Continuity equation conform fundamental diagrams of pedestrian streams

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Short Abstract. We present a local and spatio-temporally consistent measurement of speed, density and flow which ensures particle number conservation by applying a field representation and the continuity equation. The new definitions are compared with classical measurements using trajectories from laboratory experiments on uni- and bi-directional streams.

Since the beginning of the century, it has been possible to capture trajectories of pedestrian streams precisely from video recordings [1,2]. To enable measurements at high density the heads of the pedestrians are marked, tracked and thus provide a complete representation of the phase space. But classical definitions and local measurements of flow, density and velocity of pedestrian streams using trajectories (Lagrangian representation) are based on different segments in phase space. The flow is defined as an average value over time while the density is defined as the average value of an area. This inconsistency becomes a problem in central relations such as the flow equation or the fundamental diagram [3]. These have a particular effect in inhomogeneous states like states including stop and go waves, where in addition the pedestrians do not change their position in the stop phase, but the head of the body moves. In order to obtain a local and spatio-temporally consistent measurement of the quantities flow, density and velocity while ensuring particle number conservation fields (Euler representation) and the continuity equation can be used. To map trajectories of pedestrians heads parameter free and unambiguously to fields, this article introduces a method based on the Voronoi decomposition [4] in which the variables speed, density and flow are measured at a line.

The new definition of the density $\rho_{l_0}^V$ at a line l_0 follows

$$\rho_{l_0}^V(t) = \frac{1}{w}\int_{y_0}^{y_1} \boldsymbol{\rho}\,(\vec{x},t)dy \qquad (1)$$

with $\boldsymbol{\rho}(\vec{x},t)$ being the density field using a Voronoi decomposition of the space and w being the width of the corridor. The new definition of the specific flow at l_0 follows

$$J_{S,l_0}^V(t) = \frac{1}{w}\int_{y_0}^{y_1} \vec{\boldsymbol{j}}\,(\vec{x},t)\vec{n}_{l_0}dy \qquad (2)$$

with $\vec{\boldsymbol{j}}(\vec{x},t)$ being the flow field based on the Voronoi decomposition.

We present results applying the newly proposed definitions to well analysed experimental datasets of uni- and bidirectional pedestrian streams within straight corridors. Furthermore,

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we show that the new definitions of speed, density, flow and the particle number conserving flow equation are consistent with classical measurements and enable to scrutinise inconsistencies in the state of the art of pedestrian fundamental diagrams. As an example, Figure 1 shows the comparison of the fundamental diagram of unidirectional flow in a straight corridor.

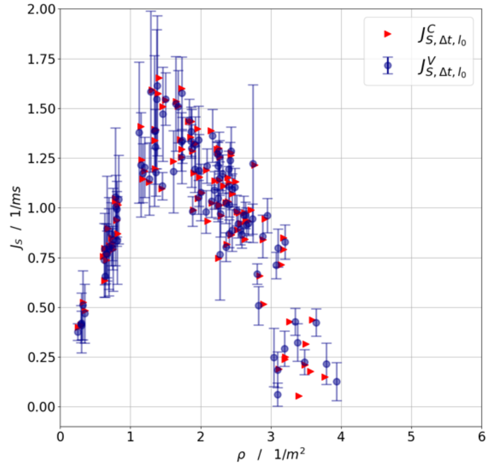

Fig. 1. Fundamental diagram for unidirectional pedestrian streams in a straight corridor as mean values within a time interval of $\Delta t = 10\ s$. Comparison between the new definitions $J^V_{S,\Delta t,l_0}$ and $\rho^V_{\Delta t,l_0}$ measured at l_0 in blue and classical measurements in red. The classical flow is measured at l_0 according to $J^C_{S,\Delta t,l_0} = {\Delta N}/{(\Delta t * b)}$ and the classical density is measured within an area A with a thickness of 2 m around l_0 according to $\rho^C_A = {N_A}/{A}$.

The experimental data is available from the Pedestrian Dynamics Data Archive of the Forschungszentrum Jülich under https://doi.org/10.34735/ped.2013.6 and https://doi.org/10.34735/ped.2013.5.

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