Studying the scaling behavior of quantum annealing to find the ground-state of the 1D Hubbard model

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Goal

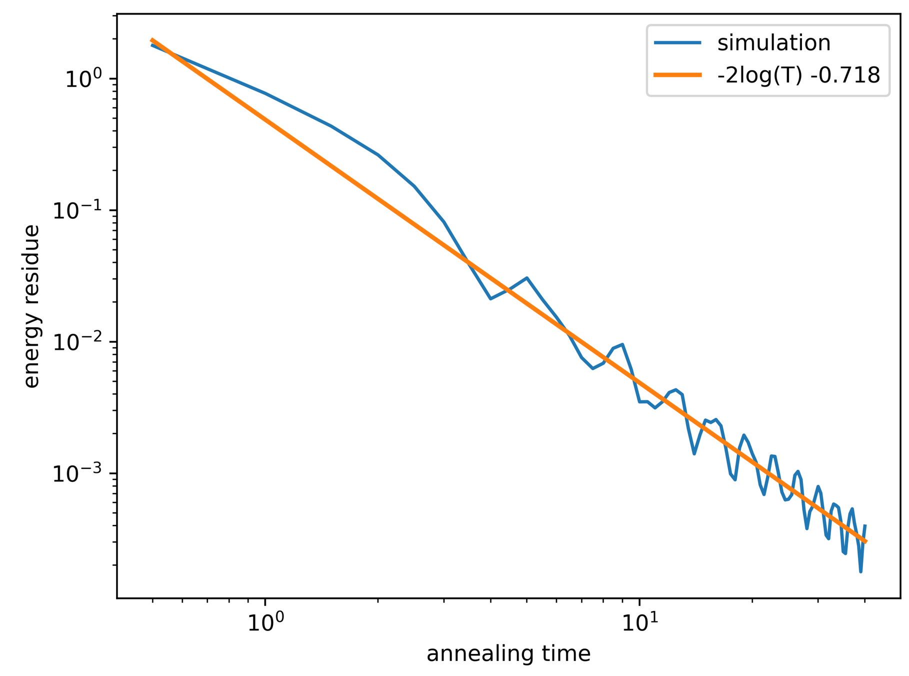
To investigate time scaling to obtain the ground-state of the Hubbard Hamiltonian for half-filled lattices using quantum annealing

$$H_H = -\frac{t}{2}\sum_{\substack{i=1\\ i\neq L}}^{2L}(X_iX_{i+1}+Y_iY_{i+1}) + \frac{u}{4}\sum_{i=1}^{L}(I-Z_i)(I-Z_{i+L})$$

How?

Use the adiabatic theorem to analyze the behavior of energy residue obtained after the annealing process for various total annealing times

- The adiabatic theorem suggests that - Transition probability $\leq \frac{\alpha}{T_A^2}$
- We can also expect the energy residue to have a similar behavior and thus the final energy residue after quantum annealing can be plotted against total annealing time
- It can be seen from the plot that the energy residue behaves according to the adiabatic theorem
- The y-intercept of the plot would give α which is a function of the gaps of the spectrum and in turn a function of the lattice size



The figure shows how the energy residue decreases for longer and longer annealing times for a problem with L=8. This behaviour seems to agree with the adiabatic theorem having $\Delta E = \frac{\alpha}{T_A^2}$

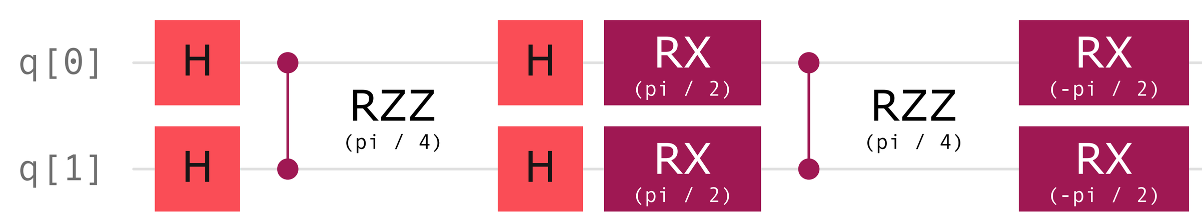
Gate-based simulation using JUQCS

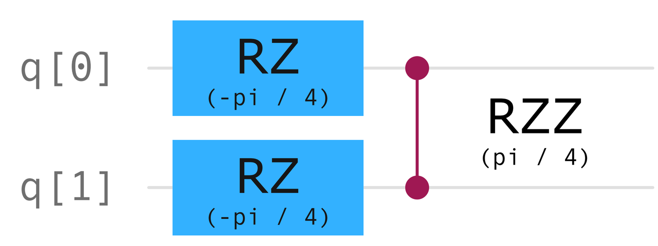
Jülich Quantum Computer Simulator is a high performance program which we use to simulate circuits for upto 40 qubits. We perform a gate-based circuit simulation of the quantum annealing process by using the Suzuki-Trotter product formula to simulate Schrödinger dynamics

This requires the implementation of a sequence of gates that perform

$\exp(\frac{it\tau}{2}(X_iX_{i+1}+Y_iY_{i+1}))$ and $\exp(\frac{iu\tau}{4}(I-Z_i)(I-Z_{i+L}))$

which can be achieved respectively by the following circuits





The initial state for quantum annealing also needs to be prepared on the quantum computer. We prepare the circuit for this as explained in [Zhang Jiang et al., PRA 9, 044036 (2018)]

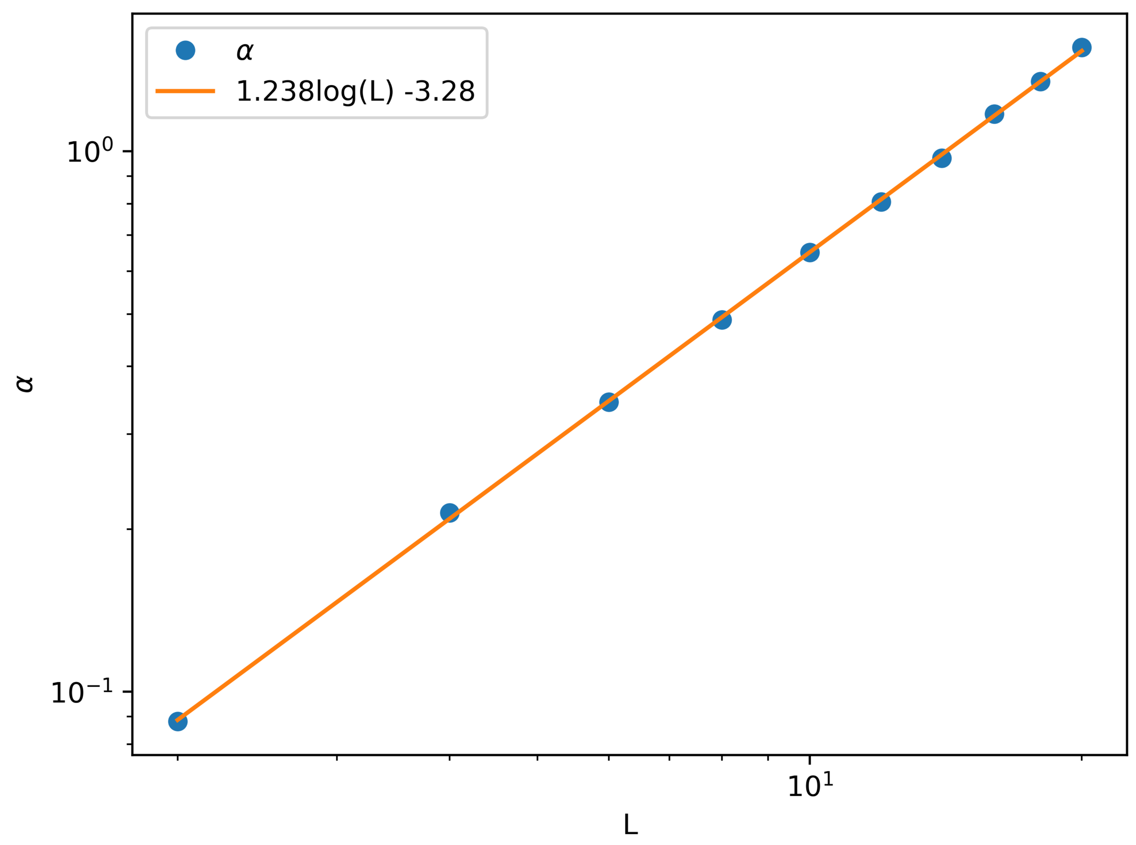
Summary and Outlook

- The simulation results show that the time required to find the ground-state of the half-filled 1D Hubbard model scales sublinearly with system size using quantum annealing.
- This is an encouraging result for adiabatic quantum computing to find ground-states of many-body Hamiltonians.
- If this trend is found to hold even for 2D systems, then it would substantially add to the promise of quantum computing.
- We plan to look further in this direction and see what quantum annealing can bring for harder many-body problems.

Scaling of annealing time

Looking at how the time required to get a reasonable estimate to the ground-state scales with system size can give us an idea about how hard the problem is for quantum annealing.

The scaling of α determines how the required annealing time would scale with system size.

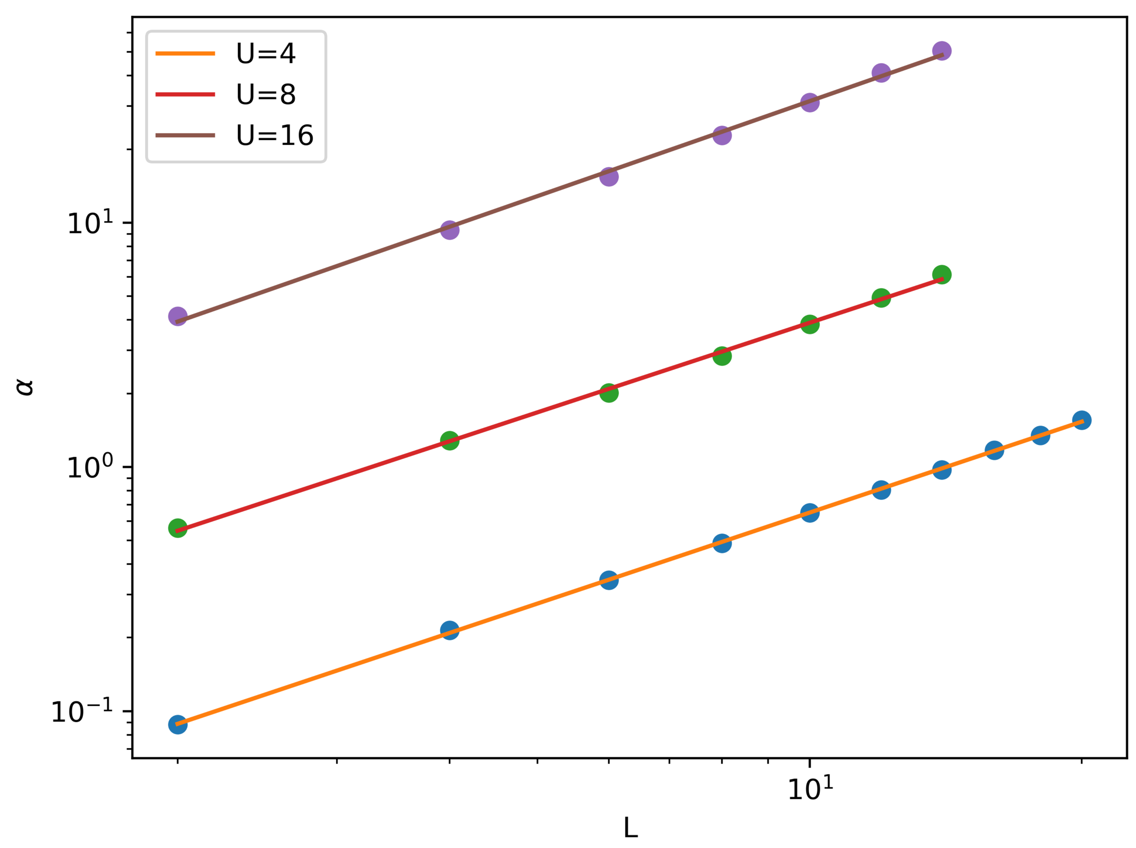


The figure shows how α grows with system size and it can be seen from the equations below that this suggests a sublinear scaling of required annealing time to obtain a reasonable estimate to the ground state of the Hubbard model

$$\alpha \propto L^{1.238}$$

$$\Rightarrow \Delta E \propto \frac{L^{1.238}}{T_A^2}$$

$$\Rightarrow T_A \propto L^{0.62}$$



The figure shows the scaling of α for parameter regimes of the Hubbard model with strong coupling. It can be seen that the behaviour holds even for high coupling strengths

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