Statistical Methods in the Search for Electric Dipole Moments at COSY

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on behalf of the JEDI collaboration

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Motivation - Matter/Anitmatter Asymmetry

- **Big Bang**: Equal amount of matter & antimatter
- **Early universe**: Asymmetric annihilation process $B + \bar{B} \rightarrow \gamma\gamma + \cdots$

- **Today**: Asymmetry $(n_B - n_{\bar{B}})/n_\gamma$; with no antimatter left AMS-02)
  - model prediction from SCM not sufficient: $\approx 10^{-18}$
  - measured from cosmic background radiation spectrum (satellites): $\approx 10^{-10}$
  - **COBE** (1989-1993), **WMAP** (2001-2010) and **Planck** (2009-2013)

- According to the three Sakharov Criteria
- $\Rightarrow$ Search for CP violation **beyond SM**

Sakharov Criteria:

1. Baryon number violation
2. No thermal equilibrium
3. C and CP-Violation

Electric Dipole Moment (EDM)

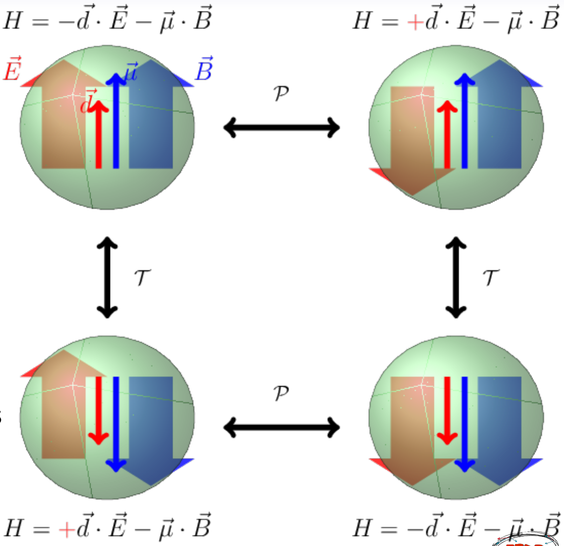
- **EDMs** and **MDMs** are fundamental properties of particles

$$\vec{d} = d \cdot \vec{S} = \eta_{\mathrm{EDM}} \frac{q}{2mc} \vec{S}$$

$$\vec{\mu} = \mu \cdot \vec{S} = g \frac{q}{2m} \vec{S}$$

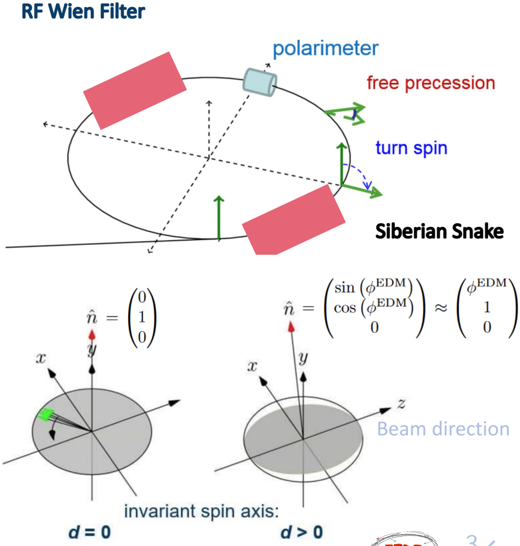
- EDM violates both **P** and **CP** symmetry assuming **CPT Theorem** holds

- EDMs extremely small, only upper limits until now (probe beyond SM physics)



Measurement Principle of EDM

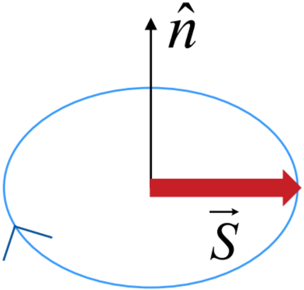
- **1) Injection** of vertically polarized beam
- **2) Rotation** of beam polarization into accelerator plane by **Solenoid**
- **3) Accumulation** of vertical polarization, signal buildup by **RF Wien Filter**.
- **4) Observation** of time dependent polarization by **polarimeter (JePo)**

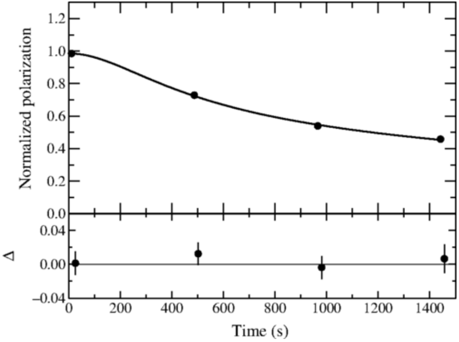


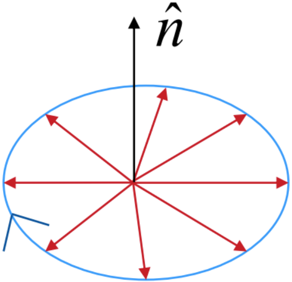
- Polarization vector **precesses** in plane around **invariant spin axis** $\hat{n}$
- EDM: Tilts $\hat{n}$ in radial direction ($\hat{\eta}_x \approx \phi^{\mathrm{EDM}}$)

Spin Coherence Time (SCT)

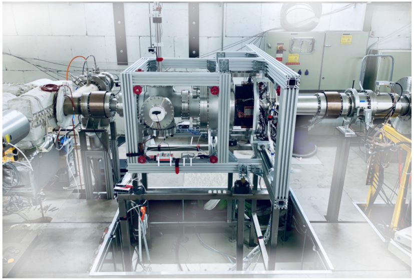
- Simple model: $P(t) \approx P_0 \exp\left(-\frac{t}{\tau}\right)$
- SCT τ: time after which polarization drops to $1/e$
- Buildup time for vertical polarization limited by SCT
- SCT $> 1000\,$s achieved at COSY for $\approx 10^9$ deuterons (2016) [PhysRevLett.117.054801]

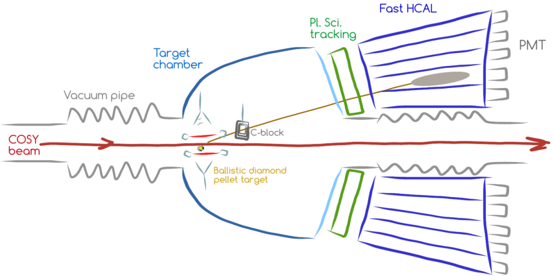






Jedi Polarimeter at COSY

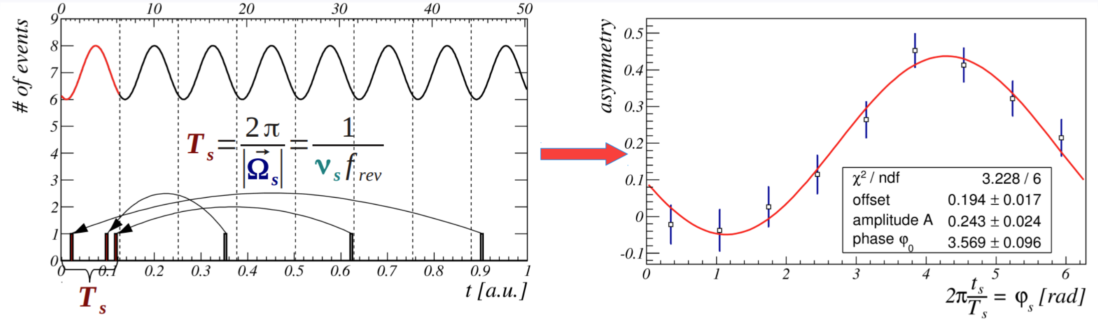




- JePo (Jedi Polarimeter), since 2019, based on LYSO crystals

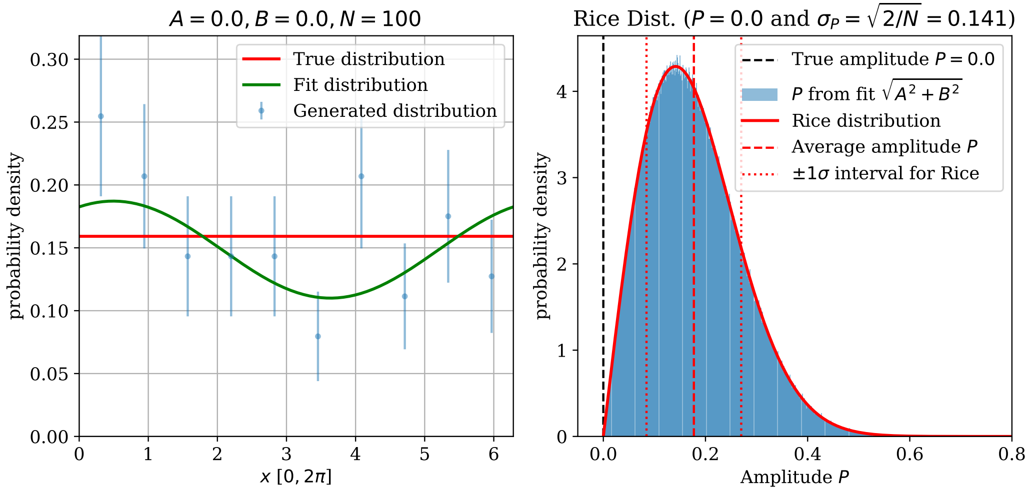
- Left-Right Asymmetry indicates vertical Polarization
- Up-Down Asymmetry indicates horizontal Polarization

Polarization measurement from Data



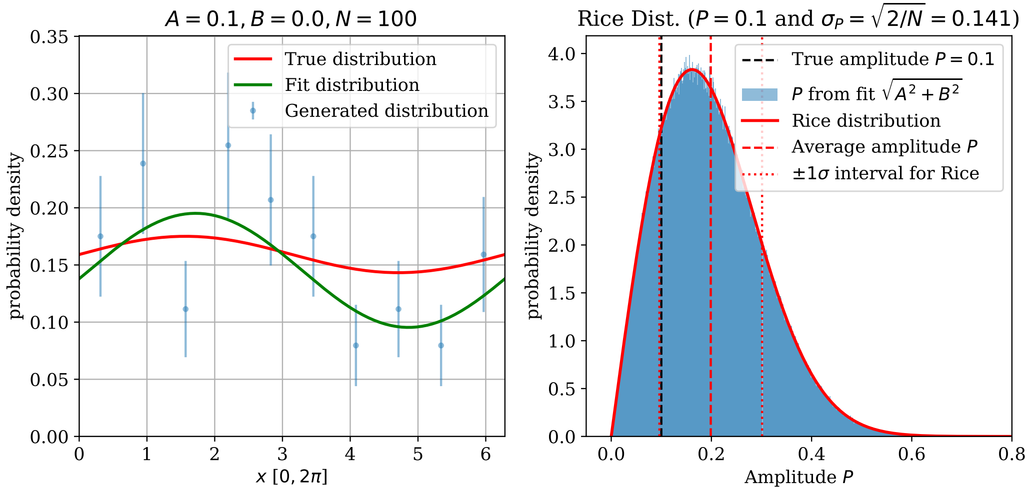
- Spin precession much faster than event rate in detector
- $\Rightarrow$ Solution is **mapping** of data into one **spin precession period**
- $\Rightarrow$ data can be fitted with as sine/cosine
- Asymmetry $= \frac{N_{\text{up}} - N_{\text{down}}}{N_{\text{up}} + N_{\text{down}}} \propto [A \sin(x - \varphi_0) + \text{offset}] \propto (1 + P\cos(x - \phi))$

Amplitude Overestimation in Sine Fits (no signal case)



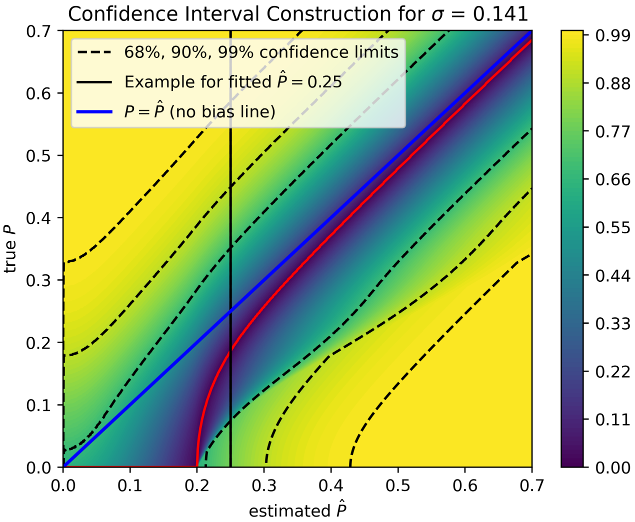
simulated data: $f(x) \propto (1 + A\sin(x) + B\cos(x)) = (1 + P\cos(x - \phi))$

Amplitude Overestimation in Sine Fits (small signal case)



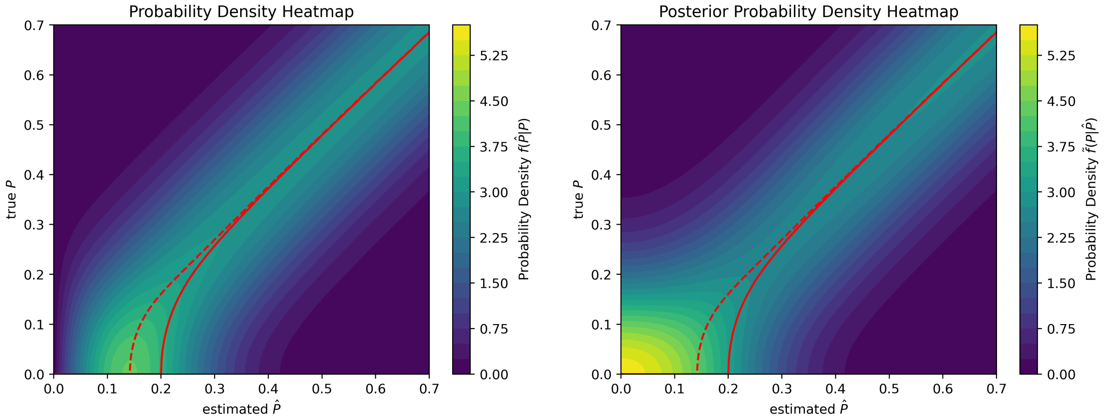
simulated data: $f(x) \propto (1 + A\sin(x) + B\cos(x)) = (1 + P\cos(x - \phi))$

Feldman-Cousins Confidence Intervals



- Feldman-Cousins approach to construct confidence limits
- uses **likelihood ratios** compared to the best assumption (solid red line)
- for given amplitude $\hat{P}$ (fitted from data) the corresponding true P limits can be read off
- also relevant in Axion search at COSY [PhysRevX.13.031004]

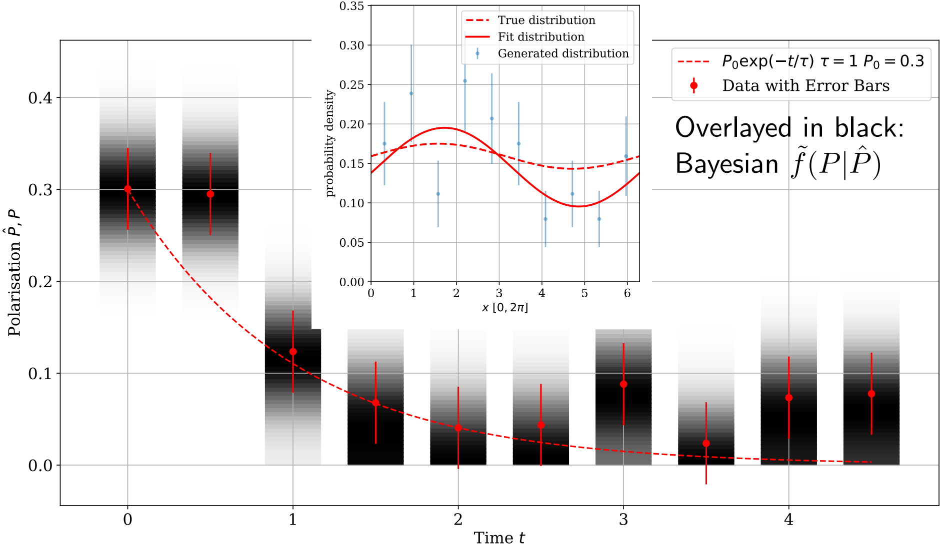
Bayes' Theorem (constant prior) for Rice Distribution



$$f(\hat{P}|P) = \mathrm{Rice}(\hat{P}; P, \sigma) \quad \sigma = \sqrt{2/N}$$

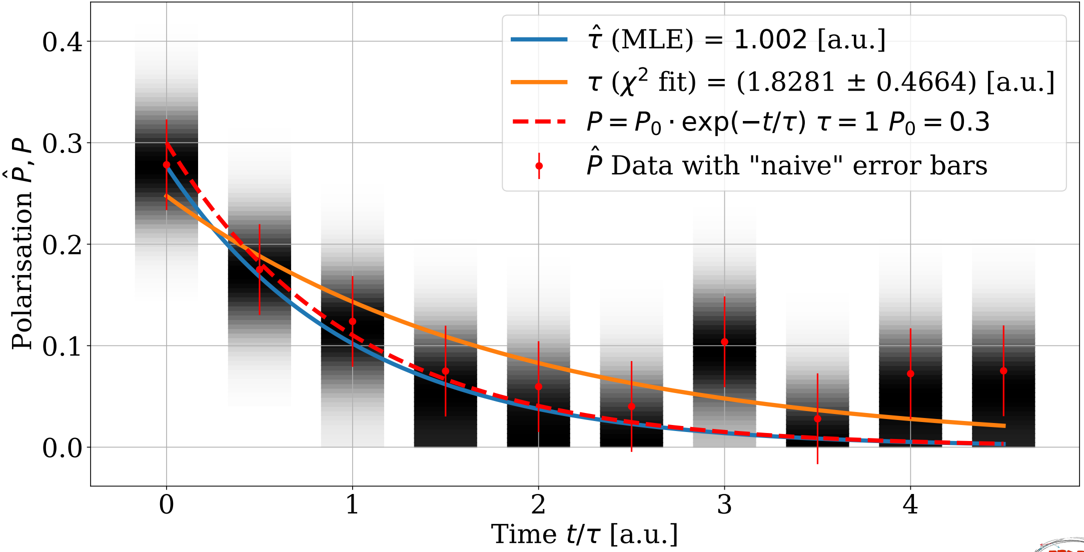
$$\tilde{f}(P|\hat{P}) = f(\hat{P}|P) \,/\, \int_0^\infty f(\hat{P}|P)\mathrm{d}P$$

Exponential Decrease of Amplitude

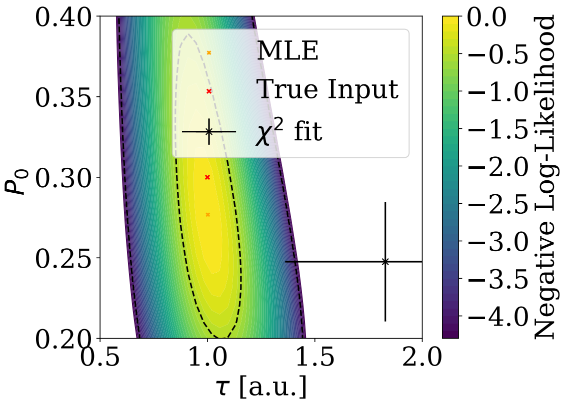


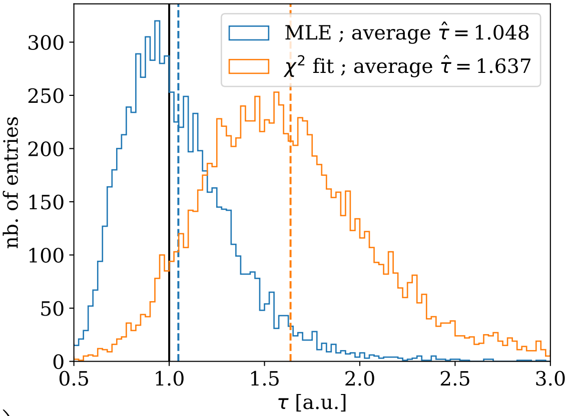
Overlayed in black:
Bayesian $\tilde{f}(P|\hat{P})$

Fitting Example for Exponential Decrease of Amplitude



Maximum Likelihood Fits for Exponential Decrease of Amplitude





minimize: $$L = \prod_{i=1}^{N_{\text{bin}}} \tilde{f}\left(P_0 \exp\left(\frac{-t_i}{\tau}\right) \middle| \hat{P}_i\right)$$

Summary

- Estimates of parameters from **least squares fits** assuming **Gaussian errors** can often be **biased**, even for relatively simple scenarios

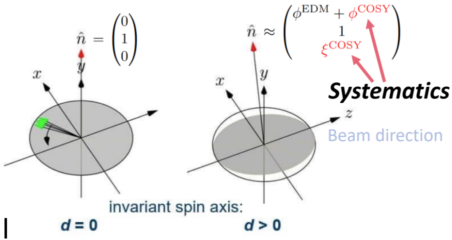
- **Feldman-Cousins** approach to set **confidence limits** based on measurements

- Fitting example: **amplitude** of a sine-function & application to **Spin Decoherence**
- Study of simple **simulated** model and corresponding Rice distribution
- Bias does occur for χ^2-fit: can be reduced by **Bayesian** and MLE approach

Backup

Spin Precession in Electric and Magnetic Fields

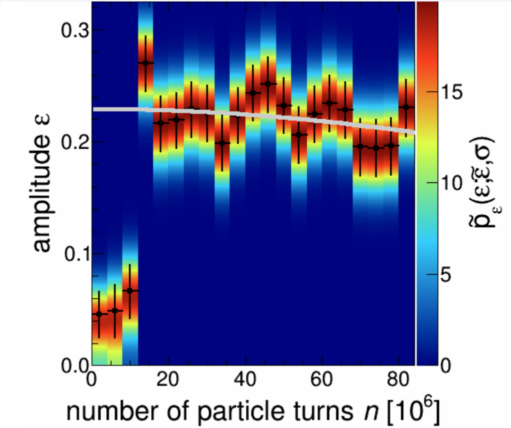
- **Goal**: Determination of the orientation of $\hat{n}$
- Reality: systematic effects in **radial** (x) and **longitudinal** (z) direction

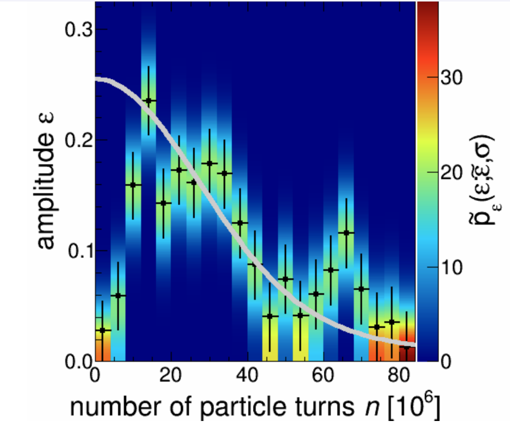


- Thomas-BMT Equation in ideal magnetic ring:
  $\vec{\omega}_s = -\frac{q}{m}\left[G\vec{B} + \frac{\eta_{\text{EDM}}}{2}\left(\vec{\beta} \times \vec{B}\right)\right]$
- G anomalous magnetic moment, η_{EDM} is very small

- consider a cyclotron: $\omega_{\text{cyc}} = \frac{q}{\gamma m} B$ with $\vec{\omega}_s \approx -\frac{q}{m} G\vec{B} = \gamma G \vec{\omega}_{\text{cyc}}$
- **Spin tune** $\nu_s = \frac{\text{spin revolutions}}{\text{particle turns}} = \frac{|\vec{\omega}_s|}{\omega_{\text{cyc}}} \approx \gamma G \approx 0.1609$
- approx 6 particle revolutions per 2π precession (beam momentum $= 0.97\,$GeV/c)

Application of MLE to COSY Data (2015)





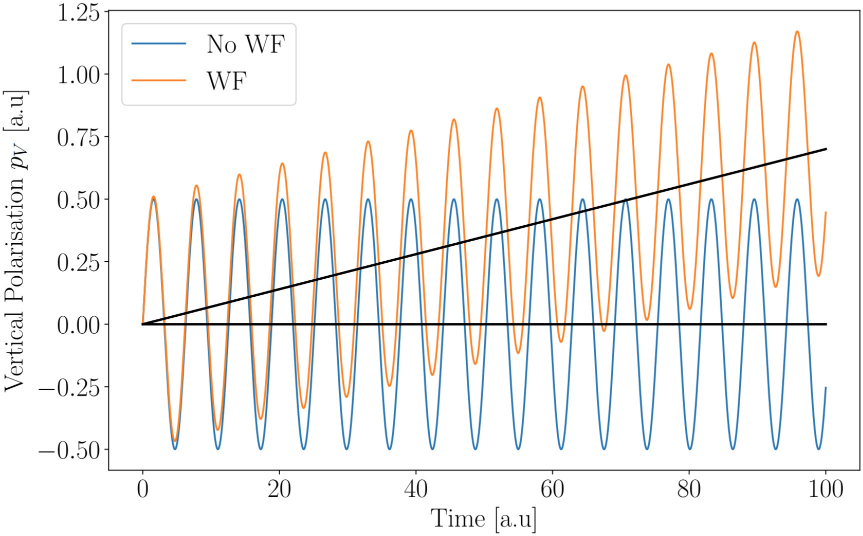
long SCT measurement
$\Rightarrow \chi^2$ fit and MLE are similar

short SCT measurement
$\Rightarrow$ maximum likelihood fit necessary

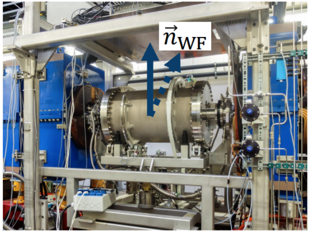
COSY Data for EDM

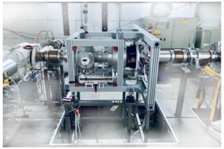
- **Precursor 1 (2018)**
  - Measured $\hat{n}_x^{\mathrm{WF}}$ and $\hat{n}_z^{\mathrm{WF}}$
  - Measured $\hat{n}_z^{\mathrm{Snake}}$
- **Precursor 2 (2021)**
  - Measured $\hat{n}_x^{\mathrm{WF}}$ and $\hat{n}_z^{\mathrm{WF}}$ (again)
  - Measured $\hat{n}_z^{\mathrm{Snake}}$ (again) and $\hat{n}_z^{\mathrm{Solenoid}}$ (new)
- **Updates 2018 - 2021**
  - **Alignment campaigns** of COSY magnet system
  - **Beam Based Alignment** of quadrupoles & Siberian Snake
  - Improved matching ($\vec{F}_L = 0$) of the RF Wien Filter
  - New Jedi Polarimeter **JEPO**
  - New analysis: Pilot Bunch Method

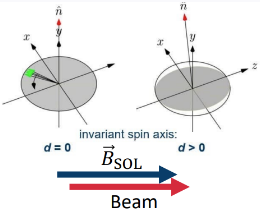
Polarization Buildup with RF Wien Filter

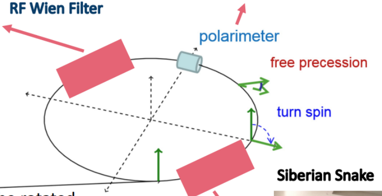


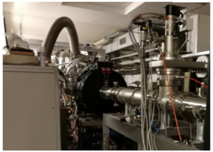
MEASUREMENT PRINCIPLE











- $\vec{E} \perp \vec{B} \perp$ Beam $\rightarrow \vec{F}_L = 0$
- **Rotational** Device: $\vec{n}_{\mathrm{WF}}$- Field can be rotated around the beam pipe by $\boldsymbol{\phi}^{\mathbf{WF}}$

$$\vec{n}_{\mathrm{WF}} = \begin{pmatrix} \sin\left(\phi^{\mathrm{WF}}\right) \\ \cos\left(\phi^{\mathrm{WF}}\right) \\ 0 \end{pmatrix} \approx \begin{pmatrix} \phi^{\mathrm{WF}} \\ 1 \\ 0 \end{pmatrix}$$

- Longitudinal $\vec{B}$ field ∥ Beam
- $\vec{B}$ - Field kicks $\hat{n}$ in **longitudinal** direction (z) by ξ^{Snake}

$$\vec{n} = \begin{pmatrix} \phi^{\mathrm{EDM}} + \phi^{\mathrm{COSY}} \\ 1 \\ \xi^{\mathrm{Snake}} + \xi^{\mathrm{COSY}} \end{pmatrix}$$

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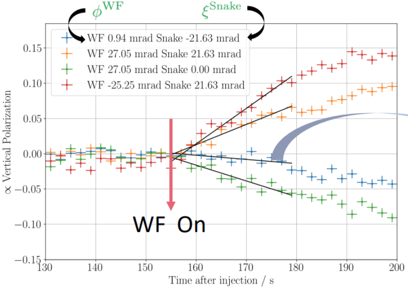
MEASUREMENT PRINCIPLE



$$\epsilon^2(\phi^{\mathrm{WF}}, \xi^{\mathrm{Snake}}) \propto |\vec{n}_{\mathrm{WF}} \times \vec{n}|^2$$

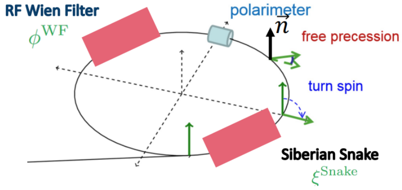
$\vec{n}_{\mathrm{WF}}$: B field axis of rf Wien filter $\quad \vec{n}$: ISA

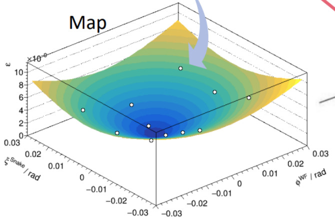
Build up rate

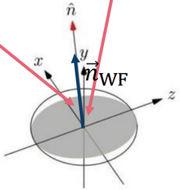
$$\approx \left| \begin{pmatrix} \phi^{\mathrm{WF}} \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} \phi_0^{\mathrm{EDM}} + \phi_0^{\mathrm{COSY}} \\ 1 \\ \xi^{\mathrm{Snake}} + \xi_0^{\mathrm{COSY}} \end{pmatrix} \right|^2$$

$$\approx \left[\left((\phi_0^{\mathrm{EDM}} + \phi_0^{\mathrm{COSY}}) - \phi^{\mathrm{WF}} \right)^2 + \left(\xi^{\mathrm{Snake}} + \xi_0^{\mathrm{COSY}} \right)^2 \right]$$

$$\epsilon \propto \frac{\mathrm{d}}{\mathrm{d}t} p_y(t)$$







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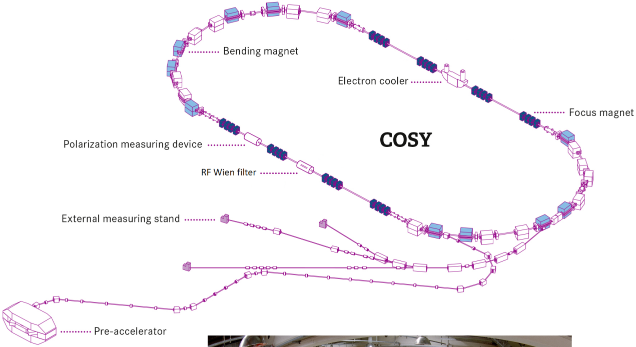
COSY at Forschungszentrum Jülich

- COSY Cooler Synchrotron ($184\,\mathrm{m}$)
- Two electron coolers ($100\,\mathrm{keV}$, $2\,\mathrm{MeV}$)
- Stochastic cooler (pickup+kicker)

- $p = 0.3 - 3.7$ GeV/c
- for protons and deuterons
- **polarized**/unpolarized beam

- built for Hadron Physics (beam on target)
- challenge: **Precision Experiments**
- **30 years of operation (until late 2023)**



Rice Distribution

Gaussian distribution:

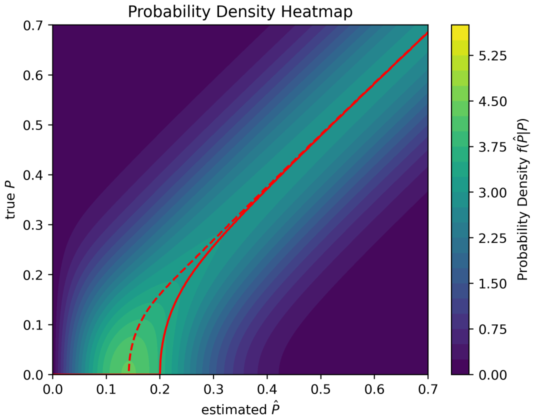
$$f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

$x \in (-\infty, +\infty)$ expected value: $\mu \neq P_{\text{True}}$

Rice distribution:

$$f(x;\nu,\sigma) = \frac{x}{\sigma^2} \exp\left(-\frac{1}{2}\frac{(x^2+\nu^2)}{\sigma^2}\right) I_0\left(\frac{x\nu}{\sigma^2}\right)$$

$\Rightarrow \quad x \in [0, +\infty) \quad \nu = P_{\text{True}} \quad \sigma = \sigma_{\text{gauss}}$



$$f(\hat{P}|P) = \text{Rice}(\hat{P}; P, \sigma) \quad \sigma = \sqrt{2/N}$$

Gaussian and Rice distribution

- Usually start from well known Gaussian distribution (approached for small σ/P_{True}):

$$f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \quad x = \hat{P} \text{ (from fit)}$$

$$x \in [-\infty, +\infty] \quad \text{expected value: } \mu \neq P_{\text{True}} \quad \text{standard deviation: } \sigma$$

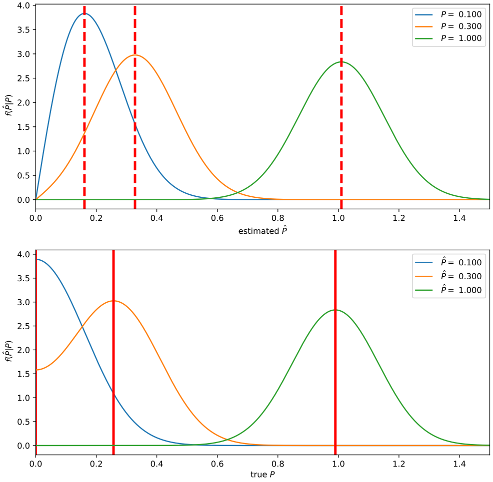
- Rice distribution probability density function [I_0 is the modified Bessel function]:

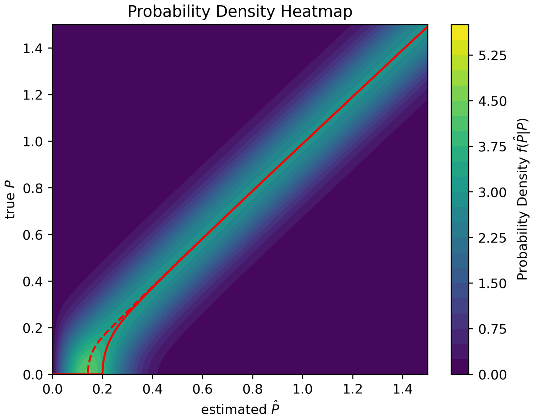
$$f(x;\nu,\sigma) = \frac{x}{\sigma^2} \exp\left(-\frac{1}{2}\frac{(x^2+\nu^2)}{\sigma^2}\right) I_0\left(\frac{x\nu}{\sigma^2}\right) \quad \Rightarrow \quad \nu = P_{\text{True}} \quad \sigma = \sigma_{\text{gauss}}$$

$$x \in [0, +\infty] \quad \text{expected value: } \sigma\sqrt{\pi/2}\, L_{1/2}(-\nu^2/2\sigma^2) \quad \text{(Laguerre polynomial)}$$

$$\text{standard deviation: } 2\sigma^2 + \nu^2 - (\pi\sigma^2/2)\, L^2_{1/2}(-\nu^2/2\sigma^2)$$

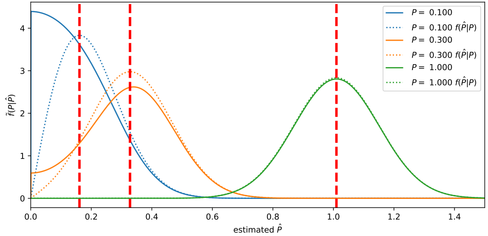
Rice Distribution Details

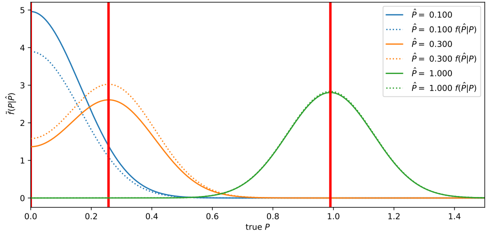


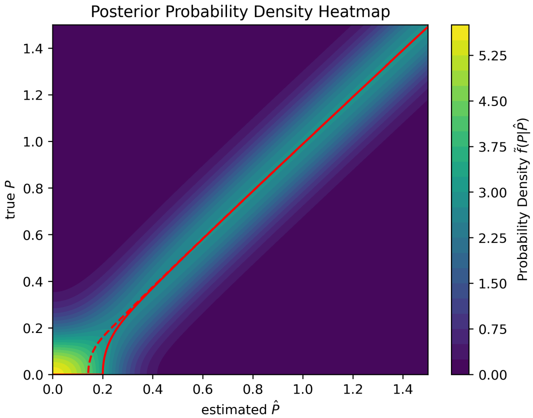


$$f(\hat{P}|P) = \mathrm{Rice}(\hat{P}; P, \sigma) \quad \sigma = \sqrt{2/N}$$

Rice Probability Density - Bayes' Theorem Details

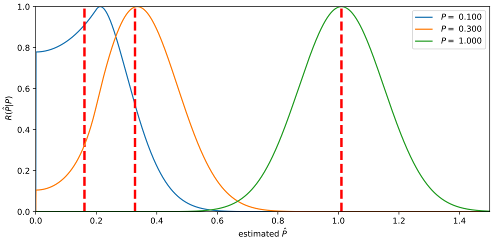


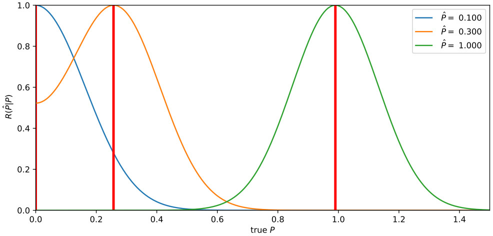


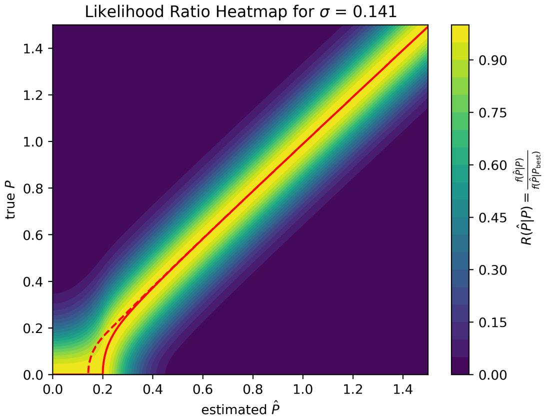


$$\tilde{f}(P|\hat{P}) = f(\hat{P}|P) \,/\, \int_0^\infty f(\hat{P}|P)\mathrm{d}P$$

Rice Distribution - Likelihood Ratio Details

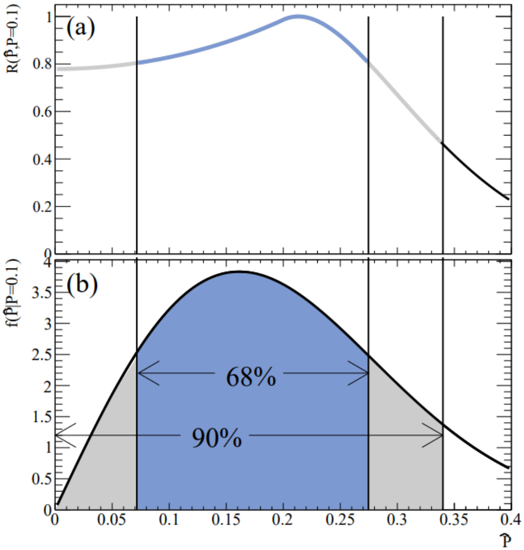






$$R(\hat{P}|P) = f(\hat{P}|P)/f(\hat{P}|P_{\text{best}})$$

Feldman-Cousins Intervals Construction



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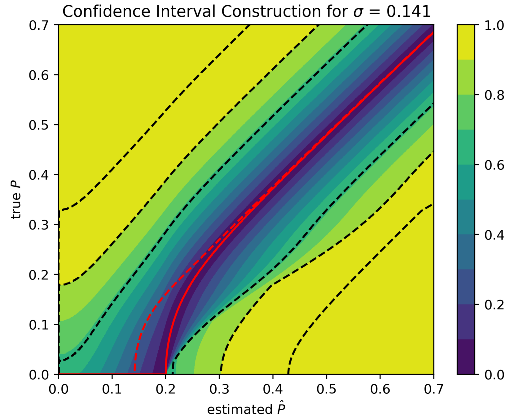
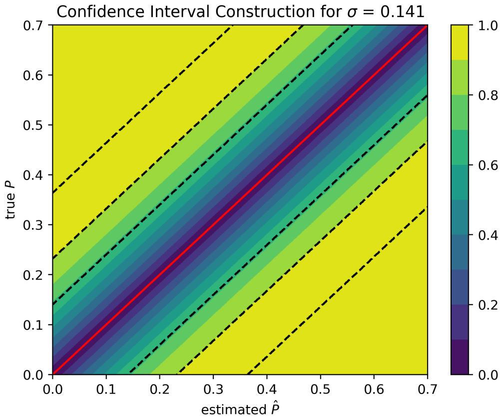
Confidence Intervals - Rice and Gaussian 2D





Small relative uncertainties $\sigma_P/P \ll 1$
$\Rightarrow$ Rice distribution **approaches Gaussian**

Gaussian distribution is perfectly symmetric:
no bias between observation and true amplitude