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# Comment on Reply to Comment on ‘Electromagnetic lensing using the Aharonov–Bohm effect’

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COMMENT

# Comment on Reply to Comment on 'Electromagnetic lensing using the Aharonov–Bohm effect'

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## Abstract
Recently, Schreiber and Wolf (2024 *New J. Phys.* **26** 118002) have questioned the validity of Stokes' theorem, one of the most important tools in the theory of electromagnetism, by stating a counterexample. In this comment it is shown that the counterexample is invalid.

## 1. Introduction

Recently, Schreiber and Wolf [1] have presented a particular loop integral of a magnetic vector potential which is claimed to not vanish although the loop lies in a field-free, simply connected region. This constitutes a counterexample to Stokes' theorem, according to which the loop integral should vanish. In this comment it is shown, however, that the counterexample is invalid.

In [1], an attempt was made to calculate the loop integral by direct integration of a series expansion. The attempt, however, was abandoned without success because this yielded expressions that were too complicated for further analysis. The reported result, namely that the loop integral does not vanish, is therefore only a conjecture.

The calculation is now carried out in a different way, without using Stokes' theorem.

## 2. Calculation of the loop integral

The particular loop integral ((2) in [1]), depending on a parameter $R$, is

$$I = \oint \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{s} = \int_0^\infty A_z(0,z)\ \mathrm{d}z + \int_0^R A_r(r,\infty)\ \mathrm{d}r + \int_\infty^0 A_z(R,z)\ \mathrm{d}z + \int_R^0 A_r(r,0)\ \mathrm{d}r, \tag{1}$$

with $r$, $\theta$, $z$ the radial, azimuthal, and axial cylindrical coordinates, respectively, $A_r$ and $A_z$ the respective components of the magnetic vector potential $\boldsymbol{A}$, and $\boldsymbol{s}$ unit tangential vectors along the sides of the rectangular loop. A notation error in the last term (compare (2) in [1]) has been corrected. In the field-free, simply connected region [1] containing the loop, the magnetic field $\boldsymbol{B} = \mathrm{rot}\boldsymbol{A}$ is vanishing, so that its azimuthal component

$$B_\theta = \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} = 0 \tag{2}$$

yields the substitution used in (4).

Now, the value of the integral (1) is determined by considering its derivative $\mathrm{d}I/\mathrm{d}R$ with respect to the parameter $R$. With

$$\frac{\mathrm{d}}{\mathrm{d}R}\left(\int_0^\infty A_z(0,z)\ \mathrm{d}z + \int_0^R A_r(r,\infty)\ \mathrm{d}r + \int_R^0 A_r(r,0)\ \mathrm{d}r\right) = 0 + A_r(R,\infty) - A_r(R,0), \tag{3}$$

$$\frac{\mathrm{d}}{\mathrm{d}R}\int_{\infty}^{0} A_z(R,z)\ \mathrm{d}z = \int_{\infty}^{0} \frac{\partial}{\partial R}A_z(R,z)\ \mathrm{d}z = \int_{\infty}^{0} \frac{\partial}{\partial z}A_r(R,z)\ \mathrm{d}z = A_r(R,0) - A_r(R,\infty)\,, \tag{4}$$

it follows $\mathrm{d}I/\mathrm{d}R = 0$, so that the loop integral $I$ is constant. Further, equation (1) obviously yields the constant $I_0 = 0$ for $R = 0$, so that $I = \oint \boldsymbol{A}\cdot\mathrm{d}\boldsymbol{s} = 0$ independent of $R$.

Remarkably, the concrete form of the magnetic vector potential is not required in this proof, which can obviously be extended to arbitrary rectangles in a field-free, simply connected region. The vanishing of the loop integral along such rectangles is illustrated in [2], figure 2, by a numerical example.

## 3. Conclusion

The above proof shows beyond doubt that the loop integral in question is vanishing, in accordance with Stokes' theorem. The conclusion $\oint \boldsymbol{A}\cdot\mathrm{d}\boldsymbol{s} \neq 0$ in [1] is incorrect, and therefore the counterexample to Stokes' theorem in [1] is invalid.

## Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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## References

[1] Schreiber M T and Wolf M 2024 Reply to Comment on 'Electromagnetic lensing using the Aharonov-Bohm effect' *New J. Phys.* **26** 118002
[2] Lentzen M 2024 Comment on 'Electromagnetic lensing using the Aharonov-Bohm effect' *New J. Phys.* **26** 118001