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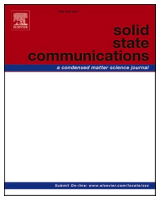

The boson-defined thermodynamics at the liquid-solid transition

Ulrich Köbler

Research Centre Jülich, Institute PGI, D-52425, Jülich, Germany

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ABSTRACT

Experimental evidence from rare gas (RG) solids, Ne, Ar, Kr and Xe, shows that a boson field orders at the liquid-solid transition. Upon ordering, all bosons condense in the lowest energy state (Bose-Einstein condensation). This is the highest possible thermodynamic order and provides an entropy argument for the surprising exclusion of the interatomic van der Waals interactions from performing the liquid-solid transition. Since the ordered boson field interacts with the atomic system, it defines the observed long-range coherent order of the crystalline state. This order is more perfect than can be expected if the locally anisotropic interatomic interactions would cause the ordering transition. Boson dynamics is evidenced by the observed universal temperature dependence of heat capacity and thermal length changes, $\Delta L/L_0$, in the critical range below the melting temperature, T_m. It is argued that the bosons are elastic quadrupole radiation. Upon ordering, boson fields confine themselves to the finite volume of a stationary unit, such as a domain or a mosaic block, and compress the ordered unit increasingly with decreasing temperature. This compression ensures the cohesion of the solids up to T_m. The low-temperature lattice parameters, therefore, are rather short and the calculated van der Waals cohesive energies are larger by a factor of ten compared to the melting temperatures, T_m. In analogy to the term magnetostriction, the thermal lattice contractions below T_m could be termed "elastostriction".

1. Introduction

In the temperature range between the Debye-temperature, Θ_D, and the melting temperature, T_m, the thermodynamic properties of solids cannot be understood by considering atomistic near-neighbour interactions alone [1]. For the most common solids, Θ_D is in the range $200<\Theta_D<500$ K but T_m is in the range $1000<T_m<1500$ K [2]. Taking Θ_D as a reasonable measure of the absolute inter-atomic interaction strength (see discussion of Fig. 7), the inter-atomic interactions are too weak to ensure the cohesion of the solids up to T_m. The aim of this study is to gain more information on the nature of the forces that hold solids together below T_m, focusing on heat capacity and thermal length change data of the rare-gas (RG) solids Ne, Ar, Kr and Xe. The analyzed experimental data are drawn entirely from the existing literature [3].

The RG solids are ideal materials to investigate the binding forces of the solids because the atomistic part of these forces can be assumed to be entirely of the van der Waals type and can be calculated to a reasonable approximation. This allows a separation of the experimentally observed total binding forces into the atomistic part and the unexplained additional binding forces. It turns out that the atomistic van der Waals interactions have a negligible binding function for the RG-solids (see discussion of Fig. 8).

It has to be noted that it is not self-evident that the only parameter of the Debye boson field, the Debye temperature, Θ_D, provides a reasonable quantitative measure of the total inter-atomic interaction strength. As we have explained elsewhere [4,5], in all solids one has to distinguish between the excitation spectrum of the mass-less sound waves and the excitation spectrum of the phonons. In the following we will call the sound-waves Debye bosons [6]. In contrast to the mass-less Debye bosons, phonons are the excitations of the massive atomic lattice and therefore have a mass. This is a condition that the phonons can be observed using inelastic neutron scattering. In other words, using neutron scattering, the phonons are sampled selectively. Moreover, the two types of excitations differ by translational symmetry that is continuous for the ballistically propagating Debye bosons but discrete-periodic for the phonons. The two translational symmetries can be considered as the generators of the two (quasi)particles. The dispersion of the Debye-bosons is a somewhat weaker than linear function of wave-vector with no upper energy limit [4,5]. This conforms to the well-known fact that sound waves can be excited, up to the highest temperatures, as decaying modes, out of thermal equilibrium. The dispersions of the acoustic phonons are given essentially by trigonometric functions of wave-vector with a clearly limited upper energy, defined by the inter-atomic interaction strength (see Fig. 7, below) [4,5]. Note, that

E-mail address: u.koebler@fz-juelich.de.

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in contrast to the ballistically propagating Debye bosons, the phonons are, in harmonic approximation, non-propagating standing waves, confined to the individual mosaic block, similar to magnons that are confined to the volume of each domain [7]. As a consequence, the phonons contribute little to the thermal conductivity [8].

The reason that the characteristic parameter of the Debye boson field, Θ_D, provides a good approximation of the absolute inter-atomic interaction strength is, that due to a strong interaction between Debye bosons and acoustic phonons, the dispersions of the two excitations attract each other. This attraction is typical of interacting systems with different (translational) symmetries. Commonly, the two dispersion relations have perfectly assimilated, at low q-values, and start with the same, linear wave-vector dependence [9]. The linear dispersion of the Debye bosons, i.e., their velocity, therefore, corresponds to the stiffness constant of the acoustic phonons, and Θ_D becomes a measure of the absolute phonon energy (see Fig. 7, below). The harder the material, the higher is Θ_D, and the higher is the upper phonon energy.

Within the group of the inert gas solids, Ne is particularly interesting since it is the only known material for which the melting temperature of $T_m = 24.6$ K is lower than $\Theta_D = 71$ K (see Fig. 7, below) [10]. One might, therefore, anticipate that the liquid-solid transition of Ne is a matter of the stronger inter-atomic interactions, and should occur at a temperature of $T_m \sim \Theta_D \sim 71$ K. This is, obviously, not the case. Apparently, the inter-atomic interactions (phonons) are generally excluded from performing the liquid-solid transition and, as a consequence, have no binding function. This puzzling observation is evidently beyond the atomistic models of the solid and provides a clear indication for the existence of energy degrees of freedom in addition to the inter-atomic interactions. As we will explain below, these, up to now, not identified binding forces are of bosonic nature and are not only responsible for the cohesion of the solids, up to T_m, they drive the liquid-solid transition as well. Since the long-range crystalline atomic order of solid Ne, realized below T_m, is identical with the crystalline order of the other RG-solids with $T_m > \Theta_D$, the long-range crystalline atomic order is generally due to the boson field that orders at the liquid-solid transition.

The surprising phenomenon that the liquid-solid transition of Ne is at a lower temperature than conforms to the inter-atomic interactions is known from magnetism [7]. The relevant bosons of the magnetic solids have been identified as magnetic dipole radiation, generated by the precessing spins [11,12]. We have called these bosons, Goldstone bosons [13]. It is generally observed that the magnetic ordering transitions, executed by the Goldstone bosons, are at a lower temperature, compared to the strength of the exchange interactions between neighbouring spins, given by the magnon energies at the zone-boundary [7]. One clear evidence, proving the exclusion of the exchange interactions from the critical spin dynamics is, that the critical exponents are identical for ferromagnets and for antiferromagnets of the same symmetry class [11]. Moreover, since integer and half-integer spins precess differently, the generated Goldstone boson fields differ, and the observed critical exponents are different in magnets of the same symmetry class but with an integer or a half integer spin [7,11]. This experimentally clearly established quantum effect, is evidently beyond the atomistic models of magnetism that consider non-quantized classical spins [14].

As a qualitative entropy argument for the paradox that the boson-defined magnetic ordering temperatures are lower than expected according to the strength of the exchange interactions between the spins, it has to be mentioned that the long-range, collinear spin order within each magnetic domain, forced by the ordered Goldstone boson field, is much more perfect than it can be expected if the locally anisotropic near-neighbour interactions would be relevant and would execute the ordering transition. Note that the long-range coherence of the ordered spin system is a consequence of the fact that the Goldstone-bosons get generated by stimulated emission. Due to stimulated emission, the wave function of the bosons remains, over a long-distance, a coherent wave. It seems to be a general principle of nature to realize the highest possible order at order-disorder phase transitions. This can be achieved only if the bosons order and determine the order of the atomistic system. The observed perfect order of the spins (or atoms), therefore, is a direct consequence of the Bose-Einstein condensation which enables all bosons to condense in the same quantum state. This is the thermodynamically highest possible order. One could imagine the ordered boson field within each domain or mosaic block as an interference pattern.

Essential for ordering of boson fields is that the generation process of the bosons is dominated, as postulated by Einstein, by stimulated emission. This leads to a collimation or amplification of particular, phase-coherent bosons in one quantum state. Ordering occurs when the density of the collimated bosons has reached the critical value for the spontaneous onset of stimulated emission. The liquid-solid transition is then a threshold induced event, similar to the ignition of a LASER and appears as a first order phase transition in the heat capacity [15]. If the production rate of the bosons is low, the density of the bosons is low and the ordering transition of the boson field occurs at a relatively low temperature. In this case, the boson-executed ordering transition can be at a lower temperature than conforms to the inter-atomic interactions. This holds for the bosons of the magnetic solids (magnetic dipole radiation) [7] and for the bosons of the lightest member of the RG-solids, of Ne (elastic quadrupole radiation). We will call the bosons that order at the melting point, MP-bosons. As a consequence, we have to assume that for solids consisting of light atoms, the elastic quadrupole moments are small and the production rate of the MP-bosons is correspondingly low. The mass dependent temperature of the liquid-solid transition (see Fig. 8, below) supports the conclusion that the MP-bosons that order at T_m are elastic quadrupole radiation. The non-relevant inter-atomic interactions define local details only, such as the coordination of the neighbouring atoms in the crystals or a parallel or antiparallel orientation of neighbouring spins in the magnetic solids. These local details have no influence on the ordering process or on the type of the long-range coherent order.

In the following analyses of published experimental data of the RG-solids, earlier findings are confirmed that at the liquid-solid transition, a boson field orders [3,11,16]. The RG-solids are a particular group of materials because they all have the cubic fcc structure and a valence of zero and, therefore, are chemically very similar such that their properties can be compared (see Fig. 8, below). Moreover, the inter-atomic interactions can be assumed to be of the van der Waals type [17,14,18]. It turns out that the equilibrium low-temperature lattice parameters of the RG-solids are surprisingly short and the calculated van der Waals cohesive energies are too high by a factor of about ten, compared to the melting temperatures [14]. In the following, it will be shown that the short low-temperature lattice parameters are caused by a theoretically unexplained compressive force, exserted by the ordered MP-boson field on each mosaic block. This new type of force is particular to all ordered boson fields. In the case of the ordered magnets, the decrease of the lattice parameter with decreasing temperature due to the ordered Goldstone boson field, is known as magnetostriction.

Since the temperature dependence of heat capacity and of $\Delta L/L_0$ of Ne is given, below T_m, as for the other RG-solids with $T_m > \Theta_D$, by the typical universal critical power functions (see Figs. 2 and 6, below), it follows that the liquid-solid transition of Ne is, as for the other RG-solids with $T_m > \Theta_D$, the ordering transition of a boson field, in spite of the much stronger absolute inter-atomic interaction strength compared to T_m. Moreover, in contrast to Ar, Kr, and Xe, there is no phonon-defined temperature range in the heat capacity of Ne (see Fig. 6, below). In other words, for all temperatures, the phonons are excluded from the dynamics of Ne and affect the absolute energy scale (the pre-factors of the universal power functions) only.

Because the bosons interact with the atomic system, the observed perfect, long-range atomic order of the crystalline RG-solids (fcc structure) is a consequence of the perfect order of the ordered MP-boson field. Note that the non-relevant atomistic van der Waals interactions would not result into the long-range and coherent atomic order in each mosaic block. The MP-boson field that orders at T_m has to be made responsible

for the cohesion of the solids up to T_m as well.

Although universality is a well-known phenomenon in the vicinity of the magnetic ordering transition, it is observed at many other phase transitions into a long-range ordered state, such as the ferroelectric transition [19], the vapor-liquid transition [20] and the liquid-solid transition. One exception is the superconducting transition at which the typical critical dynamics is missing. As a consequence, the superconducting transition is not the transition into a long-range ordered state [21]. This conforms to the fact that there is no correlation between the Cooper-pairs. Since below T_m, universality is observed in the temperature dependence of the heat capacity and of the thermal length changes for all solids [16,22], it is clear that the liquid-solid transition is generally the ordering transition of a boson field.

A very important additional condition of a boson-induced stable atomic or spin order is that the bosons do not only concentrate in energy, they confine themselves in space by forming sharply contoured stationary ordered units (domains) on ordering. This process is theoretically unexplained but experimentally evident. The long-range ordered units are resonators, self-organized by the condensed boson field. In these ordered units the bosons are in standing modes that can be assumed to be coherent with the lattice structure.

Formation of stationary domains at the magnetic ordering temperature means that the running Goldstone bosons, emitted by the magnetic dipoles, become fixed to the atomic lattice [7,23]. Note that the domains are the true signature of the long-range ordered spin system. Since magnetic dipole radiation is emitted along the precession axis of the spins, the generated radiation field is linear. The magnetic domains, therefore, are one-dimensionally ordered units. As a consequence of the perfect collinear spin order within each domain, the magnon dispersions, commonly, are as for the linear spin chain, irrespective of the three-dimensional exchange interactions [24]. This further illustrates the dominance of the bosons over the atomistic interactions. Since the magnons are confined to the individual domain, no magnons are observed for crystal directions along which there are no domains [25,26].

By a similar reason are the dispersions of the acoustic phonons of many solids as for the linear atomic chain, irrespective of the assumed three-dimensional local inter-atomic interactions, that are not relevant (see Fig. 7, below). In this case, the one-dimensional dispersion of the acoustic phonons is induced by the Debye bosons that are collimated, due to stimulated emission, along those crystallographic directions with a high density of the sources of the Debye-bosons, the atoms. Note, however, that the collimation of the Debye-bosons is perfect for $T \rightarrow 0$ only, i.e., at the "ordering temperature" of the Debye boson field, that is $T = 0$. Since the sources of the Debye bosons are elastic dipoles, the Debye bosons are, as the Goldstone bosons (magnetic dipole radiation), linear waves.

The mosaic blocks, occurring in practically all crystalline solids, have to be identified as the domains of the MP-boson field that orders at T_m. As is well known, domains and mosaic blocks can reach macroscopic dimensions [23,27]. Apparently, the interaction between neighbouring domains or mosaic blocks is weak. As a consequence, it is easily possible to transform the whole magnet from the multi-domain ground state into the mono-domain state at magnetic saturation or the poly-crystal into a single crystal, i.e., into a large mosaic block [27].

Since the atomic order, induced by the MP-bosons is three-dimensional, the MP-bosons cannot be one-dimensional waves, as the Goldstone bosons or the Debye bosons, but must be three-dimensional. As a consequence, the sources of the MP-bosons cannot be dipoles. In contrast to the sources of the Debye-bosons, that are evidently elastic dipoles, the sources of the MP-bosons must be elastic quadrupoles. One observation supporting the conclusion that the MP-bosons are excitations of the elastic solid is, that the critical behaviour of the boson-defined heat capacity at T_m is qualitatively identical for metals and for insulators [15]. In other words, Debye bosons and MP-bosons have identical properties in metals and in insulators.

Direct evidence that the MP-bosons are, as the Debye bosons, excitations of the elastic solid is, that the MP-bosons give rise to a distinguished elastic constant [16]. While the elastic stiffness constants, c_{ij}, are well-known to be defined by the velocities of the Debye bosons, the elastic compliances, s_{ij}, are a measure of the compressibility of the crystals, and are determined by the self-constricting force of the ordered MP-boson field [28,16]. Typical of the different origin of the two elastic constants is that they have different dimensions and different temperature dependencies [28]. Note that the elastic stiffness constants have the dimension of a pressure but the elastic compliances have the dimension of a reciprocal pressure.

As is known from second-order magnetic phase transitions, boson-defined thermodynamic observables (susceptibility, heat capacity) diverge at the ordering temperature of the boson field [29]. However, the two elastic constants do not diverge but reach a finite maximum only that is for the elastic stiffness constants, c_{ij}, at $T = 0$ but for the elastic compliances, s_{ij}, at T_m [16]. As a consequence, the "ordering temperature" of the Debye-boson field is $T = 0$, where the velocity of the Debye-bosons has a maximum and the ordering temperature of the MP-bosons is at T_m, where the compressibility has a finite maximum. In other words, at T_m the solids are rather soft. In contrast to the Debye bosons which do not order at a finite temperature and, therefore, are propagating waves at all temperatures, the ordered MP-bosons are in stationary states for all temperatures below T_m. The elastic compliances do not diverge at T_m, because all liquid-solid transitions are first order and, therefore, are threshold induced events [15]. This interpretation provides, for the first time, a plausible microscopic understanding of the elastic compliances, introduced phenomenologically some hundred years ago [28]. It appears that macroscopic quantities of the solids are generally defined by bosons.

The concentric force, exerted by the ordered MP-boson field on each mosaic block is a dynamic quantity that increases with decreasing temperature, whereby the volume of each mosaic block, and therefore the lattice parameter of the bulk material, decreases strongly (see Fig. 2, below) [30,16]. In the magnetic case, this phenomenon is well-known as magnetostriction, which sets in with the formation of domains at the magnetic ordering temperature [31]. Note, however, that the magnetic domains are one-dimensional objects and get compressed along their axes by the one-dimensionally ordered Goldstone boson field. On the other hand, thermal contraction of each mosaic block below T_m is concentric, caused by the three-dimensionally ordered MP-boson field. In analogy to the term magnetostriction, this bosonic phenomenon could be termed "elastostriction". This new type of bosonic force is considered as responsible for the cohesion of the solids up to T_m [16].

2. Methodology

Mass-less bosons are difficult to observe directly and must be identified by their actions. The prominent thermodynamic signature of a boson field is its heat capacity, but it is not possible to decompose the observed heat capacity into the contribution of the bosons and the contribution of the phonons because the two systems interact. However, in the critical range near an ordering transition, the bosons dominate the dynamics in that they define the temperature dependence of the heat capacity. Evidence for this is obtained by the observed universality in the critical temperature range. Universality means that not only the heat capacity but all other boson-defined thermodynamic observables, such as the thermal lattice contractions, follow, over a finite temperature range, power functions of the distance from the critical point with rational exponents (see Figs. 2 and 3, below). This qualifies the critical temperature as a particular point. The critical exponents are universal, i.e., they are identical for all solids of the same symmetry class. In other words, the critical exponents are characteristic of the boson type that gives rise to the observed universality of the thermodynamic observables. The pre-factors of the critical power functions, the critical amplitudes, are material specific and are defined by the interactions of the

bosons with all other energy degrees of freedom. Due to this interaction, the bosons are able to control the dynamics of the atomistic system. On the one hand, universality allows for an analytically simple description of the critical dynamics but, on the other hand, a quantitative understanding of the fitted critical exponents and of the fitted critical amplitudes is a very complicated and not generally solvable theoretical problem [29]. Since universality holds above and below the ordering temperature it follows that the ordering transition is also a matter of the boson field. In fact, the observed liquid-solid transition of the atomistic system is caused by the coupling of the atomistic system to the relevant bosons. Due to this coupling, the atomistic system, orders at the same temperature as the boson field. In the following it will be demonstrated that heat capacity [3] and relative thermal length change data, $\Delta L/L_0$, of the RG-solids exhibit universality over a large temperature range below the melting temperature T_m [30]. This is taken as evidence that both quantities are controlled by the bosons that order at T_m, the MP-bosons (elastic quadrupole radiation). It appears safe to conclude that at all types of order-disorder phase transitions at which universality is observed, a boson field orders. Preliminary and less conclusive interpretations of the experimental data analyzed here have been documented before [32].

3. Analysis of experimental data

The crossover from phonon dynamics, for heat capacity values of lower than the Dulong-Petit (D-P) limit, to boson dynamics, for larger heat capacity values, gives rise to an extremely sharp kink in the so-called Grüneisen plot (Fig. 1) [17,18]. In this plot, the temperature dependence of the coefficient of the linear, α, or of the cubic, $\beta = 3\cdot\alpha$, thermal lattice expansivity is plotted vs. the temperature dependence of the heat capacity. In other words, the temperature is an implicit parameter. Each data point in the Grüneisen plot is for a different temperature. As Fig. 1 illustrates, the heat capacity of cubic Xe continues to increase over the atomistic Dulong-Petit limit but with an analytically different temperature dependence (compare Fig. 3, below). The sharp kink in the Grüneisen plot is at T = 54 K, which is near to the Debye-temperature of Θ_D = 61 K (see Fig. 7, below). As we have explained, the Debye-temperature provides a reasonable temperature equivalent of the upper phonon energy. The sharp kink at the D-P-limit shows that a crossover is a functional change in the temperature dependence of the thermodynamic observables that cannot be simulated

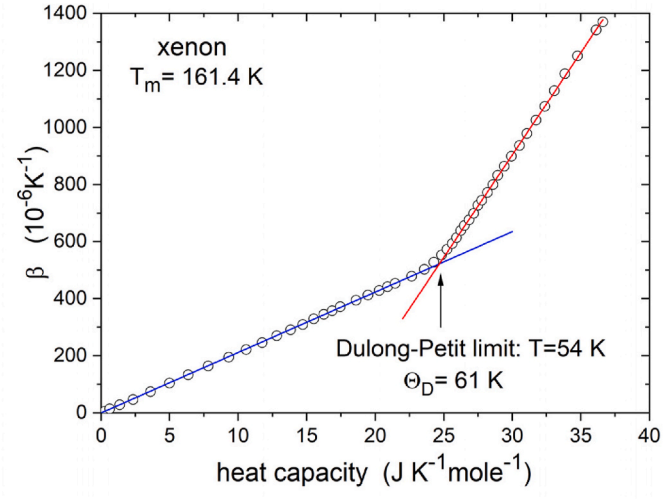

Fig. 1. Coefficient of the volume thermal expansivity, β, of solid Xe vs. heat capacity [3,30]. In this plot, each data point belongs to a different temperature. The crossover from phonon dynamics, for heat capacity values of lower than the D-P-limit, to boson dynamics for larger heat capacity values gives rise to a very sharp kink. The temperature of the kink is close to the Debye-temperature.

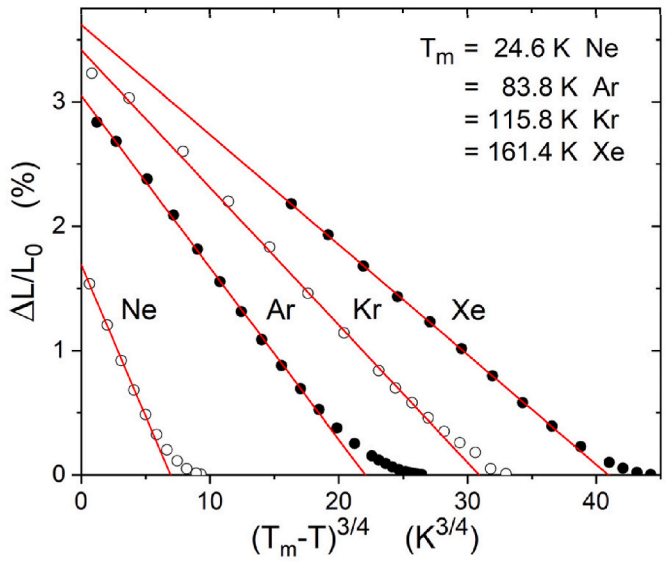

Fig. 2. Relative thermal length changes, $\Delta L/L_0$, of the RG-solids as a function of $(T_m-T)^{3/4}$ [30]. The reference value, L_0, is at T = 4.2 K. Note that T = 0 is at $(Tm-T)^{3/4}$ = 11, 28, 35 and 45 for Ne, Ar, Kr and Xe, respectively. Universality of the critical exponent of 3/4 proves that the $\Delta L/L_0$ data are caused by the boson field that orders at T_m. The crossover to phonon dynamics at T~Θ_D is at the deflection from the linear behaviour.

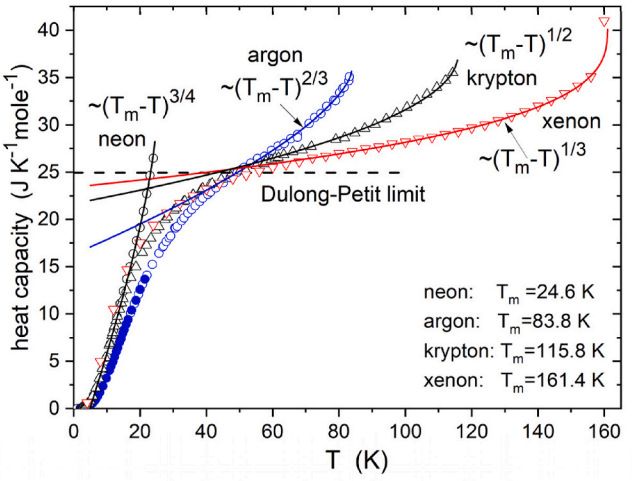

Fig. 3. Overview of the heat capacities of the RG-solids Xe, Kr, Ar and Ne as a function of temperature [3,10,33,34]. The heat capacity data of larger than the D-P-limit are due to the MP-bosons and follow universal power functions of the distance from the melting temperature T_m. Below the D-P-limit, the phonons are the relevant excitations, and the heat capacity decreases stronger with decreasing temperature. In the heat capacity of Ne, the crossover from boson dynamics to phonon dynamics is missing (see discussion of Fig. 6).

by one continuous function of temperature (see Fig. 3, below). The kink in the Grüneisen plot makes clear that there are two excitation systems that get activated (relevant) alternately. At the kink, the symmetry changes from the discrete-periodic translational symmetry of the phonons, below the D-P-limit, to the continuous (lattice structure independent) translational symmetry of the MP-bosons above the D-P-limit. Since the Grüneisen relation results from atomistic considerations, it applies to the atomistic temperature range below the D-P-limit only. Above the D-P-limit, thermal lattice parameter variations are due to the MP-bosons, and exhibit an analytically different, stronger temperature dependence than below the D-P-limit where the phonons are the relevant excitations (see Fig. 2). In other words, the ordered MP-boson field

compresses the lattice strongly with decreasing temperature but only down to the symmetry change to the phonons at the D-P-limit.

The thermal length change data, $\Delta L/L_0$, of the four RG-solids are plotted as a function of $(T_m\text{-}T)^{3/4}$ in Fig. 2 [30]. The adopted rational exponent of 3/4 is consistent with the average value of all fitted exponents of $\sim 0.767 \pm 0.02$. The identical exponent of 3/4 of all RG-solids proves universality, i.e., a boson defined, material independence. The crossover from boson dynamics, above the D-P-limit, to phonon dynamics below the D-P-limit appears as a deflection of the $\Delta L/L_0$ data from the straight-line behaviour to a weaker temperature dependence for lower temperatures in Fig. 2. Note that T→0 is on the right-hand side of Fig. 2. Below the D-P-limit, the atomistic mechanism, according to Grüneisen, is responsible for the lattice parameter variations instead of the MP-bosons [17,18]. For T→0, thermal lattice contractions of all solids assume a temperature-independent behaviour [30]. This can be understood, considering that strong thermal lattice contractions are a domain effect, i.e., a consequence of the self-constriction of the ordered boson field, and therefore occur below a boson-driven ordering transition only. Note, that in the critical range at T = 0, the Debye-bosons are the relevant excitations instead of the phonons. In this range, the heat capacity of the Debye-boson field is given by the universal power function $\sim(T\text{-}T_c)^3$ with $T_c = 0$ as critical temperature. In other words, the Debye-boson field does not order at a finite temperature, and no (additional) domains get formed upon approaching T→0. As a consequence, no significant lattice contraction occurs for T→0. No domains, means no significant lattice contractions.

As Fig. 2 shows, the lattice parameters have a minimum for T→0. Moreover, the higher T_m is, the stronger is the lattice parameter reduced for T→0. For these low-temperature lattice parameter values the cohesive energies of the RG-solids have been calculated, assuming van der Waals interactions between the RG-atoms [17,14]. From the calculated cohesive energies of $E_{coh} = 0.45$, 1.85, 2.67 and 3.83 kcal/mol for Ne, Ar, Kr and Xe, respectively [14] melting temperatures, T_m, can be evaluated by setting: $E_{coh} = k_B \cdot T_m$. In this way, melting temperatures of $T_m = 226$, 931, 1344 and 1927 K are obtained for Ne, Ar, Kr and Xe, respectively, which are larger by about a factor of ten than the actual melting temperatures, T_m (see Fig. 2). The large melting temperatures, calculated from the large low-temperature cohesive energies are a consequence of the small lattice parameters for T→0 due to the compression of the RG-solids by the ordered MP-boson field.

It should be noted that for the same reason, are the low-temperature cohesive energies, calculated for the cubic alkali halide crystals with fcc lattice structure, too high compared to the melting temperatures [16]. For this class of solids, ideal ionic bond can be assumed. The atomistic part of the cohesive energy, therefore, is exclusively of electrostatic nature and depends on the distance between the positive and negative ions alone. Using the point charge model, the cohesive energy is a function of the lattice-parameter alone [14,35]. To give an example, for NaCl a cohesive energy of $E_{coh} = -182.6$ kcal/mol has been calculated using the point charge model [14,35]. This energy corresponds to a melting temperature of $T_m = 91893$ K which is higher by a factor of 86 compared by the actual melting temperature of $T_m = 1074$ K [15]. This shows that, independent of the type of the inter-atomic interactions, the solids are strongly compressed at low temperatures by the action of the ordered MP-boson field, that is present in all solids. This compression results in a surprisingly small low-temperature lattice parameter. The calculated low-temperature cohesive energies, therefore, are too high, compared to the actual melting temperatures.

The identical critical exponent of 3/4 in the temperature dependence of $\Delta L/L_0$ below T_m of all four RG-solids (Fig. 2) proves universality of the boson dynamics. Universality can be expected to hold in the same temperature range for the heat capacity as well. In particular, one should anticipate that in the heat capacity, the critical exponent should be identical with the critical exponent in $\Delta L/L_0$ of 3/4. The argument for this is that both quantities are controlled by the heat capacity of the MP-bosons. Note that the heat capacity is the only thermodynamic observable of a boson field. However, the exponent of 3/4 is observed in the heat capacity of neon only (Figs. 3 and 6). As we have mentioned, for Ne the phonons remain completely excluded from the dynamics for all temperatures, in spite of the fact that they have a larger absolute excitation energy than conforms to T_m (see Figs. 6 and 7, below). The fact that the critical exponents of the heat capacities of the other RG-solids deviate from 3/4 could be explained by the mosaic structure that can be different for each sample of the same material [22]. The mosaic structure results in a complicated way from the temperature gradient and the cooling rate during solidification of the melt and is never explicitly communicated. We have to assume that the $\Delta L/L_0$ data are specific for the individual mosaic block and are independent of the mosaic structure, while the temperature dependence of the heat capacity seems to depend additionally on the interaction between the mosaic blocks, and therefore is different for different samples [22].

As Fig. 3 shows, the exponent ε of the critical power function $(T_m\text{-}T)^{\varepsilon}$ of the heat capacity varies in a systematic way with T_m between 1/3, 1/2, 2/3 and 3/4 for Xe, Kr, Ar and Ne, respectively. The exponent ε is the lower, the higher T_m is. It can be seen that the fitted critical power functions $\sim(T_m\text{-}T)^{\varepsilon}$ describe the heat capacity values of Xe, Kr and Ar of larger than the D-P-limit excellently (compare Fig. 5, below). Interestingly, for Ar, Kr and Xe, the heat capacity crosses the Dulong-Petit limit at nearly the same temperature. This corresponds to the very similar Debye temperatures that are 61 K, 71 K, 88 K and 71 K for Xe, Kr, Ar and Ne, respectively [3]. It should be noted, that the nearly constant Debye temperatures of all RG-solids are neither correlated with the melting temperatures nor with the calculated, low-temperature, van der Waals cohesive energies [14,35].

For Ne, the crossover from the heat capacity, defined by the MP-bosons, to a phonon-defined heat capacity is missing, as can be seen more clearly in Fig. 6. As a consequence, the phonons are completely excluded from the dynamics of Ne, in spite of the fact that they have a larger absolute energy (see Fig. 7, below).

A simple relation, linear in T_m, results from the fitted heat capacity values at T_m, $c_p(T_m)$. As can be seen in Figs. 3 and 4, the heat capacity at the melting temperature, T_m, is always larger than the atomistic D-P-limit of ~ 25 $JK^{-1}mole^{-1}$. Since all heat capacity values of larger than the atomistic D-P-limit have to be attributed to boson degrees of freedom

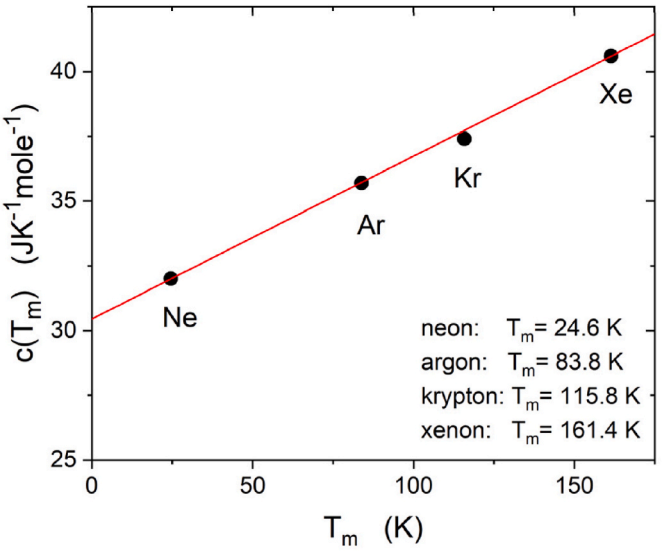

Fig. 4. Fitted heat capacity values of the RG-solids at the melting temperature, T_m, as shown in Fig. 3, as a function of T_m [3,10,33,34,36]. The extrapolated value for $T_m \to 0$ is somewhat larger than the classical D-P-limit of ~ 25 $JK^{-1}mole^{-1}$ because in the critical range at T = 0 the Debye-bosons are the relevant excitations, instead of the phonons, since they have a larger heat capacity compared to the case that the phonons would be relevant.

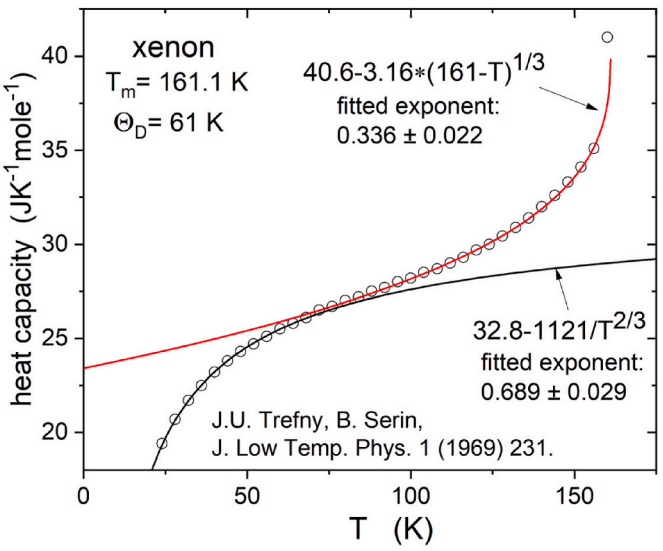

Fig. 5. Temperature dependence of the heat capacity of Xe, illustrating the functional change from phonon dynamics below the D-P-limit (~25 $JK^{-1}mole^{-1}$) to boson dynamics above the D-P-limit [33]. The phonon-defined region can well be described by a power function of absolute temperature. For more explanations, see text.

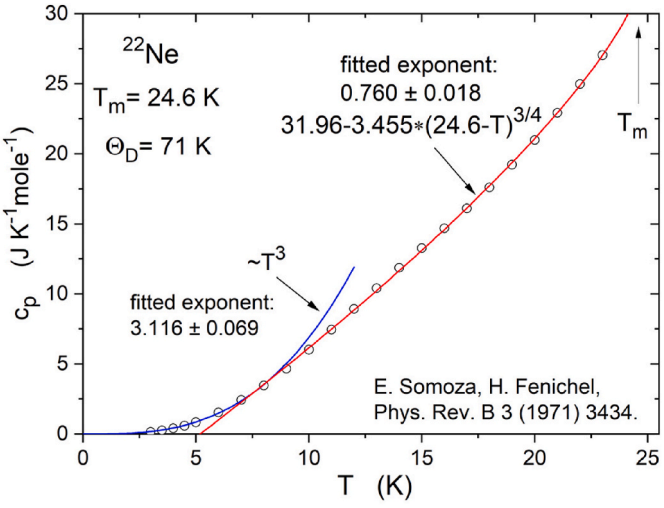

Fig. 6. Temperature dependence of the heat capacity of Ne [10]. The heat capacity is completely described by the two critical power functions at T = 0 (Debye bosons) and at T = T_m (MP-bosons). A phonon defined temperature range is missing, although the phonons have a larger absolute energy than the MP-bosons.

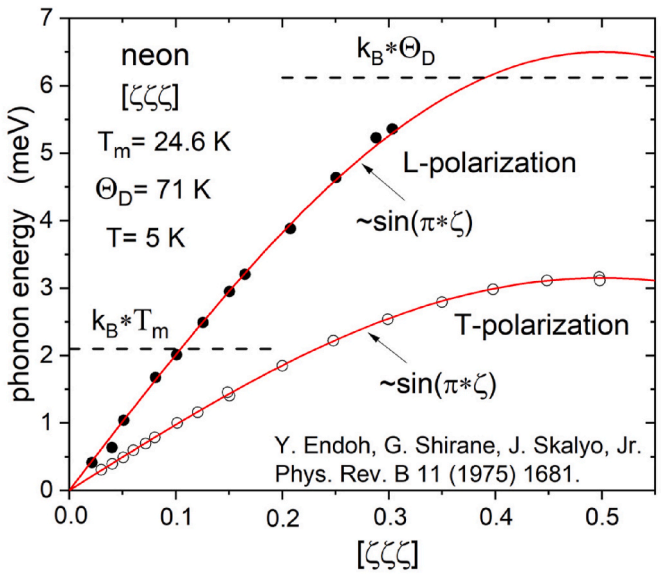

Fig. 7. Phonon dispersion of Ne along the room diagonal of the fcc unit cell, measured at T = 5 K [37]. For both polarizations the dispersion is given by a sine-function of wave-vector. For comparison, the energy equivalent of the Debye-temperature of Θ_D = 71 K and of the melting temperature of T_m = 24.6 K are indicated as dashed horizontal lines.

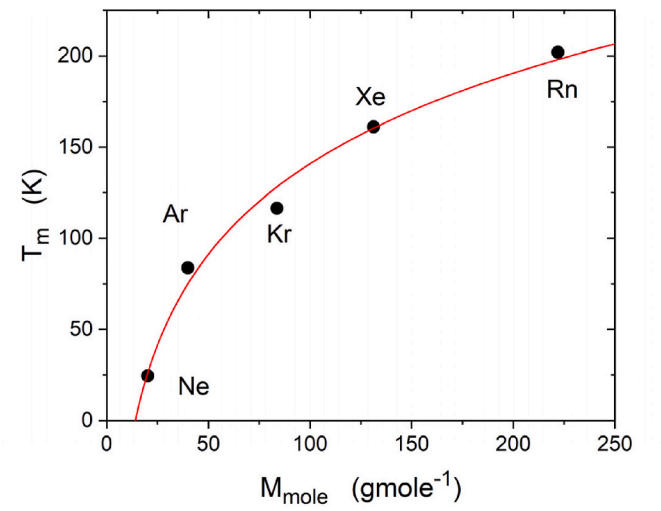

Fig. 8. Melting temperatures of the RG-solids, including the unstable Radon isotope Rn^{222}, as a function of the atomic wight, illustrating a systematic dependence on the atomic wight [38]. The red curve is a heuristic fit function. For more explanations, see text.

it follows that the liquid-solid transition is generally due to bosons [16]. However, as Fig. 4 shows, the value of $c_p(T_m)$, extrapolated to $T_m \to 0$ is larger than the classical D-P-limit of ~25 $JK^{-1}mole^{-1}$. The increased heat capacity value for $T_m \to 0$ has to be attributed to boson degrees of freedom as well, and can qualitatively be explained by the fact that in the critical range at T = 0, the Debye bosons are the relevant excitations, instead of the phonons. Condition for relevance of the Debye bosons is that they have a larger heat capacity than the phonons. In other words, the relevant excitation system must have the larger heat capacity. As a consequence, for all temperatures beyond the critical range at T = 0, the observed heat capacity remains increased. The actual heat capacity of the phonon system, therefore, saturates at a larger value than the classical D-P-limit (see discussion of Fig. 5). Fig. 4 supports the conclusion that all liquid-solid transitions are due to the MP-bosons. As a consequence, the stability of the solids, up to T_m, must be by the MP-bosons as well and cannot be by the inter-atomic interactions.

As can be seen in Fig. 3, for Ar, Kr and Xe the crossover from high-temperature boson dynamics to low-temperature phonon dynamics occurs rather precisely at the Dulong-Petit limit, i.e., at a heat capacity value of ~25 $K^{-1}mole^{-1}$. Fig. 5 gives an expanded view of this crossover for Xe. Rather surprising and not self-evident is that below the validity range of the critical power function ~$(T_m\text{-}T)^{\varepsilon}$, where the phonons are the relevant excitations, the heat capacity can well be described over a rather large temperature range by a single power function of absolute temperature according to the formula c = A-B*$T^{-\gamma}$ with a rational exponent γ. As a consequence, this power function of temperature has universal character and seems to be determined by bosons, in spite of the fact that in this temperature range the absolute heat capacity values are

given essentially by the material specific phonons [4]. This phenomenon is similar to the situation in Ne, where the MP-bosons determine the temperature dependence of the heat capacity below the D-P-limit, although the phonons dominate the absolute heat capacity values.

The dependence on absolute temperature according to the function $c = A\text{-}B*T^{-\gamma}$ indicates that the critical temperature of the responsible bosons is T = 0. As a consequence, the temperature function $c = A\text{-}B*T^{-\gamma}$ is defined by the Debye bosons, although the Debye bosons are not relevant at these high temperatures. Not relevant means that the dispersion relation of the Debye bosons is thermally no longer populated. Nevertheless, it is possible that, due to the interaction between Debye bosons and phonons the associated dispersion relations attract each other. This affects the heat capacity of the relevant phonons [4,5].

In fact, the asymptotic phonon heat capacity, obtained by extrapolation to T→∞, i.e., the fit parameter A, is much larger than the D-P-limit. For Xe (Fig. 5) is $A = 32.8\ JK^{-1}mole^{-1}$. This can be explained by the mentioned interaction between phonons and Debye bosons. Due to this interaction, the dispersion relations of the two systems attract each other. This attraction increases the parameter A and decreases the critical amplitude at $c_p(T_m)$. It can be seen in Fig. 5 that the two fitted power functions of temperature do not cross but approach tangentially. This corresponds to the different translational symmetries of the two excitations which is continuous for the MP-bosons, above the D-P-limit, but discrete-periodic for the phonons below the D-P-limit. Note that for interacting systems with different symmetries crossing of the associated dispersion relations is forbidden. The fact that, for all four RG-solids, the fitted exponent γ is, within the experimental error limits, a rational number supports the conclusion that the power function of absolute temperature, $c = A\text{-}B*T^{-\gamma}$, is boson-defined, i.e., by the Debye bosons.

The low-temperature T^3 function in the heat capacity of Ne in Fig. 6 is evidently the heat capacity of the Debye-boson field. The validity range of the T^3 function ends at the crossover to the critical power function at T_m that is a power function of the argument (T_m-T). In contrast to Ar, Kr and Xe, there is no intermediate temperature range with a phonon-defined heat capacity. Note that the two critical power functions at T = 0 and at $T = T_m$ in Fig. 6 approach tangentially but do not cross. This indicates that the Debye bosons and the MP-bosons have different symmetries and interact [16]. The different symmetries conform to the different dimensionalities of the two associated elastic constants, that is a pressure for the elastic stiffness constants, c_{ij}, (Debye bosons) but a reciprocal pressure for the elastic compliance, s_{ij}, (MP-bosons) [16]. Fig. 6 illustrates once again that the heat capacity cannot be described by one function of temperature over a temperature range including a crossover.

The phonon dispersions of all four RG-solids have been evaluated experimentally [3]. It is observed that, along the room diagonal of the fcc unit cell, the phonon dispersions are given precisely by a sine-function of wave-vector (Fig. 7). This is commonly observed for the acoustic phonons of most other cubic solids. The sine-function is the dispersion of the linear atomic chain. This is surprising for a three-dimensional solid in which each atom should interact with all neighbouring atoms. The observed linear-chain dispersion illustrates the suppression of the three-dimensional inter-atomic interactions by the bosons. We can assume that the linear chain dispersion is induced by the Debye bosons that are collimated, via stimulated emission, along those crystallographic symmetry directions with a high density of the sources of the Debye bosons, i.e., the atoms. Commonly, these are the main crystallographic symmetry axes. Note that the linear propagation mode of the Debye-bosons is used in all experimental evaluations of the elastic stiffness constants, c_{ij}, using sound waves [9]. Due to the strong interaction between the collimated Debye bosons and the acoustic phonons, the dispersions of the acoustic phonons are as for the linear atomic chain. The Debye bosons (elastic dipole radiation) have a similar effect on the dispersion of the acoustic phonons as the Goldstone bosons (magnetic dipole radiation) on the dispersion of the magnons [12]. As for the dispersion of the acoustic phonons, the magnon dispersions of many magnets are as for the linear spin-chain [25,26]. Note that the collimation of the Goldstone bosons sets-in at the crossover into the paramagnetic critical range and is perfect at the critical temperature. Since the critical temperature of the Debye bosons is T = 0 the Debye bosons propagate only at T = 0 along the main symmetry directions, exclusively. Observation of sound waves, or phonons, along other crystallographic symmetry directions then is no longer possible. In the calculation of the heat capacity of the Debye-boson field from the low-temperature sound velocities, i.e., in the calculation of the Debye-temperature, Θ_D, it is therefore sufficient to consider sound waves propagating along those directions only [9].

Fig. 7 shows the phonon dispersions of Ne along the room diagonal of the fcc unit cell for transverse and for longitudinal polarization, measured at T = 5 K [37]. As for the other RG-solids, the two phonon branches are perfectly described by sine-functions of wave-vector [3]. For comparison, the energy equivalent of the Debye temperature and of the melting temperature are indicated by dashed horizontal lines. It can be seen that the Debye temperature corresponds to a reasonable approximation to the maximum of the phonon energy and can, in fact, be taken as a measure of the absolute inter-atomic interaction strength. Neon is exceptional in that the energy equivalent of the melting temperature is much lower than the absolute phonon energy. This confirms that the strong inter-atomic interactions (phonons) are excluded from the critical dynamics at T_m and, as a consequence, have no binding function. Evidently, the phonons are standing modes, confined to the volume of the individual mosaic block. The dimensions of the mosaic-blocks are defined by the ordered MP-boson field as well.

The ordering transition of the MP-bosons, i.e., the liquid-solid transition, occurs when the density of the phase coherent MP-bosons has reached the threshold for the spontaneous onset of stimulated emission. The liquid-solid transition, therefore, is a threshold induced event and appears as a first-order transition in the heat capacity [15]. The density of the MP-bosons depends essentially on the production rate of the bosons. Since the sources of the MP-bosons are assumed to be elastic quadrupoles, it can be understood that the production rate of the MP-bosons is the larger the larger the elastic quadrupole moment, and therefore the atomic mass of the RG-atom is. In fact, as Fig. 3 shows, T_m is the larger the heavier the RG-atom is. Fig. 8 visualizes the dependence of the melting temperature of the RG-solids on the atomic mass, including Radon that occurs as the unstable isotope Rn^{222} with a half-live of 3.8 days only [38]. Rn is the heaviest element of the zero-group of elements of the Periodic Table. These elements have a valence of zero and seem to be chemically very similar. This allows for a comparison of their properties as in Figs. 4 and 8. Note that the solid red curve in Fig. 8 is a heuristic power function to guide the eye. Very interesting is that for an atomic wight of lower than ~14 g/mol a melting temperature of zero is extrapolated. This conforms to the observation that for the lightest of all inert gases, Helium, with an atomic mass of four, there is no liquid-solid transition at ambient pressure. The liquid-solid transition can be forced by the application of a pressure of at least 25 bar (2.5 MPa) [39]. Since an external pressure decreases the distance between the atoms, it can be understood that an external pressure increases the elastic quadrupole moments, and therefore the production rate of the MP-bosons and, as a consequence, favours solidification. This is similar to the observation that many metals get superconducting under pressure only [40,41]. Pressure, evidently, deforms the soft zones between the metal atoms and increases the electric quadrupole moments of these zones, that have been identified as the sources of the bosons that order at the transition temperature of the conventional superconductors [40,21]. At the superconducting transition temperature these bosons condense in a spherical wave function that includes the two Cooper-pair electrons. The Cooper-pairs are the stable ordered units, generated upon ordering of these bosons (electric quadrupole radiation). Creation of Cooper-pairs fits the general behaviour, that upon ordering all types of bosons create stable stationary units.

4. Results and discussion

It has been shown that the Dulong-Petit (D-P) limit of the heat capacity marks a break in the dynamics of all solids (Fig. 1). Below the D-P-limit, the dynamics is determined by the inter-atomic interactions (phonons) but above the D-P-limit, the dynamics is determined, up to the melting point, by an ordered boson field. We have called the bosons, that order at the melting point, MP-bosons. Plausible arguments have been given that the MP-bosons are elastic quadrupole radiation. In other words, one has to distinguish between atomistic and bosonic excitations that become relevant, or activated, alternately as a function of temperature. The mutual exclusion of the two excitations from defining the dynamics is a consequence of their different translational symmetries. As is well-known, different symmetries, exclude each other. In contrast to the phonons, the MP-bosons are mass-less, and, therefore, cannot be observed using neutron scattering. The general prevalence of the bosons at order-disorder phase transitions reveals convincingly when the boson-induced ordering transition of the atomistic system is at a lower temperature than conforms to the absolute inter-atomic interaction strength. This holds for all magnetic ordering transitions [7]. Here we have shown in detail that the boson-driven liquid-solid transition of Ne at $T_m = 24.6$ K is also at a much lower temperature than corresponds to the absolute inter-atomic interaction strength, a reasonable quantitative measure of which is given by the Debye-temperature of $\Theta_D = 71$ K (Fig. 7). In other words, the bosons are able to prevent even stronger inter-atomic interactions from performing ordering transitions. The ordering transition of MP-boson field occurs when the density of the collimated and phase-coherent bosons has reached the critical value for the spontaneous onset of stimulated emission. This is a threshold induced event, similar to the ignition of a LASER, and appears as a seemingly first-order transition in the heat capacity at T_m of all solids [15]. The boson density depends essentially on the production rate of the bosons. For the lightest element of the RG-solids, Ne, the elastic quadrupole moments can be assumed to be weak and the production rate of elastic quadrupole radiation correspondingly low. The density of the MP-bosons necessary to reach the threshold for the spontaneous onset of stimulated emission, therefore, is realized at a temperature ($T_m = 24.6$ K) that is lower than the inter-atomic interactions ($\Theta_D = 71$ K). The same situation holds for all ordered magnets [7].

Since for all solids, except for Ne, the melting temperatures, T_m, are much higher than the Debye temperatures, Θ_D, the cohesion of the solids, up to T_m, cannot be by the inter-atomic interactions but is by the bosons that order at T_m, the MP-bosons (elastic quadrupole radiation). The observed order of the atomic lattice, therefore, is a direct consequence of the Bose-Einstein condensation. The boson-induced long-range order of the crystalline solids results from the fact that the bosons get generated by stimulated emission. Due to stimulated emission the bosons are in phase-coherent wave functions over large distances. The boson induced atomic order is much more perfect than the order that can be expected if the locally anisotropic inter-atomic interactions would be relevant. This is considered as a qualitative entropy argument for the dynamic dominance at the bosons at order-disorder phase transitions. The non-relevant local interactions define the coordination of the neighbouring atoms only. The long-range coherence of the atomic lattice is caused by the ordered bosons.

The bosons define not only the order of the atomistic system, but also the limits of the coherently ordered regions. We must assume that upon ordering, the running bosons, generated by magnetic or electric dipoles, become fixed to the atomic lattice. This process is theoretically unexplained. Moreover, ordered boson fields do not only assume the absolutely lowest energy they also intend to assume the smallest possible volume. This is by an unexplained self-constricting force of the ordered boson fields, that compresses the ordered unit increasingly with decreasing temperature. Due to this compression, the lattice parameter of the ordered unit decreases strongly with decreasing temperature (Fig. 2). In the case of the magnetic domains, this one-dimensional phenomenon is well-known as magnetostriction. For the same reason are the observed lattice parameters of the RG-solids surprisingly short for T→0, and the calculated van der Waals cohesive energies between the RG atoms, are about a factor of ten higher than conforms to the melting temperatures [17,14]. This shows that the atomistic van der Waals interactions have virtually no binding function. In the same way can the effective, low-temperature cohesive energies of the alkali halide crystals with the cubic fcc lattice structure be calculated, assuming ideal ionic bond [35,42]. Using the point charge model, the cohesive, or binding energy, depends on the lattice parameter alone. To give an example, a cohesive energy of $E_{coh} = 214.4$ kcal/mol has been calculated for NaF using the room-temperature lattice parameter of $a_0 = 4.620$ Å [14,35,38]. Setting E_{coh} equal to $E_{coh} = p \cdot V_{mol}$, with V_{mol} as the molar volume, the pressure p which acts on solid NaF at room-temperature can be obtained. Using $V_{mol} = 16.4$ cm^3/mol for NaF a pressure of 55 GPa or 550 kbar results. Alternatively, E_{coh} can be set equal to $E_{coh} = k_B \cdot T$. In this way a temperature of ~108000 K is obtained which is higher by a factor of 85 than $T_m = 1268$ K of NaF [15]. In other words, at room-temperature the alkali-halide crystals are also strongly compressed and the lattice parameters are surprisingly short. This has to be ascribed, as for the RG-solids, to the compression of each mosaic block by the ordered MP-boson field. As the lattice expands, with increasing temperature, the compression of each mosaic block decreases. It can be assumed that this pressure has fallen to the ambient value at the melting temperature T_m. The elastic compliances, s_{ij}, and therefore the compressibility, have a maximum at T_m.

The bosons that order at the magnetic ordering transition, the Goldstone bosons, could be shown to be magnetic dipole radiation, generated by the precessing spins [7,12]. On the other hand, in the only known two genuine Ising magnets, the 2d-Ising antiferromagnets Rb_2CoF_4 and K_2CoF_4, the spins do not precess, and no Goldstone bosons get generated [7]. In this exceptional case, the magnetic ordering transition is, in fact, due to the exchange interactions between the spins, that are, evidently, able to cause an ordering transition [43,44]. As a consequence of the relevant exchange interactions, no domains get formed at the ordering transition of Ising magnets and no perfect collinear spin structures result. The magnetic ordering transition, therefore, appears not as a sharp event in susceptibility measurements and the susceptibility along the Ising axis remains finite for T →0 [45]. Since no domains get formed, there should be no significant magnetostriction.

The bosons of the ferroelectric solids are evidently electric dipole radiation [19]. The bosons that order at the transition temperature of the conventional superconductors, could be shown to be electric quadrupole radiation [40,21]. Note that in metals there are no electric dipoles possible. The electric quadrupoles are identified as the anisotropic charge distributions in the soft zones between the hard metal atoms [40,41]. This view is supported by the observation that for many metals the superconducting transition temperatures can considerably be increased, applying an external pressure [41]. If the effect of the pressure is to deform the charge distribution in the soft zone between the hard metal atoms in such a way that the electric quadrupole moments get increased, the production rate of the electric quadrupole radiation will increase. In this way, the density of the bosons increases and the superconducting transition temperature will increase correspondingly. As for the MP-bosons (elastic quadrupole radiation), the bosons that order at the superconducting transition (electric quadrupole radiation) create three-dimensionally ordered units. It could be rationalized that at the superconducting transition, many of these bosons condense in the same spherical wave function and form an electromagnetic shell, that encapsulates the two Cooper-pair electrons and shields their charges perfectly against all other charges in the metal [40,21]. This allows the Cooper-pairs to propagate without any resistance across the metal. In other words, the Cooper-pairs have to be viewed, quite analogous to domains or mosaic blocks, as the stable ordered units created upon ordering of a boson field. The constricting force that confines the

ordered electric quadrupole radiation shell to the finite volume of a Cooper-pair increases with decreasing temperature and lets the diameter of Cooper-pair decrease strongly [40,21]. A measure of the thereby increasing binding energy between the two Cooper-pair electrons is given by the energy gap [11]. In other words, the energy gap is also a boson-defined quantity. In analogy to the term magnetostriction, the decreasing size of the Cooper-pairs with decreasing temperature could be termed electrostriction.

5. Conclusion

Experimental evidence has been presented that the atomistic concepts of solid-state physics are not sufficient to understand the dynamics of the solids in the vicinity of order-disorder phase transitions, as well as the observed perfect order in the ordered state. The atomistic models consider massive particles (atoms, electrons, spins …) and the interactions between them, exclusively. However, in terms of the Renormalization Group theory, the atomistic near-neighbour interactions are not relevant in the vicinity of ordering transitions [46]. Instead, the observed critical dynamics of the atomistic systems is caused by mass-less bosons. A direct experimental observation of mass-less bosons appears not possible. The most spectacular manifestations of the bosons are the various types of order-disorder phase transitions of the solids. As we have explained here, at each of these phase transitions, a particular boson type orders in the background. Due to a finite interaction of the bosons with the corresponding atomistic system, the atomistic system orders at the same temperature as the boson field, and it appears as if the atomistic system would order. Here we have treated on the problem that there are no inter-atomic interactions known that would be strong enough to stabilize the solids up to melting temperatures of more than ~2000 K. Moreover, the observed long-range and coherent order of the atomic lattice is much more perfect than it can be expected if the local near-neighbour interactions would perform the ordering transition. Ordering of boson fields is via Bose-Einstein condensation. This means, at the ordering temperature of the boson field, all bosons condense in the lowest energy state. This is the thermodynamically highest possible order and provides a plausible entropy argument for the exclusion of the locally anisotropic inter-atomic interactions from performing the ordering transition. Evidently, nature intends to realize the highest possible order at order-disorder phase transitions. The order realized by the inter-atomic interactions is by no means perfect [43–45]. Ordered boson fields concentrate not only in energy they concentrate in space as well. Upon ordering, the boson fields confine themselves to a finite and sharply contoured unit, such as a domain, a mosaic block or a Cooper-pair. The ordered units can assume macroscopic dimensions, and define the coherence length of the ordered bosons. Moreover, ordered boson fields intend to assume the smallest possible volume, and compress the ordered unit increasingly with decreasing temperature. A microscopic explanation of this process is a big theoretical challenge for the future. In analogy to the term magnetostriction, the lattice contraction due to the ordered MP-bosons (elastic quadrupole radiation) could be termed elastostriction. The bosons are either dipole- or quadrupole radiation. Ordering of electric- or magnetic dipole radiation results into one-dimensionally ordered units, the two examples of which are the ferroelectric and the magnetic domains. The ordered units give definite evidence that the boson field and atomistic system, are in an ordered state. Note that magnons are indicative of short-range order only, and can be observed above the magnetic ordering transition as well. Ordering of electric- or elastic quadrupole radiation results into three-dimensionally ordered units. The three-dimensional mosaic blocks have been identified as the ordered units, generated upon ordering of the MP-bosons. Consistent with the elastic nature of the MP-bosons, it could be shown that the MP-bosons define the elastic compliances, s_{ij}, of the metals and of the insulators [28,16]. As a consequence, for the first time a microscopic understanding of the elastic compliances could be presented. For the elastic stiffness constants, c_{ij}, it is long known that they are defined by elastic degrees of freedom, i.e., by the Debye bosons (elastic dipole radiation) [28].

Although our investigations were restricted to the RG-solids, we suggest that MP-bosons order at the liquid-solid transition of all other materials [42]. Definite evidence for the bosonic nature of the critical dynamics below T_m is obtained from the observed universality of the heat capacity and relative thermal length changes below T_m. Universality results from the lattice-structure independent propagation mode of the bosons and is the typical fingerprint of boson dynamics. As a consequence, a boson field orders at all order-disorder phase transitions at which universality is observed [20]. A very remarkable consequence of these findings is that the cohesion of the solids, up to T_m, is not by the inter-atomic interactions, but by a new type of bosonic force that is not only inherent in the ordered MP-boson field but in all other ordered boson fields as well. The compressing force exerted by the ordered MP-boson field on each mosaic block makes the solids solid up to T_m and can be added as a fifth force to the forces of nature discussed previously: the gravitation, the electromagnetic force, the weak and the strong nuclear interaction.

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Declaration of competing interest

I herewith confirm that the manuscript with the title: The boson-defined thermodynamics at the liquid-solid transition submitted for publication in the journal Solid State Communications is my own work. I am the only responsible author. The ideas and new findings reported in the manuscript are my own achievements. Since I got no financial or scientific support, I have nothing to declare.

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Data availability

Data will be made available on request.

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