

Are compact open-charm tetraquarks consistent with recent lattice results?

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We argue that the hypothesis that positive-parity charm meson resonances exhibit a compact tetraquark structure has some clear tension with recent lattice results for the S -wave πD system for an $SU(3)$ flavor symmetric setting. In particular, we show that such a diquark-anti-diquark tetraquark scenario would call for the presence of a state in the flavor $[\mathbf{15}]$ representation, not seen in the lattice analysis. Moreover, we show that analogous lattice data in the axial-vector channel are even more sensitive to the internal structure of these very interesting states.

I. INTRODUCTION

Since their discovery in 2003 the lightest open-charm positive-parity mesons containing strangeness remained largely a mystery—especially in light of the seemingly expected properties of their non-strange partner states. In the past, attempts were made to explain the low-lying charm-strange D_{sJ} states as $c\bar{s}$ mesons [1–5], chiral partners of the ground state D_s and D_s^* mesons [6, 7], compact $[cq][\bar{s}\bar{q}]$ tetraquark states [8], mixing of the $c\bar{s}$ and tetraquarks [9], a $D\pi$ atom for the $D_{s0}^*(2317)$ [10], and $D^{(*)}K$ hadronic molecules [11–17]. The experimental data show four features that need to be understood [18]:

- (1) *The D_{sJ} states are too light:* Both $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are much lighter than the quark model expectations [19] for the lowest scalar and axial-vector $c\bar{s}$ mesons.
- (2) *Fine-tuning:* Within only 2 MeV the relation $M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} \approx M_{D^{*+}} - M_{D^+}$ holds.
- (3) *Mass hierarchy:* One finds $M_{D_0^*(2300)} \sim M_{D_{s0}^*(2317)}$ and $M_{D_1(2430)} \sim M_{D_{s1}(2460)}$, although usually adding a strange quark leads to an increase in mass of about 150–200 MeV.
- (4) *Discrepancy theory versus experiment:* Various theoretical studies [12, 14–16, 20], later confirmed by lattice QCD [21, 22], find lower masses for D_0^* and D_1 than values reported from experiments [23].

All these find a natural explanation, if the lowest positive-parity charmed mesons are interpreted as hadronic molecules [18]. In this case, the flavor structure of this family of states is governed by the flavor decomposition representing the scattering of the lightest pseudoscalar mesons off D mesons, which are $SU(3)$

flavor octets and anti-triplets, respectively. This decomposition is given by [14, 20]

$$[\mathbf{3}] \otimes [\mathbf{8}] = [\mathbf{3}] \oplus [\mathbf{6}] \oplus [\mathbf{15}]. \quad (1)$$

The light pseudoscalar octet corresponds to the pseudo-Nambu-Goldstone (pNG) bosons of the spontaneously broken chiral symmetry of quantum chromodynamics (QCD). Note that we do not include scattering of the ninth pseudoscalar, the η' , here, since due to the action of the $U(1)_A$ anomaly it is not a pNG boson.

Non-strange isospin-1/2 multiplets appear in all three irreducible representations, however, chiral symmetry constraints dictate that only the $[\mathbf{3}]$ and the $[\mathbf{6}]$ are attractive for the charm-light meson scatterings [14, 20]. In particular, in this scheme the $D_0^*(2300)$ listed in the Review of Particle Physics [23] is interpreted as emerging from two distinct poles (resonances), one at 2105 MeV and the other at 2451 MeV, with the lower one being a member of the same $SU(3)$ multiplet as the $D_{s0}^*(2317)$, the $[\mathbf{3}]$, where the attraction is the strongest. The state at 2451 MeV is a member of the $[\mathbf{6}]$ representation of $SU(3)$, where the interaction is weaker than in the $[\mathbf{3}]$, but still sufficiently strong to generate a pole at physical quark masses sufficiently close to the physical axis to show an impact on observables [18, 24]. Chiral symmetry constraints impose that the leading order interaction of the particles in the $[\mathbf{15}]$ is repulsive and thus no resonance should be found in this channel.

Thus, a crucial test of this interpretation is connected to the existence of the $[\mathbf{6}]$ state with a pole located around 2451 MeV and the absence of a pole in the $[\mathbf{15}]$. In contrast to this, conventional quark model states with a quark composition $c\bar{u}$ or $c\bar{d}$ can appear only in the flavor $[\mathbf{3}]$ representation.

One way to unravel the $SU(3)$ structure underlying the spectrum of the lightest open charm scalar states is to unambiguously establish the existence of the state in the $[\mathbf{6}]$ by determining in the $SU(3)$ symmetric limit whether it appears as a near-threshold pole, when the pNG boson mass (i.e. the pion mass) is sizably larger than the physical value, as predicted by unitarized chiral perturbation

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theory [18, 25]. This needs to be accompanied by the absence of a state in the [15]. First results indicated that, indeed, the low-energy charm-light meson scattering in the [15] is *repulsive* and that in the [6] is *attractive* with a near-threshold pole, thus providing strong evidence for the molecular nature for these states [26]. This finding was confirmed recently [27] by a detailed Lüscher analysis imposing similar quark masses.

While these findings provide strong support for a molecular structure of the mentioned states, it remains to be studied what the compact tetraquark picture predicts for this system, with building blocks being colored (anti)-diquarks (for a review of the concept see Ref. [28]). In particular, in this work we critically review the claim of Ref. [29] that there exists a scenario where compact tetraquarks also appear only in the flavor [3] and [6] representations and not in the [15]. In contrast to this claim, we argue that as soon as one allows for axial-vector diquarks in addition to the scalar diquarks investigated in Ref. [29] additional states in the [15] emerge. This observation appears to be already at odds with the findings in lattice QCD quoted above. Moreover, we demonstrate that additional data at similarly high unphysical quark masses in the axial-vector channel should clearly show a signature of a state in the [15] representation if the compact tetraquark picture were correct, while no such state should appear in this multiplet in the molecular picture. We are thus close to unambiguously identifying the structure of heavy-light multi-quark states.

II. FLAVOR STRUCTURE OF LIGHT AND HEAVY LIGHT DIQUARKS

A detailed discussion of the SU(3) flavor structure of positive-parity open-heavy-flavor mesons within the tetraquark picture can be found in Refs. [29–32]. To be concrete, consider a tetraquark state with a heavy quark, a light quark and two light antiquarks. The flavor decomposition of the light degrees of freedom reads

$$\begin{aligned} [\mathbf{3}] \otimes [\mathbf{3}] \otimes [\mathbf{3}] &= [\mathbf{3}] \otimes ([\mathbf{3}_A] \oplus [\mathbf{6}_S]) \\ &= [\mathbf{3}_A] \oplus [\mathbf{3}_S] \oplus [\mathbf{6}_A] \oplus [\mathbf{15}_S], \end{aligned} \quad (2)$$

where the labels A and S indicate that in these flavor multiplets the two antiquarks are antisymmetric or symmetric in flavor space, respectively. Note that in contrast to Eq. (1), which contains 24 states in total, here 27 states appear. This difference arises because the η' was not included in the flavor decomposition of Eq. (1), as mentioned above. We will return to this feature below.

In Refs. [30, 31], it is argued that the observed states are tetraquarks in the flavor [3] and [15] representations—features that were deduced by employing the 't Hooft interaction at the diquark level. This scenario is clearly ruled out by the lattice data quoted above. However, these works do not discuss the role of the diquark spin.

In contrast to the studies in Refs. [30, 31], Ref. [29] takes as its key starting point that the Pauli principle imposes restrictions on the light (anti)diquark quantum numbers (see, e.g., Ref. [33]). Specifically, it emphasizes that an S -wave diquark in the color anti-triplet with zero angular momentum must have the same symmetry in both spin and flavor. When focusing on the spin-zero diquark, this implies that only the two flavor antisymmetric multiplets for tetraquarks, [3_A] and [6_A], are allowed, consistent with the findings of the lattice studies in Refs. [26, 27]. To accommodate both $D_{s0}^*(2317)$ and $D_{s1}(2460)$ in this picture, the mass difference between the spin-one and spin-zero cq diquarks, which are partners under heavy quark spin symmetry (HQSS), must be approximately equal to that of D^* and D . This requirement is not only phenomenologically sensible but also addresses the fine-tuning issue mentioned in the Introduction. Therefore, one has, at the physical point,

$$\begin{aligned} M_{cq}^{[S=1]} - M_{cq}^{[S=0]} &\approx M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} \\ &= 142 \text{ MeV}. \end{aligned} \quad (3)$$

This spin symmetry violating mass difference should be only weakly dependent on the pion mass due to the $1/m_c$ suppression, where m_c is the charm quark mass, and thus we will use it also in estimates for mass differences at heavier pion masses below.

The construction scheme of diquark models implies that not only the flavor [3_A] diquark in S -wave lives in the color anti-triplet, but also the [6_S] diquark, and thus both possess an attractive one-gluon exchange potential. If the spin-one cq diquark exists as a relevant degree of freedom, as required by a consistent diquark phenomenology, the flavor blindness of gluonic interactions would imply the existence of a spin-one light qq diquark (and $\bar{q}\bar{q}$ anti-diquark) as well, since the exchange-momentum scales for both systems are similar, $\sim \Lambda_{\text{QCD}}$. Since here the spin wave function is symmetric, it must be combined with the flavor symmetric multiplets for anti-diquarks in the first line of Eq. (2). Consequently, spin-one $\bar{q}\bar{q}$ anti-diquarks appear in the [3_S] and [15_S] representations for tetraquarks in the second line of Eq. (2). The mass of the anti-diquark contained in these can be estimated from the Σ_c - Λ_c mass difference. Thus one obtains

$$\begin{aligned} M_{\bar{q}\bar{q}}^{[S=1]} - M_{\bar{q}\bar{q}}^{[S=0]} &\approx M_{\Sigma_c} - M_{\Lambda_c} \\ &= 167 \text{ MeV} \end{aligned} \quad (4)$$

for physical quark masses. HQSS predicts that this mass difference needs to agree with that of the respective baryons (with an uncertainty of about 20 MeV due to relative corrections of order $\Lambda_{\text{QCD}}(1/m_c - 1/m_b)$), which is 191 MeV lying in the expected range. It also agrees with the estimate $\sim \frac{2}{3}(M_\Delta - M_N) \approx 180 \text{ MeV}$ in Ref. [33].

Given both spin-zero and spin-one diquarks and these mass splittings, it should be expected that additional multiplets develop bound states in the tetraquark picture, besides the ones considered in Ref. [29]. In the next

section we argue that states of the $[\overline{15}_S]$ representation should be present in the scalar and especially the axial-vector sector of the open charm states, if those states had a tetraquark structure of the diquark–anti-diquark type.

III. RESULTS

In Ref. [29], it is argued that the states in the flavor sextet and those in the anti-symmetric flavor anti-triplet should be near mass degenerate. However, this is in conflict with the results of Ref. [27], at least at the unphysically large quark mass used in the lattice calculation. On the one hand, a state in the flavor anti-triplet was found in the lattice study as a deeply bound state at a mass of about 2435 MeV, corresponding to a binding energy of slightly above 200 MeV, at a pion mass of about 700 MeV in a flavor symmetric setting. On the other hand, the flavor sextet state appeared as a virtual state with a pole between 2510 and 2610 MeV.¹ Although this pole lies 40–140 MeV below threshold, it should not be directly compared with the mass of a bound state having a similar binding energy. This is because a bound state indicates strong attraction in the respective channel—when the attraction is reduced, the bound state first moves to the threshold and only after that it becomes a virtual state.

In this work we want to investigate to what extent the lattice data are consistent with the compact tetraquark picture. To that end we need to first of all assume that the state in the sextet is a very shallow bound state and not a virtual state, since the latter property would point at a molecular structure [34], although that introduces already some tension to the lattice data.

Since the mentioned lattice analysis was performed in a flavor symmetric setting, there is no mixing between states in different flavor multiplets and the mass difference between them thus directly reflects the mass difference between the multiplets. We thus have (at least at $M_\pi \approx 700$ MeV)

$$M_{[6_A]} - M_{[\overline{3}_A]} \approx 200 \text{ MeV}. \quad (5)$$

This observation puts into question the quantitative analysis of Ref. [29].

In Ref. [30], a mechanism is presented that naturally provides a mass splitting between the mentioned multiplets driven by the 't Hooft interaction. In particular, it is argued that the 't Hooft interaction induces mass shifts that lift the naively expected degeneracy between the multiplets (see Eqs. (12–15) therein), in particular the $[\overline{3}_A]$ acquires a rather strong additional attraction, both $[\overline{15}_S]$ and $[6_A]$ a milder one, while the $[\overline{3}_S]$ feels a rather strong additional repulsion.

These considerations let us ignore the states in the $[\overline{3}_S]$ in what follows. Then we find, using the estimate of Eq. (4) for the mass difference of the spin 1 and the spin 0 diquark, the mass of the lightest axial-vector tetraquark state in the $[\overline{15}_S]$ representation with spin-zero cq diquark and spin-one light anti-diquark to be

$$M_{[\overline{15}_S],1+} \approx 2600 \text{ MeV}, \quad (6)$$

which refers to a deep bound state with a binding energy of about 50 MeV, at the SU(3) symmetric point with a 700 MeV pion mass.

The axial-vector mass in Eq. (6) is very close to the mass of the axial-vector state in the flavor $[\overline{3}_A]$ representation, which we find from adding the mass difference between the spin-one and spin-zero cq diquark, Eq. (3), to the mass of the lightest scalar found in the lattice study to find

$$M_{[\overline{3}_A],1+} \approx 2575 \text{ MeV}. \quad (7)$$

For this estimate we assumed that the mass difference between the spin-one and spin-zero cq diquarks at the SU(3) symmetric point with a large pion mass is similar to the one at the physical point. We therefore conclude that in the compact tetraquark picture their should be a pair of close by deep bound states in the axial vector sector, one in the flavor $[\overline{3}_A]$ and one in the flavor $[\overline{15}_S]$. At the same time, the states in the $[\overline{3}_S]$ should be pushed so high up that they do not matter in the region of interest—this is in fact analogous to the fate of the η' in the mesonic analysis.

As argued above, the anti-diquark in the $[\overline{15}_S]$ should have spin one. Thus, combining this with a spin-zero cq diquark one gets axial vectors for the states given in Eq. (6) above. On the other hand, combining it with a spin-one cq diquark, one gets three different possible total spins: 0, 1 and 2. All these states are grouped into two different multiplets under HQSS: $\frac{1}{2}_q \otimes 1_{\{\bar{q}\bar{q}\}} = (\frac{1}{2} \oplus \frac{3}{2})_{q\{\bar{q}\bar{q}\}}$ for the total angular momentum of the light degrees of freedom j_ℓ , where we use $\{\bar{q}\bar{q}\}$ to denote the flavor symmetric anti-diquark. Coupling this to the heavy quark spin $s_c = 1/2$, one gets two tetraquark multiplets under HQSS: one is given by

$$\frac{1}{2}_c \otimes \frac{1}{2}_{q\{\bar{q}\bar{q}\}} = (0 \oplus 1)_{(cq)\{\bar{q}\bar{q}\}}, \quad (8)$$

and the other is given by

$$\frac{1}{2}_c \otimes \frac{3}{2}_{q\{\bar{q}\bar{q}\}} = (1 \oplus 2)_{(cq)\{\bar{q}\bar{q}\}}. \quad (9)$$

Thus, there should be several $[\overline{15}_S]$ in SU(3) flavor space, and they have different total spins:

$$(0 \oplus 1)_{cq\{\bar{q}\bar{q}\}} \oplus (1 \oplus 2)_{cq\{\bar{q}\bar{q}\}},$$

where each HQSS spin doublet is paired inside parentheses.

¹ The lattice results qualitatively agree with the prediction in the molecular picture [18].

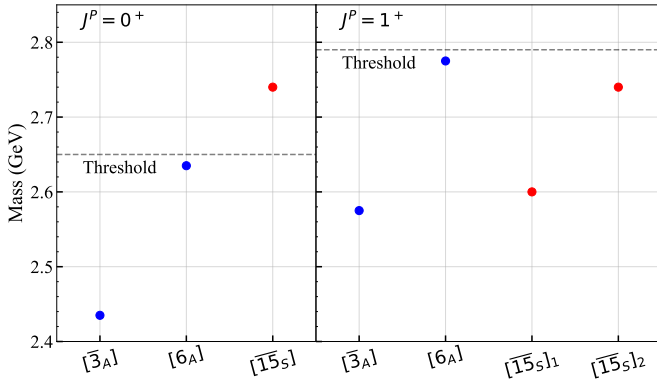


FIG. 1. Sketch of the spectrum of scalar and axial-vector tetraquarks in the diquark–anti-diquark model for a pion mass of about 700 MeV. Blue (red) symbols refer to states containing spin-zero (spin-one) light diquarks. In contrast, in the molecular picture, only the analogs of the blue states appear with the higher one being located on the unphysical sheet.

Furthermore, the HQSS spin multiplets of charmed tetraquarks containing a spin-zero light anti-diquark, being antisymmetric in flavor and denoted by $[\bar{q}\bar{q}]$, read

$$\frac{1}{2}_c \otimes \frac{1}{2}_q \otimes 0_{[\bar{q}\bar{q}]} = (0 \oplus 1)_{(cq)[\bar{q}\bar{q}]} . \quad (10)$$

This spin decomposition holds for both flavor $[\bar{3}_A]$ and $[6_A]$.

To find the mass of the corresponding scalar $[\bar{15}_S]$ state in Eq. (8), we need to add the mass in Eq. (6) to the mass difference between the spin-zero and spin-one cq diquarks provided in Eq. (3). Putting all pieces together we estimate (neglecting the hyperfine splitting for the couplings of spin-one cq and spin-one $\bar{q}\bar{q}$ in line with other studies in this context, e.g., Ref. [8])

$$M_{[\bar{15}_S],0^+} \approx 2740 \text{ MeV}, \quad (11)$$

which means that this naive estimate for the scalar $[\bar{15}_S]$ state would have a mass higher than the SU(3) symmetric threshold, about 2650 MeV. Thus it should probably show up as a resonance. However, such a state was not found in the lattice analysis of Ref. [27].

The mass spectrum for the scalar and axial-vector tetraquarks in the diquark–anti-diquark picture is summarized in Fig. 1. Our analysis shows that in the tetraquark picture the axial-vector spectrum should look strikingly different from the scalar spectrum of the lightest positive-parity open-charm mesons. Although the masses quoted above must be regarded as rough estimates given the intrinsic uncertainties for the individual ingredients and the neglected interactions, qualitatively this observation appears unavoidable, if (anti)diquarks can be regarded relevant degrees of freedom in different types of hadrons. Clearly, the spectrum shown in Fig. 1 is very much different to what is predicted in the molecular picture, where the spectra in the scalar and axial-vector sector are basically the same, because of HQSS for

charmed-meson–light-meson interactions, and there is no state present in the flavor $[\bar{15}_S]$.

Thus, given the lattice data presently available in the scalar sector, no strong case against the tetraquark picture can be made considering only the masses. Nevertheless, it is important to notice that not only the masses but also the finite-volume energy levels computed in lattice QCD and the $D\pi$ invariant mass distributions in B decays measured by LHCb can be well described in the molecular picture (for a recent concise review, see Ref. [35]). A description of such detailed data in the diquark–anti-diquark tetraquark picture is still missing.

Here, it would also be very important to get data in the axial-vector channel. There again the lightest state should be in the $[\bar{3}_A]$ multiplet emerging from the spin-one cq diquark in combination with the spin-zero light anti-diquark. However, the same quantum numbers can be reached by combining the spin-zero cq diquark with the spin-one light anti-diquark, as mentioned above. Taking all the different pieces together these two states should have roughly the same mass in the compact tetraquark model. Recently, there have been lattice results in this channel with $D^*\pi$, $D^*\eta$ and $D_s^*\bar{K}$ coupled channels at a 391 MeV pion mass [36]. Therein, the lightest state was found just below the $D^*\pi$ threshold, and one additional state arises despite of large uncertainties. This two-pole structure is consistent with the results in Ref. [18] (for a general discussion on the two-pole structures, see Ref. [37]). Yet, no state that could be assigned to the 15-plet was found in the lattice analysis.

IV. SUMMARY

In this paper we argue that especially the recent lattice data for open-charm positive-parity mesons with spin zero at a flavor symmetric point with a pion mass of the order of 700 MeV show some tension with predictions that would emerge from a compact tetraquark picture, while they are in nice agreement with expectations from the molecular ansatz.

It is also demonstrated that the differences between the predictions of the two structure assumptions get a lot more drastic when moving to the axial-vector channel. In the diquark–anti-diquark tetraquark picture a pair of two lightest states should emerge: one in the flavor $[\bar{3}_A]$ with a spin-one cq diquark combined with a spin-zero light anti-diquark, and one in flavor $[\bar{15}_S]$ with a spin-zero cq diquark combined with a spin-one light anti-diquark. Both states should appear as deeply bound states with a mass difference of less than 50 MeV. Higher up in energy, there should then be a signal in the flavor $[6_A]$ as well as another one in the $[\bar{15}_S]$. Again, relatively close together, although here the uncertainty is even larger.

In the molecular picture, there should be only one deeply bound state, namely the anti-triplet state, also for the axial-vector system showing a binding energy close to what was found in the scalar sector, a virtual state in the

flavor [6], and no pole at all in the [15].²

We thus conclude that additional lattice data in the axial-vector channel with the same lattice setting as already employed in Ref. [27] would be extremely valuable in learning whether the lightest positive-parity charmed mesons can be classified as hadronic molecules or compact tetraquarks.

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- ² At higher energies, there could be more states generated with the pNG bosons replaced by the light vector mesons [18, 38, 39].

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