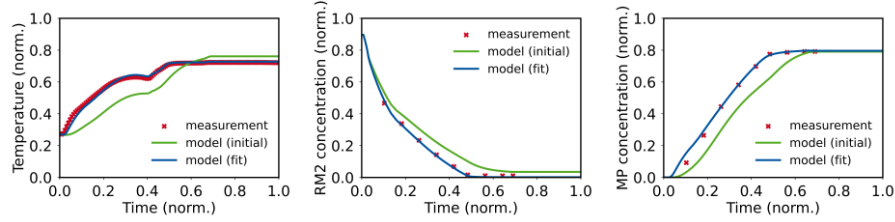


Figure 3. Comparison of the experimental results from batch run used for fitting with the simulation results obtained with the initial and the fitted model: Temperature (left) and concentrations for RM2 (middle) and MP (right).

For validation of the fitted model parameters, we conducted another experimental run of the industrial reactor at similar conditions. Figure 4 illustrates the temperature profiles and concentration measurements against the simulation results of the fitted model. We observe that the model matches the profiles closely. Some deviations regarding the MP concentration are, however, noticeable.

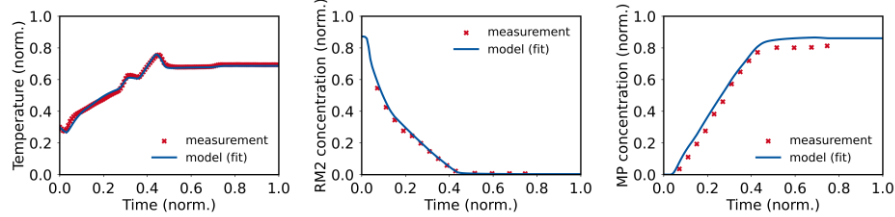


Figure 4. Comparison of the experimental results from validation batch run with the simulation results obtained with the fitted model: Temperature (left) and concentrations for RM2 (middle) and MP (right).

We also tested the model for other batch runs for which historical data on the temperature profile and the end concentrations but not all initial conditions, e.g., temperatures and impurities of raw material feed flows, was available. Here, we also observe a generally good match of the fitted model results and the experimental data, but also find some deviations, especially in the concentrations of some byproducts. We attribute the deviations to uncertainty factors in the initial conditions including BP impurities in the feed flows, limitations of the mechanistic model in capturing all characteristics of the multi-step reaction, and presumably slight overfitting of the parameters. In comparison to the initial model, the parameter fitting improves the prediction accuracy.

Overall, we conclude that the fitted model correctly captures the trends of temperature profiles and concentration measurements. In the next step, we will thus use the model with the fitted parameters for optimization of the feed profiles.

#### 4. Dynamic optimization of the reactor feeding profiles

In the optimization of the feeding profiles, we aim at identifying changes in the currently established feeding strategy to produce the same amount of MP while using fewer raw materials per batch. As a starting point for the optimization, we herein consider the feeding profiles from the fed-batch run for which we collected concentration measurements and fitted the model parameters. The objective of the optimization is to decrease the total costs of RM1 and RM3 fed to the reactor, which we formulate as

$$\begin{aligned}
 \min_{\dot{m}_i(t)} \quad & m_{RM1} \cdot c_{RM1} + m_{RM3} \cdot c_{RM3} \\
 \text{s. t.} \quad & m_{MP,cur}(t_{end}) \leq m_{MP}(t_{end}), \\
 & T_{R,min} \leq T_R \leq T_{R,max}, \\
 & V_{L,min} \leq V_L \leq V_{L,max}, \\
 & 0 \leq \dot{m}_i(t) \leq \dot{m}_{i,max}, \\
 & c_{BP,j} \leq c_{BP,j,max}, \\
 & i \in \{RM1, RM3, CW\}, j \in BPs,
 \end{aligned} \tag{1}$$

where  $c_{RM1}$  and  $c_{RM3}$  are cost factors and the mass flows,  $\dot{m}_i(t)$ , of the raw materials RM1 and RM3 and the CW are the control variables, i.e., degrees of freedom. To enforce a minimal MP production, we add a constraint for the final produced amount of MP,  $m_{MP}(t_{end})$ , to at least matching the currently established strategy (cur),  $m_{MP,cur}(t_{end})$ . We consider additional constraints that include limits on the reactor temperature,  $T_R$ , the reactor level  $V_L$ , the mass flows,  $\dot{m}_i(t)$ , and concentrations of BPs,  $c_{BP,j}$ . The upper and lower limits on the reactor temperature and level arise from safety considerations, the upper limits on the mass flows are based on plant equipment limits, and the upper limit on the BPs' concentrations results from process downstream quality limits. Furthermore, the process control system only allows for a maximum number of four step changes in the mass flow of RM1 which we formulated as a multi-stage problem with free stage duration. For solving the optimization problem (Eq. 1), we apply DyOS with the same integrator and optimizer settings as for parameter estimation (cf. Section 3).

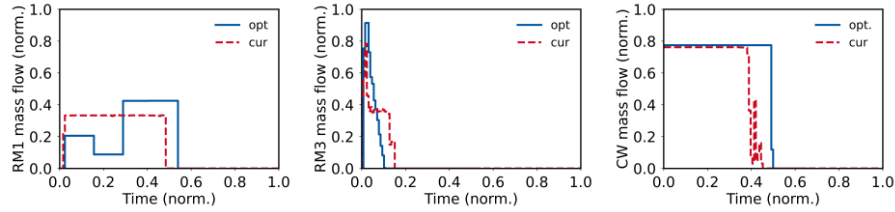


Figure 5: Optimized vs. currently established feeding strategy for RM1, RM3, and CW.

Figure 5 illustrates the currently established (dashed line) and optimized (full line) feed. According to the optimization results, RM1 should be fed in three steps with different flow rates instead one constant rate. The optimized RM3 feed is at a higher rate from the start of the batch and is then reduced almost linearly instead of in three steps. The cooling water feed is also at the maximum possible rate and changes only slightly compared to the established routine; specifically, the reactor is cooled longer. With the optimized feeding strategy, we find the produced MP amount being at the lower bound, i.e., matching the MP amount produced with the established feed profiles, which is expected as it is enforced by a constraint in the problem formulation. At the same time, the amounts of RM1 and RM3 can be reduced by 6% and 17%, respectively. This corresponds to about 10% cost savings as the same MP amount is produced by using less raw material. The main reasons for the reduced amount of required raw materials are higher main product

yield, less byproduct production, and less overfeeding of RM3. We are currently working on the experimental validation of these results.

## 5. Conclusions

We present the dynamic modeling and optimization of an industrial 20 m<sup>3</sup> fed-batch reactor. We first develop a mechanistic reactor model and then collect experimental data on the reaction process and perform parameter estimation using our in-house dynamic optimization software DyOS. Based on the developed model, we apply DyOS to identify promising feeding strategies for which the model suggests improved reaction yield and higher economic profit by 10 %. The experimental validation of the identified feeding strategies is planned as the next step with the overall goal to establish a new feeding routine for the industrial operation of the reactor.

Future work could include further model validation and potentially improvements of the model, e.g., by collecting additional experimental data that can be used for parameter fitting. Next to model refinements, more fine-grained feed profiles could be investigated to incentivize changes to the current process control system limitations. Furthermore, the presented method could be transferred to other processes with fed-batch reactors.

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