Towards the Analysis of the $T_{cc}(3875)$ Approaching the Physical Point

with 6-Stout Smeared Ensembles and Distillation

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Abstract

We present the plan for a study of exotic states using 6-stout smeared esnembles approaching the physical point. Computational resources required for the determination of the optimal distillation parameters are presented in order to maximize the signal and conserve resources in future studies employing this quark-field smearing algorithm. The runs will be preformed with resources at JSC using the latest architectures to show the expected total cost of a di-meson calculation using our machines. Our objective is to investigate scattering in a newly found resonance state, with minimal quark content $cc\bar{u}d$, the $T_{cc}(3875)$ [3]. We can sidestep problems inherent in the experimental data by showing that the lowest positive-parity charmed mesons are hadronic molecules.

Introduction

The class of exotics that we aim to explore are resonances of doubly charmed mesons in isospin channels $I = 0, 1$. The flavor content is based on the decay channel $D^0D^0\pi^+$ and has isospin 0. Our pipeline is as follows:

- Construct dimeson interpolating operators using distillation
- Construct correlation function coming from the GEVP in the right irreducible representation
- Compute the spectrum and energy shifts with respect to the DD^* threshold for a heavy quark mass close to the charm quark mass.
- Perform a Lüscher analysis to obtain the scattering amplitude.

We endeavor to confirm that the DD^* interaction is repulsive in the $I = 1$ channel and attractive in the $I = 0$ channel, which logically follows from the $I = 0$ assignment for the T_{cc}^+ state [6].

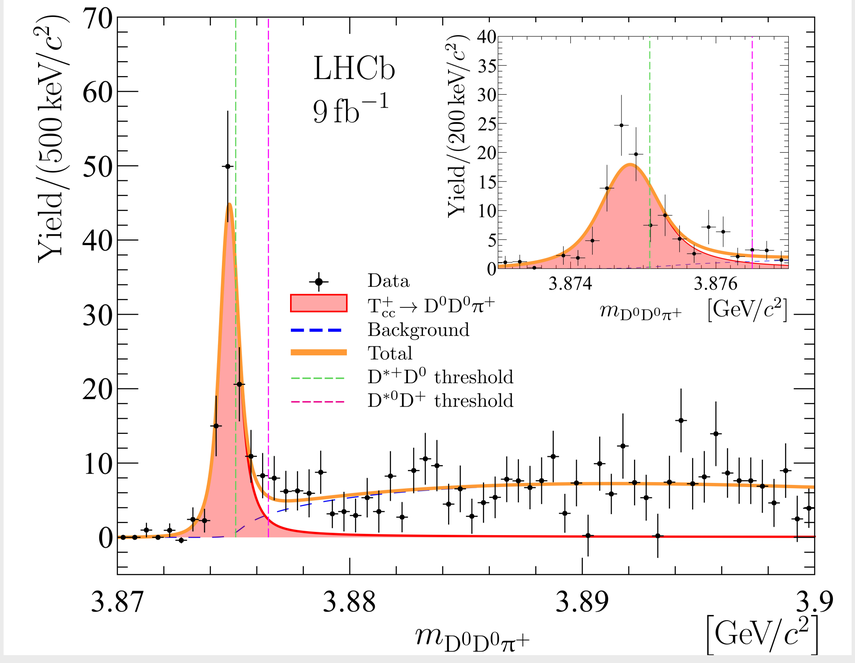

Figure 1: Distribution of $D^0D^0\pi^+$ [1]

Ensemble Details

The ensembles we generated have $N_f = 2 + 1$ quark flavors, a tree level Symanzik improved gluon action and 6-stout dynamical smeared Wilson fermions.

	m_{ud}	m_s	$L^3 \times T$	m_π [MeV]	N_{conf}
$\beta = 3.30$ a $= 0.125$[fm]	-0.1309	-0.057	$48^3 \times 64$	135	*
	-0.1291	-0.057	$32^3 \times 64$	200	*
	-0.1265	-0.057	$24^3 \times 64$	280	1000
	-0.1233	-0.057	$24^3 \times 64$	330	1000
	-0.1200	-0.057	$16^3 \times 64$	400	1000
	m_{ud}	m_s	$L^3 \times T$	m_π [MeV]	N_{conf}
$\beta = 3.57$ a $= 0.085$[fm]	-0.0498	-0.007	$64^3 \times 96$	135	*
	-0.0483	-0.007	$48^3 \times 64$	200	400
	-0.0440	-0.007	$32^3 \times 64$	300	400
	-0.0380	-0.007	$24^3 \times 64$	420	400
	m_{ud}	m_s	$L^3 \times T$	m_π [MeV]	N_{conf}
$\beta = 3.70$ a $= 0.065$[fm]	-0.02981	-0.0	$64^3 \times 96$	135	*
	-0.02855	-0.0	$64^3 \times 96$	200	*
	-0.0250	-0.0	$40^3 \times 96$	300	400
	-0.0220	-0.0	$32^3 \times 96$	380	400
	-0.0200	-0.0	$32^3 \times 96$	420	400

*** In production**

Computational Resources

Distillation is costly initially both in storage and component construction. For the di-meson system we are investigating, the contraction cost is not the dominant contribution. We will use the MultiGrid (MG) solver from `QUDA`, `Chroma` with `Superbblas` support, the `PRIMME` eigensolver, and `Numpy Einsum` for contractions. The amount of computation and storage scales with the lattice size N and the rank of the distillation basis, n. The optimal rank of the distillation basis is determined experimentally, but it is proportional to the spatial volume of the lattice [2][5].

Computation	Operations cost	Memory footprint
Distillation basis[a]	N^3Tn^3D	N^3nT
Meson elementals[b]	N^3Tn^3D	$N^3nD + n^3D$
Perambulators[c]	N^3Tn	N^3Tn
Contractions[d]	n^4T	n^3T

[a] Generate colorvector matrix elements
[b] Contract two matrices $\to$ tensor
[c] Projection of the inverse Dirac operator $\to$ square matrices
[d] Contract together matrix elements and perambulators

Distillation Framework

We have the following ingredients[4]:

1. **Solution vectors**:
$$S^{(k)}_{\alpha\beta}(\vec{x}, t'; t) = M^{-1}_{\alpha\beta}(t', t)V^k(t)$$
2. **Perambulators**:
$$\tau_{\alpha\beta}(t', t)^{kl} = V^{(k)\dagger}(t')M^{-1}_{\alpha\beta}(t', t)V^l(t)$$
The perambulator, defined by the lattice representation of the Dirac operator, M, allows us to access all spatial entries of the propagator between t_f and t_0. Perambulators are independent of the creation operators so the inversion cost is fixed by nvecs and the spatial extent of the lattice.
3. **Elementals**:
$$\Phi^{(i,j)}_{\mu\nu}(t) = \delta^{ab}(D_1\xi^i)^a(D_2\xi^j)^b(t)S_{\mu\nu}$$
$$= V^\dagger(t)[\Gamma^A(t)]_{\alpha\beta}V(t) \equiv V^\dagger(t)\mathcal{D}^A(t)V(t)S^A_{\alpha\beta}$$

Where

- $S_{\mu\nu}$ are the subduction matrices
- M is the Dirac operator
- D_n is a covariant derivative acting on nth quark of the meson interpolator.

$V(t)$ is a matrix with $4 \times N_v$ columns constructed from eigenvectors of the covariant 3d Laplace operator. It is important to note that $V(t)$ does not act on Dirac components. Thus, $V(t)$ is a block identity in Dirac space and each block contains the first N_v eigenvectors $v_i(t)$.

$$V^{(i,\alpha)}(t)_{\vec{x},t',\beta} = v_i(t)_{\vec{x}}\delta_{tt'}\delta_{\alpha\beta}$$

At this point, we need to perform contractions to obtain the correlator

$$C^{(2)}_M(t', t) = Tr[\Phi^B(t')\tau(t', t)\Phi^A(t)\tau(t, t')]$$

References

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Acknowledgements

This research has received funding from scientific work plan of work package 1 of the NRW-FAIR network funded by MKW NRW under the funding code NW21-024-A.