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Unraveling the Boundary Layers of High Rayleigh Number Convection Through Direct Numerical Simulations

Roshan J. Samuel[a,*], Prafulla P. Shevkar[a], Mathis Bode[b], Janet D. Scheel[c], Katepalli R. Sreenivasan[d] and Jörg Schumacher[a,d]

[a] *Technische Universität Ilmenau, Institute of Thermodynamics and Fluid Mechanics, 98684, Ilmenau, Germany*
[b] *Forschungszentrum Jülich GmbH, Jülich Supercomputing Centre, 52428, Jülich, Germany*
[c] *Occidental College, Department of Physics, CA 90041, Los Angeles, USA*
[d] *New York University, Tandon School of Engineering, NY 11201, New York, USA*

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ABSTRACT

We study the behavior of boundary layers in thermal convection at RAYLEIGH numbers ranging from 10^5 to 10^{12} in a box of aspect ratio $\Gamma = 4$, and periodically extended sides. We use the GPU accelerated spectral element solver, NekRS, for our simulations. At the highest RAYLEIGH number, the simulations utilize nearly 3400 A100 GPUs of the JUWELS Booster facility, to cover as many as 40 free-fall time units, enabling us to obtain reliable statistics in steady-state. This demonstrates the excellent scalability of the solver. Moreover, since the focus of the study is on the boundary layers at the top and bottom walls, we ensure that the near-wall regions are well-resolved. We observe that the boundary layers have both shear dominated and plume dominated regions, with a nearly constant area-fraction between these regions. We also study the structure of wall shear-stress fields and their connection to plume structures emanating from the walls.

1. Introduction

Thermal convection has remained a fertile benchmark for computational studies of fluid flows starting from the earliest works of [1]. The ubiquity of the phenomenon as well as the simplicity of the Rayleigh-Bénard convection (RBC) setup typically used to study natural convection has greatly impelled the progress in this field for more than half a century.

In RBC, a thin layer of fluid is confined between a pair of parallel, horizontal plates, with gravity acting along the vertical direction. The plates are isothermal, with the top plate maintained at a lower temperature than the bottom plate [2]. Ideally, the plates extend infinitely in the horizontal directions. One can numerically approximate this by using a moderately large aspect ratio for the computational domain, and by using periodic boundary conditions along the horizontal directions. This is a key design aspect of the simulations reported here.

Experiments of RBC are typically performed in closed domains with no-slip side-walls, which impact the organization of the flow. Our simulations alleviate this effect, thus eliminating the large-scale circulation (LSC) flow. This imparts a markedly different structure to the boundary layer flow, whose analysis is the focus of this work.

2. Problem Setup and Numerical Method

We solve the non-dimensionalized Navier-Stokes equations with Boussinesq approximation along with the energy equation [3],

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + T\hat{z} + \sqrt{\frac{Pr}{Ra}}\nabla^2 \mathbf{u}, \tag{1}$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{\sqrt{Ra Pr}}\nabla^2 T. \tag{2}$$

Here, $\mathbf{u}$, p and T denote the velocity, pressure and temperature fields respectively. Moreover, mass conservation is imposed in the form of the continuity equation, $\nabla \cdot \mathbf{u} = 0$. The dimensionless parameters Ra (RAYLEIGH number) and Pr (PRANDTL number) dictate the nature of the flow and its properties. While Ra represents the strength of convective forcing, Pr is the ratio between the diffusion of momentum and temperature. We keep Pr fixed at 0.7 (corresponding to

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* Corresponding author
roshan-john.samuel@tu-ilmenau.de (R.J. Samuel); prafulla-prakash.shevkar@tu-ilmenau.de (P.P. Shevkar); m.bode@fz-juelich.de (M. Bode); jscheel@oxy.edu (J.D. Scheel); krs3@nyu.edu (K.R. Sreenivasan); joerg.schumacher@tu-ilmenau.de (J. Schumacher)

ORCID(s): 0000-0002-1280-9881 (R.J. Samuel); 0000-0001-8259-4903 (P.P. Shevkar); 0000-0001-9922-9742 (M. Bode); 0000-0002-1669-4188 (J.D. Scheel); 0000-0002-3943-6827 (K.R. Sreenivasan); 0000-0002-1359-4536 (J. Schumacher)

Ra	N_e	p	grid-points	N_{BL}	dt	Δt	N_{GPUs}
10^5	$100 \times 100 \times 64$	5	80 million	71	1.33×10^{-2}	1000	24
10^6	$100 \times 100 \times 64$	7	220 million	57	5.73×10^{-3}	1000	48
10^7	$100 \times 100 \times 64$	9	467 million	42	2.45×10^{-3}	1000	100
10^8	$150 \times 150 \times 96$	7	740 million	24	1.82×10^{-3}	1000	160
10^9	$150 \times 150 \times 96$	9	1.6 billion	15	9.49×10^{-4}	400	240
10^{10}	$400 \times 400 \times 200$	7	11 billion	16	4.59×10^{-4}	100	1000
10^{11}	$500 \times 500 \times 256$	7	22 billion	16	2.73×10^{-4}	100	1440
10^{12}	$500 \times 500 \times 256$	9	46.7 billion	10	2.02×10^{-4}	40	3360

Table 1
Details of the simulations.

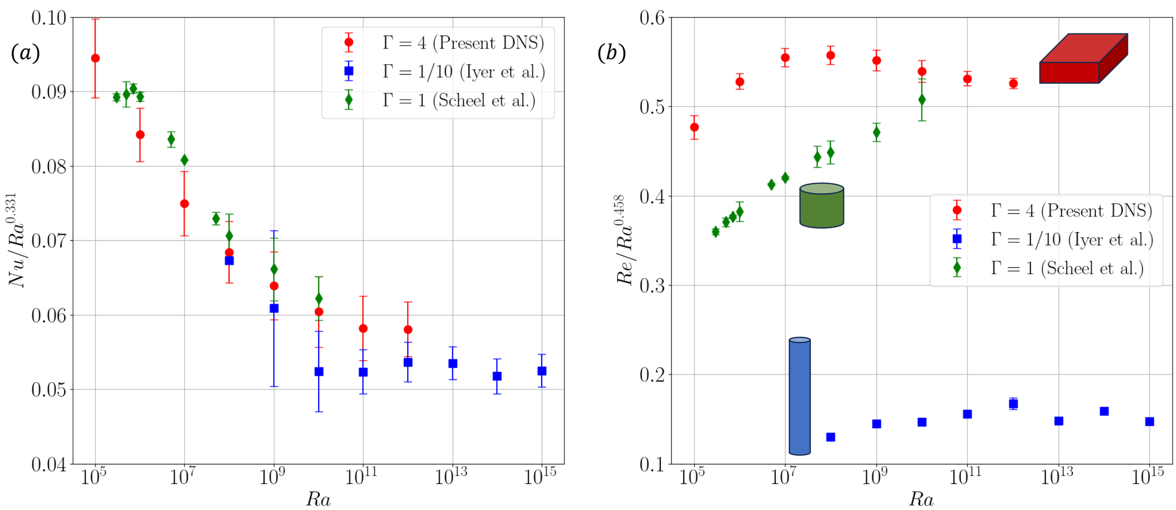

Figure 1: Scaling of Nu and Re with Ra.

air) for all our simulations, but vary Ra from 10^5 to 10^{12} (see Table 1).

The simulations are performed in a Cartesian box of length (L), width (W) and height (H) defined as $L = W = \Gamma H$, where Γ is the aspect ratio of the box. We use a constant $\Gamma = 4$ in all our simulations [4]. The top and bottom walls are isothermal, no-slip plates maintained at $T = 0$ and 1 respectively, whereas periodic boundary conditions are imposed on the sides.

Furthermore, the boundary layers (BLs) become thinner as Ra increases, which demands larger grids to resolve the BLs properly. Consequently the grid sizes also increase with Ra, going up to nearly 47 billion points at $Ra = 10^{12}$. Even at this Ra, we maintain 10 collocation points in the BL (denoted by N_{BL}). Table 1 also lists the number of spectral elements, N_e, polynomial order, p, average time-step, dt, total free-fall times in steady state, Δt, and the number of GPUs required, N_{GPUs}.

We use the spectral element method (SEM) [5, 6] to solve the governing equations. Specifically, we use NekRS [7], which is a GPU accelerated implementation of SEM, derived from the renowned Nek5000 solver. We performed our simulations on the JUWELS Booster at Jülich, utilizing almost the full-cluster (840 nodes) for the $Ra = 10^{12}$ case. This case was run for nearly 30 hours over 4 reservations, using more than 1.1 million GPU core hours.

3. Results and Discussion

Two quantities of key interest in simulations of RBC are NUSSELT number (Nu) and REYNOLDS number (Re). The rate of convective heat flux across the top and bottom plates is quantified by Nu, whereas Re is a measure of momentum transport by the convective flow. The scaling laws for both Nu and Re with respect to Ra are subjects of continuous investigation [8, 9]. Figure 1 shows the scaling of Nu (a)

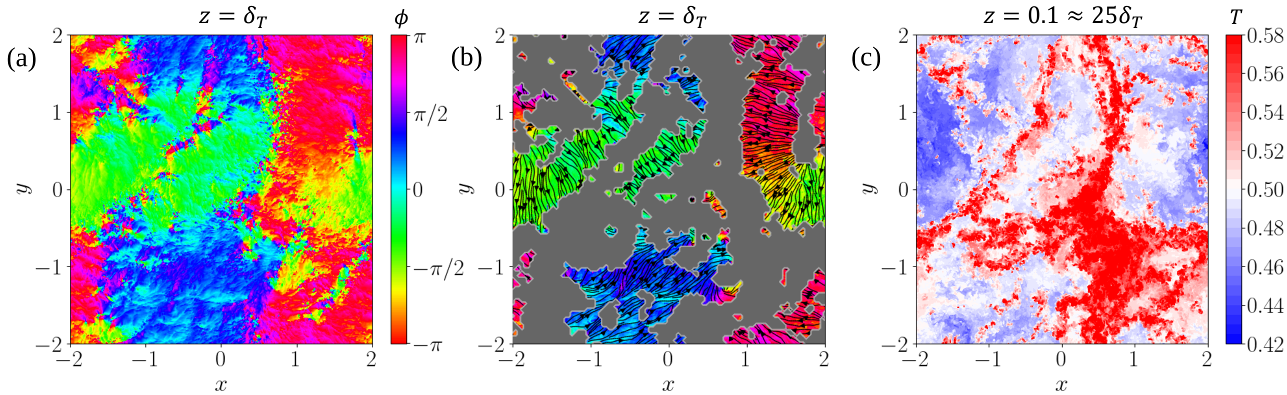

Figure 2: Structure of boundary layer flow and plumes at $Ra = 10^{10}$.

and Re (b) obtained from our simulations, compared against previous results from literature [10, 11].

The Nu scaling seems to collapse well with the classical $1/3$ scaling for $Ra \geq 10^{11}$, whereas the Re scaling shows a significant geometry dependence. The previous works of [10] and [11] used cylindrical domains of aspect ratios $1/10$ and 1 respectively, and their representative shapes are drawn in the plot for clarity.

In Fig. 2, the BL flow and structure of plumes are shown near the bottom plate for $Ra = 10^{10}$. The thermal BL thickness for this case is $\delta_T = 3.72 \times 10^{-3}$. Panel (a) shows the angle of the flow, $\phi \in [-\pi, \pi]$, in the plane parallel to the plate. Regions with smooth color variations denote patches of coherent shear flow, whereas the areas with grainy fluctuations are decoherent regions of plume upwelling. In (b), the plume dominated regions are isolated by thresholding the planar velocity, $u_h = (u_x^2 + u_y^2)^{1/2}$. The grey regions indicate areas with $u_h < U^h_{\mathrm{rms}}$, where U^h_{rms} is the root-mean-square value of u_h averaged over the whole plane. Finally the temperature field at $z = 25\delta_T$ plotted in (c) highlights the alignment between plumes and the decoherent patches on the plate.

Furthermore, we investigated the mean velocity profiles and compared them with Blasius profile to probe for signatures of flat-plate BL flow. We also compared the different measures of momentum and thermal BL thicknesses. Significantly, we do not observe a coherent large-scale circulation typically seen in RBC experiments in closed cylindrical cells of aspect ratio 1 and smaller. This has potential implications on our understanding of the boundary layers dynamics in RBC. Further details and analysis can also be found in [4].

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