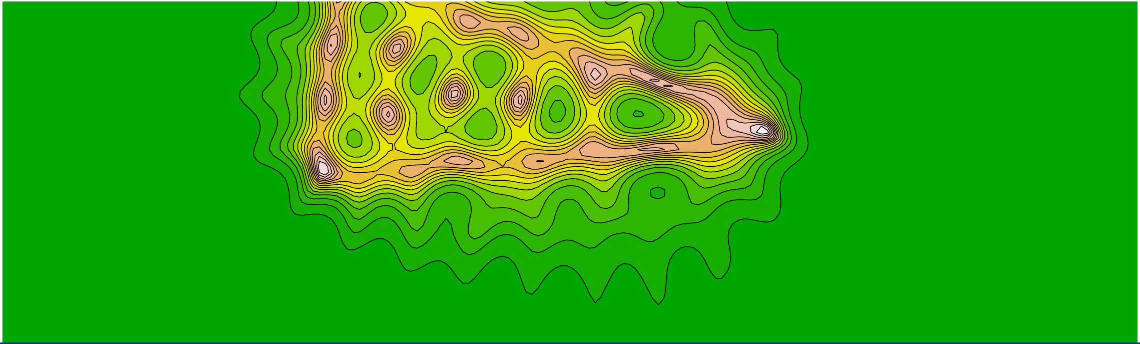


# NON-LINEAR EIGENVALUE PROBLEMS ARISING IN ACOUSTIC SCATTERING

Application-driven functionalities: Survey of functionalities

**ExaNLA 2025 Jülich** | February 11, 2025 | Andreas Kleefeld | Jülich Supercomputing Centre

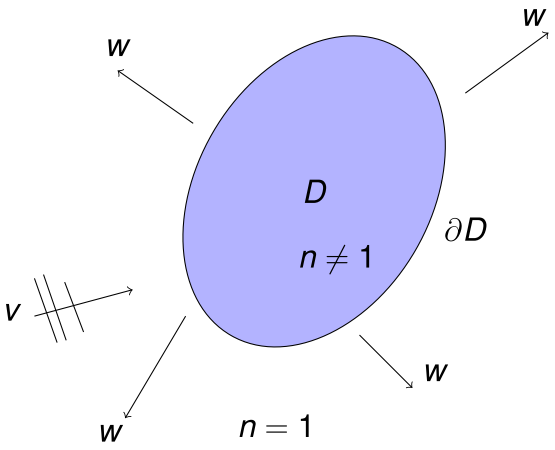


JÜLICH Forschungszentrum | JÜLICH SUPERCOMPUTING CENTRE

# WHAT MY COMMUNITY AND I DO

## Direct and inverse acoustic scattering

- Is there a wave $v$ (depends on a wave number $k$) that does not scatter?
- Leads to a coupled system of (reduced) wave equations with unbounded domain.
- Solved with boundary integral equations.
- Discretization leads to a non-linear eigenvalue problem.



# NON-LINEAR EIGENVALUE PROBLEM

## What we need?

- Non-linear eigenvalue problem with $\mathrm{M} : \mathbb{D} \subset \mathbb{C} \to \mathbb{C}^{m\times m}$, $m \gg 0$ is given by

$$\mathrm{M}(k)\vec{v} = \vec{0}\,, \quad \vec{v} \in \mathbb{C}^m\,, \quad \vec{v} \neq \vec{0}\,, \quad \mathbb{D} \subset \mathbb{C}$$

  where each entry of $\mathrm{M}(k) \in \mathbb{C}^{m\times m}$ is a holomorphic function in the open domain $\mathbb{D}$. Here, $k$ is the (non-linear) eigenvalue and $\vec{v}$ is the (right) eigenvector.
- Solved with Beyn's contour algorithm.
- **What we need:** Numerical quadrature (trapezoidal rule), solving linear systems (matrix fully populated, no structure, complex-valued entries), singular value decomposition (same here), linear eigenvalue solver (same here, but small), matrix-matrix products

# OUTLOOK

- 2D to 3D: $m \to c \cdot m^2$, where $c$ depends on the wave number
- More than one obstacle: $c \cdot m^2 \to c \cdot q \cdot m^2$ with $q$ the number of obstacles.
- Electromagnetic scattering: $c \cdot q \cdot m^2 \to 3c \cdot q \cdot m^2$ (vector-valued function)
- When the index of refraction is piece-wise constant: $3c \cdot q \cdot m^2 \to 3a \cdot c \cdot q \cdot m^2$, where $a$ are the number of different pieces.
- Evaluation of eigenvalue curves needs the solution of many non-linear eigenvalue problems (for varying material parameter such as the index of refraction $n$).

# OUTLOOK

## What I can offer

- Solid mathematical background in linear algebra and analysis.
- Specifically on numerical linear algebra and analysis with respect to solving PDEs and ODEs numerically.
- My knowledge also extends to stochastics.

- To think/discuss about: What about probabilistic algorithms for linear algebra computations (randomized NLA)?

# REFERENCES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