TOWARDS THE ANALYSIS OF THE $T_{cc}(3875)$ WITH DISTILLATION

January 23, 2025 | **Grant Bradley**, Giovanni Pederiva, Stefan Krieg | Jülich Supercomputing Center

Phenomenological Motivation for $T_{cc}^{+}(3875)$

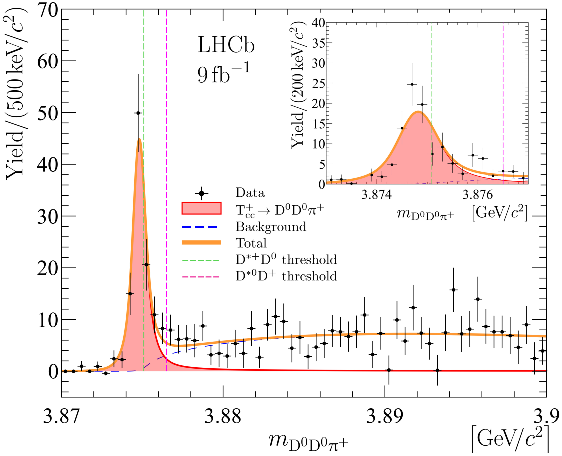
- currently longest-lived exotic hadron
- minimal quark content $cc\bar{u}\bar{d}$
- manifests as a peak in mass spectrum of $D^0 D^0 \pi^+$ mesons.
- LHCb data ends around 4000 MeV

Using **Lattice QCD**, we want to better understand nature of this exotic state:

- Is DD^* interaction repulsive in the $I = 1$ channel and attractive in the $I = 0$ channel?
- establish a pole in the corresponding scattering amplitude $t(E_{cm})$
- To avoid issues w/ left-hand cut, might need adapted Lüscher analysis (see Colins et. al 2402.14715)

Introduction

- **no clear theoretical interpretation of the experimental data**
- T_{cc} would be a bound state if the processes $D^* \to D\pi$, $T_{cc} \to DD\pi$ excluded
- Can we show that the lowest positive-parity charmed mesons are hadronic molecules?



[LHCb 2109.01038]

Steps of calculation

- ✓ **Ensemble generation:** $N_f = 2 + 1$ quark flavors, a tree level Symanzik improved gluon action and 6-stout dynamical smeared Wilson fermions
- ✓ **HPC Tasks:** Generation of distillation basis, perambulators, meson elementals using `Chroma` with `superbblas` support on the Jureca cluster at JSC
- Construct **Di-meson distilled operators** using Hadspec method of subduction coefficents and helicity operators
- ✓ Perform **contractions** of multi-hadron operators $\rightarrow$ 2pt correlators
- Construct correlation function coming from the **GEVP** in the right irreducible representation
- **Compute spectrum** and energy shifts w.r.t to the DD^* threshold for a heavy quark mass close to the charm quark mass.
- **Lüscher analysis** to obtain finite volume energies from Scattering amplitudes
- **Search for Poles** AKA when an attractive potential is not deep enough to hold a bound state

Ensemble Details

Ensembles at several a and m_π allow systematic study of m_π dependence

	m_{ud}	m_s	$L^3 \times T$	m_π [MeV]	N_{conf}
$\beta = 3.30$ a = 0.125[fm]	-0.1309	-0.057	$48^3 \times 64$	135	*
	-0.1291	-0.057	$32^3 \times 64$	200	*
	-0.1265	-0.057	$24^3 \times 64$	280	1000
	-0.1233	-0.057	$24^3 \times 64$	330	1000
	-0.1200	-0.057	$16^3 \times 64$	400	1000
	m_{ud}	m_s	$L^3 \times T$	m_π [MeV]	N_{conf}
$\beta = 3.57$ a = 0.085[fm]	-0.0498	-0.007	$64^3 \times 96$	135	*
	-0.0483	-0.007	$48^3 \times 64$	200	400
	-0.0440	-0.007	$32^3 \times 64$	300	400
	-0.0380	-0.007	$24^3 \times 64$	420	400
	m_{ud}	m_s	$L^3 \times T$	m_π [MeV]	N_{conf}
$\beta = 3.70$ a = 0.065[fm]	-0.02981	-0.0	$64^3 \times 96$	135	*
	-0.02855	-0.0	$64^3 \times 96$	200	*
	-0.0250	-0.0	$40^3 \times 96$	300	400
	-0.0220	-0.0	$32^3 \times 96$	380	400
	-0.0200	-0.0	$32^3 \times 96$	420	400

- $N_f = 2 + 1$
- Symanzik improved gluon action
- 6-stout dynamical smeared Wilson fermions
- Close to $D^0 D^0 \pi^+$ threshold, so sensitive to m_{ud}
- Important to have many ensembles!

Distillation Smearing

Smearing method that restricts the interpolators to a small subspace, called the **distillation basis**, which contains enough contributions from the relevant eigenstates for good overlap w/ phys. states

1. **Solution vectors**:

$$S^{(k)}_{\alpha\beta}(\vec{x}, t'; t) = M^{-1}_{\alpha\beta}(t', t) V^k(t)$$

2. **Perambulators**:

$$\tau_{\alpha\beta}(t', t)^{kl} = V^{(k)\dagger}(t') M^{-1}_{\alpha\beta}(t', t) V^l(t)$$

   access all spatial entries of the propagator between t_f and t_0. Perambulators are independent of the creation operators so the inversion cost is fixed by nvecs and the spatial extent of lattice.

3. **Elementals**:

$$\Phi^{(i,j)}_{\mu\nu}(t) = \delta^{ab}(D_1\xi^i)^a(D_2\xi^j)^b(t)S_{\mu\nu}$$

$$= V^\dagger(t)[\Gamma^A(t)]_{\alpha\beta}V(t) \equiv V^\dagger(t)\mathcal{D}^A(t)V(t)S^A_{\alpha\beta}$$

- $S_{\mu\nu}$: subduction matrices
- M : Dirac operator
- D_n: covariant derivative acting on nth quark of interpolator.

Ingredients for Distillation with HPC

- costly *initially* both in storage and component construction
- **Will save us time and resources in the end..**
- Software stack: MultiGrid (MG) solver from `QUDA`, `Chroma` with `Superbblas` support, the `PRIMME` eigensolver, and `Numpy Einsum` for contractions

Computation	Operations cost	Memory footprint
Distillation basis[a]	$N^3 T n^3 D$	$N^3 n T$
Meson elementals[b]	$N^3 T n^3$	$N^3 n + n^3$
Perambulators[c]	$N^3 T n$	$N^3 T n$
Contractions[d]	$n^4 T$	$n^3 T$

[a] Generate colorvector matrix elements
[b] Contract two matrices $\rightarrow$ tensor
[c] Projection of the inverse Dirac operator $\rightarrow$ square matrices
[d] Contract together matrix elements and perambulators

once a suitable set of perambulators compute, **reuse** to correlate a collection of interpolators

Signal Saturation for Mesonic correlators

Perform contractions to obtain the correlator

$$C_M^{(2)}(t',t) = Tr[\Phi^B(t')\tau(t',t)\Phi^A(t)\tau(t,t')]$$

Solve the GEVP

$$C(t)v^n(t) = \lambda_n(t)C(t_0)v^n(t) \tag{1}$$

for collection of meson elementals

- displacements
- momenta
- 1 GEVP per irrep

from the correlation matrices that are averaged over all spin and momentum polarizations and over source timeslices.

- Compute the principal correlator
- Form optimized operators

[arxiv:0905.2160v1].

Signal Saturation w/ Distillation

Advantages:

- finite momentum on both sides (src + snk)
- scattering studies need well-controlled momentum insertions
- guarantees hermiticity of operators **important for GEVP**

Study in progress:

- # tsrc vs. signal (max 48)
- # distillation vectors vs. signal
- extract the spectrum across several irreps, eg. T_1^+, A_1^-, A_2 varying # distillation vectors, $N \in \{32, 64, 96, 128\}$

Di-meson interpolating operators

Wave function of the tetraquark state T_{cc}^{+} includes two color singlet channels:

$$DD^* = \frac{1}{\sqrt{2}}(D^{0*}D^{+} - D^{+}D^{*}), \tag{2}$$

$$D^*D^* = \frac{1}{\sqrt{2}}(D^{*0}D^{*+} - D^{*+}D^{*0}) \tag{3}$$

Project to definite momentum:

$$O^{DD^*} = \sum_{k,j} A_{kj}(D\vec{p}_{1k})D^*_j(\vec{p_{2k}}), \vec{p}_{1k} + \vec{p_2 k} = \vec{P} \tag{4}$$

$$= \sum_{k,j} A_{kj}[(\bar{u}\Gamma_1 c)_{\vec{p}_{1k}}(\bar{d}\Gamma_{2j}c)_{\vec{p_{2k}}} - (\bar{d}\Gamma_1 c)_{\vec{p}_{1k}}(\bar{u}\Gamma_{2j}c)_{\vec{p_{2k}}}] \tag{5}$$

can momentum project at both ends thanks to distillation! [Madanagopalan:2022]

DD^* interpolators

- angular momentum no longer a good quantum number in discretized spacetime
- no longer have rotational symmetry of continuum

ID	$\vec{P}$	LG	Λ^P	J^P	I	interpolators: $M_1(\vec{p}_1^{\,2})M_2(\vec{p}_2^{\,2})$
1	$(0,0,0)$	O_h	T_1^+	1^+	$0,2$	$D(0)D^*(0),\ D(1)D^*(1)\ [2],\ D^*(0)D^*(0)$
2	$(0,0,0)$	O_h	A_1^-	0^-	1	$D(1)D^*(1)$
3	$(0,0,1)\frac{2\pi}{L}$	Dic_4	A_2	$0^-,1^+,2^-$	$0,1,2$	$D(0)D^*(1),\ D(1)D^*(0)$
4	$(1,1,0)\frac{2\pi}{L}$	Dic_2	A_2	$0^-,1^+,2^-,2^+$	$0,1,2$	$D(0)D^*(2),\ D(1)D^*(1)\ [2],\ D(2)D^*(0)$
5	$(0,0,2)\frac{2\pi}{L}$	Dic_4	A_2	$0^-,1^+,2^-$	$0,1,2$	$D(1)D^*(1)$

[Padmanath 2022]

Towards Analysis and Summary

1. compute spectrum for a range of:
   - center-of-mass momenta in various irreps of the octahedral group O_h^D

2. FV analysis of the discrete spectrum on several volumes and momentum frames **Note: Hadrons containing heavy quarks are prone to discretization errors thus a controlled continuum limit at finite lattice spacing is required**

3. finer lattice spacing?

4. isospin-1 P-wave scattering phase shift

5. systematic uncertainty from m_π

6. T_{cc}^+ dependence on $m_\pi \to$ phys pt.

Acknowledgements

- Thank you to my collaborators at JSC/IKP-4 for their continued guidance with pushing this project forward!
- Funding through NRW-FAIR Work Project 1
- Computing done in-house at JSC on Jureca