

On the Influence of the Blending Parameter σ in LBM WMLES With the HRR-BGK Collision Scheme

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ABSTRACT

This work investigates the influence of the blending parameter σ on the Hybrid Recursive Regularized BGK collision scheme with and without a correction term on the selection of LES and wall model. The comparison is done using the well-known turbulent channel flow test case at a friction REYNOLDS number Re_τ of 1,000. The wall is either modeled via the implicit Musker profile or explicitly with a combination of the Werner and Wengle and the Schmitt profile. The considered LES models are the Wall-Adapting Local Eddy-viscosity (WALE) and the Vreman model. For the discretization of the velocity space both, the D3Q19 and D3Q27 stencil are considered. Different resolutions of the channel are studied, leading to y_1^+ values of the first cell at the wall $y_1^+ = 12.5, 25$ and 50 . The entire range of σ values in the interval $[0.0, 1.0]$ is analyzed.

1. Introduction

Nowadays, to perform high-fidelity simulations, high performance computing (HPC) and the use of significant computational resources are indispensable. The Lattice Boltzmann Method (LBM) offers an efficient, explicit high-fidelity fluid simulation tool. However, optimal parameters for turbulence and wall modeling in the multitude of possible LBM implementations are not well understood yet. Therefore, we employ the massively-parallel LBM solver *Musubi* [1] in our investigations to extend the previous contributions: Spinelli et al. [2, 3] performed a systematic study of collision schemes, LES and wall models on accuracy and efficiency for turbulent channel flow (TCF) and flow past a circular cylinder using the LBM solver *Musubi*. Their investigations revealed that the suitability of a collision scheme for a certain test case depends on the combination in which it is used (e.g. LES and wall model). This has been observed for the Hybrid Recursive Regularized BGK (HRR-BGK) collision scheme [4] in particular. Besides, HRR-BGK makes use of a blending parameter σ . Thus, last year, Spinelli and Gericke [5] extended these investigations by evaluating the influence of σ on the HRR-BGK as it impacts the numerical dissipation of the scheme. In their investigations, they simulated the TCF by employing a fixed

combination of the Vreman LES model and the Musker wall model. They showed that adjusting σ dependent on the stencil as well as the resolution improves the accuracy. This study extends the previous research by analyzing the influence of σ in the HRR-BGK scheme on the selection of LES and wall model by comparing accuracy and performance. The two considered LES models are the Vreman [6] and the WALE [7]. The latter one tends to provide more accurate results, while the Vreman model typically is faster [3]. The wall is modeled by the implicit Musker profile [8] or the explicit combination of the Werner and Wengle [9] and the Schmitt [10] profile, called Power-Law profile (see also [3]). To further enhance the HRR-BGK scheme, the correction term as proposed by Feng et al. [11] is utilized. The velocity space is discretized with both, the D3Q19 and D3Q27 stencil.

2. Method

The method is based on the Lattice Boltzmann equation (LBE). In its post-collision expression, the LBE extended by the correction term ψ_i of Feng et al. [11] is written as

$$f_i^*(\mathbf{x}, t) = f_i^{\text{eq}}(\mathbf{x}, t) + (1 - \omega_i) f_i^{\text{neq}}(\mathbf{x}, t) + 0.5 \Delta t \psi_i. \quad (1)$$

f_i is the discrete Probability Density Function (PDF) of particles streaming from one lattice node $\mathbf{x}$ to an adjacent one $\mathbf{x} + \mathbf{c}_i \Delta t$, $\mathbf{c}_i$ is the lattice velocity and Δt the time increment. The equilibrium and non-equilibrium PDFs are given as

$$f_i^{\text{eq}}(\mathbf{x}, t) = w_i \sum_{n=0}^{q-1} \frac{1}{n!} \mathbf{a}_0^{(n)} \mathcal{H}_i^{(n)}, \quad (2)$$

$$f_i^{\text{neq}}(\mathbf{x}, t) = w_i \sum_{n=2}^{q-1} \frac{1}{n!} \mathbf{a}_1^{(n)} \mathcal{H}_i^{(n)}. \quad (3)$$

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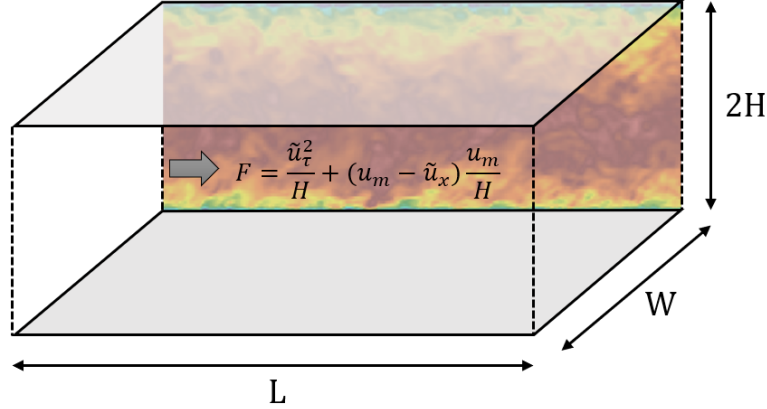


Figure 1: Computational domain for the turbulent channel flow with superimposed flow field and the force driving the flow in horizontal direction. Walls are indicated in grey.

The Hermite polynomials $\mathcal{H}_i^{(n)}$ as well as the Hermite coefficients $\mathbf{a}_0^{(n)}$ and $\mathbf{a}_1^{(n)}$ are chosen as stated in [5]. For this investigation, the HRR-BGK collision scheme introduced by Jacob et al. [4], is used. The second-order Hermite coefficient of f^{neq} with the blending parameter σ is then given given as

$$\mathbf{a}_1^{(2,\text{HRR})} = \sigma \mathbf{a}_1^{(2,\text{RR})} + (1 - \sigma) \sigma \mathbf{a}_1^{(2,\text{FD})}, \quad (4)$$

with $0 \leq \sigma \leq 1$. If $\sigma = 0$, $\mathbf{a}_1^{(2,\text{HRR})}$ is reconstructed by the velocity gradients calculated by Finite Difference (FD). This is what Spinelli et al. called Projected Recursive Regularization BGK (PRR-BGK) scheme. It is stable, but highly dissipative [2]. For $\sigma = 1.0$, $\mathbf{a}_1^{(2,\text{HRR})}$ leads to the Recursive Regularized BGK (RR-BGK) scheme proposed by Malaspinas [12].

3. Test case and results

The well-known turbulent channel flow at a Re_τ of 1,000 is used as test case [13]. Its computational domain is shown in Fig. 1.

A test case description, including boundary conditions and domain size (they also investigated the effect of the domain size) is offered in [3]. To account for the entire range of possible σ values, the following parameters are used: [0.0, 0.5, 0.9, 0.92, 0.94, 0.96, 0.98, 1.0]. For Vreman, the model coefficient is set to 0.07, for WALE to 0.5. The results are compared to the ones with Vreman and Musker of [5] extended by the runs with $\sigma = 0.0$ and $\sigma = 0.5$ to have a complete dataset. As a reference, DNS data by Lee and Moser [13] is used. Results show that the optimal σ -range lies between 0.9 and 1.0 revealing the PRR-BGK scheme is not suited for this specific test case. Using the correction term makes HRR-BGK slower but it does not

increase the accuracy. Regarding the LES model, there is no much difference in terms of accuracy if Vreman or WALE (in conjunction with Musker wall function) is used. As the Vreman model has less floating point operations compared to WALE, it is preferable. Furthermore, the choice of σ depends on the wall model: For Vreman with Musker the best results are obtained for $\sigma = 0.9$, while the optimum for Vreman with Power-Law is $\sigma = 0.94$. Finally, we can take the node-level performance of these collision schemes into account: It was recently investigated by Wendler et al. [14]. They showed that the D3Q19 is ≈ 1.5 times faster than the D3Q27, due to the reduced number of links. Furthermore, they revealed that the HRR-BGK is ≈ 1.5 times faster than the one with the correction term but only improves accuracy by 0.1 % [3]. As the HRR-BGK schemes are implemented with σ as a factor, its choice has no impact on the performance. Contrary to that, the stencil has. Thus, for the TCF test case we can conclude that in terms of balancing accuracy and computational effort, the combination of Vreman-Power-Law-HRR-BGK-D3Q19 with a σ value of 0.94 and without correction term gives the best result independent of the resolution.

4. Conclusions

We investigated the influence of the blending parameter σ on the Hybrid Recursive Regularized BGK (HRR-BGK) collision scheme with and without a correction term on the selection of LES and wall model using the turbulent channel flow test case ($Re_\tau = 1,000$). Aiming to determine the influence of σ while identifying the optimal combination of it with LES and WM that balances cost-effectiveness with accuracy we can conclude that this optimum is achieved by the Vreman-Power-Law-HRR-BGK-D3Q19 combination with a σ value of 0.94 and without the correction term. To

enable a more robust validation of the findings, future work will include additional test cases.

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