Letter

Excess quasiparticles and their dynamics in the presence of subgap states

P. B. Fischer [1,2] and G. Catelani [1,3]

[1] *JARA Institute for Quantum Information (PGI-11), Forschungszentrum Jülich, 52425 Jülich, Germany*
[2] *JARA Institute for Quantum Information, RWTH Aachen University, 52056 Aachen, Germany*
[3] *Quantum Research Center, Technology Innovation Institute, Abu Dhabi 9639, UAE*

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Material inhomogeneities in a superconductor generically lead to broadening of the density of states and to subgap states. The latter are associated with spatial fluctuations of the gap in which quasiparticles can be trapped. Recombination between such localized quasiparticles is hindered by their spatial separation and hence their density could be higher than expectations based on the recombination between mobile quasiparticles. We show here that the recombination between localized and mobile excitations can be efficient at limiting the quasiparticle density. We comment on the significance of our findings for devices such as superconducting resonators and qubits. We find that for typical aluminum devices, the subgap states do not significantly influence the quasiparticle density.

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A finite energy gap for excitations is what makes superconductors attractive materials for realizing electronic devices with low losses. At temperature T approaching the critical one (T_c), quasiparticle excitations are a significant source of losses, but because of the gap, at $T \ll T_c$ the number of quasiparticles is exponentially suppressed in thermal equilibrium. However, experimental evidence points to a low-temperature density of quasiparticles much larger than expected in devices such as qubits and resonators. Various nonequilibrium mechanisms are being investigated as sources of excess quasiparticles, such as environmental and cosmic radiation [1–3], pair-breaking photons [4–7], and phonon bursts [8,9], and the quasiparticle dynamics can be studied even in the few-quasiparticle regime [10]. The basic model for the dynamics of the quasiparticle density n as function of time t was introduced long ago by Rothwarf and Taylor (RT) [11]: quasiparticles created at a rate G recombine pairwise, leading to a rate equation $dn/dt = G - Rn^2$ with R the recombination coefficient; in the steady-state $dn/dt = 0$, the density is determined by the competition between generation and recombination, $n = \sqrt{G/R}$. More recently [12,13] it has been argued that a thorough understanding of the experimental evidence requires extending this simple model, in particular to account for inhomogeneities in the superconducting gap, for which there is direct evidence in strongly disordered superconductors [14]; in fact the devices studied in Ref. [13] were fabricated with highly disordered granular aluminum. The gap inhomogeneities can be due to spatial variations in the concentration of magnetic impurities [15] or in the strength of the pairing constant [16]. For concreteness, in this work we focus on weak magnetic impurities, but the results are straightforwardly applicable to the other case.

Magnetic impurities have long been know to suppress T_c with increasing concentration [17]; here we are interested in how they affect the density of states (DoS). According to Bardeen-Cooper-Schrieffer (BCS) theory, in the absence of magnetic impurities the DoS is characterized by a square-root divergent peak at an energy Δ. Using the approach of Abrikosov and Gorkov (AG) [18,19], extended to account for fluctuations in the concentration of impurities [20–22], it can be shown that a sufficiently high concentration of magnetic impurities weakly coupled to the conduction electrons [22] modifies the DoS in two ways (see Fig. 1 left): first, they broaden the peak by changing the square root divergence to a square root threshold at energy $E_g = \Delta(1 - \eta^{2/3})^{3/2} < \Delta$; this broadening depends on the average impurity concentration and is quantified by a dimensionless pair-breaking parameter $\eta = 1/\tau_s\Delta$, where $1/\tau_s$ is related to the exchange interaction part of the total scattering rate of electrons by impurities; we set the reduced Planck constant $\hbar$ and the Boltzmann constant k_B to one throughout. Second, spatial fluctuations in the impurity concentration cause local depressions of the energy gap where quasiparticles can be trapped (see Fig. 1 right), since the states in these local depression are subgap (that is, they have energy smaller than E_g); the subgap states add to the DoS a compressed-exponential "tail" decaying over an energy scale $\epsilon_T \ll \eta^{2/3}\Delta$. According to Ref. [12], the spatial separation between "localized" quasiparticles trapped in the subgap sates hinders their recombination; this reduction of the quasiparticle recombination rate then leads to an increase in their number above what is expected for "mobile" quasiparticles of energy $E > E_g$. In this work we revisit the role of localized quasiparticles in determining the overall quasiparticle density and discuss the implications of our results for superconducting devices.

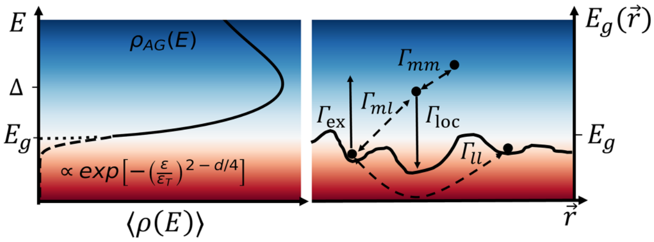

FIG. 1. Left: Average density of states as function of energy, showing the AG broadened peak above E_g and the subgap tail below it. The different background colors are used to distinguish localized (red) and mobile (blue) quasiparticle states. Right: Local value of the gap as function of position; also depicted are the various processes affecting the quasiparticle densities, namely recombination between localized (recombination coefficient Γ_{ll}) and/or mobile (Γ_{ml} and Γ_{mm}) quasiparticles, localization (rate $\Gamma_{\rm loc}$), and excitation ($\Gamma_{\rm ex}$).

A phenomenological model generalizing the RT one can be straightforwardly written down for the dynamics of the densities of localized (x_l) and mobile (x_m) quasiparticles [13,23]

$$\frac{dx_l}{dt} = \Gamma_{\rm loc}x_m - \Gamma_{\rm ex}x_l - \Gamma_{ml}x_m x_l - \Gamma_{ll}x_l^2 + g_l, \tag{1}$$

$$\frac{dx_m}{dt} = \Gamma_{\rm ex}x_l - \Gamma_{\rm loc}x_m - \Gamma_{ml}x_m x_l - \Gamma_{mm}x_m^2 + g_m. \tag{2}$$

Here, the densities $x_\alpha = n_\alpha/n_{\rm Cp}$, $\alpha = l,\ m$, are normalized by that of the Cooper pairs $n_{\rm Cp} = 2\nu\Delta$, with ν the DoS per spin at the Fermi energy, $\Gamma_{\rm ex}$ denotes the rate for excitation from a localized to a mobile state, $\Gamma_{\rm loc}$ the rate for the inverse (localization) process, $\Gamma_{\alpha\beta}$, $\alpha,\ \beta = l,\ m$, the recombination coefficient between localized (l) and/or mobile (m) quasiparticles (see Fig. 1 right), and g_α the rate at which quasiparticles are generated by some pair-breaking mechanism. In the absence of localized states ($x_l = 0 = \Gamma_{\rm loc}$), the model reduces to the RT equation. In contrast, the authors of Ref. [12] assumed that quasiparticles are generated at high energy only ($g_l = 0$) and quickly localize ($\Gamma_{\rm loc} \gg \Gamma_{ml}x_l,\ \Gamma_{mm}x_m$); with these assumptions, and if excitation can be ignored, Eqs. (1) and (2) approximately reduce to $dx_l/dt = g_m - \Gamma_{ll}x_l^2$, again taking the RT form. Although direct comparison with the approach of that work is not straightforward, its main insight can be qualitatively expressed by saying that effectively the recombination rate Γ_{ll} depends on the generation rate through the dimensionless parameter $\kappa = (n_{\rm Cp}r_c^3)^2 g_m/\Gamma_{mm}$, where r_c denotes the relevant radius of the localized states, which is expected to be of order the coherence length ξ for the disorder strengths at which, as we will show, this regime can be relevant and a few times ξ for weaker disorder. Then for $\kappa \gg \kappa_c$ Ref. [12] concludes that $\Gamma_{ll} \simeq \Gamma_{mm}$, while for $\kappa \ll \kappa_c$ the relationship becomes $\Gamma_{ll} \simeq \Gamma_{mm}g(\kappa)$ with the function $g(\kappa)$, whose concrete form is not relevant for our purposes, being always smaller than unity. The crossover value κ_c was estimated from simulations of the recombination process between localized quasiparticles to be of order $\kappa_c \simeq 10^{-4}$. The arguments in Ref. [12] were developed using formulas for bulk superconductors which can be generalized to effectively two-dimensional films of thickness less than the coherence length [23]. We note that strictly speaking Γ_{ll} accounts for recombination between quasiparticles located in different traps; recombination within a trap could be enhanced (cf. Ref. [24]), but this does not affect our arguments [23].

In considering the relevance of the results of Ref. [12] to experiments, one should examine whether the assumption of fast localization is justified, since the relaxation rate of a quasiparticle due to phonon emission decreases strongly with energy upon approaching the superconducting gap [25]. More recently, it has been shown that the absorption of low-energy photons can lead to a finite width (that is, an "effective temperature") of the quasiparticle distribution even if the phonons are assumed to be at zero temperature [26]; similarly, high-energy photons responsible for quasiparticle generation generally lead to a distribution with finite effective temperature [27]. A natural question then is: under which conditions [e.g., relation between (effective) temperature and typical energy ϵ_T of the subgap states] can localization significantly affect quasiparticle dynamics? The goal of this paper is to study such dynamics taking into account states both above and below E_g. To this end, we need microscopic estimates for the coefficients entering Eqs. (1) and (2); such estimates can be obtained using a kinetic equation approach (see for instance Ref. [28]). Here we discuss the values of the rates, presenting more details in the Supplemental Material [23].

The recombination of two quasiparticles is accompanied by the emission of a phonon. The strength of the electron-phonon interaction is typically quantified by a time τ_0 [25–27] in terms of which we have for the recombination coefficient of mobile quasiparticles $\Gamma_{mm} \simeq r$ with $r = 4(\Delta/T_c)^3/\tau_0$. In thin superconducting films the parameter r can be reduced due to so-called phonon trapping [29]; it could also be affected by the concentration of implanted impurities [30]. Therefore, we will assume that r is determined experimentally for a given material and film thickness. Moreover, using the approach of Ref. [12] we find that $\Gamma_{ml} \simeq \Gamma_{mm}$. Regarding Γ_{ll}, as discussed above we have $\Gamma_{ll} \leqslant \Gamma_{mm}$; the possible ineffectiveness of phonon-trapping for recombination involving localized quasiparticles [24] could lead to this inequality being violated, but this does not invalidate our results, as we discuss in the Supplemental Material [23]. Clearly, a finite value for Γ_{ll} can only lead to a lower density of localized quasiparticles, so an upper bound for x_l is obtained by setting $\Gamma_{ll} = 0$. Concerning generation, similarly to Ref. [12] we set $g_l = 0$; we discuss in the Supplemental Material [23], for both thermal phonons and pair-breaking photons, under which conditions $g_l \ll g_m$, so that our assumption gives a reasonable approximation.

Turning to the localization and excitation rates, we note that the latter vanishes in the limit of zero phonon temperature T and assuming that no nonequilibrium mechanism such as stray photons can give energy to the trapped quasiparticles. Below we will consider first the case $\Gamma_{\rm ex} = 0$ and then the effect of a finite excitation rate, in particular due to thermal phonons. The localization rate can be estimated from [23]

$$\Gamma_{\rm loc}x_m \simeq \frac{2}{\Delta}\int_{E_g}^{\infty} dE\ \rho_{AG}(E)f(E)\tau_{\rm loc}^{-1}(E), \tag{3}$$

where E is the energy measured from the Fermi level, ρ_{AG} is the AG DoS (cf. Fig. 1 left), and f is the quasiparticle distribution function; the energy-dependent localization time

$\tau_{\rm loc} \propto \tau_0$ accounts for the electron-phonon interaction and depends on the (disorder-averaged) density of states of the localized states (normalized by the normal-state DoS ν) [16,20–23]

$$\rho_l(\tilde{\epsilon}) \simeq \frac{a_d}{\eta^{2/3}}\left(\frac{\epsilon_T}{\Delta}\right)^{1/2}\tilde{\epsilon}^{\alpha_d}\exp[-\tilde{\epsilon}^{2-d/4}] \tag{4}$$

with $d = 2,\ 3$ the effective dimensionality of the system ($d = 2$ for films of thickness less than the coherence length), $\alpha_d = [d(10-d)-12]/8$, $a_2 \simeq 0.32$, $a_3 \simeq 0.53$, and $\tilde{\epsilon} = (E_g - E)/\epsilon_T$, where the energy scale ϵ_T depends on the strength of the disorder and its fluctuations [23]. For quasiparticles with energy close to E_g ($E_g < E \lesssim \Delta$) we estimate

$$\tau_{\rm loc}^{-1}(\epsilon) \approx r\frac{a_d}{8-d}\left(\frac{\epsilon_T}{\Delta}\right)^{7/2}\left[\Gamma\left(\frac{4(\alpha_d+1)}{8-d}\right)\epsilon^2 + 2\Gamma\left(\frac{4(\alpha_d+2)}{8-d}\right)\epsilon + \Gamma\left(\frac{4(\alpha_d+3)}{8-d}\right)\right], \tag{5}$$

where $\epsilon = (E - E_g)/\epsilon_T$ and the symbols Γ within square brackets denote the gamma function. A more general discussion of the relation between localization rate and electron-phonon interaction is given in the Supplemental Material [23].

If the time $\tau_{\rm loc}$ were independent of energy, we would simply have $\Gamma_{\rm loc} = 1/\tau_{\rm loc}$. Here for our purposes we establish an upper bound on $\Gamma_{\rm loc}$ by considering the competition between localization and relaxation of a mobile quasiparticle into mobile states. Considering again quasiparticles with energy close to E_g, the rate τ_m^{-1} for the latter process is

$$\tau_m^{-1}(\epsilon) \simeq r\frac{4}{105}\sqrt{\frac{2}{3}}\left(\frac{\epsilon_T}{\Delta}\right)^{7/2}\epsilon^{7/2}. \tag{6}$$

We define the crossover (normalized) energy ϵ_c by the equation $\tau_{\rm loc}(\epsilon_c) = \tau_m(\epsilon_c)$ [$\epsilon_c \simeq 3.55\,(2.32)$ for $d = 3\,(2)$]; quasiparticles with energy $\epsilon > \epsilon_c$ will more likely remain mobile after emitting a phonon rather than localize, while the opposite holds for $\epsilon < \epsilon_c$. Therefore, the relevant energy range determining localization can be taken between $\epsilon = 0$ ($E = E_g$) and $\epsilon \sim \epsilon_c$. Since $\tau_{\rm loc}^{-1}$ is an increasing function of energy, we conclude that $\Gamma_{\rm loc} \lesssim \tau_{\rm loc}^{-1}(\epsilon_c)$; in what follows, we will take $\Gamma_{\rm loc}$ to be given by the upper bound,

$$\Gamma_{\rm loc} \simeq b_d r(\epsilon_T/\Delta)^{7/2}, \tag{7}$$

with $b_3 \simeq 2.62$ and $b_2 \simeq 0.59$.

Next, we study the steady-state density as predicted by Eqs. (1) and (2) when $\Gamma_{\rm ex} = 0$. Let us assume that the generation rate is large enough that $\Gamma_{ll} \simeq r$; then taking the sum of the two equation we find the steady-state total density $x = x_m + x_l = \sqrt{g_m/r}$, independent of the localization rate. However, the densities of the two components depend on the latter,

$$x_m = \sqrt{\frac{g_m}{r}}\frac{1}{1+\beta}, \quad x_l = \sqrt{\frac{g_m}{r}}\frac{\beta}{1+\beta} \tag{8}$$

with $\beta = \Gamma_{\rm loc}/\sqrt{g_m r}$. When $\beta \ll 1$, most quasiparticles are mobile and the density of localized quasiparticles is small, $x_l \simeq \Gamma_{\rm loc}/r \ll x_m$. In fact, in the regime of small β the effect of Γ_{ll} can be ignored (at leading order); this can be understood by noticing that the recombination rate for mobile quasiparticles $\tau_r^{-1} = rx_m \simeq \sqrt{g_m r}$ is large compared to $\Gamma_{\rm loc}$, and therefore most quasiparticles recombine before they have a chance to localize

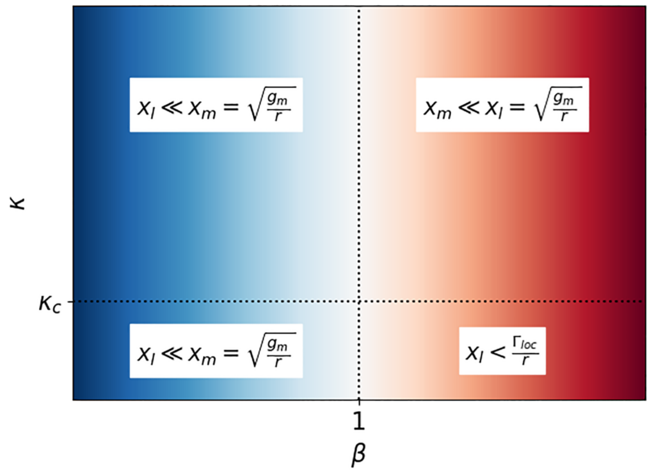

FIG. 2. Diagram representing the various regime possible depending on the values of parameters β and κ. The background color denotes quasiparticles being mostly mobile (blue, $\beta < 1$) or localized (red, $\beta > 1$). Only when $\beta > 1$ and $\kappa < \kappa_c$ (lower right quadrant), the density can be influenced by the suppression of recombination between localized quasiparticles discussed in Ref. [12].

As the generation rate decreases, the regime $\beta \gtrsim 1$ can be reached. The expressions in Eq. (8) still apply so long as $\kappa \gtrsim \kappa_c$; interestingly, when both β and κ are large, we get $x_m \simeq g_m/\Gamma_{\rm loc}$ and the localized states effectively acts as quasiparticle traps, which would beneficial for qubits [31]. If $\kappa \lesssim \kappa_c$ and $\beta \gtrsim 1$ the mechanism discussed in Ref. [12] could become effective at suppressing Γ_{ll} below r; since in this fast localization regime we expect $x_m < x_l$ and the density to decrease monotonically with decreasing generation rate, an upper limit on x_l when $\kappa \lesssim \kappa_c$ and $\beta \gtrsim 1$ is always given by its value estimated when these parameters are of order unity, namely

$$x_l \lesssim \Gamma_{\rm loc}/r. \tag{9}$$

We stress that only if the (total) density is below this upper limit, the mechanism considered in Ref. [12] could be relevant, since if the density is higher, x_l is determined by the competition between localization of mobile quasiparticles and localized-mobile recombination, while localized-localized recombination can be ignored. We summarize the possible regime for the quasiparticle densities in Fig. 2.

We now consider the effect of a mechanism exciting quasiparticles from localized to mobile states. If $\kappa \gg \kappa_c$, the results in Eq. (8) generalize to

$$x_m = \sqrt{\frac{g_m}{r}}\frac{1+\tilde{\beta}}{1+\beta+\tilde{\beta}}, \quad x_l = \sqrt{\frac{g_m}{r}}\frac{\beta}{1+\beta+\tilde{\beta}}, \tag{10}$$

where $\tilde{\beta} = \Gamma_{\rm ex}/\sqrt{g_m r}$. Not surprisingly, the excitation process increases x_m at the expense of x_l. Concerning the value of the excitation rate $\Gamma_{\rm ex}$, we note that at sufficiently low temperatures, $T \ll \epsilon_T$, phonon absorption is not effective at delocalizing quasiparticles; then absorption of photons with energy $\omega_0 \gg \epsilon_T$ should be taken into account, as done in fact in Ref. [13] for resonators. For such devices, one can set $\Gamma_{\rm ex} = \Gamma_0 \bar{n}$, where $\bar{n}$ is the average number of photons stored in the resonator and Γ_0 is in general a material- and geometry-dependent parameter; it can be related to the coupling strength $c_{\rm phot}^{\rm QP}$ [32] between the photons and the quasiparticles, $\Gamma_0 = c_{\rm phot}^{\rm QP}\sqrt{2\Delta/\omega_0}$ (here we assume $\omega_0 \gg \eta^{2/3}E_g$); for aluminum resonators, we estimate $\Gamma_0 \approx 10^{-1}$-$10^{-2}\,{\rm s}^{-1}$. As temperature increases above ϵ_T the photon absorption rate should be compared to that of thermal phonons,

$$\Gamma_{\rm ex}^T \simeq a_T\, r\left(\frac{T}{\Delta}\right)^{7/2} \tag{11}$$

with $a_T \simeq 0.76$ for $\epsilon_T \ll T \ll \Delta\eta^{2/3}$ and $a_T \simeq 0.66$ for $\Delta\eta^{2/3} \ll T \ll \Delta$ (the latter value is as for BCS superconductors [25]). Note that in writing Eq. (7) we assumed $T \ll \epsilon_T$; in the regime $\epsilon_T \ll T \ll \Delta\eta^{2/3}$, we have $\Gamma_{\rm loc} \ll \Gamma_{\rm ex}^T$ [23] and hence $\beta \ll \tilde{\beta}$, implying $x_l \ll x_m$, cf. Eq. (10).

The model in Eqs. (1) and (2) makes it also possible to calculate the relaxation rates λ_i, $i = 1, 2$, of the quasiparticle densities towards the steady state, $x_\alpha(t) \sim \bar{x}_\alpha + \delta x_{\alpha 1}(0)e^{-\lambda_1 t} + \delta x_{\alpha 2}(0)e^{-\lambda_2 t}$, where $\bar{x}_\alpha$, $\alpha = l, m$, are the steady-state densities and $\delta x_{\alpha i}(0)$ account for deviation from the steady state at time $t = 0$. Assuming $\kappa > \kappa_c$ so that all the recombination rates take the value r, by linearization around the steady state we find $\lambda_1 = 2r\bar{x}$ and $\lambda_2 = \Gamma_{\rm loc} + \Gamma_{\rm ex} + r\bar{x}$ [23]. The rate λ_1 coincides with that of the original RT model and represents the relaxation of the total density x, while λ_2 is the decay rate of the "differential" mode for which $\delta x_l = -\delta x_m$; that is, λ_2 governs the return to the steady state when quasiparticles are exchanged between localized and mobile states rather than added to the system by a pair-breaking mechanism.

Having obtained estimates for all the relevant rates, we can now address the question of which regime is relevant for current experiments in aluminum devices. The relevant material parameters are the recombination coefficient r and the localization rate $\Gamma_{\rm loc}$, while the generation rate depends on external factors such as temperature, device shielding, and filtering. The recombination coefficient in aluminum is relatively well known; for our estimates in thin-film devices we set $r \sim 10^7\,{\rm s}^{-1}$ [33]. To evaluate $\Gamma_{\rm loc}$, Eq. (7), we need first to estimate the energy scale ϵ_T; to this end, we note that it must be small compared to the broadening $\Delta - E_g \sim \eta^{2/3}\Delta$ of the peak in the DoS. The latter can be extracted from experiments in which the phenomenological Dynes parameter γ_D is used in fitting data sensitive to the peak shape using the formula $\rho_D(E) = {\rm Re}[(E/\Delta + i\gamma_D)/\sqrt{(E/\Delta + i\gamma_D)^2 - 1}]$. Although this formula predicts a finite DoS of order γ_D also at small energies $E \ll \Delta$, in general the peak broadening and the deep subgap DoS are determined by different parameters; for example, if the broadening is due to the superconductor being in tunnel contact with a normal-metal film, the formula stops being valid below a so-called minigap energy whose value depends on the property of the normal layer [34]. For this reason, we do not estimate γ_D from measurements sensitive to deep subgap states. Current-voltage measurements in relatively thick (500 nm) Al SINIS structures found $\gamma_D \simeq 3 \times 10^{-4}$ [35]; experiments on thermal transport [36,37] and thermoelectric effect [38] in Josephson junctions with Al films of thicknesses between 14 and 40 nm report γ_D between 5×10^{-4} and 5×10^{-3}. As mentioned above, for the theoretical modeling of the subgap states that we use to be consistent, the condition $\epsilon_T \ll \gamma_D\Delta$ must be satisfied, so we conservatively assume $\epsilon_T/\Delta \sim 5 \times 10^{-4}$, a value an order of magnitude larger than estimated in Ref. [20]. Then according to Eq. (9) we have $x_l \lesssim \Gamma_{\rm loc}/r < 10^{-11}$. Since in Al we have $n_{\rm Cp} \simeq 5 \times 10^6\,\mu{\rm m}^3$, this normalized density corresponds to less than one quasiparticle even in large devices such as 3D transmons [39] or coplanar waveguide resonators [40] with volume on the order of a few times $10^3\,\mu{\rm m}^3$. Therefore we must conclude that localization does not contribute to excess quasiparticles in typical aluminum devices. Given our estimate above for ϵ_T, we additionally note that in typical experiments at $T \simeq 10\,{\rm mK}$, we have $T/\epsilon_T \sim 8$; even though the excitation rate in Eq. (11) is slow, $\Gamma_{\rm ex} \sim 0.03\,{\rm Hz}$, in rough agreement with the estimate in Ref. [20], it is nonetheless much faster than both the zero-temperature localization rate from Eq. (7) and its finite temperature counterpart; therefore, we expect that for $T \gtrsim \epsilon_T$ excitation prevails over localization, again pointing to the conclusion that localization is not relevant in determining the quasiparticle density. This conclusion also applies to devices made with β-Ta films, since both $T_c \sim 0.87$ K and $\epsilon_T/\Delta \sim 1.5 \times 10^{-4}$ have values similar to that of Al, according to a recent work [24]. Interestingly, the density measured there follows the thermal equilibrium expectation, while the decay rate is argued to be the excitation rate $\Gamma_{\rm ex}^T$ and therefore compatible with that of the differential mode [23].

In the preceding paragraph we have considered typical thin Al films. Even keeping aluminum as the main superconducting material, the value of γ_D and hence possibly the ratio ϵ_T/Δ can be increased in several ways, for example by Mn doping [35] and by proximity effect with a normal metal [34] such as Cu [38], methods that, however, suppress the gap Δ. Alternatively, oxygen doping can increase Δ [41,42] (up to a point) as well as γ_D [43]; in fact, the quasiparticle dynamics in granular aluminum resonators appears to be affected by localization [13]. Similar behavior has been recently reported in resonators incorporating tungsten silicide [44]. Other materials widely used in qubits and resonators such as Nb ($\gamma_D \sim 5 \times 10^{-2}$ [45]), NbN ($\gamma_D \sim 2.4$-4×10^{-2} [46]), or being explored for spintronics applications [47] such as TaN ($\gamma_D \sim 7 \times 10^{-2}$ [48]), also have larger broadening (in units of Δ) than Al. Increasing ϵ_T/Δ by an order of magnitude would push up the bound on x_l by more than three orders of magnitude, and also make possible experiments in the regime $T \lesssim \epsilon_T$ in which excitation by thermal phonon is slower than localization. Moreover, materials with higher gap generally have faster recombination (larger r) than Al [25], making it easier to reach the regime $\beta > 1$ for a given generation rate g, since $\beta \simeq (\epsilon_T/\Delta)^{7/2}\sqrt{r/g_m}$. Therefore, exploring devices fabricated with properly chosen materials could shed further light on the role of subgap states in their performances. On the theoretical side, our considerations could be extended to cleaner materials by building on the approach presented in Ref. [49]. In a complementary direction, one could also consider the case of more strongly coupled magnetic impurities, such that an impurity band of localized states is formed with energy well below the gap [22]; then localization and mobile-localized quasiparticle recombination could be enhanced [50], effects that have been used to interpret experiments in superconducting tunnel junction detectors [51].

Note added. Recently, similar conclusions were reached in Ref. [52] based on current-voltage measurements in Al- and Nb-based junctions.

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Data availability. No data were created or analyzed in this study.

[1] A. P. Vepsäläinen, A. H. Karamlou, J. L. Orrell, A. S. Dogra, B. Loer, F. Vasconcelos, D. K. Kim, A. J. Melville, B. M. Niedzielski, J. L. Yoder, S. Gustavsson, J. A. Formaggio, B. A. VanDevender, and W. D. Oliver, Impact of ionizing radiation on superconducting qubit coherence, Nature (London) **584**, 551 (2020).

[2] L. Cardani, F. Valenti, N. Casali, G. Catelani, T. Charpentier, M. Clemenza, I. Colantoni, A. Cruciani, G. D'Imperio, L. Gironi, L. Grünhaupt, D. Gusenkova, F. Henriques, M. Lagoin, M. Martinez, G. Pettinari, C. Rusconi, O. Sander, C. Tomei, A. V. Ustinov, M. Weber, W. Wernsdorfer, M. Vignati, S. Pirro, and I. M. Pop, Reducing the impact of radioactivity on quantum circuits in a deep-underground facility, Nat. Commun. **12**, 2733 (2021).

[3] C. D. Wilen, S. Abdullah, N. A. Kurinsky, C. Stanford, L. Cardani, G. D'Imperio, C. Tomei, L. Faoro, L. B. Ioffe, C. H. Liu, A. Opremcak, B. G. Christensen, J. L. DuBois, and R. McDermott, Correlated charge noise and relaxation errors in superconducting qubits, Nature (London) **594**, 369 (2021).

[4] M. Houzet, K. Serniak, G. Catelani, M. H. Devoret, and L. I. Glazman, Photon-assisted charge-parity jumps in a superconducting qubit, Phys. Rev. Lett. **123**, 107704 (2019).

[5] S. Diamond, V. Fatemi, M. Hays, H. Nho, P. D. Kurilovich, T. Connolly, V. R. Joshi, K. Serniak, L. Frunzio, L. I. Glazman, and M. H. Devoret, Distinguishing parity-switching mechanisms in a superconducting qubit, PRX Quantum **3**, 040304 (2022).

[6] X. Pan, Y. Zhou, H. Yuan, L. Nie, W. Wei, L. Zhang, J. Li, S. Liu, Z. H. Jiang, G. Catelani, L. Hu, F. Yan, and D. Yu, Engineering superconducting qubits to reduce quasiparticles and charge noise, Nat. Commun. **13**, 7196 (2022).

[7] C. H. Liu, D. C. Harrison, S. Patel, C. D. Wilen, O. Rafferty, A. Shearrow, A. Ballard, V. Iaia, J. Ku, B. L. T. Plourde, and R. McDermott, Quasiparticle poisoning of superconducting qubits from resonant absorption of pair-breaking photons, Phys. Rev. Lett. **132**, 017001 (2024).

[8] R. Anthony-Petersen, A. Biekert, R. Bunker, C. L. Chang, Y.-Y. Chang, L. Chaplinsky, E. Fascione, C. W. Fink, M. Garcia-Sciveres, R. Germond, W. Guo, S. A. Hertel, Z. Hong, N. Kurinsky, X. Li, J. Lin, M. Lisovenko, R. Mahapatra, A. Mayer, D. N. McKinsey, S. Mehrotra, N. Mirabolfathi, B. Neblosky, W. A. Page, P. K. Patel, B. Penning, H. D. Pinckney, M. Platt, M. Pyle, M. Reed, R. K. Romani, H. S. Queiroz, B. Sadoulet, B. Serfass, R. Smith, P. Sorensen, B. Suerfu, A. Suzuki, R. Underwood, V. Velan, G. Wang, Y. Wang, S. L. Watkins, M. R. Williams, V. Yefremenko, and J. Zhang, A stress-induced source of phonon bursts and quasiparticle poisoning, Nat. Commun. **15**, 6444 (2024).

[9] E. Yelton, C. P. Larson, K. Dodge, K. Okubo, and B. L. T. Plourde, Correlated quasiparticle poisoning from phonon-only events in superconducting qubits, Phys. Rev. Lett. **135**, 123601 (2025).

[10] E. T. Mannila, P. Samuelsson, S. Simbierowicz, J. T. Peltonen, V. Vesterinen, L. Grönberg, J. Hassel, V. F. Maisi, and J. P. Pekola, A superconductor free of quasiparticles for seconds, Nat. Phys. **18**, 145 (2022).

[11] A. Rothwarf and B. N. Taylor, Measurement of recombination lifetimes in superconductors, Phys. Rev. Lett. **19**, 27 (1967).

[12] A. Bespalov, M. Houzet, J. S. Meyer, and Y. V. Nazarov, Theoretical model to explain excess of quasiparticles in superconductors, Phys. Rev. Lett. **117**, 117002 (2016).

[13] L. Grünhaupt, N. Maleeva, S. T. Skacel, M. Calvo, F. Levy-Bertrand, A. V. Ustinov, H. Rotzinger, A. Monfardini, G. Catelani, and I. M. Pop, Loss mechanisms and quasiparticle dynamics in superconducting microwave resonators made of thin-film granular aluminum, Phys. Rev. Lett. **121**, 117001 (2018).

[14] B. Sacépé, C. Chapelier, T. I. Baturina, V. M. Vinokur, M. R. Baklanov, and M. Sanquer, Disorder-induced inhomogeneities of the superconducting state close to the superconductor-insulator transition, Phys. Rev. Lett. **101**, 157006 (2008).

[15] I. O. Kulik and O. Y. Itskovich, Effect of concentration non-uniformity on the properties of superconductors with paramagnetic impurities, Sov. Phys. JETP **28**, 102 (1968).

[16] A. I. Larkin and Y. N. Ovchinnikov, Density of states in inhomogeneous superconductors, Sov. Phys. JETP **34**, 1144 (1972).

[17] B. T. Matthias, H. Suhl, and E. Corenzwit, Spin exchange in superconductors, Phys. Rev. Lett. **1**, 92 (1958).

[18] A. Abrikosov and L. Gorkov, Contribution to the theory of superconducting alloys with paramagnetic impurities, Sov. Phys. JETP **12**, 1243 (1961).

[19] S. Skalski, O. Betbeder-Matibet, and P. R. Weiss, Properties of superconducting alloys containing paramagnetic impurities, Phys. Rev. **136**, A1500 (1964).

[20] A. Silva and L. B. Ioffe, Subgap states in dirty superconductors and their effect on dephasing in Josephson qubits, Phys. Rev. B **71**, 104502 (2005).

[21] M. A. Skvortsov and M. V. Feigel'man, Subgap states in disordered superconductors, J. Exp. Theor. Phys. **117**, 487 (2013).

[22] Y. V. Fominov and M. A. Skvortsov, Subgap states in disordered superconductors with strong magnetic impurities, Phys. Rev. B **93**, 144511 (2016).

[23] See Supplemental Material at http://link.aps.org/supplemental/10.1103/jj9b-27qs for details on the generalized RT

model and the subgap density of states, which includes Refs. [53–56].

[24] S. A. H. de Rooij, R. Fermin, K. Kouwenhoven, T. Coppens, V. Murugesan, D. J. Thoen, J. Aarts, J. J. A. Baselmans, and P. J. de Visser, Recombination of localized quasiparticles in disordered superconductors, arXiv:2410.18802.

[25] S. B. Kaplan, C. C. Chi, D. N. Langenberg, J. J. Chang, S. Jafarey, and D. J. Scalapino, Quasiparticle and phonon lifetimes in superconductors, Phys. Rev. B **14**, 4854 (1976).

[26] G. Catelani and D. M. Basko, Non-equilibrium quasiparticles in superconducting circuits: photons vs. phonons, SciPost Phys. **6**, 013 (2019).

[27] P. B. Fischer and G. Catelani, Nonequilibrium quasiparticle distribution in superconducting resonators: Effect of pair-breaking photons, SciPost Phys. **17**, 070 (2024).

[28] A. I. Larkin and Y. N. Ovchinnikov, Vortex motion in superconductors, in *Nonequilibrium Superconductivity*, Modern Problems in condensed matter science, edited by D. Langenberg and A. I. Larkin (North-Holland Publishing Company, Amsterdam, 1986).

[29] J. Chang and D. Scalapino, Nonequilibrium superconductivity, J. Low Temp. Phys. **31**, 1 (1978).

[30] R. Barends, S. van Vliet, J. J. A. Baselmans, S. J. C. Yates, J. R. Gao, and T. M. Klapwijk, Enhancement of quasiparticle recombination in Ta and Al superconductors by implantation of magnetic and nonmagnetic atoms, Phys. Rev. B **79**, 020509(R) (2009).

[31] R.-P. Riwar, A. Hosseinkhani, L. D. Burkhart, Y. Y. Gao, R. J. Schoelkopf, L. I. Glazman, and G. Catelani, Normal-metal quasiparticle traps for superconducting qubits, Phys. Rev. B **94**, 104516 (2016).

[32] P. B. Fischer and G. Catelani, Nonequilibrium quasiparticle distribution in superconducting resonators: An analytical approach, Phys. Rev. Appl. **19**, 054087 (2023).

[33] C. Wang, Y. Y. Gao, I. M. Pop, U. Vool, C. Axline, T. Brecht, R. W. Heeres, L. Frunzio, M. H. Devoret, G. Catelani, L. I. Glazman, and R. J. Schoelkopf, Measurement and control of quasiparticle dynamics in a superconducting qubit, Nat. Commun. **5**, 5836 (2014).

[34] A. Hosseinkhani and G. Catelani, Proximity effect in normal-metal quasiparticle traps, Phys. Rev. B **97**, 054513 (2018).

[35] G. C. O'Neil, D. R. Schmidt, N. A. Tomlin, and J. N. Ullom, Quasiparticle density of states measurements in clean superconducting AlMn alloys, J. Appl. Phys. **107**, 093903 (2010).

[36] A. Fornieri, C. Blanc, R. Bosisio, S. D'Ambrosio, and F. Giazotto, Nanoscale phase engineering of thermal transport with a Josephson heat modulator, Nat. Nanotechnol. **11**, 258 (2016).

[37] A. Fornieri, G. Timossi, P. Virtanen, P. Solinas, and F. Giazotto, 0-π phase-controllable thermal Josephson junction, Nat. Nanotechnol. **12**, 425 (2017).

[38] G. Germanese, F. Paolucci, G. Marchegiani, A. Braggio, and F. Giazotto, Bipolar thermoelectric Josephson engine, Nat. Nanotechnol. **17**, 1084 (2022).

[39] H. Paik, D. I. Schuster, L. S. Bishop, G. Kirchmair, G. Catelani, A. P. Sears, B. R. Johnson, M. J. Reagor, L. Frunzio, L. I. Glazman, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf, Observation of high coherence in Josephson junction qubits measured in a three-dimensional circuit QED architecture, Phys. Rev. Lett. **107**, 240501 (2011).

[40] P. J. de Visser, D. J. Goldie, P. Diener, S. Withington, J. J. A. Baselmans, and T. M. Klapwijk, Evidence of a nonequilibrium distribution of quasiparticles in the microwave response of a superconducting aluminum resonator, Phys. Rev. Lett. **112**, 047004 (2014).

[41] R. C. Dynes and J. P. Garno, Metal-insulator transition in granular aluminum, Phys. Rev. Lett. **46**, 137 (1981).

[42] F. Valenti, F. Henriques, G. Catelani, N. Maleeva, L. Grünhaupt, U. von Lüpke, S. T. Skacel, P. Winkel, A. Bilmes, A. V. Ustinov, J. Goupy, M. Calvo, A. Benoît, F. Levy-Bertrand, A. Monfardini, and I. M. Pop, Interplay between kinetic inductance, nonlinearity, and quasiparticle dynamics in granular aluminum microwave kinetic inductance detectors, Phys. Rev. Appl. **11**, 054087 (2019).

[43] R. C. Dynes, J. P. Garno, G. B. Hertel, and T. P. Orlando, Tunneling study of superconductivity near the metal-insulator transition, Phys. Rev. Lett. **53**, 2437 (1984).

[44] T. F. Q. Larson, S. G. Jones, T. Kalmar, P. A. Sanchez, S. P. Chitta, V. Verma, K. Genter, K. Cicak, S. W. Nam, G. Fulop, J. Koch, R. W. Simmonds, and A. Gyenis, Localized quasiparticles in a fluxonium with quasi-two-dimensional amorphous kinetic inductors, arXiv:2504.07950 (2025).

[45] M. R. Nevala, S. Chaudhuri, J. Halkosaari, J. T. Karvonen, and I. J. Maasilta, Sub-micron normal-metal/insulator/superconductor tunnel junction thermometer and cooler using Nb, Appl. Phys. Lett. **101**, 112601 (2012).

[46] S. Chaudhuri, M. R. Nevala, and I. J. Maasilta, Niobium nitride-based normal metal-insulator-superconductor tunnel junction microthermometer, Appl. Phys. Lett. **102**, 132601 (2013).

[47] M. Müller, R. Hoepfl, L. Liensberger, S. Geprägs, H. Huebl, M. Weiler, R. Gross, and M. Althammer, A superconductor free of quasiparticles for seconds, Mater. Quantum. Technol. **1**, 045001 (2021).

[48] S. Chaudhuri and I. J. Maasilta, Superconducting tantalum nitride-based normal metal-insulator-superconductor tunnel junctions, Appl. Phys. Lett. **104**, 122601 (2014).

[49] S. E. de Graaf, L. Faoro, L. B. Ioffe, S. Mahashabde, J. J. Burnett, T. Lindström, S. E. Kubatkin, A. V. Danilov, and A. Y. Tzalenchuk, Two-level systems in superconducting quantum devices due to trapped quasiparticles, Sci. Adv. **6**, eabc5055 (2020).

[50] A. G. Kozorezov, A. A. Golubov, J. K. Wigmore, D. Martin, P. Verhoeve, R. A. Hijmering, and I. Jerjen, Inelastic scattering of quasiparticles in a superconductor with magnetic impurities, Phys. Rev. B **78**, 174501 (2008).

[51] R. A. Hijmering, A. G. Kozorezov, A. A. Golubov, P. Verhoeve, D. D. E. Martin, J. K. Wigmore, and I. Jerjen, Modeling of local trapping states in superconductor tunnel junctions with Kondo impurities, IEEE Trans. Appl. Supercond. **19**, 423 (2009).

[52] E. Gurra, D. A. Bennett, S. M. Duff, M. R. Vissers, and J. N. Ullom, Can slow recombination in ordered superconductors explain the excess quasiparticle population? arXiv:2507.03217.

[53] F. Giazotto, T. T. Heikkilä, A. Luukanen, A. M. Savin, and J. P. Pekola, Opportunities for mesoscopics in thermometry and

refrigeration: Physics and applications, Rev. Mod. Phys. **78**, 217 (2006).

[54] A. Golubov and E. Houwman, Quasiparticle relaxation rates in a spatially inhomogeneous superconductor, Physica C **205**, 147 (1993).

[55] Y. Savich, L. Glazman, and A. Kamenev, Quasiparticle relaxation in superconducting nanostructures, Phys. Rev. B **96**, 104510 (2017).

[56] J. Zittartz and J. S. Langer, Theory of bound states in a random potential, Phys. Rev. **148**, 741 (1966).