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# Using the Transition-Density Formalism in the First Computation of 4He Compton Scattering

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**Abstract.** The method and results of the first theory description of $^4$He Compton scattering at nuclear energies is presented, with a focus on figures. An upcoming publication [1] contains details and a comprehensive list of references.

Compton scattering on light nuclei up to about the first resonance region serves a dual purpose: to extract the neutron polarisabilities; and to explore the part of the nuclear interactions mediated by charged pion-exchange currents. Indeed, these two are interwoven: Neutron properties are more easily extracted when bound (and therefore stable inside the nucleus), but only if the nuclear binding effects are accounted for by model-independent and accurate theory. In Compton scattering from 50 to about 130 MeV, the nucleon's stiffness against deformation in electric and magnetic fields is measured by the parameters of its two-photon response: the electric and magnetic dipole polarisabilities $\alpha_{E1}$ and $\beta_{M1}$ reveal the extent to which charge and current distributions of the nucleon's constituents shift in external electromagnetic fields, characterising the induced radiation dipoles [2]. The very same constituents are also responsible for one major part of the interactions which bind nucleons into nuclei.

Chiral Effective Field Theory ($\chi$EFT) is the model-independent and systematically improvable framework in which such questions can be systematically answered from data, namely with credible estimates of residual theory uncertainties. Indeed, a large-scale international effort at a new generation of high-precision facilities aims to understand low-energy Nuclear Physics by extracting nucleon polarisabilities from Compton scattering experiments, in close collaboration between theory and experiment; see also the contributions by G. Feldman [3] and D. Hornidge [4] reporting on the HI$\gamma$S and MAMI A2 parts, respectively. So far, the isoscalar (isospin-averaged) polarisabilities are found from deuteron data [5, 6]:

$$\alpha_{E1}^{(s)} = 11.1 \pm 0.6_{\rm stat} \pm 0.2_{\rm B\Sigma R} \pm 0.8_{\rm th} \ , \ \beta_{M1}^{(s)} = \ 3.4 \mp 0.6_{\rm stat} \pm 0.2_{\rm B\Sigma R} \pm 0.8_{\rm th} \ . \qquad (1)$$

To reduce the combined uncertainties so that they become commensurate to those of the proton polarisabilities of ±0.5, both high-quality data and high-accuracy theory are needed. This will provide a better understanding of the degree to which proton and neutron polarisabilities

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differ and check a prediction based on a Cottingham Sum Rule evaluation of the self-energy correction to the proton-neutron mass difference: $\alpha_{E1}^{(p)} - \alpha_{E1}^{(n)} = [-1.7 \pm 0.4]$ [7].

Different nuclear targets test different linear combinations of proton and neutron polarisabilities. For example, the deuteron and $^4$He are sensitive to the isospin averages, while $^3$He probes $2\alpha_{E1}^{(p)} + \alpha_{E1}^{(n)}$ etc, allowing for cross-validation of high-accuracy measurements and theory. Furthermore, rates increase with atomic number. The deuteron theory is well-understood [2], and $^3$He computations are now available as well [8–11]. On top of available deuteron, $^4$He and $^6$Li data, new experiments are approved for these targets and $^3$He [3, 4].

As a target, $^4$He has several endearing features. For experimentalists, it is cheap, inert, safe to handle, liquefies at relatively high temperatures, and its high dissociation energy allows clear differentiation between elastic and inelastic events even with detectors of modest energy resolution. For theorists, this perfect scalar-isoscalar target allows extractions of the scalar-isoscalar polarisabilities which are free of contamination from spin polarisabilities, and for a high-accuracy test of the charged-meson exchange currents in a tightly bound system.

Figure 1 summarises the key result, with the isoscalar polarisabilities of eq. (1) as input [1]. The blue band produced by chiral "semilocal momentum-space regularised" (SMS) potentials at different cutoffs [12] suggests that a $\chi$EFT treatment at order $O(e^2\delta^3)$ [N$^3$LO] has less-than-10% uncertainties from residual dependence on the details of chiral 2N and 3N interactions. Order-by-order convergence indicates an even smaller lower-bound uncertainty of about ±6% at the highest energies. Agreement between theory and the 50 available data from HI$\gamma$S,

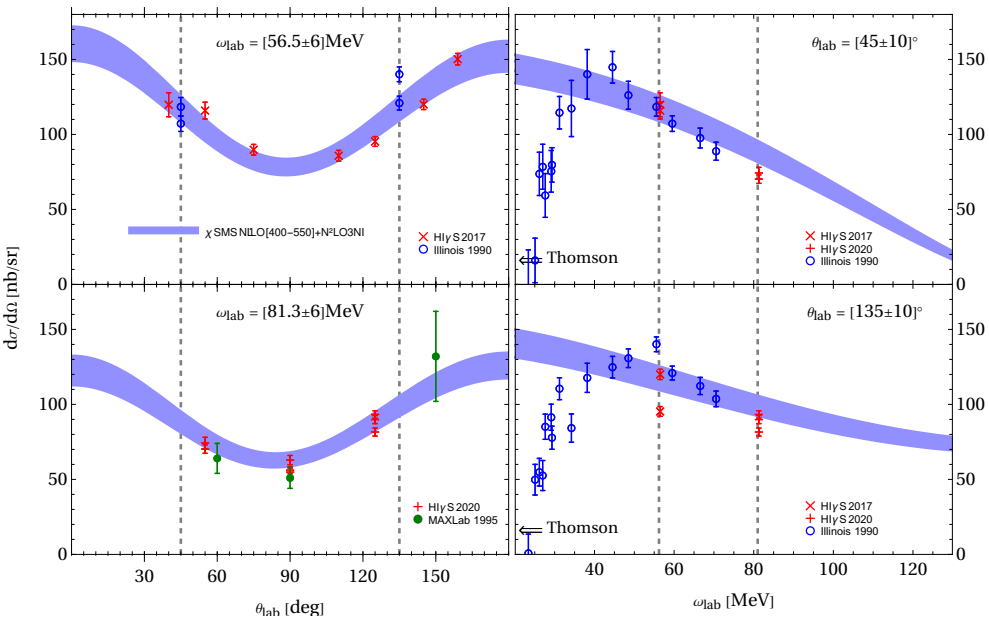


**Figure 1.** (Colour on-line) $^4$He Compton cross sections at $O(e^2\delta^3)$ [N$^3$LO] at $\omega_{lab}$ = 56.5 MeV (top left) and 81.3 MeV (bottom left), and $\theta_{lab}$ = 45° (top right) and 135° (bottom right) [1]. The band is generated with nucleon densities from $\chi$SMSN$^4$LO +N$^2$LO3NI potentials with cutoffs {400; 450; 500; 550} MeV [12]. Data from Illinois [13], MAXlab [14] and HI$\gamma$S [15, 16] are included without accounting for differences between nominal and actual energies or angles. Central polarisability values as in eq. (1). Lack of agreement $\lesssim$ 50 MeV expected; see ref. [1] for details.

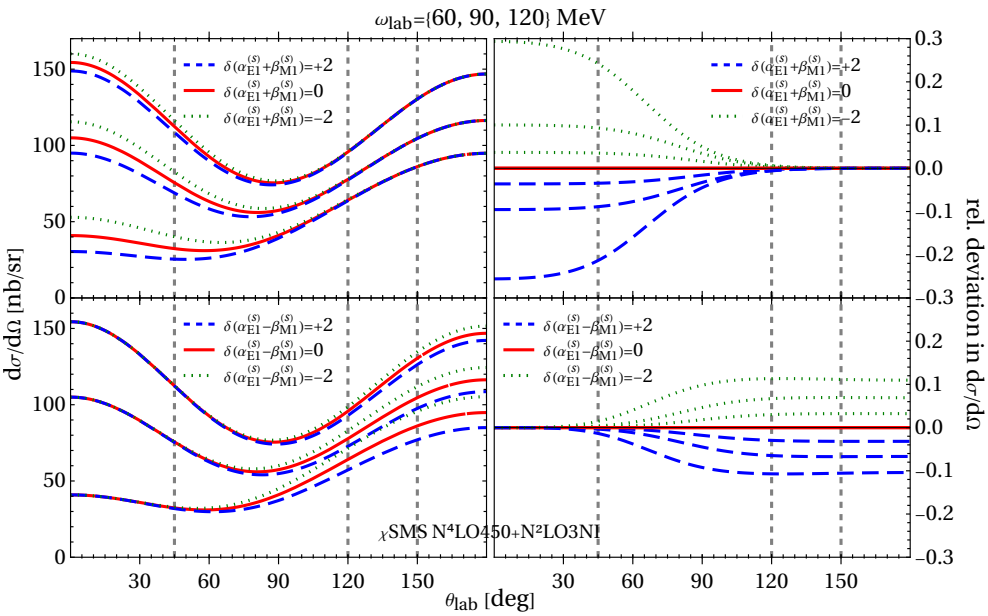


**Figure 2.** (Colour on-line) Sensitivity of the cross section to varying the scalar-isoscalar polarisabilities around their central values (solid line) of eq. (1) by +2 (blue dashed) and −2 (green doted) units for a "mean" potential $\chi$SMSN$^4$LO+450MeV+N$^2$LO3NI at 60, 90 and 120 MeV (top to bottom) [1]. Left: differential cross section. Right: Relative deviations from central values increase with energy.

MAXlab and the University of Illinois Tagged Photon Facility [13–16] is good within the experimental and theoretical uncertainties *in the range where the assumptions of the present theoretical description hold*: namely for those $\omega \sim m_\pi$ at which the intermediate four-nucleon system predominantly propagates incoherently, with only minor rescattering effects. These only become dominant as $\omega \to 0$ to restore the Thomson limit. Informed by these data and accounting for uncertainties of both theory and experiment, it is safe to conclude that the incoherent-propagation assumption is justified at $\omega \gtrsim 50$ MeV. A thorough discussion of uncertainties and convergence of the $\chi$EFT expansion is left to an upcoming publication [1].

For planning experiments, theory uncertainties are mitigated because they are mostly angle-independent, whereas fig. 2 demonstrates that the absolute and relative *sensitivities* to varying the scalar-isoscalar polarisability combinations $\alpha^{(s)}_{E1} \pm \beta^{(s)}_{M1}$ have a rather strong angular dependence: $\alpha^{(s)}_{E1} + \beta^{(s)}_{M1} = 14.5 \pm 0.4$ is fairly well constrained by the Baldin Sum Rule, while the uncertainties in $\alpha^{(s)}_{E1} - \beta^{(s)}_{M1}$ dominate eq. (1). Thus, these results are sufficiently reliable to be useful for an exploratory study of magnitudes and sensitivities to the nucleon polarisabilities to advance current planning of experiments – as previously argued for $^3$He [11]. Polarisability extractions, on the other hand, should address residual theoretical uncertainties more diligently, as for the proton and deuteron [17]. That work is under way, most notably to include rescattering effects [18] and update the Compton kernels to $O(e^2\delta^4)$ [N$^4$LO].

These results are obtained using the *Transition-Density Method* introduced in ref. [19]. It factorises the interaction of a probe with a nucleus of $A$ nucleons into an *interaction kernel* between the probe and the $n$ *active nucleons* which directly interact with it, and a backdrop of $A - n$ *spectator nucleons* which do not. The effect of the latter is described by a *n-body den-*

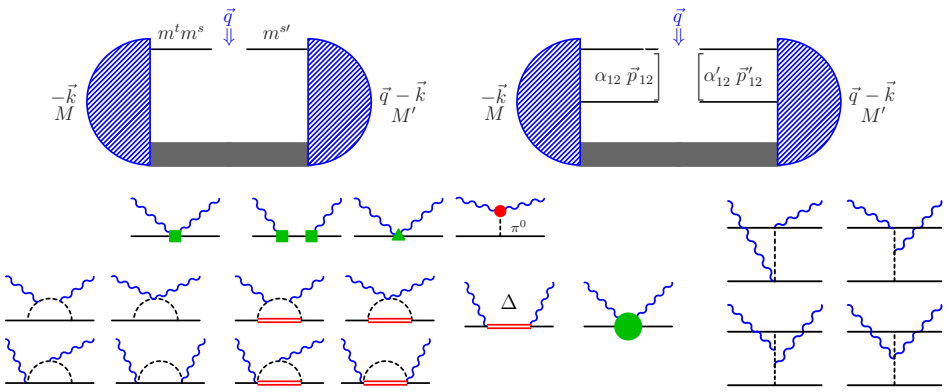


**Figure 3.** (Colour on-line) Top: Definition of one-body (left) and two-body (right) densities in the $A$-nucleon bound state in the cm system [1]. One-nucleon quantum numbers are the spin and isospin projections $m^s$ and $m^t$ of the active nucleon. The two-nucleon quantum numbers are collectively denoted as $\alpha_{12}$ and relative momentum $\vec{p}_{12}$. The spin projection of the incident nucleus is $M$. Primed variables for outgoing states. Bottom: One-body (left) and two-body (right) Compton kernels up to and including order $O(e^2\delta^3)$ [N$^3$LO] [8–11]. Crossed and permuted diagrams are not shown.

*sity*, namely a transition probability density amplitude that $n$ active nucleons with a specific set of quantum numbers are found inside the nucleus before an interaction which transfers momentum $\vec{q}$ and changes the initial quantum numbers. This re-arranges the active nucleons into another specific set of quantum numbers after the interaction. Figure 3 (top) illustrates this separation, with the interaction kernel depicted as an arrow. The one- and two-body densities generated from a number of chiral potentials as well as the AV18+UIX potential involving the $^3$He, $^3$H and $^4$He system are available at https://datapub.fz-juelich.de/anogga.

But what about interactions with more than 2 active nucleons? It is a fundamental advantage of $\chi$EFT that it provides a well-defined procedure to predict a hierarchy of $n$-body interaction kernels. As discussed in refs. [11, 20–23] and summarised in [2, sect. 5.2], only kernels with one and two active nucleons contribute in Compton scattering up to and including $O(e^2\delta^4)$ [N$^4$LO] at $\omega \sim m_\pi$. Therefore, three-or-more-body densities do not need to be considered in the present $O(e^2\delta^3)$ [N$^3$LO] investigation.

Another advantage of this factorisation is that densities of the same nucleus and momentum transfer $\vec{q}$ can be recycled for different interaction kernels, while the same interaction kernels can be recycled in different nuclei. In Compton scattering on $^4$He, we use the same kernels of fig. 3 as for our $^3$He results [8–11] which originate in the deuteron and proton/neutron kernels [20–23]. Thus, the few-body Compton calculations share a common analysis framework. We also recycle the same $^3$He and $^4$He densities in (for example) pion scattering and neutral pion production [24]. This split reduces the computational effort by orders of magnitude: Densities are produced using well-developed modern numerical few-body techniques and only once, while the kernel convolutions only involve sums and integrals over the undetected one- and two-nucleon quantum numbers and momenta. The summation over one-body quantum numbers is near-instantaneous. For two-body matrix elements, integration over the relative momenta $\vec{p}_{12}$ and $\vec{p}'_{12}$ and sum over quantum numbers amount to less than a CPU hour per energy and angle on a (s)lowly desktop for better-than-0.7% numerical accuracy. We reproduced the "traditional" results for $^3$He [8–11], and the reduced computational cost allowed for more extensive explorations of numerical convergence [19].

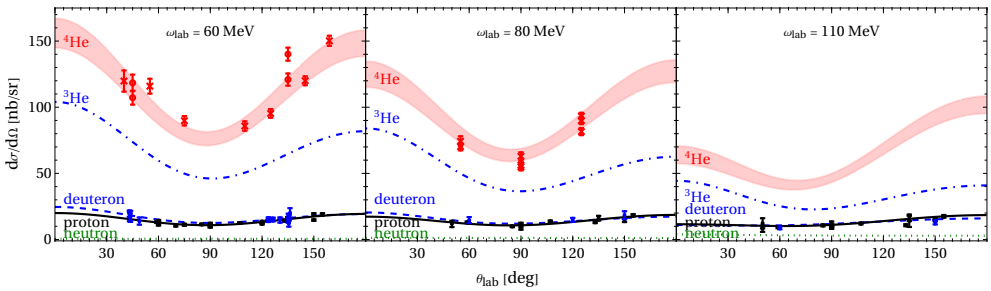


**Figure 4.** (Colour on-line) $\chi$EFT predictions and data at $\omega_{\rm lab} = \{60; 80; 110\}$ MeV for a proton (black), neutron (green dotted), deuteron (blue dashed), $^3$He (blue dot-dashed) and $^4$He (band) target [1].

Let us turn now to the Compton kernel itself [8–11]; see fig. 3 (bottom). Its one-nucleon part consists of the contributions in which photons interact with a point nucleon ("Born terms": top row on left), and of the "structure terms" (other rows on left) which give rise to nonzero polarisabilities because the photons couple to the $\Delta(1232)$ resonance and the pion cloud around the nucleon and $\Delta$. The two-nucleon part (on the right) describes the coupling of photons to charged meson-exchange currents. Both kernels are complete up to and including order $O(e^2\delta^3)$ [N$^3$LO], where the small expansion parameter is $\delta = \sqrt{m_\pi/\Lambda_\chi} \approx \Delta/\Lambda_\chi \approx 0.4$ with $\Delta \approx 300$ MeV the $\Delta$-N mass splitting and $\Lambda_\chi \approx 700$ MeV the breakdown scale of $\chi$EFT.

As fig. 4 shows, Compton cross sections on few-nucleon targets do not scale with powers of the target charge $Z$ in the region of interest for extracting polarisabilities, $\omega \in [50; 120]$ MeV. While the proton and deuteron cross sections ($Z = 1$) are roughly of similar size, the $^3$He and $^4$He differ by a factor of about two although they are both $Z = 2$ targets. At 60 (120) MeV, the average $^4$He nearly 7 (5) times that of the proton and deuteron. Rather, it appears that scaling is related to the number of charged-pion currents (*i.e.* of pn pairs).

An upcoming publication will provide details of formalism, uncertainties and results, sensitivity studies of the photon-beam asymmetry of $^4$He, and a comparison to Compton scattering off lighter nuclear targets [1]. $^4$He is an excellent choice because of the expected high rates and clean signal. Work is under way to improve the theory side to allow the step from predicting sensitivities which are adequate for experimental planning, to extracting polarisabilities from high-accuracy data. Indeed, data taking is scheduled at HI$\gamma$S in the very near future. The theory group is also applying the transition-density formalism to Compton scattering on other targets as well as to other processes on $^3$He, $^3$H, $^4$He – and beyond [24].

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## References

[1] H. W. Griesshammer, J. Liao, J. A. McGovern, A. Nogga and D. R. Phillips, Eur. Phys. J. A **60** (2024) no.6, 132 [arXiv:2401.16995 [nucl-th]].

[2] H. W. Grießhammer, J. A. McGovern, D. R. Phillips and G. Feldman, Prog. Part. Nucl. Phys. **67** (2012) 841 doi:10.1016/j.ppnp.2012.04.003 [arXiv:1203.6834 [nucl-th]].

[3] G. Feldman, these proceedings.

[4] D. Hornidge, these proceedings.

[5] L. S. Myers *et al.* [COMPTON@MAX-lab Collaboration], Phys. Rev. Lett. **113** (2014) 262506 doi:10.1103/PhysRevLett.113.262506 [arXiv:1409.3705 [nucl-ex]].

[6] L. S. Myers *et al.*, Phys. Rev. C **92** (2015) 025203 doi:10.1103/PhysRevC.92.025203 [arXiv:1503.08094 [nucl-ex]].

[7] J. Gasser, M. Hoferichter, H. Leutwyler and A. Rusetsky, Eur. Phys. J. C **75** (2015) 375 [Erratum: Eur. Phys. J. C **80** (2020) 353] doi:10.1140/epjc/s10052-015-3580-9 [arXiv:1506.06747 [hep-ph]].

[8] D. Choudhury, A. Nogga and D. R. Phillips, Phys. Rev. Lett. **98** (2007) 232303 doi:10.1103/PhysRevLett.98.232303 [nucl-th/0701078] [Erratum: Phys. Rev. Lett. **120** (2018) 249901] doi:10.1103/PhysRevLett.98.232303 [arXiv:1804.01206 [nucl-th]].

[9] D. Shukla, A. Nogga and D. R. Phillips, Nucl. Phys. A **819** (2009) 98 doi:10.1016/j.nuclphysa.2009.01.003 [arXiv:0812.0138 [nucl-th]].

[10] D. Choudhury, *PhD thesis*, Ohio University (2006) http://rave.ohiolink.edu/etdc/view?acc_num=ohiou1163711618.

[11] A. Margaryan, B. Strandberg, H. W. Grießhammer, J. A. Mcgovern, D. R. Phillips and D. Shukla, Eur. Phys. J. A **54** (2018) 125 doi:10.1140/epja/i2018-12554-x [arXiv:1804.00956 [nucl-th]].

[12] P. Reinert, H. Krebs and E. Epelbaum, Eur. Phys. J. A **54**, no. 5, 86 (2018) doi:10.1140/epja/i2018-12516-4 [arXiv:1711.08821 [nucl-th]].

[13] D. P. Wells, PhD thesis University of Illinois at Urbana-Champaign (1990).

[14] K. Fuhrberg and M. Schumacher, Nucl. Phys. A **591** (1995) 1 doi:10.1016/0375-9474(95)00177-3.

[15] M. H. Sikora, M. W. Ahmed, A. Banu, C. Bartram, B. Crowe, E. J. Downie, G. Feldman, H. Gao, H. W. Grießhammer and H. Hao, *et al.* Phys. Rev. C **96** (2017) 055209 doi:10.1103/PhysRevC.96.055209.

[16] X. Li *et al.*, Phys. Rev. C **101** (2020) 034618 doi:10.1103/PhysRevC.101.034618 [arXiv:1912.06915 [nucl-ex]].

[17] H. W. Grießhammer, J. A. McGovern and D. R. Phillips, Eur. Phys. J. A **52** (2016) 139 doi:10.1140/epja/i2016-16139-5 doi:10.22323/1.253.0104 [arXiv:1511.01952 [nucl-th]].

[18] N. R. Walet, J. Singh, J. Kirscher, M. C. Birse, H. W. Grießhammer and J. A. McGovern, Few-B. Sys. **64** (2023) 56 doi:10.1007/s00601-023-01824-x [arXiv:2303.09361 [nucl-th]].

[19] H. W. Grießhammer, J. A. McGovern, A. Nogga and D. R. Phillips: Few-B. Sys. **61** (2020) 48 doi:10.1007/s00601-020-01578-w [arXiv:2005.12207 [nucl-th]].

[20] S. R. Beane, M. Malheiro, D. R. Phillips and U. van Kolck, Nucl. Phys. A **656** (1999) 367 doi:10.1016/S0375-9474(99)00312-7 [nucl-th/9905023].

[21] S. R. Beane, M. Malheiro, J. A. McGovern, D. R. Phillips and U. van Kolck, Nucl. Phys. A **747** (2005) 311 doi:10.1016/j.nuclphysa.2004.09.068 [arXiv:nucl-th/0403088].

[22] R. P. Hildebrandt, *PhD thesis*, TU München (2005) [nucl-th/0512064].
[23] R. P. Hildebrandt, H. W. Grießhammer and T. R. Hemmert, Eur. Phys. J. A **46** (2010) 111 doi:10.1140/epja/i2010-11024-y [nucl-th/0512063].
[24] A. Long, H. W. Grießhammer and A. Nogga, in progress.