Energy Policy 208 (2026) 114866

Contents lists available at ScienceDirect

Energy Policy

journal homepage: www.elsevier.com/locate/jepo

Managing conflicting interests in subsidy design: A bi-level optimization approach for heating technology subsidies

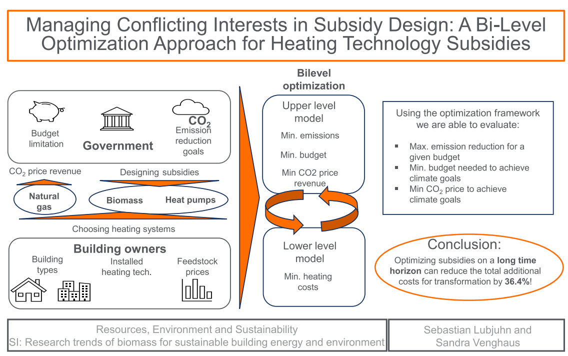
Sebastian Lubjuhn [a,*], Sandra Venghaus [a,b]

[a] RWTH Aachen, School of Business and Economics, Kackertstraße 7, Aachen, 52062, Germany
[b] Forschungszentrum Jülich, Jülich Systems Analysis, Jülich, 52425, Germany

HIGHLIGHTS

- Heat pumps will dominate the private heating.
- Germany is on track to achieve climate goals in the heating sector.
- Optimal subsidies reduce the required budget to €80 billion.
- Subsidies without CO_2 pricing are the least effective for climate mitigation.
- Combining CO_2 pricing with subsidies strategically nearly halves the required CO_2 price.

GRAPHICAL ABSTRACT



ARTICLE INFO

Keywords:
Bilevel optimization
Linear optimization
Heat sector
Subsidies
Heat pumps
Biomass

ABSTRACT

Germany's heating sector emits 15 % of its total CO_2 emissions. Heating technologies like biomass heating and heat pumps are vital to reduce emissions in the heating sector, but a limited number of studies have addressed long-term optimization of subsidies to support diffusion of renewable technologies in heating. This challenge is compounded by the conflicting interests of the German government, which aims to minimize total emissions via subsidy allocation, and property owners, who seek to minimize costs. We propose a bi-level optimization model utilizing black box optimization at the upper level and linear programming at the lower level. Our findings suggest that Germany can achieve its climate targets under current circumstances and subsidies. However, this comes at high government spending of 469 Billion €. By adopting strategic subsidy policies that leverage the different phases of technological diffusion, comparable levels of CO_2 reduction can be achieved while significantly reducing governmental spending—from double the revenue generated by the CO_2 price to nearly half. This approach provides a more cost-efficient and resilient solution, better equipped to address fluctuations in overall governmental expenditures and withstand potential budget crises. Additionally, we are able to minimize greenhouse gas emissions, the required total governmental budget, or the required price of CO_2 emissions.

1. Introduction

In 2021, the global greenhouse gas (GHG) emissions from the operations of buildings rose to 10 $GtCO_2$ UN (2022), which makes up 27 % of the total energy related emissions in the world IEA (2025). In 2015, 195 countries signed the Paris Agreement, pledging to limit the increase in average global temperatures to well below 2 °C above preindustrial levels. As the European Union's largest economy and emitter

* Corresponding author.
Email address: sebastian.lubjuhn@socecon.rwth-aachen.de (S. Lubjuhn).

https://doi.org/10.1016/j.enpol.2025.114866
Received 12 December 2024; Received in revised form 18 July 2025; Accepted 28 August 2025
Available online 12 September 2025
0301-4215/

Nomenclature

Sets

A	Set of different technologies that receive subsidies $a \in A \subset I$
$Build$	Set of technologies i in the building sector $i \in Build \subset I$
I	Set of heating technologies $i \in I$
I_s	Set of technologies that are feeding heat to sub-sector s $i \in I_s \subset I$
R	Set of residual types $r \in R$
S	Set of sub-sectors, which represent different heat demands $s \in S$
T	Set of years $t \in T$
T^*	Set of years in which the government can decide on subsidies, budgets or taxes $t^* \in T^* \subset T$

Parameters

c_{it}^{inv}	Cost of investing in a unit of heating technology i at time t
c_{it}^{var}	Variable costs of operating heating technology i at time t
c_{it}^{varimp}	Variable costs, when the feedstock is imported for operating heating technology i at time t
c_{it}^{fix}	Yearly fixed costs of heating technology i at time t
C_{it}	Capacity of heat that can be produced in one year t by heating technology i
d_t	Discount factor at time t
dec_{it}	Number of heating technology units i that are phased out at time t
dem_{st}	Heat demand of sub-sector s at time t
em_{it}	Emission factor of the heat produced by technology i at time t
$\text{em}_{it}^{\text{imp}}$	Emission factor of the heat produced by technology i at time t, when the feedstock is imported
G_t^{Build}	Maximum number of heating systems that can be installed
$\textbf{in}_{ir}$	Amount of residual r used in heating concept i per unit of heat.
life_i	Expected lifetime of technology i in the building sector in year t
$\text{Res}_{r,t}$	Biomass residual r that can be used for heat production at t
Start_i	Start capacity of technology i in the start year

Decision variables

Upper level model (ULM)

β_{t^*}	Budget for subsidies at t^*
σ_{at^*}	Incentive on investment costs for technologies a at t^*
τ_{t^*}	CO_2 price at t^*

Lower level model (LLM)

$nCap_{it}^{\text{sub}}$	Subsidized number of units of technology i installed at time t
$nCap_{it}$	Unsubsidized number of units of technology i installed at time t
$xHeat_{it}$	Heat produced by technology i at time t
$xHeat_{it}^{\text{imp}}$	Heat produced by technology i at time t using imported feedstock

Abbreviations

BEG	Federal Subsidies for Efficient Buildings
BEHG	German National Emissions Trading System
BGT	ULM: minimizing the budget for the subsidies
ETS	EU Emissions Trading System
GHG	Greenhouse gases
GHG	ULM: minimizing greenhouse gas emissions
HP	Heat pump
PV	Photovoltaic
TAX	ULM: minimizing the revenue of the CO_2 price

of greenhouse gases (GHG), Germany has set ambitious GHG emissions reduction goals in order to combat climate change. These goals aim to reduce GHG emissions by 65 % compared to 1990 levels by 2030 and achieve climate-neutrality by 2045.

Therefore, a transition from fossil fuel-based technologies to renewable ones is needed. While emissions reduction already seems to be on track in the energy and industrial sectors, decarbonizing the building sector, which primarily contributes to private and commercial heating, seems to be more challenging, as Germany missed its self-set goals for the building sector in 2020, 2021, and 2022 BMWK (2024). Accounting for 15 % of total German emissions the building sector plays a central role in decarbonization BMWK (2024).

In addition, the war in Ukraine has had a significant impact on the security of natural gas, oil, and coal supply, resulting in high prices for both industrial and private consumers Adolfsen et al. (2022). It has especially influenced the costs of heating, as it is currently dominated by fossil fuels, mostly natural gas (50 %) and heating oil (25 %).

The change to low emission heating technologies usually comes at high investment costs, which are borne by the households directly or indirectly through increased rents. This leads to the conflicting interests between the government which aims to reduce the CO_2 emissions and the dependency on fossil fuels and the households which aim to reduce their costs.

There are many different policies that Germany and the EU have already implemented to decrease the dependency on fossil fuels. At the EU level, it is planned to introduce an emissions trading system for the heating and transport sectors Graichen and Ludig (2024). At the national level, a price for CO_2 emissions from fossil feedstocks like natural gas and heating oil was implemented in 2021 and is planned to be merged in 2027 with the European Emissions Trading Scheme on heating and transport Graichen and Ludig (2024).

However, high carbon prices can lead to social inequities. Therefore, a more efficient and targeted subsidy policy is needed to reduce the burden of the transformation costs on the households, while promoting the adoption of renewable heating technologies.

Subsidies for insulation and heating systems were introduced in Germany and combined into the Federal Subsidies for Efficient Buildings (Bundesförderung für effiziente Gebäude (BEG)), which came into effect in January 2021 BMWK (2021).

Additionally, in the Building Energy Law (Gebäude Energy Gesetz (GEG)), the German government decided that the newly-installed heating system should use at least 65 % of the primary energy from renewable sources as soon as the municipal heating plan is finalized, i.e., from July 2028 at the latest Bundestag (2020).

However, shifts in government and changing expectations have led to frequent modifications in these subsidies and laws, establishing inconsistent incentives and preventing reliable conditions for long-term investments. For example, after the first draft of the GEG was announced, the number of newly sold gas boilers increased significantly in Q2 and Q3 of 2023, because people anticipated that these could be forbidden in the future while, on the other hand, consumers were awaiting higher subsidies for heat pumps BDH (2023).

The German government implemented the Schuldenbremse (debt brake), which is a constitutional rule in Germany designed to limit the amount of new public debt that the federal government and states can incur. Introduced in 2009 and fully implemented by 2016, the

Schuldenbremse is a key fiscal policy tool aimed at ensuring long-term fiscal stability and sustainability.

On the other hand, it means that the budget that can be devoted to transforming the heating sector is limited, and the promised subsidies are not guaranteed. This can lead to unexpected stops of subsidies, which can slow down the transition process. An example where this happened was an incentive for new electric cars (Förderung des Absatzes von elektrisch betriebenen Fahrzeugen) that was unexpectedly rescinded one year earlier BAFA (2024) due to budgeting problems within the government.

Therefore, long-term planned subsidies are desirable as they would ensure less uncertainty for households, industries, and governments and can leverage all phases of the technological diffusion.

One way to calculate the adoption of new heating systems based on long-term subsidy scenarios is linear optimization, which enables an analysis of emission reductions, the necessary governmental spending, and the additional consumer costs for a given subsidy scenario.

To find efficient subsidies, we propose bi-level models aiming to minimize GHG emissions, the total required government spending, or the total consumer costs of CO_2 to achieve the climate goals at the upper level and minimizing the transformation costs at the lower level.

1.1. Literature review

Various qualitative research has been conducted on different policy measures in the heating sector. For example, Xia-Bauer et al. (2024) presents a comparative analysis of decarbonization policies for residential buildings in the EU, China, and India and OECD (2024) examines policy objectives, measures, and trends, of 28 countries in Africa, America, Asia, and Europe, which offer valuable insights into how countries are developing effective strategies for decarbonizing buildings. In the OECD Global Survey on Buildings and Climate OECD (2024) it was found that the majority of countries are imposing standards on new buildings that align with near zero emission goals, for which affordability of decarbonization measures is the largest barrier. In the case of existing buildings the challenges lie in standardising methodologies, reducing the economic burden on building owners and resolving conflicts of interest. 87 % of the more than 140 countries in the survey OECD (2024), are using financial incentives like subsidies to lower these barriers.

Quantitative research has also been done for determining optimal subsidies to achieve environmental goals in other sectors. For example, Bigerna et al. (2019) developed a theoretical model to evaluate a monopolistic firm's investment decisions in renewable energy systems with the support of subsidies, considering the adjustments needed to meet policy targets. The model was applied to Italy to analyze the effects of government subsidy policies, ultimately proposing an intermediate subsidy level that aligned with EU policy objectives. Although this study provides significant insights into the general interactions between firms and governmental subsidies, the model is highly theoretical and focuses on monopolistic firms' decisions, making it unsuitable for finding optimal subsidization strategies for the heating market.

While previous research has addressed specific aspects of the heating markets, a gap in the research remains regarding the optimization of CO_2 prices and subsidies.

Cost minimization was used in a linear optimization in König (2011) to identify cost-effective biomass technologies in the heat, power, and transport sectors. The authors showed that the use of solid biomass was cost-efficient in heat production and, to a lesser extent, in CHP generation.

A model of a cost-optimal heating sector intended to fulfill Germany's climate goals was presented by Jordan et al. (2019), with a subsequent robustness analysis showing that solid biomass would be especially competitive in the high-temperature industry Jordan et al. (2020), whereas heat pumps would dominate the private sector.

In Lubjuhn and Venghaus (2024), the chemicals and fuels sectors were added. Using multiple scenarios, it was found that subsidizing private biomass heating instead of only heat pumps may be ineffective or even negative for reducing GHG emissions. Rather than systematically elaborating on the optimal design of subsidies, these studies only analyzed one or a selective set of subsidy scenarios. In addition, the considered subsidies were constant over all time steps, and budgets for subsidies were not considered. Furthermore, these models do not account for the opposing interests of property owners, who seek to minimize costs through investments in heating technologies, and the government, which aims to adopt more renewable technologies to minimize GHG emissions. However, this is highly important due to the additional costs often accompanying renewable technologies.

One tool to account for these opposing interests is bi-level optimization, which has been used in different contexts. Ziliaskopoulos and Papalamprou (2022) employed a bi-level optimization model for subsidies in the agricultural sector in which the government decides in the upper model on subsidies for each crop type to minimize their total environmental impact. At the same time, the farmers maximize their profits in the lower-level model.

Similarly, Wei et al. (2014) optimized the carbon tax for ten coal-fired power plants. They used the Karush-Kuhn-Tucker (KKT) optimality criteria to reduce the bilevel model into a single-level mixed integer program. This is possible when the lower-level problem is convex, and the Slater condition holds, leading to a strong duality Colson et al. (2005).

Zhou et al. (2011), in turn, developed a bi-level model for renewable energy incentive policies, the upper level of which minimized the incentive intervention, subject to a constraint ensuring the targeted share of renewable energy production. The lower level utilized a generation expansion planner intended to minimize costs. Given the non-linearity and non-convexity of the upper level, coupled with the mixed integer lower-level problem, the authors designed a heuristic algorithm for the bi-level program. Their findings suggest that simultaneously implementing both taxes and incentives is significantly more effective than using either instrument alone.

Similarly, Milyani and Kirschen (2018) proposed a bi-level programming framework in which the government could use tax revenue from CO_2 taxes for subsidies. In this bi-level optimization model, the upper level represents the government, which aims to minimize the total subsidy cost, while the lower level represents the day-ahead market-clearing process with the objective of providing the necessary load at minimal costs. The model provides the optimal subsidy and tax for each generator at each time step. The nonlinear bi-level model was subsequently transformed into a mixed-integer linear programming problem and solved using commercial software. It was shown that combining taxes and using tax revenue for subsidies can significantly reduce the required tax rate.

A bi-level fuzzy programming method for Beijing's energy planning under uncertainty, in which the leader aims to minimize CO_2 emissions and the follower focuses on minimizing system costs, was introduced by Jin et al. (2018).

Olsen et al. (2018) and Pereira et al. (2019) formulated the optimization of the carbon tax as a bi-level program, in which the upper level minimizes the tax rate, with the greenhouse gas emissions target set as a constraint, and the lower level minimizes costs in the power system. This problem, which includes binary variables at the lower level, is addressed using the weighted sum bisection method.

Zhao and You (2019) used a bi-level program to optimize the incentive policy on bioenergy in the dairy sector in the state of New York. At the upper level, the government decides on waste disposal fees, subsidies on bioelectricity generation, and refunds of capital investment while imposing a constraint on the targeted share of renewable energy production and minimizing the spending costs per unit of bioenergy produced. At the lower level, dairy farm owners maximize their net present values (NPVs) subject to the proposed bioenergy incentive policy. Because the non-linear and non-convex upper and lower level models are mixed

Table 1
Overview of the literature that used bi-level optimization approaches to optimize subsidies or taxes. LLM: lower level model; ULM: upper level model; envir. impact: environmental impact; sub: subsidies; tax: GHG tax (often CO_2 tax); elec. sources (%): shares of electricity sources of the total electricity mix; NPV: net present value.

Location	LLM objective	ULM objective	decision variables of ULM	timestep length	time frame	source
Greece	max. profits	min. envir. impact	sub	yearly	1 year	Ziliaskopoulos and Papalamprou (2022)
China	min. costs	min. GHG emissions	tax	yearly	1 year	Wei et al. (2014)
USA	min. costs	min. gov. intervention	sub and tax	yearly	1 year	Zhou et al. (2011)
USA	min. costs	min. budget	sub	hourly	1 day	Milyani and Kirschen (2018)
China	min. costs	min. GHG emissions	elec. sources (%)	5 year steps	15 years	Jin et al. (2018)
USA	min. costs	min. tax	tax	daily	5 days	Olsen et al. (2018)
Chile	min. costs	min. tax	tax	yearly	9 years	Pereira et al. (2019)
New York	max. NPV	min. gov. intervention	sub and tax	yearly	1 year	Zhao and You (2019)
Buildings, Urban District	min. costs	min. gov. intervention	sub and tax	yearly	1 year	Martelli et al. (2020)
Germany	min. costs	min. GHG emissions, min. budget, min. gov. intervention	sub, sub, sub and tax	yearly	30 years	this study

integers, the authors had to develop a tailored global optimization algorithm to solve the problem by integrating the parametric algorithm, in addition to a projection-based reformulation and decomposition algorithm. Their results indicate that simultaneously imposing taxes and incentives is much more efficient than employing one policy at a time.

Martelli et al. (2020) used black-box optimization to calculate optimal subsidies for a university campus, a building housing university offices, a hospital, and an urban district. While the study provided interesting insights, the model was only calculated for a very short time-span. Because black box optimization requires much calculation, this approach has not yet been used for the bi-level optimization of energy systems or heating system models.

Most of these optimize the subsidies for one or only a few steps because of the high computational time inherent in the bi-level problems they present. However, this is insufficient for a long-term strategy for subsidies and taxes due to either a large timespan between the steps or a short total timespan, which results in a loss of detail.

Table 1 shows previous studies and the number of timesteps considered.

Therefore, for the first time, this study introduces a black-box optimization approach to determine efficient subsidy and CO_2 pricing strategies for the heating market, with yearly steps from 2020 to 2050.

2. Methodology

To identify effective and long-term subsidy strategies, we employ a bi-level optimization model. This approach involves two interrelated levels of decision-making:

- In the upper level model (ULM), the government decides on the share of investment costs for sustainable technologies that are to be subsidized. The set of different technologies that receive subsidies is provided by A. In this analysis, we consider this to be the set of heat pumps and biomass heating. These subsidies can either be provided by a designated budget from the government (exogenously) or with the resulting income from the CO_2 tax (or emissions trading system)(endogenously). To model both possibilities, we examine different scenarios.
- In the lower level model (LLM), the households are represented, in which the household owners seek to minimize their costs. The total budget for subsidies is shared among households. Once the budget is fully used, the households can still purchase unsubsidized heating systems.

2.1. Upper level model

Depending on the aim of the policy-maker, we selected different objective functions and decision variables, which are covered by the upper level models

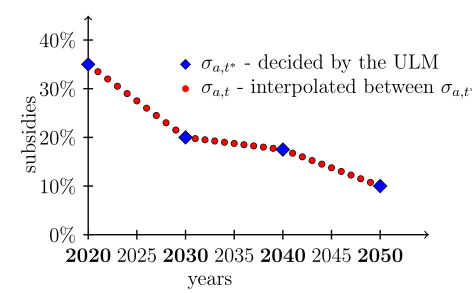


Fig. 1. Visualization of the decision variable σ_{a,t^*} of the ULM and the subsidies $\sigma_{a,t}$ used in the LLM. The same approach was adopted for the budget β_{t*} and β_t as well as the CO_2 price τ_{t^*} and τ_t.

- minimizing greenhouse gas emissions–from now on referred to as **GHG**,
- minimizing the budget for the subsidies–from now on referred to as **BGT**,
- minimizing the revenue of the CO_2 price–from now on referred to as **TAX**

that are given by Eqs. (1), (3), and (6).

The upper-level agent aims to find the best subsidies s_{at^*} for all subsidized technologies $i \in A \subset I$ and timesteps $t^* \in T^*$. In order to design realistic subsidies and CO_2 price plans, we choose timesteps $t^* \in T^* \subset T$ and use linear interpolation to derive subsidies σ_{it} for all timesteps $t \in T$ as can be seen in Fig. 1. This interpolation approach greatly decreases the complexity of decision-making by reducing the total number of variables in the upper agent and in addition makes the resulting strategies more realistic because the year-to-year changes are less extreme.

2.1.1. Minimizing greenhouse gas emissions–GHG

In **GHG**, the upper level model aims to choose subsidies σ_{t^*} in order to minimize the GHG emissions.

In Eq. (1) the objective is given by the weighted sum of the produced heat $xHeat_{it}$ with the emission factor em_{it} per unit of heat for technology i in year t and the produced heat using imported biomass $xHeat_{it}^{\text{imp}}$ multiplied by their respective emission factors $\text{em}_{it}^{\text{imp}}$.

The emission factors are calculated using the emission factors of feedstock and processes from Jordan et al. (2022).

The produced heat $xHeat_{it}$ and produced heat using imported feedstock $xHeat_{it}^{\text{imp}}$ as well as the unsubsidized $nCap_{it}$ and subsidized installed capacities $nCap_{it}^{\text{sub}}$ are defined in Eq. (2) by the solution of the lower level model $LLM(\sigma, \tau, \beta)$. Where subsidies σ, CO_2 prices τ and the budget β are inputs. The budget β for the subsidies and CO_2 price τ are fixed and exogenous.

GHG:

$$\min_{\sigma_{at^*}} \sum_{i \in I, t \in T} \mathrm{em}_{it} \cdot xHeat_{it} + \mathrm{em}^{\mathrm{imp}}_{it} \cdot xHeat^{\mathrm{imp}}_{it} \tag{1}$$

$$s.t. (xHeat_{it}, xHeat^{\mathrm{imp}}_{it}, nCap_{it}, nCap^{\mathrm{sub}}_{it}) = \underset{xHeat, xHeat^{\mathrm{imp}}, nCap, nCap^{\mathrm{sub}}}{\arg\min} LLM(\sigma, \tau, \beta). \tag{2}$$

We assume that €15 billion would be the maximum budget for heating technologies. This is based on the total spending of the German government in 2023 in the BEG, which amounted to €16.8 billion including all measures, of which the heating systems (BEG EM) subsidies were only a part BMF (2023).

The selection of CO_2 prices is detailed in Section 2.2.1.

Considering the assumptions on CO_2 tax development and the budget, the solution for **GHG** is the best choice of subsidies to reduce GHG emissions.

2.1.2. Minimizing the budget for the subsidies–BGT

In addition to deciding on subsidies σ_{t^*}, the upper-level model of **BGT**, adds the decision to pick a budget β_{t^*} for each timestep $t^* \in T^*$. Similar to the case of the subsidies, the budgets β_{t^*} are linearly interpolated to obtain budgets β_t for all timesteps $t \in T$.

The objective function presented in Eq. (3) is given by the discounted sum of the budget used, which is calculated as the product of the subsidies σ_{it}, the investment costs c^{inv}_{it} and the number of subsidized installed heating systems $nCap^{\mathrm{sub}}_{it}$. The discount factor is represented by d_t.

The constraint in Eq. (4) ensures that the budget β and the subsidies σ fulfill the climate goals for each year $t \in T$. The yearly emissions are calculated using the same emission factors $\mathrm{em}_i t$ and $\mathrm{em}^{\mathrm{imp}}_{it}$ as in **GHG** and the allowed emissions are given by GHG_t.

The definition of the variables from the solution of the lower level model in Eq. (5) is the same as in **GHG**.

BGT:

$$\min_{\sigma_{at^*}, \beta_{t^*}} \sum_{i \in I, t \in T} d_t \cdot \sigma_{it} c^{\mathrm{inv}}_{it} nCap^{\mathrm{sub}}_{it} \tag{3}$$

$$s.t. \sum_{i \in Build} (\mathrm{em}_{it} \cdot xHeat_{it} + \mathrm{em}^{\mathrm{imp}}_{it} \cdot xHeat^{\mathrm{imp}}_{it}) \leq GHG_t, \quad \forall t \in T \tag{4}$$

$$(xHeat_{it}, xHeat^{\mathrm{imp}}_{it}, nCap_{it}, nCap^{\mathrm{sub}}_{it}) = \underset{xHeat, xHeat^{\mathrm{imp}}, nCap, nCap^{\mathrm{sub}}}{\arg\min} LLM(\sigma, \tau, \beta). \tag{5}$$

In order to discount the budget we assumed a discount rate of 3 %.

Taking into account the exogenous prices of CO_2, the solution of **BGT** represents the lowest needed budget for the subsidies and provides the subsidies necessary to meet the climate goals.

2.1.3. Minimizing the revenue of the CO_2 price–TAX

Motivated by the goal of finding a solution to decarbonize the heating sector independent of additional government money to make it robust against budget cuts due to crises in total governmental spending, we add the **TAX** upper level model, in which we assume the building sector's CO_2-price income is used for the subsidies. Therefore the model decides on the subsidies σ_{at^*} and on the CO_2 τ^*_t price in order to minimize the total revenue of the CO_2 price.

The objective function in Eq. (6) is set by the discounted sum of the CO_2 price revenue, which is calculated as the sum of the yearly emissions times the CO_2 price. The budget β_t is defined in Eq. (8) as the revenue of the CO_2 price. The constraint in Eq. (7) ensures that the solution of **TAX** aligns with the climate goal.

The same input values used in the GHG and BGT scenarios are also applied to the TAX scenario.

TAX:

$$\min_{\sigma_{at^*}, \tau_{t^*}} \sum_{\substack{it \\ i \in Build}} \mathrm{d}_t \cdot \tau_t (\mathrm{em}_{it} xHeat_{it} + \mathrm{em}^{\mathrm{imp}}_{it} xHeat^{\mathrm{imp}}_{it}) \tag{6}$$

$$s.t. \sum_{i \in Build} (\mathrm{em}_{it} xHeat_{it} + \mathrm{em}^{\mathrm{imp}}_{it} xHeat^{\mathrm{imp}}_{it}) \leq GHG_t, \quad \forall t \in T \tag{7}$$

$$\tau_t \cdot \sum_{i \in Build} (\mathrm{em}_{it} xHeat_{it} + \mathrm{em}^{\mathrm{imp}}_{it} xHeat^{\mathrm{imp}}_{it}) = \beta_t, \quad \forall t \in T \tag{8}$$

$$(xHeat_{it}, xHeat^{\mathrm{imp}}_{it}, nCap_{it}, nCap^{\mathrm{sub}}_{it}) = \underset{xHeat, xHeat^{\mathrm{imp}}, nCap, nCap^{\mathrm{sub}}}{\arg\min} LLM(\sigma, \tau, \beta). \tag{9}$$

By fixing τ_t instead of making it a decision variable, it is also possible to find robust subsidy scenarios for a given CO_2 price scenario.

2.2. Lower level model

To represent the household's decisions for cost-minimal heating, we chose a modified version of the linear optimization model BIOPT used in Jordan et al. (2019) and Millinger et al. (2022) where agents have perfect foresight. To account for different heat demands, we divide the complete heating sector into building, grid-bound, and industrial heat. The building heat sector consists of nine subsectors for private heating and five commercial sectors representing different building types. Meanwhile, the industrial heat sector consists of three subsectors: low-temperature heat (less than 200 °C), medium-temperature heat (200-500 °C), and high-temperature heat (more than 500 °C).

Each subsector has different heating concepts, consisting of predefined combinations of heating technologies suitable for that subsector's building type and also appropriate for providing hot water and peak heat demand. For each subsector, we include:

- fossil fuel concepts that primarily consist of natural gas or heating oil technologies,
- electrical technologies that primarily consist of electric heating and heat pumps, often also including PV systems,
- biomass based heating, consisting of wood pellet or logwood heating,
- and hybrids of the heating concepts above.

The lower level model (LLM) is shown in Eq. (10). The variables $xHeat_{it}$ and $xHeat^{\mathrm{imp}}_{it}$ represent the produced heat of process i in year t, where $xHeat^{\mathrm{imp}}_{it}$ represents those that use imported feedstocks, which, due to the additional transport requirements, entail higher prices and emissions. $nCap_{it}$ and $nCap^{\mathrm{sub}}_{it}$ represent the unsubsidized capacity and subsidized capacity by technology i in the year t.

In the building sector $xHeat_{it}$, $xHeat^{\mathrm{imp}}_{it}$, $nCap^{\mathrm{sub}}_{it}$ and $nCap_{it}$ are chosen to be continuous. Although the number of heating systems is discrete, the high number does not significantly change the result. For the industrial sector, however, $nCap_{it}$ was chosen to be an integer variable. This is because the grid and industry heating plants tend to have a high capacity and, therefore, are not simple to split.

The objective function is to minimize households' and industry's total heating costs for all years using the investment costs c^{inv}_{it}, variable costs c^{var}_{it}, when local feedstock is used, variable costs c^{varimp}_{it} when imported feedstock is used and fixed costs c^{fix}_{it}.

Investment costs are also discounted according to the recommendations of Steinbach and Dan (2015), a discount rate of 4 % was chosen for private investors and 7.6 % for all other sectors.

The variable cost consists of the costs of the feedstock necessary to produce the heat. This includes the production costs in the case of biomass based on Jordan et al. (2022) and market-typical prices for fossil fuels Harthan et al. (2024). The transport of solid fuels includes both the transportation costs of the feedstock and the CO_2 costs of transport.

Table 2

Overview of Energy Prices, CO_2 Price Scenarios, Transport Costs, and Household Ownership. [1] In this table, we provide values only for the years 2020 and 2050. The development in the intervening years is provided by Harthan et al. (2024). [2] The national emissions trading system is planned to be incorporated into the EU ETS2 in 2027, which will also cover emissions in the building sector, therefore we still refer to it as BHEG in the study. [3] Transport distances of woody biomass in the residential sector are assumed to be 50 km, and, for the industrial sector and its higher demand, 100 km. [4] It is assumed that the first two quintiles have an income lower than €40,000 per year. Therefore, the share is calculated as $20\ \% \cdot 20\ \% + 20\ \% \cdot 37\ \% = 11.4\ \%$.

Name	Values (2020-2050)	Source
Natural gas prices[1]		
Market price	27.72–47.00 €/MWh	Bundesnetzagentur and Bundeskartellamt (2023), Harthan et al. (2024)
Consumers (< 20 MWh/year)	86.29–109.24 €/MWh	Bundesnetzagentur and Bundeskartellamt (2023), Harthan et al. (2024)
Consumers (20–200 MWh/year)	61.54–84.49 €/MWh	Bundesnetzagentur and Bundeskartellamt (2023), Harthan et al. (2024)
Consumers (> 200 MWh/year)	58.21–81.16 €/MWh	Bundesnetzagentur and Bundeskartellamt (2023), Harthan et al. (2024)
Non-industrial companies	46.42–65.70 €/MWh	Bundesnetzagentur and Bundeskartellamt (2023), Harthan et al. (2024)
Industry	36.34–55.62 €/MWh	Bundesnetzagentur and Bundeskartellamt (2023), Harthan et al. (2024)
Coal prices[1]		
Coal	9.00–18.41 €/MWh	Bundesnetzagentur and Bundeskartellamt (2023), Harthan et al. (2024)
Electricity prices[1]		
Consumers	335–279 €/MWh	Bundesnetzagentur and Bundeskartellamt (2023), Harthan et al. (2024)
Non-industrial companies	231–183 €/MWh	Bundesnetzagentur and Bundeskartellamt (2023), Harthan et al. (2024)
Industry	139–128 €/MWh	Bundesnetzagentur and Bundeskartellamt (2023), Harthan et al. (2024)
CO_2 Price scenarios[1]		
EU Emissions Trading System (EU-ETS)	25–157.49 €/tCO_{2eq}	Harthan et al. (2024)
German national emissions trading system (BHEG[2])	20–425 €/tCO_{2eq}	Harthan et al. (2024)
Transport[3]		
Transport cost	0.83 €/tkm	Elbersen et al. (2015)
Transport emissions	62 gCO_{2eq}/tkm	ECTA (2011)
Households–self-ownership		
Houses (self-owned)	39 %	Ewald et al. (2023)
Apartments (self-owned)	9 %	Ewald et al. (2023)
Self-owned in 1st income quintile	20 % (4 % of all)	Voigtländer et al. (2019)
Self-owned in 2nd income quintile	37 % (7.4 % of all)	Voigtländer et al. (2019)
Self-owned and low income[4]	11.4 % of all	Voigtländer et al. (2019)

The costs of natural gas and electricity were calculated according to the monitoring report of the Federal Network Agency Bundesnetzagentur and Bundeskartellamt (2023) while taking the projected market prices of Harthan et al. (2024) for future years into account, as well as consumption in the private, industrial, and commercial subsectors. The prices are presented in Table 2.

Some heating technologies also produce electricity. Therefore, we accounted for a credit in the variable costs, when it is used internally in the building sector, and in the industrial sector, whether it is internally used or fed into the grid.

The capacity constraint is provided in Eq. (11) and ensures that the amount of heat in t can only be produced by heating technologies i that were installed before or during the same year and that have not yet reached the end of their lifetimes. In this equation, C_{it} represents the total heat in one year that one unit of heating concept i can produce in year t. The expected lifetime of a heating concept i is given by life_i. To account for the age of installed capacity prior to the model's starting point, we define dec_{it} as the number of heating units i that fail in year t due to exceeding their lifetimes.

To ensure that the heat demand dem_{st} is fulfilled each year t and for each subsector s, the demand constraint Eq. (13) is applied. The index i_s is used to ensure that only heating concepts $i_s \in I$ feed heat to subsector $s \in S$.

The demand dem_{st} was calculated by means of the B-STar (Building Stock Transformation Model) Koch et al. (2018).

Since the adoption of heating technologies is limited by the industry that installs the heating technologies, a growth constraint was added to Eq. (14), where G_t^{Build} represents the number of heating systems that can be installed in the building sector in year t.

We selected the roadmap of the German Heat Pump Association BWP (2021) for the years until 2030 as upper bounds for the heat pump technologies and linearly increased it to 4 million, which is an upper limit based on the number of installers in Germany and a survey conducted by Altermatt et al. (2023).

In Eq. (16), we ensure that the model starts with the same installed heating concepts Start_i as in 2020.

For this, we used the same starting value as in Lubjuhn and Venghaus (2024), which was calculated using the current distribution of feedstocks used in 2020.

The amount of biomass residues that could be used for heat production is limited by Res_{rt} for each residual type $r \in R$ and year $t \in T$ in Eq. (15). The parameter $\mathrm{in}_{i_r,r}$ represents the amount of residual $r \in R$ is input per unit of heat produced by heat concept $i_r \in I$. The index i_r was used to indicate that only heating technologies $i_r \in I$ which use residual $r \in R$ are added in the sum.

We assumed the available residues to be the same as in the 95 % scenario outlined in Thrän et al. (2020).

In the LLM, the decision-makers can also use imported pellets with $xHeat_{it}^{\mathrm{imp}}$. However, this comes with higher emissions and costs due to the more extensive transportation requirements.

$LLM(\sigma, \tau, \beta)$:

$$\min_{nCap, nCap^{\mathrm{sub}}, xHeat, xHeat^{\mathrm{imp}}} \sum (1-\sigma_{it}) c_{it}^{\mathrm{inv}} nCap_{it}^{\mathrm{sub}} + \sum c_{it}^{\mathrm{inv}} nCap_{it} + \sum c_{it}^{\mathrm{fix}} (nCap_{it} + nCap_{it}^{\mathrm{sub}}) + \sum c_{it}^{\mathrm{var}} xHeat_{it} + \sum c_{it}^{\mathrm{varimp}} xHeat_{it}^{\mathrm{imp}} \tag{10}$$

$$s.t.\ xHeat_{it} + xHeat_{it}^{\mathrm{imp}} \leq \sum_{t-\mathrm{life}_i \leq t' \leq t} \mathrm{C}_{it'} \cdot (nCap_{it'}^{\mathrm{sub}} + nCap_{it'} - \mathrm{dec}_{it'}) \quad \forall (i,t) \in (I,T) \tag{11}$$

$$\sum_i \sigma_{it} c_{it}^{\mathrm{inv}} nCap_{it}^{\mathrm{sub}} \leq \beta_t \quad \forall t \in T \tag{12}$$

$$\sum_{i_s} (xHeat_{i_s t} + xHeat_{i_s t}^{\mathrm{imp}}) \geq \mathrm{dem}_{st} \quad \forall (t,s) \in (T,S) \tag{13}$$

$$\sum_{\substack{it \\ i \in Build}} (nCap_{it}^{\text{sub}} + nCap_{it}) \leq \text{G}_t^{Build} \qquad \forall t \in T \tag{14}$$

$$\sum_{i_r} \text{in}_{i_r} \cdot xHeat_{i_r t} \leq \text{Res}_{rt} \qquad \forall (t,r) \in (T,R) \tag{15}$$

$$nCap_{it_0} = \text{Start}_i \qquad \forall i \in I \tag{16}$$

The budget for the transition is considered in Eq. (12). The ULM provides the budget β_t. In the case of **GHG** and **BGT**, it is an exogenous parameter for LLM. However, in the **TAX** scenario, the budget is also endogenously determined in the LLM and consists of the sum of the total CO_2 tax revenue for the specific year, as it is calculated using $xHeat_{it}$ and $xHeat_{it}^{\text{imp}}$, as can be seen in Eq. (17):

$$\beta_t := \tau_t \cdot \sum_{i \in Build} (\text{em}_{it} xHeat_{it} + \text{em}_{it}^{\text{imp}} xHeat_{it}^{\text{imp}}) \qquad \forall t \in T. \tag{17}$$

2.2.1. Scenarios

We considered multiple scenarios to derive solutions that provide a sufficient overview of the general problem of optimizing subsidies. These scenarios are labeled using the following notation:

1. The body text presents the main characteristics of each scenario.
   - **REF**: It is the reference scenario with no subsidies and no CO2 price. It serves as the baseline to calculate the cost of renewable transition and the emissions that would be saved.
   - **POL**: It represents the reference for the current subsidy policies. The composition of these subsidies is described in more detail in Section 2.2.2.
   - **GHG**: It presents the case where the greenhouse gas emissions are minimized by ULM.
   - **BGT**: It presents the case where the budget is minimized by ULM.
   - **TAX** It presents the case where the tax revenue is minimized by ULM.

   As one can notice, the **GHG**, **BGT** and **TAX** scenarios are named after ULM objectives. The details of ULM are already provided in Section 2.1.
2. The superscript specifies the CO_2 price.
   - $^{\text{non}}$: None,
   - $^{\text{ETS}}$: EU-ETS projections Harthan et al. (2024),
   - $^{\text{BHEG}}$: BHEG projections Harthan et al. (2024),
   - $^{\text{dec}}$: CO_2 price as the decision variable in the ULM.
3. The subscript indicates the budget used for subsidies.
   - $_{\text{non}}$: None,
   - $_{\text{cur}}$: Current budget,
   - $_{\text{rev}}$: CO_2 tax revenue,
   - $_{\text{dec}}$: Budget as the decision variable in the ULM.

The overview of the analyzed scenarios is shown in Table 3.

It should be noted that we used the EU-ETS projection to price the industry emissions in all scenarios, while the price of the BEHG is expected to be applicable for the building sector and is therefore used in most scenarios. However, as can be seen from Table 2, the BEHG CO_2 price scenario shows remarkably high prices, reaching up to 425 €/tCO_2eq, compared to EU-ETS scenario. Therefore, both EU-ETS and BEHG projections were considered in scenarios for building sector to account for the different CO_2 price expectations.

2.2.2. Subsidies

Germany introduced the federal subsidy for efficient buildings in 2021 as part of a German federal program to promote energy efficiency BMWK (2021). It provides financial incentives for residential and non-residential buildings to encourage renovations and new constructions that meet high energy efficiency standards.

Another area that will be a focus of this study is the incentives for the investment costs of new heating systems. The current and historical subsidies for these are displayed in Table 4.

Table 3
Overview of the different scenarios. non: none; cur: current; dec: decision; rev: revenue.

Scenario	ULM	Subsidies	CO_2-price	Budget	Constraints
$\mathbf{REF}^{\text{non}}_{\text{non}}$	–	none	none	–	–
$\mathbf{GHG}^{\text{ETS}}_{\text{cur}}$	**GHG**	decision	ETS	cur. budget	–
$\mathbf{GHG}^{\text{BEHG}}_{\text{cur}}$	**GHG**	decision	BEHG	cur. budget	–
$\mathbf{BGT}^{\text{ETS}}_{\text{dec}}$	**BGT**	decision	ETS	decision	GHG goals
$\mathbf{BGT}^{\text{BEHG}}_{\text{dec}}$	**BGT**	decision	BEHG	cur. budget	GHG goals
$\mathbf{TAX}^{\text{dec}}_{\text{rev}}$	**TAX**	decision	decision	tax revenue	GHG goals
$\mathbf{TAX}^{\text{dec}}_{\text{non}}$	**TAX**	none	decision	none	GHG goals
$\mathbf{BGT}^{\text{non}}_{\text{dec}}$	**BGT**	decision	none	decision	GHG goals
$\mathbf{TAX}^{\text{BEHG}}_{\text{rev}}$	**TAX**	decision	BEHG	tax revenue	GHG goals
$\mathbf{POL}^{\text{BEHG}}_{\text{cur}}$	–	cur.	BEHG	cur. budget	–
$\mathbf{POL}^{\text{ETS}}_{\text{cur}}$	–	cur.	ETS	cur. budget	–

For all scenarios, we chose five timesteps for T^*.

The climate speed bonus and income bonus introduced in 2024 are additional subsidies for households that occupy self-used properties and the income bonus further requires that such households have a low income BMWK (2021).

In the $\mathbf{POL}^{\text{BEHG}}_{\text{cur}}$ and $\mathbf{POL}^{\text{ETS}}_{\text{cur}}$ scenarios, we account for the climate speed bonus by multiplying the subsidies by the share of people living in their self-used property; based on a survey presented in Ewald et al. (2023), this amounts to 39 % for houses and 9 % for apartments.

For the income bonus, we multiply the bonus by the share of self-used building owners who earn less than €40,000 (before taxes) per year. We obtain a value of 11.4 %[1] of all households that feature self-used building owners who satisfy the condition Voigtländer et al. (2019).

2.3. Blackbox optimization

Although other scholars use Karush-Kuhn-Tucker (KKT) optimality conditions to reformulate the bi-level model into a single-level problem, this is not possible in our approach due to integer variables in the lower level, which lead to the lower-level problem being nonconvex Moore and Bard (1990). There exist exact approaches to solving bi-level programs using mixed programs at the lower level. For example, Lozano and Smith (2017) proposed an algorithm based on value-function reformulation for problems with mixed integer upper levels.

However, because of the large size of the lower model and the limited number of variables at the upper level, we decided to use black-box optimization. Therefore, we assume the lower-level problem is a black-box function that returns the cost-optimal heat production and capacities for each subsidy, budget, and tax configuration.

The lower level was solved using Gurobi Gurobi Optimization, LLC (2024) (a commercial MILP solver). Because of nonsmoothness (due to integer variables in the LLM) and noise (because of convergence tolerance of the MILP), a robust and derivative-free algorithm was chosen.

We used Hexaly 13.0 Hexaly (2024) for the optimization. Although Particle Swarm Optimization Kennedy and Eberhart (1995), Complex Method Andersson (2001), and PGS-COM (Particle Generating Set–Complex Algorithm) Martelli and Amaldi (2014) were tested as well, Hexaly 13.0 was chosen to generate the results due to its superior performance in terms of run time and convergence.

3. Results

The results of the different scenarios are presented in Table 5, whereas the explicit solutions with the produced heat and subsidized

[1] This is based on the survey conducted by Voigtländer et al. (2019) which provides the share of self-used building owners across the different quintiles of income. We assumed that households in the lower two quintiles earn less than €40,000 per year. Therefore the share is calculated as 20 % · 20 % + 20 % + 37 % = 11.4 %.

Table 4
Subsidies of the BEG EM for heating systems. Biomass heating technologies marked with ^{b} receive an extra 5 % in 2021 and 2022 when the particulate matter emissions are below 2.5 mg/m^3, whereas heat pumps ^{h} receive an extra 5 % if they use water, soil, or wastewater as heat sources or if a natural refrigerant is used. The climate speed bonus decreases over time and adheres to the following steps: 20 % by 2028; from 2029 to 2030: 17 %; from 2031 to 2032: 14; from 2033 to 2034: 11 %; and from 2035 to 2036: 8 %.

	2021-2022	2023	2024
Heatings systems:	% of investment costs		
Condensing gas boiler	20		
Hybrid gas *(with at least 25 % renewable)*	30		
Solar thermal	30	25	30
Biomass	35^b	10	30
Heating pumps	35	25^h	30^h
Renewable hybrids	35^b		30
Fuel cells		25	30
Boni (cumulative):			
Replacement of heating oil	10		
Replacement of oil, coal, storage heater or natural gas if older than 20 years		10	
Income boni *(Boni for people with low income)*			30
Climate speed boni *(only on self-used property)*			max 20
H2-ready of gas technologies *(only on additional costs)*			30

Table 5
Costs and emission reduction of the different scenarios. 'Consumer' refers to costs that consumers must pay, including the CO_2 tax. 'All' costs refer to the consumer costs and budget. 'Tax redist.' refers to the consumer costs and budget without the CO_2 prices and represents the costs for the consumers when the CO_2 price revenue is redistributed and the budget is paid for with other taxes.

Scenario	Additional costs % of $\mathbf{REF}^{non}_{non}$			Transformation costs € per ton CO_2eq				GHG red. % of $\mathbf{REF}^{non}_{non}$	Budget % of all costs	Tax rev.
	Consumer	All	Tax redist.	Consumer	All	Tax redist.	Govern.			
$\mathbf{GHG}^{ETS}_{cur}$	−0.92 %	24.72 %	14.99 %	−10.58	285.00	172.75	183.33	−52.41 %	20.56 %	7.81 %
$\mathbf{GHG}^{BEHG}_{cur}$	3.64 %	26.92 %	16.66 %	39.60	293.01	181.34	141.73	−55.51 %	18.34 %	8.08 %
$\mathbf{BGT}^{ETS}_{dec}$	6.15 %	19.45 %	7.82 %	83.81	265.00	106.49	22.67	−44.35 %	11.13 %	9.74 %
$\mathbf{BGT}^{BEHG}_{dec}$	16.06 %	19.98 %	5.44 %	210.01	261.31	71.17	−138.84	−46.19 %	3.27 %	12.12 %
$\mathbf{TAX}^{dec}_{rev}$	9.56 %	20.05 %	8.27 %	121.96	255.81	105.55	−16.41	−47.36 %	8.47 %	9.81 %
$\mathbf{TAX}^{dec}_{non}$	23.78 %		9.10 %	275.80		105.47	−170.33	−52.10 %	0.00 %	11.87 %
$\mathbf{BGT}^{non}_{dec}$	−9.11 %	13.60 %		−130.00	193.96		323.96	−42.36 %	19.99 %	0.00 %
$\mathbf{TAX}^{BEHG}_{rev}$	13.82 %	21.95 %	9.91 %	163.23	259.00	116.12	−46.18	−51.2 %	5.81 %	9.87 %
$\mathbf{POL}^{BEHG}_{cur}$	3.21 %	26.22 %	15.25 %	36.26	295.94	172.13	135.87	−53.54 %	18.23 %	8.69 %
$\mathbf{POL}^{ETS}_{cur}$	0.53 %	21.52 %	11.14 %	6.43	262.21	135.78	129.35	−49.59 %	17.28 %	8.54 %

shares of investment costs and CO_2 taxes are shown in Fig. 2, Fig. 3, and Fig. 4, with more details in the supplementary file.

3.1. Heat pumps

As can be seen in Figs. 2, 3 and 4, in all of the solutions calculated, heat pumps provide the major share of renewable heat in the building sector. This is especially the case in scenarios that apply the $BHEG$-CO_2 price. They experience a large adoption of heat pumps between 2025 and 2032.

With the high CO_2 prices that rise from 55€ to 155€ per ton CO_2, heat pumps become cost competitive with the fossil technologies in most of the building subsectors. The high subsidies further promote the earlier phase out of the fossil technologies, as can be seen by comparing a solution with rather high subsidies like the current policies $\mathbf{POL}^{BEHG}_{cur}$ (Fig. 2c) with the solution of scenario $\mathbf{BGT}^{BEHG}_{dec}$ (Fig. 3e), which has rather low subsidies.

The importance of the CO_2 price and the subsidies becomes evident when comparing the solutions with the solution of the reference scenario $\mathbf{REF}^{non}_{non}$, which neither has subsidies nor a CO_2 price. As Fig. 2a) shows, heat pumps are only used in a few subsectors while natural gas boilers still dominate in 2050.

In all scenarios, a high adoption of heat pumps is accompanied by a decrease in subsidies, as heat pumps are competitive in these cases, eliminating the need for further increased subsidies. However, the high level of implementation still results in substantial costs to the budget.

3.2. Biobased subsidies

In all scenarios that apply subsidies together with taxes (Fig. 2d, 3e-g), the subsidies range from 0 % to 13 % of the investment costs for biomass heating systems in 2028.

This corresponds to the results of Lubjuhn and Venghaus (2024) and Jordan et al. (2024), who found that biomass heating in the building sector is ineffective for GHG emission reduction because of the higher cost reduction potential of using biomass in high temperature industrial heat and processes.

The large difference in subsidies also holds in Scenario $\mathbf{BGT}^{non}_{dec}$ (Fig. 3 f), which aims at fulfilling the GHG emission targets only with subsidies. The subsidies are 38 % for biomass heating and 61 % for heat pumps in 2028.

As can be seen in Fig. (3 f), and Fig. 3f,g, the subsidy shares get higher in the last years; this can be explained by the already high adoption of sustainable heating technologies and, therefore, the low costs of only adding a few of these, allowing high subsidies for the few that can still be installed.

3.3. Evaluation of current policies

With the exception of the Reference ($\mathbf{REF}^{non}_{non}$) (Fig. 2a), all solutions achieve the GHG reduction goals for the building sector. The solution of the ULM that aims to minimize GHG emissions $\mathbf{GHG}^{BEHG}_{cur}$ (Fig. 2d), defines a bound, that determines how much emissions can be saved by optimally setting the subsidies under the assumptions of budget limitations and CO_2 price. This bound is found to be 44.49 % of the

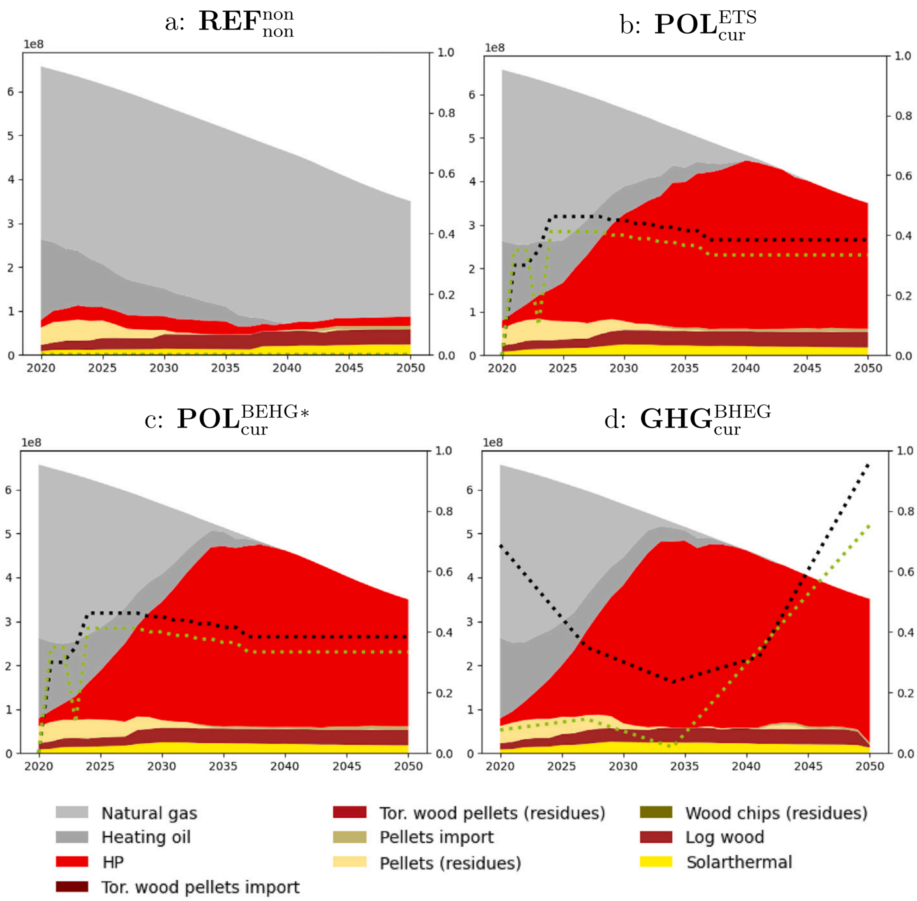


Fig. 2. Produced heat in MWh by feedstocks in the building sector (left axis) and the subsidy shares on the investment costs in % (right axis). The dashed black line represents the heat pump subsidies and the dashed green line the biomass heating system subsidies. * the subsidies shares in the $\mathbf{POL}^{BHEG}_{cur}$ scenario show the subsidies that are assumed in the one and two-family houses. The subsidies for flats are slightly lower as described in Section 2.2.2. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

reference emissions. The current policies $\mathbf{POL}^{BEHG}_{cur}$, in turn, reduce the total emissions by 70 Mt CO_2eq less.

However, the total transformation costs of the current policy scenario $\mathbf{POL}^{BEHG}_{cur}$ are 2€ per ton of CO_2eq higher than the ones from $\mathbf{GHG}^{BEHG}_{cur}$. This finding appears counterintuitive, given that marginal abatement costs typically rise with greater CO_2 emission reductions. It may point to inefficiencies in the current structure of subsidies.

Both, $\mathbf{GHG}^{BEHG}_{cur}$ and $\mathbf{POL}^{BEHG}_{cur}$ scenarios have a total subsidy budget of 18.34 % and 18.24 % of the total cost, whereas the revenue from the CO_2 price will only be around 8.08 % and 8.69 %, of the total costs. This demonstrates the large deficit of these subsidy plans, which amounts to €266 billion in the $\mathbf{GHG}^{BEHG}_{cur}$ and €246 billion in $\mathbf{POL}^{BEHG}_{cur}$.

3.4. Funding of the subsidies budget

The upper level models **BGT** and **TAX** provide solutions, that minimize the budget for the subsidies (or the CO_2 price revenue which is used as the budget) such that the climate goals are achieved. With the solutions of $\mathbf{BGT}^{BEHG}_{dec}$ and $\mathbf{TAX}^{dec}_{rev}$ (Fig. 3e and g), we even obtained solutions in which the tax revenue was higher than the subsidy budget but still satisfied the targets for greenhouse gas emissions.

The solution of $\mathbf{BGT}^{BEHG}_{dec}$, shown in Fig. 3e) defines a lower bound of the needed budget to fulfill the climate goals, when the CO_2 price of BEHG is applied. The lower bound is found to be €80 billion for the total budget, far less than the €470 billion in the current policy scenario $\mathbf{POL}^{BEHG}_{cur}$.

However, with the lower budget, the additional costs for the consumer increase to 16.06 % and 9.56 % in the $\mathbf{BGT}^{BEHG}_{dec}$ and $\mathbf{TAX}^{dec}_{rev}$, respectively, compared to the reference scenario ($\mathbf{REF}^{non}_{non}$). When the tax revenue is redistributed, however, the solutions of $\mathbf{BGT}^{BEHG}_{dec}$ and $\mathbf{TAX}^{dec}_{rev}$ become the cheapest, with €71.17 and €105.55, respectively, per ton CO_2eq of reduction.

3.5. Influence of subsidies on the climate goals needed CO_2 price

With the **TAX** upper level model, we allowed the model to decide on the taxes such that the climate goals are achieved. Therefore the solution of scenario $\mathbf{TAX}^{dec}_{non}$ (Fig. 4i) shows the pathway of CO_2 prices that minimize the total CO_2 price revenue.

In $\mathbf{TAX}^{dec}_{non}$ (Fig. 4i), the shares of the needed CO_2 price, go up to €715/tCO_2eq in 2031, which is far higher than the maximum of €325/tCO_2eq in $\mathbf{TAX}^{dec}_{rev}$ (Fig. 3h) and the €425/tCO_2eq in the BEHG projection. This aligns with the findings of Milyani and Kirschen (2018), who also showed that the needed CO_2 price for GHG mitigation decreases significantly when taxes and subsidies are used. Furthermore, compared to the BEHG CO_2 price, the lower CO_2 price in $\mathbf{TAX}^{dec}_{rev}$ underscores the fact that a lower CO_2 price is sufficient for emissions reduction in the building sector. However, as the BEHG is also coupled with the transport sector and is planned to be linked to the European Emission Trading System 2 (ETS2) for building and transport, this might not provide a realistic option as the ETS2 price can not be simply decided by the German policy makers.

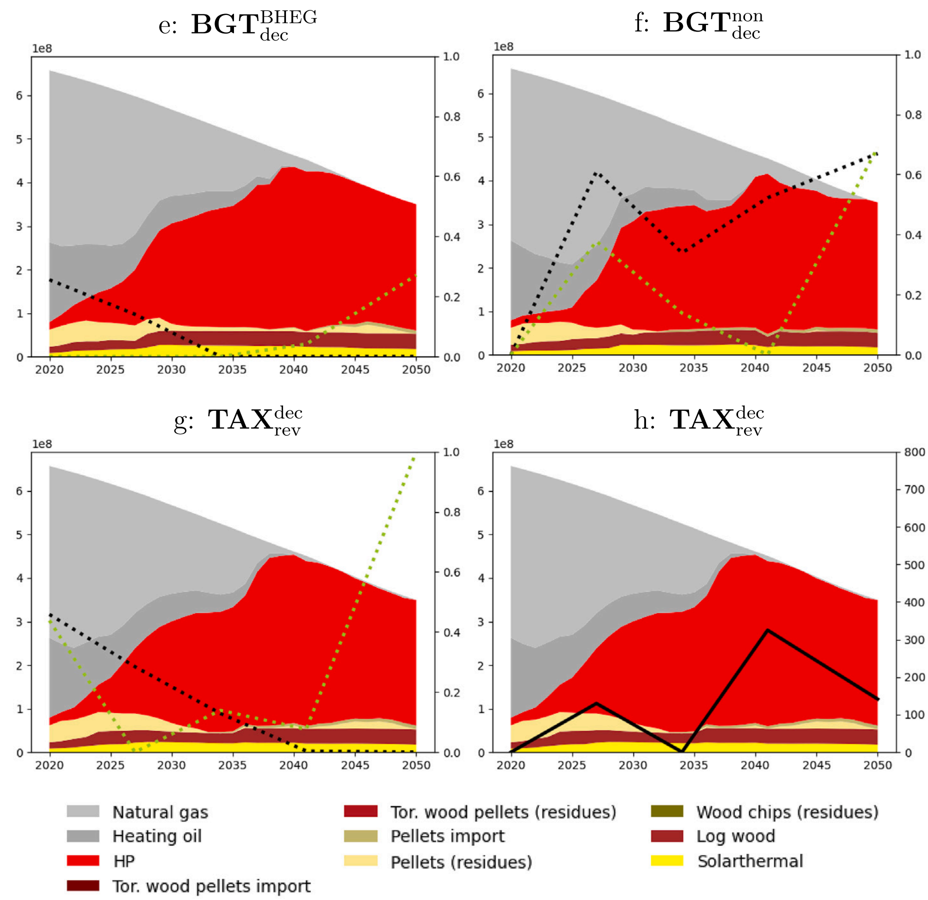


Fig. 3. Produced heat in MWh by feedstocks in the building sector (left axis) and the subsidy shares on the investment costs in % (right axis) or the CO_2 price (black line, right axis). The dashed black line represents the heat pump subsidies and the dashed green line the biomass heating system subsidies. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

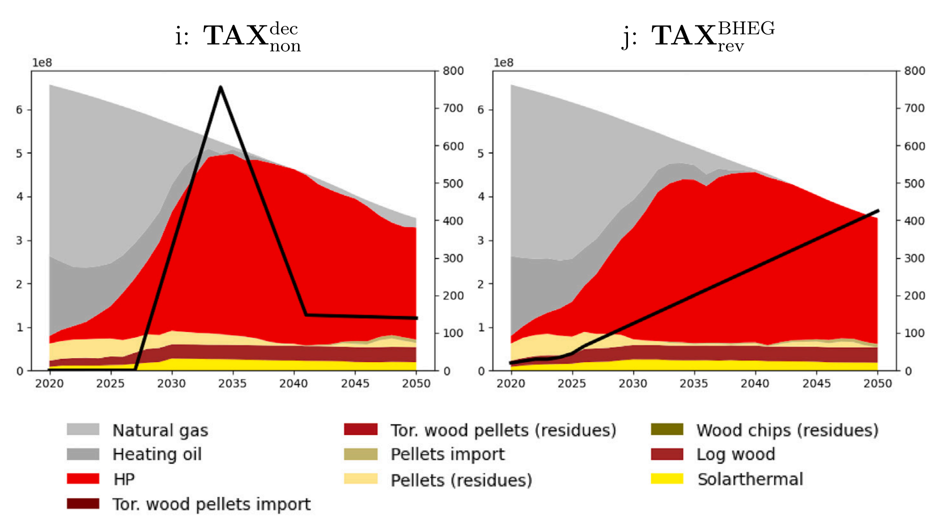


Fig. 4. Produced heat in MWh by feedstocks in the building sector and the CO_2 price in € for each tCO_2eq (black line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

While the only CO_2 price scenario $\mathbf{TAX}^{\mathrm{dec}}_{\mathrm{non}}$ reduces the emissions by 52.1 % compared to the reference ($\mathbf{REF}^{\mathrm{non}}_{\mathrm{non}}$), the scenario $\mathbf{BGT}^{\mathrm{non}}_{\mathrm{dec}}$ (Fig. 3f) which aims to fulfill the climate goals without a CO_2 price but by subsidies alone, leads to the most emissions and the highest transformation costs, when the high governmental spending is funded by other taxes. This shows that subsidies alone are ineffective for climate change reduction in the heating sector.

3.6. Compromise solution–low emissions, low heating expenditures, and low fiscal burdens

Although the current policy scenario $\mathbf{POL}_{\mathrm{cur}}^{\mathrm{BEHG}}$ is suitable regarding climate change mitigation, the results show that the deficit of €246 billion provided by the subsidies is exceptionally high. Situations of government budget crises, may lead to short-term cuts in subsidies, which could in turn undermine reaching the climate goals.

The solution of the minimizing the total budget ULM, $\mathbf{BGT}_{\mathrm{dec}}^{\mathrm{BEHG}}$ (Fig. 3e) could be more robust in that regard; however, the design leads to a solution that will barely reach the climate goals (i.e., 46.2 % emission reduction instead of 53.5 % reduction in the current policies). Therefore, we also considered a scenario $\mathbf{TAX}_{\mathrm{rev}}^{\mathrm{BEHG}}$ (Fig. 4j) that uses the **TAX** ULM combined with the BEHG for the CO_2 price. This resulted in solution that saved 11 % (170 Mt CO_2eq) more GHG emissions than the $\mathbf{BGT}_{\mathrm{dec}}^{\mathrm{BEHG}}$ while not having a deficit between the governmental spending for subsidies and CO_2 price revenue like the $\mathbf{POL}_{\mathrm{cur}}^{\mathrm{BHEG}}$ (B€ 80 surplus instead of B€ 245 deficit).

The solution of $\mathbf{TAX}_{\mathrm{rev}}^{\mathrm{BEHG}}$ provides subsidies starting high in the early adoption phase in 2020 with 30 % for heat pumps and 20 % for biomass. Then during the main adoption the subsidies decrease from 22 % and 8.7 % in 2028 to 6.45 % and 0 % in 2036. When the adoption is fully completed nearly no subsidies are applied anymore with 0 % and 0 % in 2042 before they then rise again up to 22.6 % and 28.9 % in 2050.

4. Limitations

Because of the complexity of calculating the subsidies for climate speed and income boni, some assumptions had to be made that influenced the results of the reference scenario. In addition, in its current form, the model does not account for the regional variability of biomass availability, as it relies on a national average, as outlined in the methodology. Although incorporating a more detailed supply chain model could be a viable enhancement, due to the high computational time, existing models typically focus on municipal or regional scales rather than national ones Atashbar et al. (2016). Incorporating regionalized data could enable the identification of regions with high potential, where bio-based heating technologies might be more cost-competitive under specific local conditions. This could also apply to heating technologies that are not cost-competitive on a national scale. However, it is unlikely that these would greatly influence the results. In the case study, no hydrogen for heating was considered; however, in the current setting of the LLM, it is also not expected that the agents would use hydrogen due to the high expected costs for hydrogen in private heating applications and uncertainties around the necessary infrastructure Rosenow (2024). A further limitation is that we did not account for market imperfections and preferences beyond costs. These do, however, exist in the context of product preferences, especially in the private heat sector. For example, the use of log wood stoves not only provides heating but also creates a comfortable atmosphere, thus generating a preference and willingness to pay.

5. Conclusion and policy implications

Using the different ULM configurations, we obtained valuable insights for evaluating current policies. While scenario analysis for subsidies of single level models can only provide a comparison of defined subsidy scenarios, the bi-level modeling framework can find the optimal subsidy pathways considering the objective function and constraints of the policy maker.

Three ULM were investigated:

- The **GHG** ULM is used to identify an upper bound on what CO_2 emissions reduction is possible given the proposed budget and CO_2 price, which allows us to investigate if the current policies are effective for climate mitigation. We found that the current policy scenario $\mathbf{POL}_{cur}^{BEHG}$ reduces the GHG emissions by 2 % less than the reference GHG optimal solution $\mathbf{GHG}_{cur}^{BEHG}$ while also having a higher transformation costs of 2€ per ton of CO_2, which indicates that the current subsidy design is distributing the subsidies budget inefficiently.
- By applying the **BGT** ULM we calculate the budget needed to reach the climate goal for a given CO_2 price scenario, providing a lower bound on the budget necessary to achieve the climate goals. We found this to be €80 billion, which is far less than the €470 billion of the current policy scenario.
- By using the **TAX**, it is possible to find solutions in which the CO_2 price revenue pays for the subsidies, making them independent of future budget shortages. **TAX** can also be used to calculate a solution for a fixed CO_2 price to calculate subsidies that do not produce a deficit while also over-satisfying the climate goals, providing a compromise between GHG minimization and budget minimization, as we did in the $\mathbf{TAX}_{\mathrm{rev}}^{\mathrm{BEHG}}$ scenario.

While the total emission savings would be 80 Mt CO_2eq. less by applying the solution given by $\mathbf{TAX}_{\mathrm{rev}}^{\mathrm{BEHG}}$, the transformation costs for the consumer, when the tax surplus is redistributed, would go down to 116.12€ per tCO_2 instead of 172.13€ per tCO_2 in the current scenario. That would save around 1,313€ per inhabitant.[2]

Further it would provide a robust subsidy pathway in case of future budgeting problems in the government as the deficit of €246 Billion in the current policy scenario is replaced by a surplus of €80 Billion in the $\mathbf{TAX}_{\mathrm{rev}}^{\mathrm{BEHG}}$ scenario.

Therefore the solution of $\mathbf{TAX}_{\mathrm{rev}}^{\mathrm{BEHG}}$ could provide a great compromise with low emissions, low heating expenditures and low fiscal burdens.

The results of the study further suggest the following policy recommendations:

- The subsidies should mainly focus on heat pumps, since subsidies for biomass heating seem to be ineffective in the early and main adoption phases.
- An effective subsidy scheme should be responsive to the adoption phase of renewable technologies. When adoption rates are high, continued generous subsidies may be inefficient, as the technologies are already competitive and the marginal benefit of support is lower.

Black-box optimization can be an efficient tool for strategic policy making, and the methodology applied in the German context is transferable to other nations. Even though the heuristic approach cannot guarantee an optimal solution, the solutions obtained from the different scenarios improve the objective values compared to the $\mathbf{POL}_{\mathrm{cur}}^{\mathrm{BEHG}}$ with respect to emissions reduction or transformation costs.

In future research, the current LLM could be replaced by other types of models, such as an agent-based model that could be used to also include non-monetary factors of decision making such as greater comfort due to a log wood stove or incorporate more household-specific decision factors that significantly influence the decision-making process, such as proximity to log wood and personal heating requirements. Finally, more complex subsidy ideas that account for households' income and living situations, such as the income bonuses and bonuses that only account for self-owned households, could be optimized.

CRediT authorship contribution statement

Sebastian Lubjuhn: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Sandra Venghaus:** Writing – review & editing, Supervision, Resources, Project administration, Funding acquisition.

[2] The current policy scenario costs in addition to the reference, when the CO_2 price is redistributed and the budget is paid by other taxes 311.3 € billion, while $\mathbf{TAX}_{\mathrm{rev}}^{\mathrm{BEHG}}$ costs 202.3 € billion. Divided by 83 million inhabitants in Germany.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the authors used DeepL and ChatGPT to improve language and readability. After using these tools, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered potential competing interests:

Sebastian Lubjuhn reports that financial support was provided by German Research Foundation. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy—Cluster of Excellence 2186 "The Fuel Science Center"—ID: 390919832.

Appendix A. Supplementary data

Supplementary data to this article can be found online at doi:10.1016/j.enpol.2025.114866.

Data availability

Data will be made available on request.

References

Adolfsen, J.F., Kuik, F., Lis, E.M., Schule, T., 2022. Economic Bulletin Issue 4, 2022. Technical Report. European Central Bank (ECB). https://www.ecb.europa.eu/pub/pdf/ecbu/eb202204.en.pdf

Altermatt, P.P., Clausen, J., Brendel, H., Breyer, C., Gerhards, C., Kemfert, C., Weber, U., Wright, M., 2023. Replacing gas boilers with heat pumps is the fastest way to cut German gas consumption. Commun. Earth Environ. Nature Publishing Group, 4, 1–8. https://doi.org/10.1038/s43247-023-00715-7. https://www.nature.com/articles/s43247-023-00715-7

Andersson, J., 2001. Multiobjective Optimization in Engineering Design: Applications to Fluid Power Systems (ph.D. Thesis). Sweden: Department of Mechanical Engineering, Linköpings universitet.

Atashbar, N.Z., Labadie, N., Prins, C., 2016. Modeling and optimization of biomass supply chains: a review and a critical look. IFAC-Papersonline 49, 604–615. https://doi.org/10.1016/j.ifacol.2016.07.742

BAFA, 2024. Energieeffizienz in der Elektromobilität. Bundesamt für Wirtschaft und Ausfuhrkontrolle. https://www.bafa.de/DE/Energie/Energieeffizienz/Elektromobilitaet/Neuen_Antrag_stellen/neuen_antrag_stellen.html

BDH, 2023. Marktentwicklung: Heizungsindustrie steuert auf Rekordjahr zu. Bundesverband der Deutschen Heizungsindustrie. https://www.bdh-industrie.de/presse/pressemeldungen/artikel/marktentwicklung-heizungsindustrie-steuert-auf-rekordjahr-zu

Bigerna, S., Wen, X., Hagspiel, V., Kort, P.M., 2019. Green electricity investments: environmental target and the optimal subsidy. Eur. J. Oper. Res. 279, 635–644. https://doi.org/10.1016/j.ejor.2019.05.041. https://www.sciencedirect.com/science/article/pii/S0377221719304710

BMF, 2023. 29. Subventionsbericht des Bundes–Bundesfinanzministerium. Technical Report. Bundesministerium der Finanzen. https://www.bundesfinanzministerium.de/Content/DE/Downloads/Broschueren_Bestellservice/29-subventionsbericht.html

BMWK, 2021. Bundesförderung für effiziente Gebäude. Accessed:(2022-Oct-19). https://www.bafa.de/DE/Energie/Effiziente_Gebaeude/Foerderprogramm_im_Ueberblick/foerderprogramm_im_ueberblick_node.html. As last amended 2022-09-15.

BMWK, 2024. Germany on track for 2030 climate targets for the first time. https://www.bmwk.de/Redaktion/EN/Pressemitteilungen/2024/03/20240315-germany-on-track-for-2030-climate-targets-for-the-first-time.html

Bundesnetzagentur, 2023. Monitoringbericht der Bundesnetzagentur: Marktbeobachtung. Monitoring Report, Bundesnetzagentur and Bundeskartellamt, Bonn, Germany. https://www.bundesnetzagentur.de/DE/Fachthemen/ElektrizitaetundGas/Monitoringberichte/start.html

Bundestag, 2020. Gebäudeenergiegesetz (GEG). Deutscher Bundestag. https://www.gesetze-im-internet.de/geg/__72.html. As last amended by 20. Juli 2022.

BWP, 2021. Der Weg zur Dekarbonisierung des Gebäudesektors (The Path to Defossilising the Building Sector). Technical Report. Bundesverband Wärmepumpe (BWP), Berlin, Germany, https://www.waermepumpe.de/politik/waermemarkt/

Colson, B., Marcotte, P., Savard, G., 2005. Bilevel programming: a survey. 4OR 3, 87–107. https://doi.org/10.1007/s10288-005-0071-0

ECTA, 2011. Guidelines for Measuring and Managing CO_2 Emission from Freight Transport Operations. Technical Report. European Chemical Transport Association, https://ecta.com/wp-content/uploads/2021/03/ECTA-CEFIC-GUIDELINE-FOR-MEASURING-AND-MANAGING-CO2-ISSUE-1.pdf

Elbersen, B., Dalla Longa, F., Ruiz, P., Thiel, C., Sgobbi, A., Hengeveld, G., Kober, T., Nijs, W., 2015. The Jrc-eu-times Model: Bioenergy Potentials for EU and Neighboring Countries. Publications Office, https://doi.org/10.2790/39014. https://data.europa.eu/doi/10.2790/39014

Ewald, J., Hünnemeyer, V., Kempermann, H., Meeßen, F., Pekka Sagner, P., Zink, B., 2023. Wohnen in Deutschland 2023–Sparda Studie. Technical Report. Verband der Sparda-Banken e.V. URL: https://sparda-wohnen2023.de/

Graichen, J., Ludig, S., 2024. Supply and Demand in the ETS 2. Umweltbundesamt, https://www.umweltbundesamt.de/publikationen/supply-demand-in-the-ets-2

Gurobi Optimization, LLC, 2024. GUROBI optimizer reference manual. https://www.gurobi.com

Harthan, R.O., Förster, H., Borkowski, K., Böttcher, H., Braungardt, S., Bürger, V., Emele, L., Görz, W.K., Hennenberg, K., Jansen, L.L., Jörß, W., Kasten, P., Loreck, C., Ludig, S., Matthes, F., Mendelevitch, R., Moosmann, L., Nissen, C., Repenning, J., Scheffler, M., Steinbach, I., Bei der Wieden, M., Wiegmann, K., Brugger, H., Fleiter, T., Mandel, T., Rehfeldt, M., Rhode, C., Yu, S., Steinbach, J., Deurer, J., 2024. Projektionsbericht 2023 Für Deutschland. Technical Report.

Hexaly, 2024. Hexaly–commercial solver, for routing, scheduling, packing, and more. version 13.0. https://www.hexaly.com/docs/last/index.html. https://www.hexaly.com/docs/last/index.html

IEA, 2025. Global Energy Review 2025. Technical Report. IEA, Paris, https://www.iea.org/reports/global-energy-review-2025. licence: CC BY 4.0.

Jin, S.W., Li, Y.P., Xu, L.P., 2018. Development of an integrated model for energy systems planning and Carbon dioxide mitigation under uncertainty–tradeoffs between two-level decision makers. Environ. Res. 164, 367–378.

Jordan, M., Lenz, V., Millinger, M., Oehmichen, K., Thrän, D., 2019. Future competitive bioenergy technologies in the German heat sector: findings from an economic optimization approach. Energy (oxf.) 189, 116194. https://doi.org/10.1016/j.energy.2019.116194

Jordan, M., Meisel, K., Dotzauer, M., Schindler, H., Schröder, J., Cyffka, K.F., Dögnitz, N., Naumann, K., Schmid, C., Lenz, V., Daniel-Gromke, J., de Paiva, G.C., Esmaeili Aliabadi, D., Szarka, N., Thrän, D., 2024. Do current energy policies in Germany promote the use of biomass in areas where it is particularly beneficial to the system? analysing short- and long-term energy scenarios. Energy. Sustain. Soc. 14, https://doi.org/10.1186/s13705-024-00464-1

Jordan, M., Millinger, M., Thrän, D., 2020. Robust bioenergy technologies for the German heat transition: a novel approach combining optimization modeling with sobol' sensitivity analysis. Appl. Energy 262, 114534. https://doi.org/10.1016/j.apenergy.2020.114534

Jordan, M., Millinger, M., Thrän, D., 2022. BENOPT-Heat: an economic optimization model to identify robust bioenergy technologies for the German Heat transition. SoftwareX 18, 101032. https://doi.org/10.1016/j.softx.2022.101032. (accessed: 2022-Sep-23).

Kennedy, J., Eberhart, R., 1995. Particle swarm optimization. In: Proceedings of ICNN'95–International Conference on Neural Networks, vol. 4. pp. 1942–1948, https://doi.org/10.1109/ICNN.1995.488968

Koch, M., Hennenberg, K., Hünecke, K., Haller, M., Hesse, T., 2018. Rolle der Bioenergie im Strom- und Wärmemarkt bis 2050 unter Einbeziehung des Zukünftigen Gebäudebestandes. Technical Report. Ôko-Institut e.V. https://www.oeko.de/fileadmin/oekodoc/Rolle-Bioenergie-im-Strom-Waermemarkt-bis-2050.pdf

König, A., 2011. Cost efficient utilisation of biomass in the German energy system in the context of energy and environmental policies. Energy Policy 39, 628–636. https://doi.org/10.1016/j.enpol.2010.10.035

Lozano, L., Smith, J.C., 2017. A value-function-based exact approach for the Bilevel mixed-integer programming problem. Oper. Res. 65, 768–786. https://doi.org/10.1287/opre.2017.1589

Lubjuhn, S., Venghaus, S., 2024. Unlocking the potential of the bioeconomy for climate change reduction: the optimal use of lignocellulosic biomass in Germany. J. Ind. Ecol. 28, 144–159. https://doi.org/10.1111/jiec.13455

Martelli, E., Amaldi, E., 2014. Pgs-Com: a hybrid method for constrained non-smooth black-box optimization problems. Comput. Chem. Eng. 63, 108–139. https://doi.org/10.1016/j.compchemeng.2013.12.014

Martelli, E., Freschini, M., Zatti, M., 2020. Optimization of renewable energy subsidy and Carbon tax for multi energy systems using Bilevel programming. Appl. Energy 267, 115089. https://doi.org/10.1016/j.apenergy.2020.115089

Millinger, M., Tafarte, P., Jordan, M., Musonda, F., Chan, K., Meisel, K., Aliabadi, D.E., 2022. A model for cost- and greenhouse gas optimal material and energy allocation of biomass and Hydrogen. SoftwareX 20, 101264. https://doi.org/10.1016/j.softx.2022.101264

Milyani, A.H., Kirschen, D.S., 2018. Optimally designed subsidies for achieving Carbon emissions targets in electric power systems. In: 2018 IEEE Power and Energy Society General Meeting (PESGM), pp. 1–5, https://doi.org/10.1109/PESGM.2018.8586055

Moore, J.T., Bard, J.F., 1990. the mixed integer linear Bilevel programming problem. Oper. Res. 38, 911–921. http://www.jstor.org/stable/171050

OECD, 2024. Global Monitoring of Policies for Decarbonising Buildings. OECD.

Olsen, D.J., Dvorkin, Y., Fernández-Blanco, R., Ortega-Vazquez, M.A., 2018. Optimal Carbon taxes for emissions targets in the electricity sector. IEEE Trans. Power Syst. 33, 5892–5901. https://doi.org/10.1109/TPWRS.2018.2827333

Pereira, A., Sauma, E., Montero, J., 2019. Impact of Carbon tax flexibility on the Chilean power system expansion planning. In: 2019 IEEE Milan PowerTech, IEEE, pp. 1–6.

UN, 2022. Global Status Report for Buildings and Construction: Towards a Zero-Emission, Efficient and Resilient Buildings and Construction Sector. Technical Report. United Nations Environment Programme.

Rosenow, J., 2024. A meta-review of 54 studies on Hydrogen heating. Cell Rep. Sustain. 1, 100010. https://doi.org/10.1016/j.crsus.2023.100010

Steinbach, J.S., Staniaszek, D., 2015. Discount Rates in Energy Systems Analysis. Buildings Performance Institute Europe (BPIE) with assistance of Franhofer ISI.

Thrän, D., Szarka, N., Haufe, H., Lenz, V., Majer, S., Oehmichen, K., Jordan, M., Millinger, M., Schaldach, R., Schüngel, J., 2020. BioplanW: Systemlösungen Bioenergie Im Wärmesektor Im Kontext Zukünftiger Entwicklungen: Schlussbericht. DBFZ Deutsches Biomasseforschungszentrum.

Voigtländer, D.M., Sagner, P., Buesink, W., 2019. Analyse der Wohneigentumsbildung. Technical Report. Institut der deutschen Wirtschaft. https://www.iwkoeln.de/studien/hanno-kempermann-agnes-ricci-pekka-sagner-thorsten-lang-wohnen-in-deutschland-2019.html

Wei, W., Liang, Y., Liu, F., Mei, S., Tian, F., 2014. Taxing strategies for Carbon emissions: a bilevel optimization approach. Energies, number: 4 Publisher: Multidisciplinary Digital Publishing Institute 7, 2228–2245. https://doi.org/10.3390/en7042228

Xia-Bauer, C., Gokarakonda, S., Guo, S., Filippidou, F., Thomas, S., Maheshwari, J.R., Vishwanathan, S.S., 2024. Comparative analysis of residential building decarbonization policies in major economies: insights from the EU, China, and India. Energy Effic. 17. https://doi.org/10.1007/s12053-024-10225-w

Zhao, N., You, F., 2019. Dairy waste-to-energy incentive policy design using Stackelberg-game-based modeling and optimization. Appl. Energy 254, 113701. https://doi.org/10.1016/j.apenergy.2019.113701

Zhou, Y., Wang, L., McCalley, J.D., 2011. Designing effective and efficient incentive policies for renewable energy in generation expansion planning. Appl. Energy 88, 2201–2209. https://doi.org/10.1016/j.apenergy.2010.12.022

Ziliaskopoulos, K., Papalamprou, K., 2022. A Bilevel linear programming model for developing a subsidy policy to minimize the environmental impact of the agricultural sector. Sustainability, Multidisciplinary Digital Publishing Institute 14, 7651. https://doi.org/10.3390/su14137651