



## Short Communication

Spin dynamics of the  $E_8$  particles

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Unlike the classical phase transition driven by thermal fluctuations, the quantum phase transition arises at zero temperature when the system is tuned by a non-thermal parameter [1]. For a continuous quantum phase transition, novel physics with higher symmetry may emerge at the quantum critical point (QCP). As the system is driven away from the QCP with a relevant perturbation, exotic physics may further emerge due to the strong renormalization of the almost infinite low-lying excitations [2]. The transverse-field Ising chain (TFIC) is a paradigmatic model [1–3]: when an Ising chain is tuned to its QCP by a magnetic field applied transverse to its Ising anisotropy, a central charge 1/2 conformal field theory emerges with corresponding scaling exponents falling into the class of Ising universality (Fig. 1) [3,4]. Surprisingly, when it is further perturbed by a longitudinal field parallel to the Ising direction, the quantum  $E_8$  integrable model emerges — a massive relativistic quantum field theory containing eight massive  $E_8$  particles (Table S1 online). The physics of the model could be described by the scattering of the  $E_8$  particles as the maximal exceptional Lie  $E_8$  algebra [2,5,6]. Hence, exploring quantum  $E_8$  field theory can extend the frontier of quantum statistical field theory, and may even inspire the studies in high-energy physics.

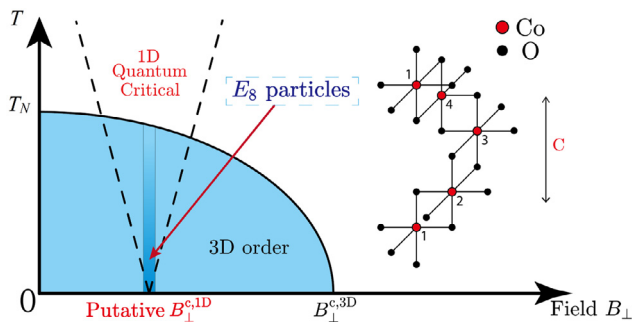
Two conditions need to be satisfied for experimental observation of exotic  $E_8$  physics: Accessing the Ising universality and the presence of a small perturbation field along the Ising direction.

An inelastic neutron scattering (INS) experiment on the ferromagnetic chain compound  $\text{CoNb}_2\text{O}_6$  provided evidence of the existence of the lightest two  $E_8$  particles whose mass ratio was given by the Golden ratio [7]. However, an apparent deviation was presented in the spectrum continuum region between the terahertz experiment [8,9] and the analytical result [6], which implies more efforts are needed for confirming the existence of the  $E_8$  physics in the material  $\text{CoNb}_2\text{O}_6$  [10]. The quasi-1D Heisenberg-Ising antiferromagnet (AFM)  $\text{BaCo}_2\text{V}_2\text{O}_8$  is believed to be another  $E_8$  candidate [11–14]. Although all  $E_8$  particles were found by INS [5], the measurement was confined to only the AFM zone center where several combinations of the  $E_8$  particles and zone folding were also observed. Therefore, a full investigation of the relativistic dispersion of the  $E_8$  particles in the whole Brillouin zone is necessary. In this work, we combine and compare experimental, analytical, and numerical approaches for the lightest three branches of excitations and unambiguously demonstrate the  $E_8$ -particles in  $\text{BaCo}_2\text{V}_2\text{O}_8$ .

$\text{BaCo}_2\text{V}_2\text{O}_8$  has space group is  $I41/acd$  (No. 142) with the lattice constants of  $a = b = 12.40 \text{ \AA}$  and  $c = 8.375 \text{ \AA}$  [11]. The magnetic  $\text{Co}^{2+}$  ions ( $S_{\text{eff}} = 1/2$ ) are arranged in edge-sharing  $\text{CoO}_6$  octahedra forming 4-fold screw chains along the  $c$ -axis (inset of Fig. 1). The  $\text{Co}^{2+}$  ions are coupled by Heisenberg-Ising AFM interactions where the Ising axis is the  $c$ -axis, while weak interchain interactions give rise to long-range magnetic order below Néel temperature,  $T_N = 5.5 \text{ K}$ . Under an external transverse magnetic field perpendicular to the spin direction (e.g., parallel to the  $b$ -axis), a three-dimensional (3D) quantum phase transition occurs at the magnetic field,  $\mu_0 H_{\perp}^{c,3D} \approx 10.3 \text{ T}$  (Fig. 1) as the spins flop from the  $c$ - to  $a$ -axes

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**Fig. 1.** (Color online) Schematic phase diagram of  $\text{BaCo}_2\text{V}_2\text{O}_8$  in a transverse magnetic field. The light and deep blue indicates the AFM phase and the  $E_8$ -particle location around the 1D QCP, respectively. The inset shows one of the  $\text{CoO}_6$  screw chains.

[5,11]. Meanwhile, the one-dimensional (1D) quantum phase transition was also discovered at  $\mu_0 H_{\perp}^{\text{c,1D}} = 4.7 \text{ T}$  [5] and lied hidden within the dome of the 3D magnetic order which provided the (staggered) longitudinal magnetic field for the  $E_8$  quantum particles.

The INS experiment was performed using the LET neutron time-of-flight spectrometer at  $T = 0.3 \text{ K}$  and  $\mu_0 H_{\perp}/b = \mu_0 H^{\text{c,1D}} (= 4.7 \text{ T})$ . Fig. 2a presents the excitation spectrum along the chain,  $(0, 0, L)$ . A series of constant-momentum and energy scans were measured over wavevector  $Q = (0, 0, 1.5)$  to  $(0, 0, 2.5)$  and the energy from  $E = 0$  to  $5 \text{ meV}$  on the IN12 triple-axis spectrometer at  $T = 1.5 \text{ K}$  ( $\ll T_N$ ) and  $\mu_0 H = 4.7 \text{ T}$ . Fig. 2e presented the combined into an energy-wavevector map.

According to theory, the ratios of the energies of the eight  $E_8$ -particles have precise values given in Table S1 (online). Fig. 2b shows the energy scans through the LET and IN12 datasets at  $Q = (0, 0, 2)$ . As the first peak is at  $1.26 \text{ meV}$ , the theoretically expected peak positions are obtained by multiplying the  $E_8$ -particle ratios by  $1.26 \text{ meV}$ . The first three peaks in the LET and IN12 experimental data found by fitting Gaussians (observed at  $m_1 = 1.26$ ,  $m_2 = 2.04$ ,  $m_3 = 2.52 \text{ meV}$  and at  $m_1 = 1.26$ ,  $m_2 = 2.05$ ,  $m_3 = 2.49 \text{ meV}$ , respectively) clearly lie at the expected positions of the first three  $E_8$  peaks. Furthermore, it should be noted that the multi-particle threshold  $2m_1$  is almost the same as that with  $m_3 \approx 1.99m_1$  and a zone-folding peak is close to  $m_2$  at  $1.98 \text{ meV}$  (indicated by  $F_0$  along  $[0, 0, L]$ ). Another zone-folding peak lies at  $2.75 \text{ meV}$  (indicated by  $F_1$ ). The fourth  $E_8$  excitation at  $m_4 = 3.04 \text{ meV}$  is very weak, while the feature at  $3.30 \text{ meV}$  is the lower boundary of the  $m_1 + m_2$  multi-particle continuum. At higher energies, it is difficult to distinguish the  $E_8$  peaks due to overlapping continua and zone-folding modes.

As high-precision data over a wide range of wavevectors were collected, we now have the opportunity to observe the behavior of the  $E_8$ -particles as well as the other excitations with a function of wavevector and energy. Returning to the energy-wavevector plots in Fig. 2a, e, the three lowest  $E_8$  excitations clearly form dispersive modes with parabolic curvature and a minimum at  $Q = (0, 0, 2)$ . The zone-folding modes are now easily identified, such as the dispersive excitations which have minima at the incommensurate wavevectors  $(0, 0, 1.875)$  and  $(0, 0, 2.15)$  at  $E = 1.9 \text{ meV}$  and overlap with  $m_2$  at  $(0, 0, 2)$ . Due to the multi-particle continua and overlapping modes, broad diffuse scattering is observed above  $3 \text{ meV}$ .

We performed state-of-the-art theoretical calculations of the spin dynamic structure factor (DSF) directly from quantum  $E_8$  integrable field theory [6]. The analytical result for the dispersion of the lightest three  $E_8$  particles is shown in Fig. 2c. iTEBD simulations were presented in Fig. 2d. These simulations take account of the

screw chain structure, which gives rise to the zone folding effects and intensity modulations resulting in a spectrum similar as the actual experiment result, Fig. 2e.

The  $E_8$ -particle dispersions are expected to follow the theoretical expression:

$$\frac{E_i^2}{\Delta_1^2} = \frac{\Delta_i^2}{\Delta_1^2} + \frac{p_i^2}{(\Delta_1/c)^2} = \frac{\Delta_i^2}{\Delta_1^2} + \left( \frac{\hbar(L-2)\pi/2d}{m_1 c} \right)^2, \quad (1)$$

where  $p_i = \hbar(L-2)\pi/2d$  is the momentum transfer at  $Q = (0, 0, 2)$  and  $d = 8.4192/4 = 2.105$  is the nearest neighbor distance between  $\text{Co}^{2+}$  ions projected onto the chain direction.  $c = (1.441 \pm 0.096) \times 10^3 \text{ m/s}$ , is the “speed of light” of the real material by from fitting the three lowest energy commensurate modes of the iTEBD simulation (dashed red lines in Fig. 2d) simultaneously with Eq. (1). For the IN12 data, the fitted dispersions (red lines in Fig. 2e) show good agreement with the experiment yielding  $c \approx (1.643 \pm 0.041) \times 10^3 \text{ m/s}$ , while the LET data presents  $c \approx (1.794 \pm 0.008) \times 10^3 \text{ m/s}$ . The speed of light from both experimental data and numerical simulations exhibits agreement with each other.

Comparing experimental, analytical, and numerical data with constant-momentum and energy scans (Fig. 2f, g), both analytical and iTEBD data show excellent agreement with the experimental data. There is about 1% deviation in momentum for the peak position corresponding to  $1\% m_1$  energy shift. This possibly is from the transverse field applied in the experiment which is slightly smaller than the exact critical field, and hence, slightly heavier  $E_8$  particles and a slightly larger minimum gap  $m_1$  are implied. A  $1\% m_1$  upward shift corresponds to about  $0.1$  to  $0.2 \text{ T}$  downward shift of the critical field whose value lies in the range of the putative QCP ( $4.7 \pm 0.3 \text{ T}$ ) identified by NMR [5]. Thus, the true critical field is considered to be  $(4.8 \pm 0.2) \text{ T}$  [9,14].

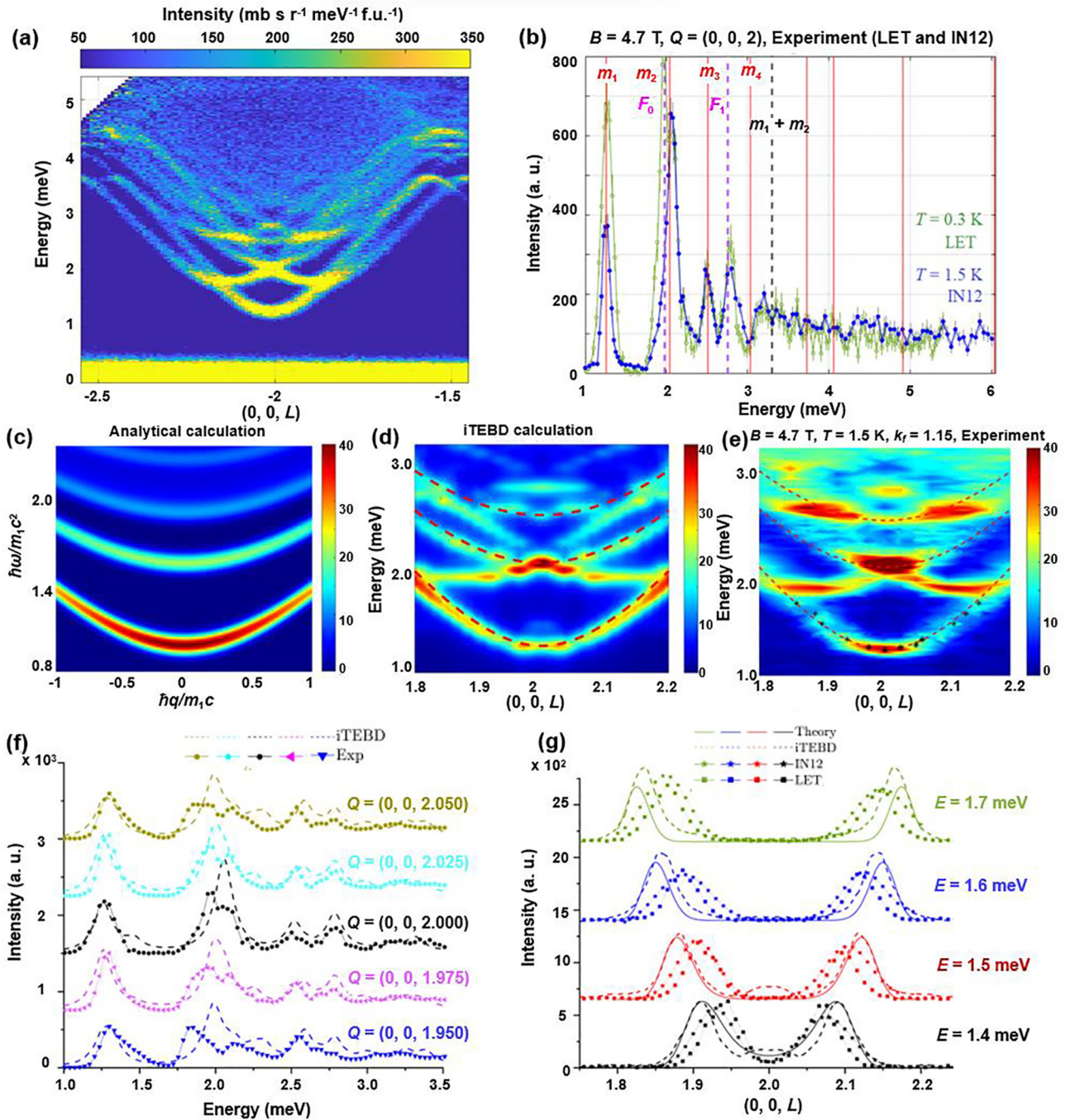
The quasi-1D antiferromagnet  $\text{BaCo}_2\text{V}_2\text{O}_8$  is a remarkably important material in the field of quantum magnetism with unique properties: Ising-like anisotropy, large intrachain versus weak but non-negligible interchain interactions, an anisotropic g-tensor producing easy-axis anisotropy, and effective staggered fields under the application of an external magnetic field. Combining the INS experiments, theoretical, and numerical iTEBD simulations, we precisely studied the  $E_8$  excitation spectrum at the 1D quantum critical point of  $B_c^{\text{1D}} = 4.7 \text{ T}$ . The observed dispersion of the first three  $E_8$  particles and several multi-particles modes provided smoking gun signatures for exploring quantum  $E_8$  physics in other strongly correlated materials.

## Conflict of interest

The authors declare that they have no conflict of interest.

## Acknowledgments

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**Fig. 2.** (Color online) The INS spectrum, theoretical, and numerical iTEBD simulations of  $\text{BaCo}_2\text{V}_2\text{O}_8$  at QCP. (a) The INS spectra under a transverse magnetic field,  $\mu_0 H_{\perp}^{\text{c,1D}} = 4.7$  T at  $T = 0.3$  K using the LET spectrometer. The data are displayed in absolute units as a function of wavevector  $Q = (0, 0, L)$  and energy, for neutron incident energy  $E_i = 6$  meV (integration range:  $-1.0 \leq H \leq 1.0$  &  $-2 \leq K \leq 2$ ). (b) Energy scans measured at the wavevector  $Q = (0, 0, 2)$  on LET and IN12, with the theoretically predicted energies of the first  $E_8$  excitations indicated by the red solid lines. Identified peaks are labelled  $m_n$  (single  $E_8$  excitations),  $m_n + m_m$  (multi- $E_8$  excitations) and  $F_n$  (zone-folding peaks). (c) Analytical calculation of dispersion of three lightest  $E_8$  particles. (d) Numerical simulation of dispersion using the iTEBD method (Eq. (S4) online). The dashed lines illustrate dispersions of the three lightest  $E_8$  particles given by Eq. (1). (e) Energy-wavevector map of the magnetic excitations constructed from constant-energy and constant-wavevector scans measured on the IN12 spectrometer at  $\mu_0 H_{\perp}^{\text{c,1D}} = 4.7$  T and  $T = 1.5$  K. The black symbols are the peak positions extracted from fitting the individual scans, and the dashed lines are the simulated dispersions of the lowest three  $E_8$  particles by using Eq. (1). (f, g) Comparison of DSF results between the iTEBD data, the analytical data, and the experimental IN12 and LET data for constant-momentum and constant energy cuts, respectively. The presented LET experimental constant momentum and constant energy cuts are shifted by an offset of 0.004 r.l.u (relative lattice unit) from experimental conditions. All DSF intensities are normalized up to the maximum intensity of experimental data.

## Author contributions

Jianda Wu, Jie Ma, and Bella Lake conceived and coordinated the project. Masaaki Matsuda, Akira Okutani, Masayuki Hagiwara, Jie Ma, Konrad Puzniak, and Bella Lake synthesized the crystals. Jie

Ma, Konrad Puzniak, Bella Lake, Karin Schmalzl, and Christian Balz designed the experiment. Xiao Wang and Jianda Wu carried out analytical and iTEBD calculation and provided theoretical analysis. Xiao Wang, Konrad Puzniak, Jie Ma, Jianda Wu, and Bella Lake wrote the manuscript.



## Appendix A. Supplementary materials

Supplementary materials to this short communication can be found online at <https://doi.org/10.1016/j.scib.2024.07.040>.

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